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On the Identification of Wiener Systems Having Saturation-like Functions with Positive Slopes

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Abstract. The aim of the given paper is the development of an approach for parametric identification of Wiener systems with piecewise linear nonlinearities, i.e., when the linear part with unknown parameters is followed by a saturation-like function with unknown slopes. It is shown here that by a simple data reordering and by a following data partition the problem of identification of a nonlinear Wiener system could be reduced to a linear parametric estimation problem. Afterwards, estimates of the unknown parameters of linear regression models are calculated by processing respective particles of input-output data. A technique based on ordinary least squares (LS) is proposed here for the estimation of parameters of linear and nonlinear parts of the Wiener system, including the unknown threshold of piecewise nonlinearity, too. The results of numerical simulation and identification obtained by processing observations of input-output signals of a discrete-time Wiener system with a piecewise nonlinearity by computer are given.

Key words: nonlinear systems, system identification, Wiener systems, parameter estimation.

1. Introduction

A lot of physical systems are naturally described as Wiener systems, i.e., when the linear system is followed by a static nonlinearity (Billings and Fakhouri, 1977; Bloemen *et al.*, 2001; Glad and Ljung, 2000; Greblicki, 1994; Hagenblad, 1999; Hunter and Korenberg, 1986; Kalafatis *et al.*, 1997; Ljung, 1999; Pupeikis *et al.*, 2003; Roll, 2003; Wigren, 1993). A special class of such systems is piecewise affine (PWA) systems, consisting of some subsystems, between which occasional switchings happen at different time moments (Hagenblad, 1999; Hansen and Seo, 2002; Roll, 2003). Assuming the nonlinearity to be piecewise linear, one could let the nonlinear part of the Wiener system be represented by different regression functions with some parameters that are unknown beforehand. In such a case, observations of an output of the Wiener system could be partitioned into distinct data sets according to different descriptions. However the boundaries of sets of observations depend on the value of the unknown threshold a – observations are divided into regimes subject to whether the some observed threshold variable is smaller or larger than a (Hagenblad, 1999; Hansen and Seo, 2002). Therefore the problem of identification of unknown parameters of nonlinear and linear blocks of the Wiener systems could be solved, if a simple way of partitioning the available data sets were found in the case of unknown a. Thus, there arises a problem, first, to find a way to partition the available data, second, to calculate the estimates of parameters of regression functions by processing particles of observations to be determined, and, third, to get the unknown threshold.

The next section introduces the statement of the problem to be solved. In Section 3, we solve the problem using the data rearrangement by the following reconstruction of the unknown intermediate signal. In Section 4, simulation results are presented. Section 5 contains conclusions.

2. Statement of the Problem

The Wiener system consists of a linear part $G(q, \mathbf{\Theta})$ followed by a static nonlinearity $f(\cdot, \eta)$ with the vector of parameters η. The linear part of the PWA system is dynamic, time invariant, causal, and stable. It can be represented by a time invariant dynamic system (LTI) with the transfer function $G(q, \mathbf{\Theta})$ as a rational function of the form

$$G(q, \mathbf{\Theta}) = \frac{b_1 q^{-1} + \ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots + a_m q^{-m}} = \frac{B(q, \mathbf{b})}{1 + A(q, \mathbf{a})} \tag{1}$$

with a finite number of parameters

$$\mathbf{\Theta}^T = (b_1, \ldots, b_m, a_1, \ldots, a_m), \quad \mathbf{b}^T = (b_1, \ldots, b_m), \quad \mathbf{a}^T = (a_1, \ldots, a_m), \tag{2}$$

that are determined from the set $\mathbf{\Omega}$ of permissible parameter values $\mathbf{\Theta}$. Here q is a time-shift operator (Ljung, 1999), the set $\mathbf{\Omega}$ is restricted by conditions on the stability of the respective difference equation. The unknown intermediate signal

$$x(k) = \frac{B(q, \mathbf{b})}{1 + A(q, \mathbf{a})} u(k) + v(k), \tag{3}$$

generated by the linear part of the PWA system (1) as a response to the input $u(k)$ and corrupted by the additive noise $v(k)$, is acting on the static nonlinear part $f(\cdot, \eta)$ (Fig. 1), i.e.,

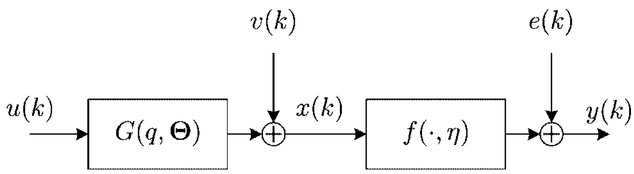

Fig. 1. The PWA system with the process noise $v(k)$ and that of the measurement $e(k)$. The linear dynamic part $G(q, \mathbf{\Theta})$ of the PWA system is parametrised by $\mathbf{\Theta}$, while the static nonlinear part $f(\cdot, \eta)$ – by η. Signals: $u(k)$ is input, $y(k)$ is output, $x(k)$ is an unmeasurable intermediate signal.

