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Evaluation Ranges of Functions using Balanced Random Interval Arithmetic

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Abstract. The results of experimental testing of balanced random interval arithmetic with typical mathematical test functions and practical problem are presented and discussed. The possibility of evaluation ranges of functions using balanced random interval arithmetic is investigated. The influence of the predefined probabilities of standard and inner interval operations to the ranges of functions is experimentally investigated.

Key words: interval arithmetic, global optimization, multidimesional scaling.

1. Introduction

Many problems in engineering, physics, economic may be reduced to global optimization problems. Mathematically the problem is formulated as

$$f^* = \min_{x \in D} f(x),$$

where $f(x)$ is a nonlinear objective function of continuous variables $f\colon \Re^n \to \Re$, $D \subseteq \Re^n$ is a multidimensional feasible region, n is the number of variables. Besides the global minimum f^* one or all global minimizers $x^*\colon\ f(x^*) = f^*$ should be found.

Interval global optimization methods are based on interval arithmetic proposed by Moore (1966). The lower and upper bounds for the function values in the subregion are estimated applying interval operations with intervals instead of the real operations with real variables in the algorithm calculating the function values. The bounds are useful to detect the subregions of the feasible region not containing a global minimizer.

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The disadvantage of interval methods is the dependency problem (Hansen, 1992): when a given variable occurs more than once in an interval computation, it is treated as a different variable in each occurrence. This causes widening of computed intervals making it more difficult to obtain tight intervals. One should always be aware of this shortcoming and take appropriate steps to reduce its effect. However it is not always possible to overcome the problem, when the objective function is defined by means of a computer code.

In random interval arithmetic proposed by Alt and Lamotte (2001) the standard or newly defined inner interval operations are chosen randomly with the same probability at each step of the computation producing comparatively tight bounds (although it cannot guarantee that all values of the objective function in the subregion are within bounds). Random interval arithmetic has been applied to compute ranges of some functions over small intervals. Alt and Lamotte (2001) have shown that random interval arithmetic provides ranges of functions over small intervals which are much closer to the exact range than the standard interval arithmetic. However random interval arithmetic provides too narrow bounds when intervals are wide, therefore it can not be applied to global optimization directly.

Balanced random interval arithmetic proposed by Zilinskas and Bogle (2003) is obtained extending the ideas of random interval arithmetic by choosing standard and inner interval operations at each step of the computation randomly with predefined probability. The influence of the probabilities of the standard and inner interval operations to the ranges of functions is experimentally investigated on a practical problem. The value used for the probability depends on the balance required between efficiency and robustness. The preliminary test results seem promising for the construction of global optimization algorithms based on these ideas of probabilistic generalized interval methods. In this paper the experimental testing results on more functions are presented.

2. Interval Global Optimization Methods

Interval global optimization methods are based on interval arithmetic proposed by Moore (1966). Interval arithmetic operates with real intervals $X = [x_1, x_2] = \{x \in \Re \,|\, x_1 \leqslant x \leqslant x_2\}$, where x_1 and x_2 are real numbers. For any real arithmetic operation $\{x \text{ op } y\}$ the corresponding interval arithmetic operation $\{X \text{ op } Y\}$ is defined, whose result is an interval containing every possible number produced by $\{x \text{ op } y\},\ x \in X,\ y \in Y$. Denoting $[a \vee b] = [\min(a,b), \max(a,b)]$, $x_c = \min(|x_1|, |x_2|)$ and $x_d = \max(|x_1|, |x_2|)$, the standard interval arithmetic operations are defined as:

$$X + Y = [(x_1 + y_1) \vee (x_2 + y_2)],$$

$$X - Y = [(x_1 - y_2) \vee (x_2 - y_1)],$$

$$X \times Y = \begin{cases} [(x_c y_c) \vee (x_d y_d)], & 0 \notin X,\ 0 \notin Y, \\ [(x_1 y_d) \vee (x_2 y_d)], & 0 \in X,\ 0 \notin Y, \\ [\min\{x_1 y_2, x_2 y_1\},\ \max\{x_1 y_1, x_2 y_2\}], & 0 \in X,\ 0 \in Y, \end{cases}$$

$$X/Y = \begin{cases} [(x_c/y_d) \vee (x_d/y_c)], & 0 \notin X, \ 0 \notin Y, \\ [(x_1/y_c) \vee (x_2/y_c)], & 0 \in X, \ 0 \notin Y. \end{cases}$$

