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Fuzzy Matrix Games Multi-Criteria Model for Decision-Making in Engineering

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Abstract. When handling engineering problems associated with optimal alternative selection a researcher often deals with not sufficiently accurate data. The alternatives are usually assessed by applying several different criteria. A method takes advantage of the relationship between fuzzy sets and matrix game theories can be offered for multicriteria decision-making. Practical investigations have already been discussed for selecting the variants water supply systems.

Key words: alternative, fuzzy sets, game theory, multiple criteria, decision-making in engineering, case study.

1. Introduction

Problems of civil engineering, industry and other areas are taking on dimensions that no longer allow satisfactory solution by currently employed methods. These are complex and interrelated problems the solution of which depends on the goals pursued by different interested parties. Attempts to interpret such problems as conflict situations which could be addressed by games theory have already been reported in the literature (Brams, 1994; Čyras and Vakriniene, 2001; Ghose *et al.*, 2002; Mitkus, 2001; Muschik and Müller, 1986; Peldschus *et al.*, 1983; Peldschus, 1986; Peldschus and Zavadskas, 1997; Peldschus, 2001; Vorobjov, 1967; Winand, 1987; Zavadskas *et al.*, 2002; Zavadskas *et al.*, 2003; Zavadskas *et al.*, 1994; Zavadskas *et al.*, 2004; Zavadskas, 2000). In those cases, use was made of well-known solution procedures assuming definite gain functions. However, it should be noted, that it is often quite difficult to obtain precise information for practical applications. In many cases, it is only possible to get rough values. However, since precise information is required, the lack of accuracy will affect the quality of the solution obtained. In the following discussion, the theory of matrix games is used alongside the theory of fuzzy sets, which offers the possibility to take into account the phenomenon known as fuzziness.

A number of articles dealing with fuzzy matrix games can be found in the literature.

In the work of L. Campos (1989), a solution of two-person zero-sum game is offered for a matrix with fuzzy pay-offs. The suggested approach draws on a commonly used method for solving a classic game and is referred to as fuzzy linear programming (FLP). To solve FLP problems, some additional well-known models aimed to assess fuzzy numbers are also offered.

In the paper of L. Campos *et al.* (1992), a general method of solving fuzzy matrix game is presented. The above method may be used when players choose their fuzzy number ranking procedures in a wide class that may be represented by linear ranking functions. Two cases are studied. In the first case, both players use the same criterion to rank fuzzy numbers, while in the second case, each player uses different criteria.

C.R. Bector *et al.* (2003) consider a problem of solving a matrix game with fuzzy pay-offs based on the principle of duality in linear programming.

In the paper of Takashi Maeda (2003), two-person zero-sum games with fuzzy pay-offs are considered. The criterion of minimax is used. Three kinds of concepts of minimax equilibrium strategies are defined and their properties are investigated. It is shown that the equilibrium strategies considered may be characterized as Nash equilibrium strategies (belonging to a family of parametric bi-matrix games with crisp pay-offs). The properties of values of fuzzy matrix games are investigated by means of possibility and necessity measures.

In the present paper, a model of a multicriteria fuzzy game is offered for solving various engineering problems.

2. Classical Matrix Games

In their classical form (Neumann and Morgenstern, 1943), matrix games are described with respect to two nonempty sets S_1 and S_2, the strategy sets of players I and II, and the gain function $A(S_1, S_2)$ defined for the Cartesian product $S_1 \times S_2$. Use is generally made of the symbolic notation

$$\Gamma = (S_1, S_2, A). \tag{1}$$

For a solution, the two players orientate themselves with respect to the payoff bounds, namely:

$$\underline{a}(S_1) = \inf_{s_2 \in S_2} a(s_1, s_2) \quad \text{warranty bound for } s_1 \in S_1,$$

and

$$\overline{a}(S_2) = \sup_{s_1 \in S_1} a(s_1, s_2) \quad \text{warranty bound for } s_2 \in S_2.$$

Choosing the optimum warranty bounds, we obtain:

$$a_*(S_1, S_2) = \max_{s_1 \in S_1} \inf_{s_2 \in S_2} a(s_1, s_2),$$

and

$$a^*(S_1, S_2) = \min_{s_2 \in S_2} \sup_{s_1 \in S_1} a(s_1, s_2).$$

To arrive at a solution, it is necessary to rely on equilibrium that manifests itself as a saddle point.