$$y(k) = f\big(x(k), \eta\big) + e(k). \tag{4}$$

Here the nonlinear part $f(\cdot, \eta)$ of the PWA system is a saturation-like function of the form (Hagenblad, 1999; Hansen and Seo, 2002)

$$f(x(k), \eta) = \begin{cases} c_0 + c_1 x(k) & \text{if } x(k) \leqslant -a, \\ x(k) & \text{if } -a < x(k) \leqslant a, \\ d_0 + d_1 x(k) & \text{if } x(k) > a, \end{cases} \tag{5}$$

that could be partitioned into three functions. These functions are: $f\{x(k; \mathbf{\Theta}), \mathbf{c}, a\} = c_0 + c_1 x(k)$, $f\{x(k; \mathbf{\Theta}), a\} = x(k)$, and $f\{x(k; \mathbf{\Theta}), \mathbf{d}, a\} = d_0 + d_1 x(k)$. The function $f\{x(k; \mathbf{\Theta}), \mathbf{c}, a\}$ has only negative values, when $x(k) \leqslant -a$, $f\{x(k; \mathbf{\Theta}), a\}$ has arbitrary positive, as well as negative values, when $-a < x(k) \leqslant a$, and $f\{x(k; \mathbf{\Theta}), \mathbf{d}, a\}$ has only positive values, when $x(k) > a$. Here $x(k; \mathbf{\Theta}) \equiv x(k)$, $\mathbf{c}^T = (c_0, c_1)$, $c_0 = -a(1 - c_1)$, $0 < c_1 < a$, $\mathbf{d}^T = (d_0, d_1)$, $d_0 = a(1 - d_1)$, $0 < d_1 < a$.

The process noise $v(k) \equiv \xi(k)$ and the measurement noise $e(k) \equiv \zeta(k)$ are added to an intermediate signal $x(k)$ and the output $y(k)$, respectively, $\xi(k)$, $\zeta(k)$ are mutually noncorrelated sequences of independent Gaussian variables with $E\{\xi(k)\} = 0$, $E\{\zeta(k)\} = 0$, $E\{\xi(k)\xi(k+\tau)\} = \sigma_\xi^2 \delta(\tau)$, $E\{\zeta(k)\zeta(k+\tau)\} = \sigma_\zeta^2 \delta(\tau)$; $E\{\cdot\}$ is a mean value, $\sigma_\zeta^2, \sigma_\xi^2$ are variances of ζ and ξ, respectively, $\delta(\tau)$ is the Kronecker delta function.

The aim of the given paper is to estimate parameters (2) of the linear part (1) of the PWA system, parameters $\eta = (c_0, c_1, d_0, d_1)^T$ of the nonlinear part (5), and the threshold a of nonlinearity (5) by processing N pairs of observations $u(k)$ and $y(k)$.

3. The Data Reordering

At first, let us rearrange the data $y(k)\ \forall k \in \overline{1, N}$ in an ascending order of their values. Thus, the observations of the reordered output $\tilde{y}(k)$ of the PWA system should be partitioned into three data sets: left-hand side data set (N_1 samples) with values lower than or equal to negative a, middle data set (N_2 samples) with values higher than negative a but lower or equal to a, and right-hand side data set (N_3 samples) with values higher than a. Here $N = N_1 + N_2 + N_3$. From the engineering point of view it is assumed that no less than 50% observations are concentrated on the middle-set and approximately by 25% or less on any side set. Hence, the observations with the highest and positive values will be concentrated on the right-hand side set, while the observations with the lowest and negative values on the left-hand side one. The observations of the middle data set of $\tilde{y}(k)$ are coincident with the respective observations of the intermediate signal $x(k)$ in the absence of the measurement noise $e(k)$. In such a case, one could get these observations simply by choosing the upper interval bound lower than the 75 percentage and the lower interval bound higher than the 25 percentage of the sampled reordered observations of $\tilde{y}(k)$.

Next, let us reconstruct an unmeasurable intermediate signal $x(k)$, using the middle data set $\tilde{y}(k)\ \forall k \in \overline{N_1 + l_1, N_2 - l_2}$ that is, really, reordered in an ascending order of their values $y(k)$ with small portions of missing observations within it that belong to the left-hand and right-hand side sets of the data. Here arbitrary integers $l_1, l_2 > 0$. To calculate an auxiliary signal $\hat{x}(k)$ (the estimate of unmeasurable $x(k)$) $\forall k \in \overline{1, N}$ one could approximate the model of the linear part of the PWA system (1) by the finite impulse response (FIR) system of the form

$$\tilde{y}(k) = \beta_0 + \beta_1 \tilde{u}(k) + \beta_2 \tilde{u}(k-1) + \cdots + \beta_\nu \tilde{u}(k-\nu+1) + \tilde{e}(k) \tag{6}$$

$\forall k \in \overline{N_1 + l_1, N_2 - l_2}$, or the expression in a matrix form

$$\widetilde{\mathbf{Y}} = \mathbf{\Lambda}\beta; \tag{7}$$

$$\widetilde{\mathbf{Y}} = \big(\tilde{y}(N_1 + l_1), \tilde{y}(N_1 + l_1 + 1), \ldots, \tilde{y}(\mathrm{T})\big)^T \tag{8}$$

is the $(L-\nu)\times 1$ vector of the middle data set of $\tilde{y}(k)$, $L = N_2 - l_2$, $\mathrm{T} = N_1 + N_2 + l_1 - l_2$,