The guaranteed lower and upper bounds for the function values can be estimated applying standard interval operations with the intervals instead of the real operations in the algorithm to calculate the function values. The bounds are useful to detect the subregions of the feasible region not containing a global minimizer. Such subregions may be discarded from the further search. If the objective function is differentiable it is possible to compute the intervals of the derivatives and discard the subregions where the objective function is monotone. If the objective function is twice continuously differentiable it is possible to compute the intervals of the second derivatives and discard the subregions where the objective function is concave. If the objective function is twice differentiable the special interval Newton method can be applied to reduce the subregions, and discard the subregions where there are no stationary points (Hansen, 1992).

The first version of the interval global optimization algorithm was oriented to minimization of a rational function by bisection of sub-domains (Skelboe, 1974). Interval methods for global optimization were further developed in (Moore, 1977; Hansen, 1978a, 1978b), e.g., the interval Newton method and the test of strict monotonicity were introduced. A thorough description including theoretical as well as practical aspects can be found in (Hansen, 1992) where the very efficient interval global optimization method involving monotonicity and nonconvexity tests and special interval Newton method is proposed. The method assumes that the objective function is twice continuously differentiable. The mathematical expressions of the functions should be available. If the monotonicity and nonconvexity tests and interval Newton method are not used the method can minimize even noncontinuous functions, but then it is not so efficient.

A branch and bound technique is usually used to construct interval global optimization algorithms. An iteration of a classical branch and bound algorithm processes a node in the search tree representing a not yet explored subspace of the solution space. Iteration has three main components: selection of the node to process, bound calculation, and branching. The rules of selection, branching and bounding differ from algorithm to algorithm. All interval global optimization branch and bound algorithms use the hyper-rectangular partitions and branching is usually performed bisecting the hyper-rectangle into two. The bounding rule describes how the bounds of minimum are found. In interval global optimization methods, bounds are estimated using interval arithmetic.

Let *UB* denote upper bound of f^* over feasible region D: $UB \geqslant \min_{x \in D} f(x)$, lower and upper bounds of function values are evaluated using interval arithmetic: $[f^L(X), f^U(X)] = f(X)$, C is the candidate set. The branch and bound scheme aims to reduce C and make it converge to X^*. The general interval branch and bound algorithm is shown in Algorithm 1.

Algorithm 1. General interval branch and bound algorithm
Initialization: $C \Leftarrow \{D\}, UB \Leftarrow f^U(D)$.
While C contains not only solutions
  Select $B \in C, C \Leftarrow C \backslash \{B\}$.
  If $f^L(B) \leqslant UB$
    Branch B: $B = \bigcup_{j=1}^{p} T_j$.
    For $j = 1$ **to** p
      If $f^L(T_j) \leqslant UB$
        $UB \Leftarrow \min(UB, f^U(T_j))$.
        $C \Leftarrow C \cup \{T_j\}$.

The interval methods have been combined with searches implemented in real number arithmetic. Jansson and Knüppel (1995) have proposed the global unconstrained minimization method involving a combination of local search, branch-and-bound technique and interval arithmetic. In this method derivatives are not required. Numerical results for well-known problems and comparisons with other methods are given in (Jansson and Knüppel, 1995).

Gorges and Ratschek (1999) have applied interval techniques of global optimization to the approximate local solution obtained from the local search in order to determine guaranteed error bounds and to improve the solution if necessary.