For the game $\Gamma = (S_1, S_2, A)$ a saddle point will be obtained, if and only in the expressions

$$\max_{s_1 \in S_1} \inf_{s_2 \in S_2} a(s_1, s_2) \quad \text{and} \quad \min_{s_2 \in S_2} \sup_{s_1 \in S_1} a(s_1, s_2)$$

exist and are equal, i.e., if

$$\max_{s_1 \in S_1} \inf_{s_2 \in S_2} a(s_1, s_2) = \min_{s_2 \in S_2} \sup_{s_1 \in S_1} a(s_1, s_2).$$

Since the strategy sets are finite, the expressions do always exist. (Accordingly, it is possible to replace inf and sup by min and max, respectively).

The equilibrium strategies of player 1 are those strategies $s_1 \in S_1$, for which the s_2 infimum reaches the maximum relative to s_1. Similarly, the equilibrium strategies for player 2 are those strategies $s_2 \in S_2$, for which the s_1 supremum reaches the minimum relative to s_2.

The criterion

$$\max_{s_1 \in S_1} \min_{s_2 \in S_2} a(s_1, s_2) = \min_{s_2 \in S_2} \max_{s_1 \in S_1} a(s_1, s_2) = \nu \tag{2}$$

is a well-known min-max principle (Neumann and Morgenstern, 1943), with the value v being the game value.

3. Fuzzy Sets

The theory of fuzzy sets, which is based upon the investigation reported by Zadeh (Zadeh, 1965), involves a mathematical description of vague (inexact, fuzzy) elements, with the vagueness of information resulting not from the stochastic character of the systems, but from the lack of uniqueness or selectivity thereof. Accordingly, the answer to the question whether an element is associated with a fuzzy set will not be in the form of a YES-OR-NO decision but it will require a carefully graded judgment of its association. The degree of association of defined elements is determined by an association function that must come within the scope of the particular mathematical definitions, axioms, and operational rules.

A fuzzy set A in X is a set of ordered pairs.

$$A = \left\{ x,_{/} u_A(x)/x \in X \right\}. \tag{3}$$

In the above expression, $_{/}u_A(x)$ is the degree of association of x with the fuzzy set A. $_{/}u_A$: $X \rightarrow R$ means that an association function is a real-value function.

Usually, the range of values of $_{/}u_A$ is restricted to the closed interval (Zadeh, 1965).

For a fuzzy decision, the association function $_{/}u_A(x)$ indicates the degree to which each element x will satisfy the respective requirements.

An element $x \in A$ signifies an optimum fuzzy decision if x possesses the maximum degree of association with A.

A widely held view ascribable to Bellmann and Zadeh (1970) is that a fuzzy decision is defined as an average of the fuzzy sets for fuzzy objectives Z and fuzzy restrictions R.

For the incomplete set of two fuzzy sets Z and R the association function is defined, pointwise, by the operator

$$_{/}u_A(x) = \mathrm{Min}[_{/}u_Z(x),_{/} u_R(x)]. \tag{4}$$

4. Association Function

For constituting the association function, the totality of values $_{/}u_A(x)$ for all the elements x from X should be taken into account.

Various concepts can be used to determine the association function. Frequently, piecewise linear association functions are considered.

In the interval $x_m > x_0$ we obtain:

$$_{/}u_A(x) = \left\{ \begin{array}{ll} 0 & \text{for } x \leqslant x_0, \\ 1 - \frac{x - x_0}{x_m - x_0} & \text{for } x_0 \leqslant x \leqslant x_m, \\ 1 & \text{for } x_m \leqslant x. \end{array} \right\}. \tag{5}$$