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & \tilde{u}(N_1 + l_1) & \ldots & \tilde{u}(N_1 + l_1 - \nu + 2) & \tilde{u}(N_1 + l_1 - \nu + 1) \\ 1 & \tilde{u}(N_1 + l_1 + 1) & \ldots & \tilde{u}(N_1 + l_1 - \nu + 3) & \tilde{u}(N_1 + l_1 - \nu + 2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \tilde{u}(\mathrm{T}) & \ldots & \tilde{u}(\mathrm{T} - \nu + 2) & \tilde{u}(\mathrm{T} - \nu + 1) \end{bmatrix} \tag{9}$$

is the full rank $L \times (\nu + 1)$ regression matrix, consisting only of observations of the non-noisy input $\tilde{u}(k)$;

$$\beta^T = (\beta_0, \beta_1 \ldots, \beta_\nu) \tag{10}$$

is a $(\nu + 1) \times 1$ vector of unknown parameters, ν is the order of the FIR filter that can be arbitrarily large but fixed (Er-Wei Bai, 2002), $\tilde{u}(k)$ are observations of $u(k)$ associated with their own $\tilde{y}(k)$, $\tilde{e}(k) = v(k) + e(k)$.

The reasons for the use of the FIR model are as follows. In this case, the dependence of some regressors on the process output will be facilitated, and the assumption of the ordinary LS that the regressors depend only on the non-noisy input signal, will be satisfied (Eykhoff, 1974). This is the main consequence of replacing the initial transfer function $G(q, \mathbf{\Theta})$ of the linear part of the PWA system by the FIR filter (6). Besides, by applying the FIR model one avoids the influence of some missing regressors, appearing in the regression matrix $\mathbf{\Lambda}$, if the infinite impulse response (IIR) system is used. Then, the parametric estimation technique, based on ordinary LS, could be applied in the estimation of parameters (10) of the given FIR system (6), using the reordered observations of the middle data-set $\tilde{y}(k)$ $\forall k \in \overline{N_1 + l_1, N_2 - l_2}$, because the rearrangement of observations does not influence the accuracy of LS estimates to be calculated. In (Pupeikis *et al.*, 2003) ARMA and FIR models have been analysed by numerical simulation provided for distinct types of nonlinearities. Finally, the decision has been made: the parameter estimation results, obtained for Wiener systems using the FIR model, are more accurate than those based on the ARMA model. In (Bloemen *et al.*, 2001), the FIR model is used for the identification and predictive control of a distillation column.

To estimate the parameters β, one can use the expression

$$\hat{\beta} = (\mathbf{\Lambda}^T\mathbf{\Lambda})^{-1}\mathbf{\Lambda}^T\widetilde{\mathbf{Y}}, \tag{11}$$

where

$$\hat{\beta}^T = (\hat{\beta}_0, \hat{\beta}_1 \ldots, \hat{\beta}_\nu) \tag{12}$$

is a $(\nu + 1) \times 1$ vector of the estimates of parameters (10).

It ought to be noted that all proofs based on the deterministic regression matrix are valid here, too.

The estimate $\hat{x}(k)$ of the intermediate signal $x(k)$ could be determined using Eq. 6, where, instead of the true values (10), their estimates $\hat{\beta}$ are substituted, i.e.,

$$\hat{x}(k) = \hat{\beta}_0 + \hat{\beta}_1 u(k) + \hat{\beta}_2 u(k-1) + \ldots + \hat{\beta}_\nu u(k - \nu + 1) \tag{13}$$

$\forall k \in \overline{\nu, N}$. Thus, the result of this step is the auxiliary signal $\hat{x}(k)$ that is a reconstructed version of the intermediate signal $x(k)$. It will be used to calculate the estimates of parameters (2) at the next step.

Now, let us calculate the estimates of the parameters (2) of the transfer function $G(q, \mathbf{\Theta})$ according to

$$\widehat{\mathbf{\Theta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{U}. \tag{14}$$

Here

$$\widehat{\mathbf{\Theta}}^T = (\hat{\mathbf{b}}, \hat{\mathbf{a}})^T, \quad \hat{\mathbf{b}}^T = (\hat{b}_1, \ldots, \hat{b}_m), \quad \hat{\mathbf{a}}^T = (\hat{a}_1, \ldots, \hat{a}_m) \tag{15}$$

are $2m \times 1$, $m \times 1$, $m \times 1$ vectors of the estimates of parameters, respectively,

$$\mathbf{X} = \begin{bmatrix} u(m+\nu) & \ldots & u(\nu) & -\hat{x}(m+\nu) & \ldots & -\hat{x}(\nu) \\ u(m+\nu+1) & \ldots & u(\nu+1) & -\hat{x}(m+\nu+1) & \ldots & -\hat{x}(\nu+1) \\ \vdots & & \vdots & \vdots & & \vdots \\ u(N-1) & \ldots & u(N-m) & -\hat{x}(N-1) & \ldots & -\hat{x}(N-m) \end{bmatrix} \tag{16}$$

is the $(N - m - \nu - 1) \times 2m$ matrix, consisting of observations of the input $u(k)$ and the auxiliary signal $\hat{x}(k)$, and $\mathbf{U} = (\hat{x}(m + \nu + 1), \hat{x}(m + \nu + 2), \ldots, \hat{x}(N))^T$ is the $(N - m - \nu - 1) \times 1$ vector, consisting of the observations of $\hat{x}(k)$.