Csallner *et al.* (2000) have investigated variants of interval branch-and-bound algorithms for global optimization where the bisection was substituted by the subdivision of subregions into many subregions in a single iteration step. The convergence properties have been investigated in detail. An extensive numerical study is presented in (Markot *et al.*, 2000).

A disadvantage of interval methods is the dependency problem: when a given variable occurs more than once in an interval computation, it is treated as a different variable in each occurrence. This causes widening of computed intervals making it more difficult to obtain tight intervals and increasing time required to solve problem using interval global optimization algorithms. Standard interval arithmetic provides guaranteed bounds but they are often too pessimistic. Standard interval arithmetic is used in global optimization providing guaranteed solutions, but there are problems for which the time of optimization is too long.

3. Balanced Random Interval Arithmetic

Random interval arithmetic proposed by Alt and Lamotte (2001) is obtained by choosing standard or inner interval operations randomly with the same probability at each step of the computation. The inner interval operations are defined as:

$$X + Y = [(x_1 + y_2) \vee (x_2 + y_1)],$$

$$X - Y = [(x_1 - y_1) \vee (x_2 - y_2)],$$

$$X \times Y = \begin{cases} [(x_c y_d) \vee (x_d y_c)], & 0 \notin X, \ 0 \notin Y, \\ [(x_1 y_c) \vee (x_2 y_c)], & 0 \in X, \ 0 \notin Y, \\ [\max\{x_1 y_2, x_2 y_1\}, \min\{x_1 y_1, x_2 y_2\}], & 0 \in X, \ 0 \in Y, \end{cases}$$

$$X/Y = \begin{cases} [(x_c/y_c) \vee (x_d/y_d)], & 0 \notin X, \ 0 \notin Y, \\ [(x_1/y_d) \vee (x_2/y_d)], & 0 \in X, \ 0 \notin Y. \end{cases}$$

A number of sample intervals are evaluated using random interval arithmetic. It is assumed that the standard deviation of the centers of the evaluated intervals is small and the distribution of radii of the evaluated intervals is normal. An approximate range of the function

$$[\mu_{centers} - \mu_{radii} - \alpha\sigma_{radii}, \ \ \mu_{centers} + \mu_{radii} + \alpha\sigma_{radii}] \tag{1}$$

is evaluated using the mean value of the centers $\mu_{centers}$, the mean value of the radii μ_{radii} and the standard deviation of the radii σ_{radii}. α is between 1 and 3 depending on the number of samples and the desirable probability that the exact range is included in the estimated range. Alt and Lamotee (2001) suggest that a compromise between efficiency and robustness can be obtained using $\alpha = 1.5$ and 30 samples. Random interval arithmetic provides bounds closer to the exact range when intervals are small, but it provides too narrow bounds when intervals are wide, therefore it can not be applied to global optimization directly.

Balanced random interval arithmetic proposed by Zilinskas and Bogle (2003) is obtained by choosing standard and inner interval operations at each step of the computation randomly with predefined probabilities. The balanced random interval arithmetic provides wider or narrower bounds depending on the predefined probabilities.

A number of sample intervals are evaluated using balanced random interval arithmetic. It is assumed that the distributions of centers and radii of the evaluated intervals are normal. An approximate range of the function is evaluated using the mean values and the standard deviations of centers and radii of the evaluated intervals.

Balanced random interval arithmetic with different probabilities was used to evaluate ranges of the objective function of a multidimensional scaling problem over random intervals. Experiments have shown that the distributions of centers are normal, but the standard deviations are not small, as they were in (Alt and Lamotte, 2001) when intervals were small. Therefore, instead of using Eq. 1, the standard deviation of the centers should be used when the range of a function is evaluated:

$$[\mu_{centers} - \alpha\sigma_{centers} - \mu_{radii} - \alpha\sigma_{radii}, \mu_{centers} + \alpha\sigma_{centers} + \mu_{radii} + \alpha\sigma_{radii}] \tag{2}$$

$\alpha = 3$ and 30 samples are used in experiments.