In addition to the linear slope of an association function, consideration is also given to an S-shaped behaviour. The advantage of nonlinear association functions lies in the fact that the transition to the "nonassociated" and "completely associated" ranges takes place far more harmoniously. The interpolating cubic spline function proved itself extremely useful for practical examples within the framework of multicriterion decision problems (Albrycht and Matloka, 1985). Using the supporting points $(x_0, 0),\ (x_D, 0.5),\ (x_M, 1)$ and the boundary conditions $_{/}u_A^{'} = (x_0) =_{/} u_A^{'}(x_M) = 0$ we obtain two third degree polynomials which are joined together in x_D in a twice continuously differentiable form. This gives the following set-up for $x_0 < x_M$:

$$_{/}u_A(x) = \left\{ \begin{array}{ll} 0 & \text{for } x \leqslant x_0, \\ Ax^3 + Bx^2 + Cx + D & \text{for } x_0 \leqslant x \leqslant x_D, \\ Ex^3 + Fx^2 + Gx + H & \text{for } x_D \leqslant x \leqslant x_M, \\ 1 & \text{for } x_M \leqslant x. \end{array} \right\}. \tag{6}$$

The coefficients $(A, \dots, H)$ are calculated from a system of equations (7) derived taking into account the requirements of $_{/}u_A(x)$ such as continuity, existence of the first and second derivatives, and choice of supporting points and boundary conditions.

$$\begin{array}{ll} (G_1) & Ax_0^3 + Bx_0^2 + Cx_0 + D = 0, \\ (G_2) & Ax_D^3 + Bx_D^2 + Cx_D + D = 0.5, \\ (G_3) & Ex_D^3 + Fx_D^2 + Cx_D + H = 0.5, \\ (G_4) & Ex_M^3 + Fx_M^2 + Cx_M + H = 1, \\ (G_5) & 3Ax_0^2 + 2Bx_0 + C = 0\,, \\ (G_6) & 3Ex_M^2 + 2Fx_M + G = 0, \\ (G_7) & 3Ax_D^2 + 2Bx_D + C - 3Ex_D^2 - 2Fx_D - G = 0, \\ (G_8) & 6Ax_D + 2B - 6Ex_D - 2F = 0. \end{array} \tag{7}$$

This system of equations $\{(G_1), \dots, (G_8)\}$ has a unique solution.

For $_{/}u_A(x)$ to be $C[0,1]$ and monotonic in x_j, it is still necessary for the condition

$$-1 + \sqrt{2} \leq \frac{|x_M - x_D|}{|x_D - x_0|} \leqslant 1 + \sqrt{2} \tag{8}$$

to be satisfied, which is usually the case. Problems will be encountered only if x_D is in the region of x_0 or x_M, respectively.

5. Fuzzy Matrix Games

The games described in Section 2 of this paper will now be considered in terms of the theory of fuzzy sets. Similar approaches have already been reported in, for example, (Albrycht and Matloka, 1985; Kraft, 1979; Menges, 1981; Schwab, 1983). However, the lack of operationalization has not yet allowed them to become practically used. The classical theory of games assumes that interpersonal conflict situations can be precisely described mathematically. The assumption made in this context is that the elements of a particular game can be represented as sharply defined sets. This involves an analysis of given mathematical expressions. However, for more stringent requirements to modeling the existence of clearly defined sets can not be postulated.

The elements of the game are affected by various sources of fuzziness. The gain or payoff function is not always defined numerically or sharply, respectively. It is fomulated semantically and, at the same time, fuzzily, in such terms as excellent, good, or sufficiently reliable, durable, resistant etc. The strategics employed by players are usually marked by different levels of significance and intensity. These and other conditions account for the need to include the theory of fuzzy sets in the solution concept of the theory of games.

For two players employing the defined strategy sets that are wholly or partially comprised of fuzzy information the fuzzy game $\Gamma_{/}u$ can be written as follows:

$$\Gamma_{/}u = \left\{(S_{1i,/}\, u_{1i}); (S_{2i,/}\, u_{ij}); (a_{ij,/}\, \tilde{u})\right\}. \tag{9}$$

With S_{1i}: for $i = 1, \ldots, m$ Strategies of player I

${}_/u_{1i}$: for $i =, \ldots, m$ Association function for the strategies of player I

S_{2i}: for $j = 1, \ldots, n$ Strategies of player II

${}_/u_{ij}$: for $i = 1, \ldots, m;\ j = 1, \ldots, n$ Association function for the strategies of player II with respect to the strategies of player I

a_{ij}: for $i = 1, \ldots, m;\ j = 1, \ldots, n$ Payoff or gain function

${}_/u_{ij}$: for $i = 1, \ldots, m;\ j = 1, \ldots, n$ Association function for the payoff function

The transition from game Γ to game $\Gamma_/u$ is accomplished in three steps.