Estimates of the parameters c_0, d_0 and c_1, d_1 are calculated by the ordinary LS, too. In such a case, the sums of the form

$$I(c_0, c_1) = \sum_{i=1}^{N_1 - l_3} \left[\tilde{y}(i) - c_0 - c_1\tilde{\hat{x}}(i)\right]^2 = \min!, \tag{17}$$

$$I(d_0, d_1) = \sum_{j=N_2+l_4}^{N} \left[\tilde{y}(j) - d_0 - d_1\tilde{\hat{x}}(j)\right]^2 = \min!, \tag{18}$$

are to be minimized in respect of the parameters c_0, c_1 and d_0, d_1, respectively, using side-set data particles of $\tilde{y}(k)$ and associated observations of the auxiliary signal $\hat{x}(k)$. Here $\tilde{\hat{x}}(k)$ are the observations of the signal $\hat{x}(k)$ that are rearranged in accordance with $\tilde{y}(k)$, arbitrary integers $l_3, l_4 > 0$.

The estimates of parameters c_1, d_1 and c_0, d_0 are calculated according to (Malinvaud, 1969)

$$\hat{c}_1 = \frac{\sum_{i=1}^{N_1-l_3} \tilde{y}(i)\tilde{\hat{x}}(i)}{\sum_{i=1}^{N_1-l_3} \tilde{\hat{x}}^2(i)}, \quad \hat{d}_1 = \frac{\sum_{j=1}^{N_3-l_4} \tilde{y}(j)\tilde{\hat{x}}(j)}{\sum_{j=1}^{N_3-l_4} \tilde{\hat{x}}^2(j)}, \tag{19}$$

$$\hat{c}_0 = \frac{\sum_{i=1}^{N_1-l_3}[\tilde{y}(i) - \hat{c}_1\tilde{\hat{x}}(i)]}{N_1 - l_3}, \quad \hat{d}_0 = \frac{\sum_{j=1}^{N_3-l_4}[\tilde{y}(j) - \hat{d}_1\tilde{\hat{x}}(j)]}{N_3 - l_4}, \tag{20}$$

respectively, but using side-sets data particles of $\tilde{y}(k)$ and associated observations of the auxiliary signal $\hat{x}(k)$, that are reordered in accordance with $\tilde{y}(k)$.

At last, the estimates of the threshold a on the right-hand side and left-hand side sets are found according to

$$\hat{a} = \hat{d}_0/(1 - \hat{d}_1), \quad \hat{a} = \hat{c}_0/(1 - \hat{c}_1), \tag{21}$$

respectively.

If N_1 and N_3 are unknown beforehand then an approach used in robust estimation could be applied here, too (Er-Wei Bai, 2002). It could be assumed that instead of the PWA system (1)–(5) one deals with the LTI system (3) (Fig. 1). In such a case, one can suppose having N measurements with some portions of N_1 and N_3 outliers. The estimates of respective parameters could be determined after rejecting these $\widehat{N}_1 + \widehat{N}_3$ samples from the initial set of observations where $\widehat{N}_1$, $\widehat{N}_3$ are the estimates of N_1 and N_3, respectively. After checking all $N!/(N - (\widehat{N}_1 - \widehat{N}_3))!(\widehat{N}_1 + \widehat{N}_3)!$ variants, the "best" estimates have been found. A more efficient approach is worked out here for dynamic systems observed in an noisy environment (Er-Wei Bai, 2002).

The same problem could be solved as the nonlinear filtering one, forming the likelihood function and taking the maximum likelihood estimator to estimate unknown parameters. However, it should be noted that the simple output and associated input data reordering with a following reconstruction of an intermediate signal that is really unknown, allow us to turn the nonlinear problem to a linear one where linear estimators based on the ordinary LS, are efficient. The presented algorithm is not only adapted to the specific nonlinearity considered with a limited general interest, – the procedure used in data reordering could be applied to robust parametric identification of LTI dynamic systems, by processing input and noisy output observations in the presence of lonely or patchy outliers of large magnitude.

4. Numerical Simulation

The true intermediate signal $x(k)$ $k = \overline{1, N}$, of the PWA system (Figs. 2b, 3b) is given by (3). The true output signal (Figs. 2c, 3c) is described by

$$y(k) = \begin{cases} -0.9 + 0.1x(k) & \text{if } x(k) \leqslant -1, \\ x(k) & \text{if } -1 < x(k) \leqslant 1, \\ 0.9 + 0.1x(k) & \text{if } x(k) > 1 \end{cases} \tag{22}$$

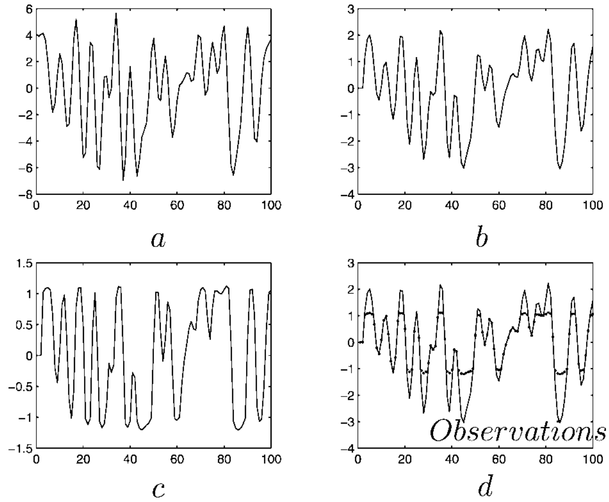

Fig. 2. The signals of the simulated PWA system with a piecewise nonlinearity (22): input $u(k)$, calculated by (23)(a), intermediate signal $x(k)$ (b), output $y(k)$ (c), intermediate and output (dotted line) signals (d).