4. Experimental Study

The aim of the experiments is to evaluate the possibility of construction of the global optimization algorithms based on the ideas of probabilistic generalized interval methods. The assumption, that the distributions of centers and radii of the evaluated balanced random intervals are normal, has to be verified. The experimental testing results with some functions are presented.

Usually mathematical test functions defined by means of analytical formulas are used to test global optimization algorithms and to evaluate their performance. Test functions have different dimensions and different numbers of local and global minimizers. Balanced random interval arithmetic with different probabilities of standard and inner interval operations was used to evaluate ranges of some test functions over random intervals.

Rosenbrock test function is defined as $f(x) = 100\left(x_0^2 - x_1\right)^2 + (x_0 - 1)^2$, $n = 2$, $D = [-2, 2]^2$. The histograms of the centers and radii of the 10000 intervals evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over one random subregion are shown in Fig. 1. The centers and radii of the standard and inner intervals over the same subregion are shown as vertical lines. The centers and radii of the evaluated balanced random intervals are equal or close to the centers and radii of either standard or inner intervals. The probability, that the center or radii of the evaluated balanced random interval are equal or close to the center or radius of the standard interval, is

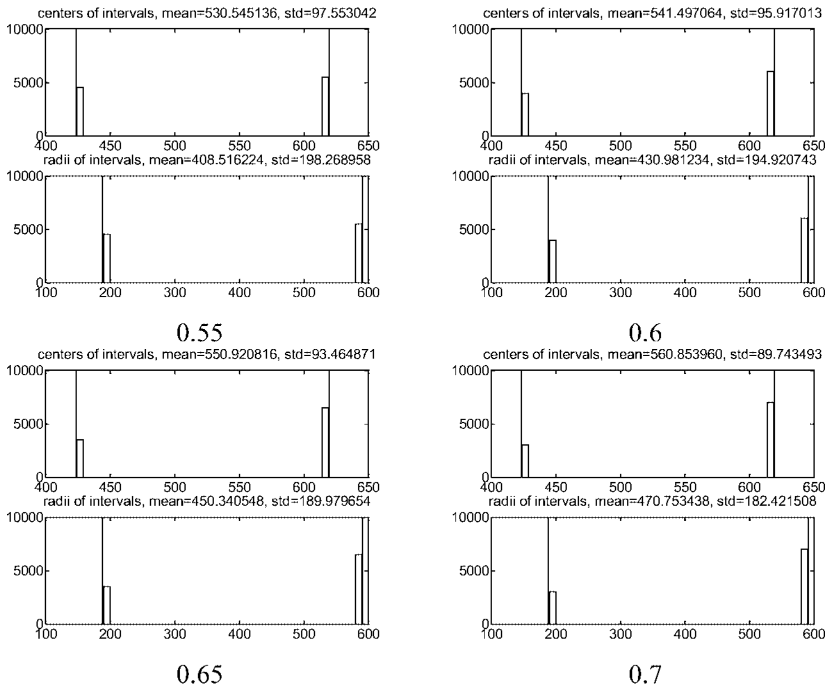


Fig. 1. The histograms of the centers and radii of the intervals of Rosenbrock test function evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over a random subregion.

equal to the probability of standard interval operations in each step of computations. The exact range of the function over the same subregion is close to the standard interval. The assumption, that the distributions of centers and radii of the evaluated intervals are normal, is wrong for this function. Therefore the balanced random interval arithmetic could not be used to evaluate ranges of this mathematical test function and global optimization algorithms based on balanced random interval arithmetic would not be applicable to optimize it.