Step 1:

A fuzzy set is defined for the set of strategies of player I. The set of strategies and the criteria quantitatively describing the strategies are assumed to be known. An association function (6) is calculated for each of the criteria, i.e., standard values are relativized to give the values of association. Thus, we obtain, for each strategy of player I, a value of association for different criteria. A set of values of association is expressed as an arithmetic mean (Laplace criterion).

$$ {}_/u_{1i} = \frac{1}{L} \sum_{i=1}^{L} {}_/u_{il}. \tag{10} $$

The values ${}_/u_{li}$ are calculated in the matrix (11).

	K_1	K_2	...	K_1	...	K_L	${}_/u_{1i}$
S_{11}	${}_/u_{11}$	${}_/u_{12}$	...	${}_/u_{1l}$	...	${}_/u_{1L}$	${}_/u_{11}$
S_{12}	${}_/u_{21}$	${}_/u_{22}$	...	${}_/u_{21}$	...	${}_/u_{2L}$	${}_/u_{12}$
...	...	...	...	...	...	...	...
S_{1i}	${}_/u_{i1}$	${}_/u_{i2}$	...	${}_/u_{il}$	...	${}_/u_{il}$	${}_/u_{1i}$
...	...	...	...	...	...	...	...
S_{1m}	${}_/u_{m1}$	${}_/u_{m2}$	...	${}_/u_{ml}$	...	${}_/u_{mL}$	${}_/u_{1m}$

(11)

Step 2:

Step 2 is concerned with the strategies for player II.

Fuzzy sets are defined for the set of strategies of player II, and the values of association are calculated according to (6). The mapping of sets is in the form of a matrix, initially signifying a basic matrix (12) for the game to be resolved. Whereas the matrix in Step 1 was used for an additive purpose, the basic matrix is to be interpreted in terms of the games theory.

	S_{21}	S_{22}	...	S_{2j}	...	S_{2n}
S_{11}	${}_/u_{11}$	${}_/u_{12}$	...	${}_/u_{1j}$	...	${}_/u_{1n}$
S_{12}	${}_/u_{21}$	${}_/u_{2j}$	...	${}_/u_{2j}$	...	${}_/u_{2n}$
...	...	...	...	...	...	...
S_{1i}	${}_/u_{i1}$	${}_/u_{i2}$	...	${}_/u_{1j}$	...	${}_/u_{in}$
...	...	...	...	...	...	...
S_{1m}	${}_/u_{m1}$	${}_/u_{m2}$	...	${}_/u_{mj}$	...	${}_/u_{mn}$

(12)

Step 3:

Step 3 is a summary of Steps 1 and 2. This is an average of the strategy sets of players I and II, with Min being chosen as a logic operator as mentioned above (4).

$$_{/}\tilde{u}_{ij} = \mathrm{Min}(_{/}u_{1i},_{/}u_{ij}). \tag{13}$$

As a result, the fuzzy game matrix (14) is obtained. Resolution is based on the min-max principle (2) taken over from the classical theory of games.

$$\begin{array}{c|ccccccc} & S_{21} & S_{22} & \dots & S_{2j} & \dots & S_{2n} \\ \hline S_{11} & _{/}\tilde{u}_{11} & _{/}\tilde{u}_{12} & \dots & _{/}\tilde{u}_{1j} & \dots & _{/}\tilde{u}_{1n} \\ S_{12} & _{/}\tilde{u}_{21} & _{/}\tilde{u}_{2j} & \dots & _{/}\tilde{u}_{2j} & \dots & _{/}\tilde{u}_{2n} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ S_{1i} & _{/}\tilde{u}_{i1} & _{/}\tilde{u}_{i2} & \dots & _{/}\tilde{u}_{1j} & \dots & _{/}\tilde{u}_{in} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ S_{1m} & _{/}\tilde{u}_{m1} & _{/}\tilde{u}_{m2} & \dots & _{/}\tilde{u}_{mj} & \dots & _{/}\tilde{u}_{mn} \end{array} \tag{14}$$