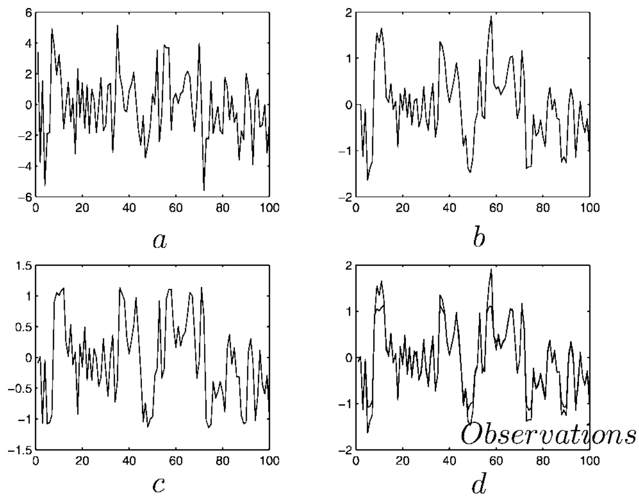

Fig. 3. The signals of the simulated PWA system with a piecewise nonlinearity (22): input $u(k)$ is white Gaussian noise (a), intermediate signal $x(k)$ (b), output $y(k)$ (c), intermediate and output signals (d).

with the sum of sinusoids (Fig. 2a)

$$u(k) = \frac{1}{20}\sum_{i=1}^{20} \sin(i\pi k/10 + \phi_i) \tag{23}$$

and white Gaussian noise with variance 4.5 (Fig. 3a) as inputs to the linear block

$$G(q, \mathbf{\Theta}) = \frac{b_1 q^{-1}}{1 + a_1 q^{-1}}. \tag{24}$$

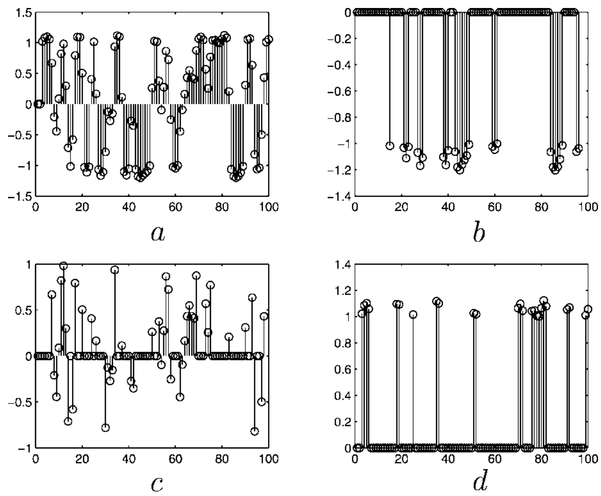

Fig. 4. Samples of $y(k)$ (a) (see Fig. 2c) and its data sets: left (b), middle (c), right (d)(here the observations, that belong to the other data set, are equal to zeros). Input $u(k)$ of the form (23) (see Fig. 2a).

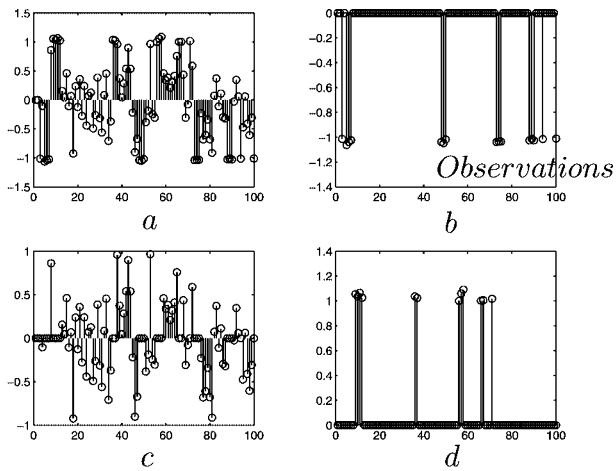

Fig. 5. Samples of signal $y(k)$ (a) (see Fig. 3c) and its data sets: left (b), middle (c), right (d). Input $u(k)$ is white Gaussian noise (see Fig. 3a).