Six Hump Camel Back test function is defined as $f(x) = 4x_0^2 - 2.1x_0^4 + \frac{1}{3}x_0^6 + x_0x_1 - 4x_1^2 + 4x_1^4$, $n = 2$, $D = [-5, 5]^2$. The histograms of the centers and radii of the 10000 intervals evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over one random subregion are shown in Fig. 2. The centers of the evaluated balanced random intervals are equal or close to the centers of either standard or inner intervals. The probability, that the center of the evaluated balanced random interval is equal or close to the center of the standard interval, is equal to the probability of standard interval operations in each step of computations. The distribution of radii of the evaluated balanced random intervals is far from normal. The exact range of the function over the same subregion is close to inner interval. The assumption, that the distributions of centers and radii of the evaluated intervals are normal, is wrong for this function. Therefore balanced random interval arithmetic could not be used to evaluate ranges of this mathematical test function and global optimization algorithms based on

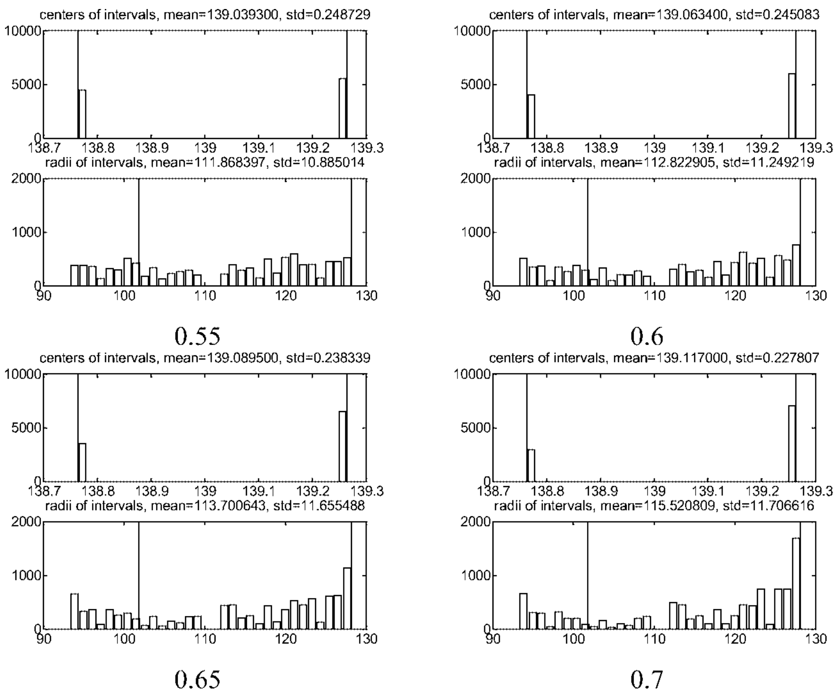


Fig. 2. The histograms of the centers and radii of the intervals of Six Hump Camel Back test function evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over a random subregion.

balanced random interval arithmetic would not be applicable to optimize it.

Goldstein and Price test function is defined as

$$f(x) = \left[1 + (x_0 + x_1 + 1)^2(19 - 14x_0 + 3x_0^2 - 14x_1 + 6x_0x_1 + 3x_1^2\right] \\ \times \left[30 + (2x_0 - 3x_1)^2(18 - 32x_0 + 12x_0^2 + 48x_1 - 36x_0x_1 + 27x_1^2)\right],$$

$n = 2$, $D = [-2, 2]^2$. The histograms of the centers and radii of the 10000 intervals evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over one random subregion are shown in Fig. 3. The distributions of centers and radii of the evaluated balanced random intervals are far from normal. The assumption, that the distributions of centers and radii of the evaluated intervals are normal, is wrong for this function. Therefore balanced random interval arithmetic could not be used to evaluate ranges of this mathematical test function and global optimization algorithms based on balanced random interval arithmetic would not be applicable to optimize it.