6. Case Study

Because of constantly growing demand for drinking water, the inpmprovement of water supply is needed. This may be achieved by joining up a 38.8 km-long pipeline to the existing water-supply system. To analyse the available alternatives, the above-mentioned algorithm was used.

When laying the pipeline, the following requirements had to be taken into account: to take a the free flow of water in distributing it in order to save the electric power; to include the existing reservoirs to bypass settlements, nature reserves and ground water; to reduce changes in routing to a minimum and avoid sharp turns.

The above requirements were too rigorous imposing a great number of limitations and not allowing for developing the alternatives of pipeline lay-out on a particular territory. Therefore, the task was only to choose adequate pipes and engineering equipment.

The alternatives could be developed for building materials (reinforced concrete, steel pipes covered by cement mortar on the inside and ordinary steel pipes) and for the pipe diameters. Therefore, a designer was faced with a choise among various grades of materials and tube diameters under the conditions of hydraulic durability. Special attention had to be paid to the areas of ground settlement where only steel tubes could be used. Finally, the following alternatives were obtained:

alternative 1 – gravity pipeline, 1000 and 800 mm prestressed concrete, 18.9 % steel with the inner cement mortar coating;
alternative 2 – gravity pipeline, 1000 and 800 mm steel with the inner cement mortar coating;
alternative 3 – gravity pipeline, 1000 and 800 mm steel;
alternative 4 – with a pump house, 800 mm prestressed concrete, 18.9 % steel with the inner cement mortar coating;
alternative 5 – with a pump house, 800 mm steel with the inner cement mortar coating;
alternative 6 – with a pump house, 1000 and 800 mm steel;
alternative 7 – with a pump house, 1000, 800, 600 mm prestressed concrete, 18.9% steel with the inner cement mortar coating;
alternative 8 – with a pump house, 1000, 800, 600 mm steel with the inner cement mortar coating;
alternative 9 – with a pimp house, 1000, 800, 600 mm steel.

To evaluate the alternative, various internal parameters had to be considered. In particular, territorial parmeters including the location of the pipeline and the construction site were analysed.

The environment was taken into account indetermining territorial parameters and those relating to the location of the pipeline. In the case studied, it was the use of agricultural areas for laying the pipeline.

The parameters relating to the construction work include the elements associated with building products. Labour force, equipment, etc. In our case, they included terms of construction, expenses, the amount of steel needed and the required number of pipes. The values obtained are given in Table 1.

Table 1

Internal factors (parameters)

No alternative	Parameters				
	location	technological			
	K_1 (ha)	K_2 (%)	K_3 (mln. Euro)	K_4 (t)	K_5 (kg/m)
A1	133.81	1.14	150.90	2946.1	671.29
A2	110.64	1.10	161.60	9651.4	306.33
A3	110.64	1.00	152.39	9651.4	248.75
A4	130.25	1.14	146.11	2550.9	582.82
A5	107.09	1.10	155.40	7760.0	245.60
A6	108.03	1.00	151.72	7760.0	212.88
A7	131.83	1.14	142.63	2233.2	604.83
A8	109.73	1.10	152.65	8182.8	259.30
A9	110.16	1.00	144.70	8413.1	216.83

Note: K_1 – usable area, hectare; K_2 – variation of construction period, %; K_3 – investment, mln. Euro; K_4 – steel demand, t; K_5 – pipe mass, kg/m.

Technological parameters include the cost of pipes to be written off and cost of construction and power. According to the experimental data, pipes to be written off annually will make 2.67% for steel pipeline, while for the pipes made of prestressed concrete or steel with cement coating they will make 1.66%.

The data obtained were transformed into a Fuzzy version by using the formula (6), and value the value $\mu 1$ was calculated according to (10) (Table 2).