Here $b_1 = 0.3$, $a_1 = -0.5$; in (23) the stochastic variables ϕ_k with a uniform distribution on $[0, 2\pi]$ were chosen. First of all, $N = 100$ data points have been generated without additive process and measurement noises (Figs. 4, 5). Afterwards, the LS problem (11) was solved, using 40 and 70 rearranged observations of the output, respectively (Figs. 6c, 7c), excluding zeros. The whole number of FIR filter parameters $\nu = 14$ was chosen based on the estimation results (Tables 1, 2), obtained for different ν in the absence of process and measurement noises. The estimate $\hat{x}(k)$ of the intermediate signal $x(k)$ was reconstructed according to (13), replacing unknown true values of parameters by their estimates. The reconstructed versions of the intermediate signal $x(k)$ are shown in Figs. 8a, 9a. The estimates $\widehat{\mathbf{\Theta}}^T = (b_1, a_1)$ of parameters $\mathbf{\Theta}$ of the transfer function $G(q, \mathbf{\Theta})$ were calculated by Eq. 14, using the observations of the auxiliary signal $\hat{x}(k)$. Afterwards, the estimate $\hat{x}_1(k)$ of the intermediate signal $x(k)$ was recalculated by

$$\hat{x}_1(k) = \hat{b}_1\, u(k-1) + \hat{a}_1 \hat{x}_1(k-1), \quad \forall k = \overline{2, 100}, \tag{25}$$

Table 1

The dependence of estimates of the parameters $b_1, a_1, c_0, c_1, d_0, d_1$, and thresholds a, $-a$ on the number of the FIR parameters ν. Input: the periodical signal (23)

Estimates	$\nu = 5$	$\nu = 10$	$\nu = 14$
$\hat{b}_1$	0.28	0.29	0.3
$\hat{a}_1$	−0.49	−0.5	−0.5
$\hat{c}_0$	−0.89	−0.89	−0.9
$\hat{c}_1$	0.09	0.1	0.1
$\hat{d}_0$	0.89	0.9	0.9
$\hat{d}_1$	0.09	0.1	0.1
$\hat{a}$	0.92	1	1
$-\hat{a}$	−0.95	−1	−1

Table 2

The values and notation are the same as in Table 1. Input: the Gaussian white noise

Estimates	$\nu = 5$	$\nu = 10$	$\nu = 14$
$\hat{b}_1$	0.29	0.3	0.3
$\hat{a}_1$	−0.49	−0.49	−0.5
$\hat{c}_0$	−0.89	−0.89	−0.9
$\hat{c}_1$	0.09	0.09	0.1
$\hat{d}_0$	0.89	0.9	0.9
$\hat{d}_1$	0.09	0.1	0.1
$\hat{a}$	0.96	0.98	1
$-\hat{a}$	−0.97	−0.99	−1

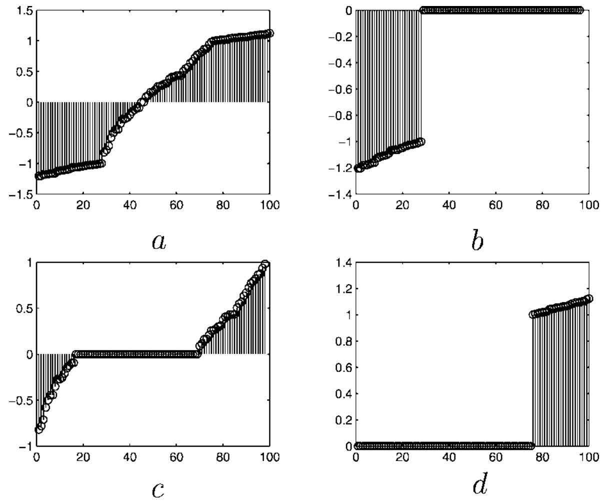

Fig. 6. The reordered in an ascending order of their values signal $y(k)$ (a) (see Figs. 2c, 4a) and its rearranged data sets: left (b), middle (c), right (d) (here the observations, that belong to the other data set, are equal to zeros). Input $u(k)$ of the form (23) (see Fig. 2a).

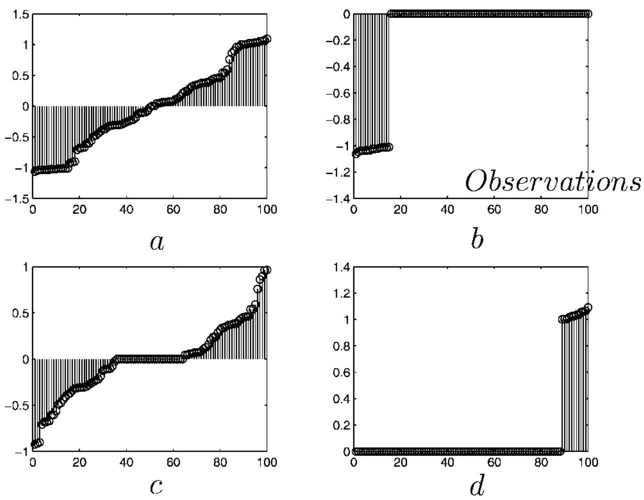

Fig. 7. The reordered in an ascending order of their values signal $y(k)$ (a) (see Figs. 3c, 5a) and its rearranged data sets: left (b), middle (c), right (d). Input $u(k)$ is white Gaussian noise (see Fig. 3a).

using $\hat{b}_1, \hat{a}_1$ and $\hat{x}_1(1) = 0$. In such a case, the estimates $\hat{b}_1, \hat{a}_1$ were equal to the true parameters: $b_1 = 0.3$, $a_1 = 0.5$. The reconstructed versions of the intermediate signal $x(k)$, calculated by Eq. 25 are shown in Figs. 8b, 9b.