Similar results have been obtained with some other simple mathematical test functions (Shekel 5, Shekel 7, Shekel 10) which involve small number of computations. For all investigated simple mathematical test problems the distributions of centers and radii of the evaluated balanced random intervals are not normal. Possibly this is because test functions are too simple and the number of involved computations is too small, which

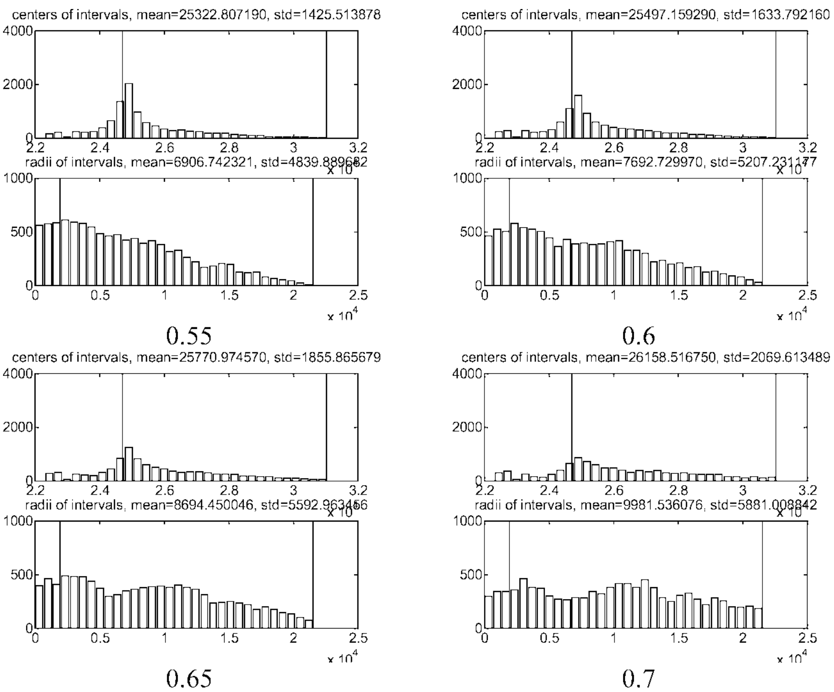


Fig. 3. The histograms of the centers and radii of the intervals of Goldstein and Price test function evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over a random subregion.

makes distributions of centers and radii of evaluated intervals not normal. The assumption, that the distributions of centers and radii of the evaluated intervals are normal, is wrong for the simple mathematical test functions. Therefore balanced random interval arithmetic could not be used to evaluate ranges of simple mathematical test functions and global optimization algorithms based on balanced random interval arithmetic would not be applicable to optimize them.

For some cases simple mathematical test problems are too simple or their dimensionality is too small. One of the ways to increase the hardness and dimensionality of test functions is the generalization to multidimensional space. For example, the Generalized Rosenbrock function could be defined as

$$f(x) = \sum_{i=1}^{n-1} \left[100\left(x_i^2 - x_{i+1}\right)^2 + (x_i - 1)^2\right], \quad D = [-n, n]^n.$$

Balanced random interval arithmetic with different probabilities was used to evaluate ranges of the Generalized Rosenbrock function with $n = 30$ over random intervals. The histograms of the centers and radii of the 10000 intervals evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over one random subregion are shown in Fig. 4. The centers and radii of the standard and inner intervals over the same subregion are shown as vertical lines. The mean values of the centers and