Then, the external (Environmental and industrial) parameters were considered. The environmental parameters include harmful environmental effects on the pipeline which may cause its deformations. To avoid them, it was necessary to determine the resistance of the materials of the pipes (of various diameters) to soil slips, settlement or deformation. For this purpose, the estimation scale based on scoring was used in considering the alternatives. It was assumed that elastic steel is more resistant to deformations than hard prestressed concrete and steel pipes with the inner coating of cement mortar are more stable and, therefore, more preferable than ordinary steel pipes. The resistance of pipes of smaller diameters made of various materials is different. Thus, the resistance of prestressed concrete pipes decreases proportionally to the decrease of their diameters, because, in this case, pipe coupling plays an important role. Steel pipes of large diameters may carry higer loads without deformation, because they are bent on a large radius.

The cost of production including annual expenses on pipeline maintenance should also include the cost of power supplied to the pipe laying and commissiong sectors, cost of fuel for transport facilities and means of maintenance and repair. As a result, the following data were obtained:

The data obtained were transformed into a Fuzzy version by formula (6) and the main matrix (12) was generated.

By using the formula (13) from Tables 2 and 4 we get a Fuzzy game matrix (14).

By applying the minmax principle, the alternative 8 was obtained as the most rational decision. According to this alternative, the gravity pipeline is rather long and has one pump station. The steel pipes covered with cement mortar on the inside are used. Pipes of 1000 mm in diameter are layed at the section of 14.35 km, while at the next section

Table 2

Determining u_i values

No alternative	K_1	K_2	K_3	K_4	K_5	Laplace criterion
A1	0	0.0086	0.6203	0.9726	0.0008	0.3203
A2	0.9511	0.5000	0	0	0.5143	0.3930
A3	0.9511	1.0000	0.5228	0	0.9816	0.6911
A4	0.0308	0.0086	0.8901	0.9937	0.1069	0.4060
A5	1.0000	0.5000	0.2036	0.1485	0.8007	0.5305
A6	0.9815	1.0000	0.5668	0.1485	1.0000	0.7392
A7	0.0010	0.0086	1.0000	1.0000	0.0709	0.4161
A8	0.9565	0.5000	0.5215	0.0921	0.7057	0.5551
A9	0.9520	1.0000	0.9403	0.0608	0.9993	0.7904

Table 3

External factors (parameters)

No alternative	Parameters		
	Environmental	Production/Economical	
	Y_1(scale)	Y_2(mln. Euro)	Y_3(T Euro)
A1	9	2.50	850.0
A2	2	2.68	850.0
A3	3	4.07	850.0
A4	8	2.42	1333.9
A5	3	2.58	1333.9
A6	4	4.05	1333.9
A7	7	2.36	1013.2
A8	4	2.53	1013.2
A9	5	3.86	1080.3

Note: Y_1 – resistance to deformations, scale; Y_2 – cost of pipes to be written of, mln. Euro; Y_3 – cost of production and power, T Euro.

Table 4

Basic matrix

$S_1 \setminus S_2$	$Y_1(S_{21})$	$Y_2(S_{22})$	$Y_3(S_{23})$
$x_1(S_{11})$	0.0280	0.9807	1.0000
$x_2(S_{12})$	0.9700	0.9011	1.0000
$x_3(S_{13})$	0.8900	0.0005	1.0000
$x_4(S_{14})$	0.1070	0.9872	0.0054
$x_5(S_{15})$	0.8900	0.9343	0.0054
$x_6(S_{16})$	0.7770	0.0021	0.0054
$x_7(S_{17})$	0.1510	0.9967	0.6455
$x_8(S_{18})$	0.7770	0.9552	0.6455
$x_9(S_{19})$	0.6420	0.0485	0.3736