It ought to be noted that the accuracy of estimates of the intermediate signal, calculated by formulas (13) and (25), is more or less similar except for the first 15 observations, when the FIR model (13) was used. If $\hat{x}(k)$ has been obtained, then it is simple to separate different particles of observations that belong to the respective side-sets. The estimates of

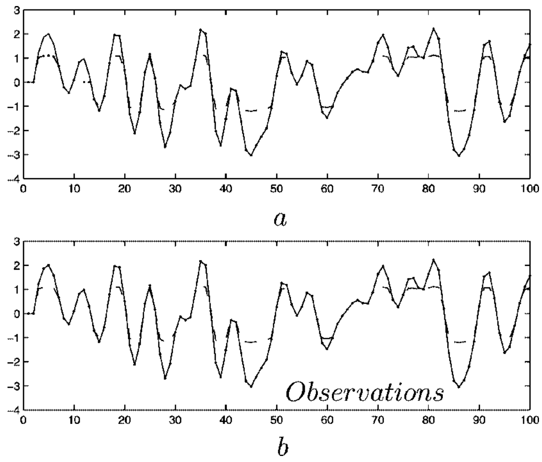

Fig. 8. The intermediate signal $x(k)$ (continuous line), the output signal $y(k)$ (dashed line), the reconstructed versions of $x(k)$ (dotted line), calculated using Eq. 13 (a) and Eq. 25 (b).

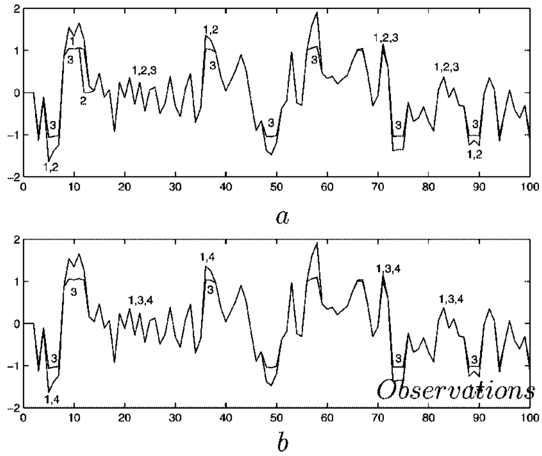

Fig. 9. The intermediate signal $x(k)$ (curve 1), the output signal $y(k)$ (curve 3), the reconstructed versions of $x(k)$ (curves 2, 4), calculated using Eq. 13 (a) and Eq. 25 (b), respectively.

parameters c_1, d_1 and c_0, d_0 are calculated according to formulas (19) and (20), respectively. In such a case, the rearranged observations of $\hat{x}(k)$ and $y(k)$ were substituted in formulas (19) and the estimates of c_1 and d_1 were determined: $\hat{c}_1 = \hat{d}_1 = 0.1$. Then, the estimates $\hat{c}_0$ and $\hat{d}_0$ were calculated by (20). Their values are also coincidental with the values of true coefficients: $\hat{c}_0 = -0.9$, while $\hat{d}_0 = 0.9$. It should be noted that $N_1 = 31$, $N_3 = 29$ for the periodical signal (23) (Fig. 2a) and $N_1 = 15$, $N_3 = 12$ for the Gaussian white noise (Fig. 3a) were used to calculate the estimates $\hat{c}_0, \hat{c}_1, \hat{d}_0, \hat{d}_1$, respectively. The estimates of the threshold were established by Eqs. 21. The values of estimates $\hat{a}$ were equal to the true value $a = 1$.

Table 3

Averaged estimates of the parameters $b_1, a_1, c_0, c_1, d_0, d_1$, and thresholds a, $-a$ with their confidence intervals. Input: the periodical signal (23). $\mathrm{SNR}^v = 100$

Estimates	$\mathrm{SNR}^e = 1$	$\mathrm{SNR}^e = 10$	$\mathrm{SNR}^e = 100$
$\hat{b}_1$	0.28 ± 0.07	0.3 ± 0.00	0.3 ± 0.00
$\hat{a}_1$	-0.52 ± 0.06	-0.5 ± 0.00	-0.5 ± 0.00
$\hat{c}_0$	-0.85 ± 0.31	-0.89 ± 0.04	-0.9 ± 0.00
$\hat{c}_1$	0.16 ± 0.21	0.1 ± 0.02	0.1 ± 0.00
$\hat{d}_0$	1 ± 0.5	0.89 ± 0.04	0.9 ± 0.00
$\hat{d}_1$	0.04 ± 0.4	0.1 ± 0.02	0.1 ± 0.00
$\hat{a}$	0.97 ± 0.25	1 ± 0.02	1 ± 0.00
$-\hat{a}$	-1 ± 0.22	-1 ± 0.02	-1 ± 0.00

Table 4

The values and notation are the same as in Table 1. Input – the Gaussian white noise