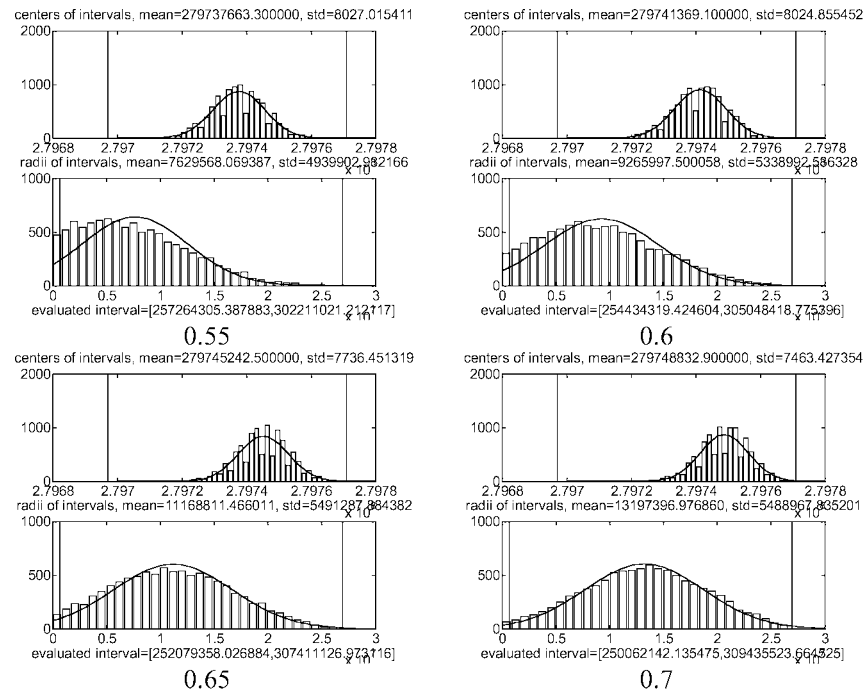


Fig. 4. The histograms of the centers and radii of the intervals of the Generalized Rosenbrock function evaluated using balanced random interval arithmetic with probabilities 0.55, 0.6, 0.65 and 0.7 over a random subregion.

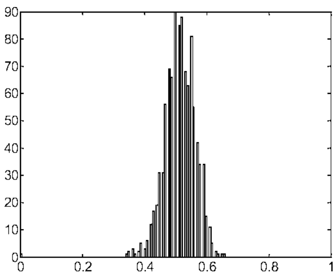
radii moves towards the center and radius of the standard interval when the probability of standard interval operations is increasing. Normal distributions with evaluated means and standard deviations are also shown. The distributions of the centers and radii are normal. Therefore the ranges of a function over a subregion could be evaluated using (2). Balanced random interval arithmetic can be used to evaluate ranges of this function and global optimization algorithms based on balanced random interval arithmetic would be applicable to optimize it.

The ranges of the Generalized Rosenbrock function in 1000 random subregions have been evaluated using balanced random interval arithmetic with different probabilities of standard and inner interval operations. The ranges were evaluated using means and standard deviations of centers and radii of 30 balanced random intervals. $\alpha = 3$ was used. The histogram of smallest probabilities for which the evaluated ranges include the function values at 2000 uniformly distributed random points is shown in Fig. 5a. For 96.4% of subregions the smallest probability for which the evaluated ranges include function values at random points is less than 0.6. For 99.8% of subregions the smallest probability for which the evaluated ranges include function values at random points is less than 0.65. The mean ratio between widths of evaluated ranges and standard intervals depending on the probability of standard interval operations is shown in Fig. 5b. When the probability is 0.6, the mean ratio is 0.865, which means that evaluated ranges are 13.5% tighter.

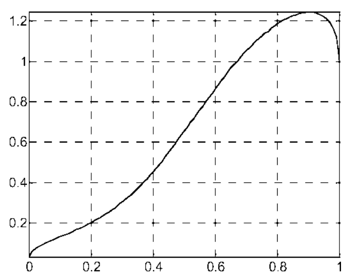
Similar results have been obtained with the objective function of a multidimensional scaling (MDS) problem with data from soft drinks testing (Mathar, 1996):

$$f(X) = \sum_{j<i} \left(\sqrt{\sum_{k=1}^{2} (x_{i,k} - x_{j,k})^2} - \delta_{ij} \right)^2,$$

where $x_{i,1}, x_{i,2}$ are the coordinates of the ith object ($i = 1 \ldots 10$ and $j = 1 \ldots 10$) in two-dimensional space, δ_{ij} are the data for the problem – dissimilarities between soft drinks. Balanced random interval arithmetic with different probabilities was used to evaluate ranges of the MDS function over random intervals and it was shown in (Zilinskas and Bogle, 2003) that the distributions of the centers and radii of intervals evaluated using balanced random interval arithmetic are normal. Therefore the ranges of a function over a

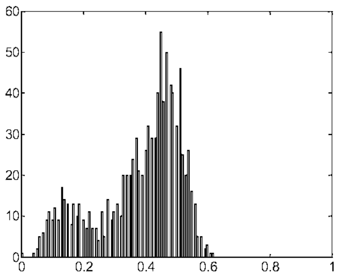


a.