Table 5

Fuzzy game matrix

$S_1 \setminus S_2$	$Y_1(S_{21})$	$Y_2(S_{22})$	$Y_3(S_{23})$	min max
$(S_{11}; 0.3203)$	(0.0280 \ 0.0280)	(0.3203 \ 0.9807)	(0.3203 \ 1.0000)	0.0280
$(S_{12}; 0.3930)$	(0.3930 \ 0.9700)	(0.3930 \ 0.9011)	(0.3930 \ 1.0000)	0.3930
$(S_{13}; 0.6911)$	(0.6911 \ 0.8900)	(0.0005 \ 0.0005)	(0.6911 \ 1.0000)	0.0005
$(S_{14}; 0.4060)$	(0.1070 \ 0.1070)	(0.4060 \ 0.9872)	(0.0054 \ 0.0054)	0.0054
$(S_{15}; 0.5305)$	(0.5305 \ 0.8900)	(0.5305 \ 0.9349)	(0.0054 \ 0.0054)	0.0054
$(S_{16}; 0.7392)$	(0.7392 \ 0.7770)	(0.0021 \ 0.0021)	(0.0054 \ 0.0054)	0.0021
$(S_{17}; 0.4161)$	(0.1510 \ 0.1510)	(0.4161 \ 0.9967)	(0.6455 \ 0.4161)	0.1510
$(S_{18}; 0.5551)$	(0.7770 \ 0.5681)	(0.5551 \ 0.9552)	(0.5551 \ 0.6455)	0.5551
$(S_{19}; 0.7904)$	(0.6420 \ 0.6420)	(0.0485 \ 0.0485)	(0.3736 \ 0.3736)	0.0485

of 12 km, 800 mm pipes are installed, and at the last 12.45 km section of the pipeline 600 mm pipes are layed.

7. Concluding Remarks

The algorithm developed for fuzzy matrix games is a fuzzy concept for multi-criteria decisions fuzzy matrix games multi-criteria model for decision-making in engineering. This concept was developed in order to take into consideration both internal and external influential variables. Internal influential variables have an experiential character and will be effective until the system is made use of (building or manufacturing phase, respectively). External influential variables describe a new quality.

They have a predictive character and represent the phase of utilization. Thus, an algorithm is available which also enables quality features having a hierarchical structure to be aggregated, with different phases being allowed.

Practical investigations have already been discussed for selecting the variants water supply systems.

The strategies of player I include the constructional variants. These are studied with due consideration of the following aspects: Territorial and layout parameters such as space requirements, absence of crossings, possible connections and building parameters such as time and amount of building, possible extensions, capital costs. They represent what is known as internal influential variables. Interbalancing of parameters is allowed so that the application of a compensatory operator (10) is justifiable. The result of Step 1 of the fuzzy matrix game $\Gamma_{/}u$ is used to assess the strategies of player 1 by values of association. This serves to express the data obtained in the strategies employed by player I. The strategies of player II include the use-related influences, i.e., resistence to failure, depreciation, and operating and power costs. They represent what is known as external influential variables.

Interaction of the strategies of player I with the strategies of player II is by the agency of the minimum operator (13). As a result, a matrix describing the fuzzy game $\Gamma_{/}u$ is obtained.

The resolution of game $\Gamma_{/}u$ has a strategic character. It is used for the above-discussed examples of the selection of an optimum variant with due consideration of several criteria and the satisfaction of practical conditions that are beset by uncertainty, lack of information, and fuzziness.

Fuzzy matrix games provide numerous new possibilities of handling practical engineering, economic, investment planning, and other problems. The resolution of fuzzy matrix games constitutes a new quality of decisions representing a high degree of complexity.

Special software environment has been created for testing by MS EXCEL. Based on testing results it is planned to develop in future general-purpose programs available for a wide range of users.

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Daugiakriterinis inžinerinių sprendimų modelis remiantis neapibrėžtų matricų lošimais

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Nagrinėjant inžinerines problemas susijusias su optimalaus varianto pasirinkimu, tyrinėtojas dažnai susiduria su nepakankamai tiksliais duomenimis. Variantai dažniausiai vertinami pagal įvairius kriterijus. Šis metodas remiasi ryšiais tarp neapibrėžtų aibių ir matricinių lošimų teorijų ir yra siūlomas daugiakriteriniams sprendimams priimti.

Siūlomo modelio panaudojimo galimybės parodomos sprendžiant vandentiekio trasos statybos uždavinį.