Estimates	$\mathrm{SNR}^e = 1$	$\mathrm{SNR}^e = 10$	$\mathrm{SNR}^e = 100$
$\hat{b}_1$	0.31 ± 0.04	0.3 ± 0.00	0.3 ± 0.00
$\hat{a}_1$	-0.39 ± 0.08	-0.5 ± 0.01	-0.5 ± 0.00
$\hat{c}_0$	-0.5 ± 0.86	-0.84 ± 0.06	-0.89 ± 0.00
$\hat{c}_1$	0.42 ± 0.72	0.1 ± 0.05	0.1 ± 0.00
$\hat{d}_0$	1.07 ± 0.51	0.91 ± 0.06	0.9 ± 0.00
$\hat{d}_1$	0.04 ± 0.44	0.15 ± 0.05	0.1 ± 0.00
$\hat{a}$	1.03 ± 0.4	0.99 ± 0.02	1 ± 0.00
$-\hat{a}$	-1.21 ± 0.19	-1 ± 0.02	-1 ± 0.00

In order to determine how realizations of different process- and measurement noises affect the accuracy of estimation of unknown parameters, we have used the Monte Carlo simulation with 10 data samples, each containing 100 pairs of input-output observations (Hagenblad, 1999). 10 experiments with the same realization of the process noise $v(k)$ and different realizations of the measurement noise $e(k)$ with different levels of its intensity have been carried out. The intensity of noises was assured by choosing respective signal-to-noise ratios SNR (the square root of the ratio of signal and noise variances). For the process noise, SNR^v was equal to 100, and for the measurement noise, SNR^e: 1, 10, 100. As inputs for all given nonlinearities the periodical signal (23) and white Gaussian noise with variance 1 were chosen. In each ith experiment the estimates of parameters were calculated. During the Monte Carlo simulation averaged values of estimates of the parameters and of the threshold and their confidence intervals were calculated. In Tables 3 and 4, for each input the averaged estimates of parameters and the threshold a of the simulated PWA system (Fig. 1) with the linear part (24) ($b_1 = 0.3;\ a_1 = -0.5$) and the piecewise nonlinearity (22) ($c_0 = -0.9$, $c_1 = 0.1$, $d_0 = 0.9$, $d_1 = 0.1$) with their confidence intervals are presented. It ought to be noted that in each experiment here the value of SNR^v was fixed and was the same, while the values of SNR^e were varying due to different realizations of $e(k)$. The Monte Carlo simulation (Tables 3, 4) implies that the accuracy of parametric identification of the PWA system depends on the intensity of measurement noise.

5. Conclusions

One of important types of hybrid systems met in practice is piecewise affine Wiener systems. As a rule, piecewise nonlinearities include a saturation-like function. They cannot be described by polynomials and are usually noninvertible. On the other hand, it is known that the PWA system consists of some subsystems, among which switchings occur at occasional time moments. They could be presented by different threshold regression models, including the model of the linear part of the PWA system. Indeed, for the parametric identification of such systems, the ordinary LS can be applied, if it is known how to recognize observations that belong to different subsystems. Really, observations do not possess distinct signs that could help us to attribute them to different data sets. Therefore, frequently a problem of parametric identification of the PWA system is solved as a problem of the Wiener system with a noninvertible nonlinearity with all well-known consequences beforehand. Here the nonlinear filtering approach based on the extended Kalman filter or mixed-integer programming method are used, too (Bloemen *et al.*, 2001; Hagenblad, 1999; Roll, 2003).

It is shown here that a problem of identification of PWA systems could be essentially reduced by a simple data rearrangement in an ascending order according to their values. Thus, the available data are partitioned into three data sets that correspond to distinct threshold regression models. Afterwards, the estimates of unknown parameters of linear regression models can be calculated by processing respective sets of the rearranged output and associated input observations. A technique, based on ordinary LS, is proposed here for estimating the parameters of linear and nonlinear parts of the Wiener system, including the unknown threshold of the piecewise nonlinearity, too. During successive steps the unknown intermediate signal is reconstructed and the missing values of observations of respective data particles are replaced by their estimates. Various results of numerical simulation (Figs. 2–9), including that of Monte Carlo (Tables 3, 4) prove the efficiency of the proposed approach for the parametric identification of PWA systems. The procedure could be applied in robust parametric identification of LTI dynamic systems by processing input and noisy output observations in the presence of outliers, too.

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Apie Vinerio sistemų, turinčių dalimis tiesišką netiesiškumą su teigiamais nuožulnumais, identifikavimą

Rimantas PUPEIKIS

Straipsnyje nagrinėjamas Vinerio sistemų laipsniškas tiesinės dalies, aprašomos skirtumine lygtimi su nežinomais koeficientais ir dalimis tiesiško netiesiškumo su nežinomais nuožulnumais bei nežinomų slenksčių, junginys. Parodyta, kad pertvarkius išėjimo signalo stebėjimus pagal didėjančias jų reikšmes, galima išskirti vidurinę stebėjimų dalį, atitinkančią nestebimo tarpinio signalo stebėjimus. Pasiūlytas pilno tarpinio signalo atstatymo būdas pagal įėjimo signalo ir išėjimo signalo vidurinės dalies stebėjimus. Nežinomų tiesinės Vinerio sistemos dalies koeficientų ir dalimis tiesiško netiesiškumo parametrų bei slenksčio įverčiai gaunami mažiausiųjų kvadratų metodo algoritmais, apdorojant stebimų įėjimo, pertvarkyto išėjimo bei atkurto tarpinio signalų duomenis. Pateikti modeliavimo rezultatai.