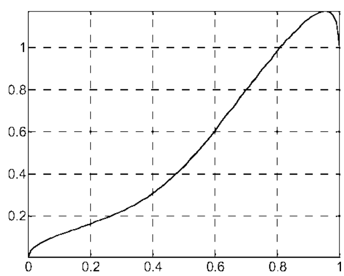


b.

Fig. 5. Results of experiments with the Generalized Rosenbrock function.



a.



b.

Fig. 6. Results of experiments with MDS function.

subregion could be evaluated using (2). Balanced random interval arithmetic can be used to evaluate ranges of this function and global optimization algorithms based on balanced random interval arithmetic would be applicable to optimize it.

The ranges of MDS function in 1000 random subregions have been evaluated using balanced random interval arithmetic with different probabilities of standard and inner interval operations. The ranges were evaluated using means and standard deviations of centers and radii of 30 balanced random intervals. $\alpha = 3$ was used. The histogram of smallest probabilities for which the evaluated ranges include the function values at 2000 uniformly distributed random points is shown in Fig. 6a. For 99.5% of subregions the smallest probability for which the evaluated ranges include function values at random points is less than 0.6. The mean ratio between widths of evaluated ranges and standard intervals depending on the probability of standard interval operations is shown in Fig. 6b. When the probability is 0.6, the mean ratio is 0.606 – evaluated ranges are 39.4% tighter.

Results of experiments with the Generalized Rosenbrock function and the practical MDS problem show that the distributions of centers and radii of the evaluated balanced random intervals are normal. The assumption, that the distributions of centers and radii of the evaluated intervals are normal, is right for these functions. Therefore balanced random interval arithmetic can be used to evaluate ranges of difficult functions. Balanced random interval arithmetic provides not guaranteed but much tighter ranges than standard interval arithmetic. Therefore it seems promising to construct global optimization algorithms based on ideas of probabilistic generalized interval methods and apply them to solve practical global optimization problems.

5. Conclusions

The results of experimental testing of balanced random interval arithmetic with simple mathematical test functions have shown that the assumption, that the distributions of centers and radii of the evaluated balanced random intervals are normal, is wrong for such functions. Therefore balanced random interval arithmetic could not be used to evaluate ranges of them and global optimization algorithms based on balanced random interval arithmetic would not be applicable to optimize simple mathematical test functions.

However test results with the Generalized Rosenbrock test function and the practical MDS problem have shown that the distributions of centers and radii of the evaluated balanced random intervals are normal. The assumption, that the distributions of centers and radii of the evaluated intervals are normal, is right for these functions. Therefore balanced random interval arithmetic can be used to evaluate ranges of difficult functions and application of global optimization algorithms based on ideas of probabilistic generalized interval methods to solve such problems seems promising.

The influence of the predefined probabilities of the standard and inner interval operations in each step of computation to the ranges of functions is experimentally investigated. The value used for the probability depends on the balance required between efficiency and robustness of global optimization algorithm. For the Generalized Rosenbrock function the value of 0.6 would give a 96.4% success rate and 13.5% tighter ranges. For the MDS function the value of 0.6 would give a 99.5% success rate and 39.4% tighter ranges.

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Funkcijų rėžių skaičiavimas naudojant balansuotą atsitiktinę intervalų aritmetiką

Julius ŽILINSKAS, Ian David Lockhart BOGLE

Pateikiami ir aptariami balansuotos atsitiktinės intervalų aritmetikos testavimo matematinėmis testo funkcijomis ir praktine užduotimi rezultatai. Ištirta funkcijų rėžių skaičiavimo, panaudojant balansuotą atsitiktinę intervalų aritmetiką, galimybė. Eksperimentiškai ištirta nustatomų standartinės ir vidinės intervalų aritmetikų operacijų tikimybių įtaka įvertinamiems funkcijų rėžiams.