INFORMATICA, 1992, Vol.3, No.3, 378–384

CHECKING FIDELITY OF THE SYSTEMS WITH FAULT DETECTION CIRCUITS

Kazys SERAPINAS and Kostas ŽUKAUSKAS

Institute of Mathematics and Informatics
2600 Vilnius, Akademijos St.4, Lithuania

Abstract. The checking fidelity of the systems with built-in fault detection circuits is investigated here taking into account the failure rates of functional units, also checking and transmission facilities of right and wrong information on the state of functional units. When fault detection circuits or information transmission facilities fail, then the information given by them may cause false or undetected failures in comparison with the real state of the corresponding functional unit.

Key words: checking fidelity, fault detection, false failure, undetected failure.

Introduction. The checking fidelity is a main criterion in solving the question how much the checking data represent the real state of the system. According to Kudrickij *et al.* (1977) the expression

$$D = 1 - \alpha(t) - \beta(t)$$

is a quantitative measure of estimation of the checking fidelity, where $\alpha(t)$ and $\beta(t)$ are the probabilities of false and undetected failures of the system respectively and t is the systems operating time after its complete restoration.

We estimate here the influence of reliability of the built-in fault detection circuits and signal transmission devices on the checking fidelity of the system.

1. Assumptions and definitions. In order to calculate the checking fidelity of a system let us introduce the following assump-

- the system consists of n functional units which in sense of reliability are series connected, i.e., there is no technical, functional or informational redundancy in the system except all built-in fault detection circuits and information transmission facilities;
- each functional unit has its own built-in fault detection circuit;
- the fault detection circuit of the given functional unit neither is self-checked nor it is checked by checking facilities of other units;
- at the starting point (when $t = 0$) the system is checked completely and it is brought to the absolutely well-operating state;
- each fault detection circuit check up only a part of its functional unit, however, this mentioned part is completely checked;
- the failure rate of each unit or its separate part is a constant quantity (not depending on time);
- signals are transmitted from fault detection circuits to display devices by a chain of transmitting units, whose number may vary in dependence on each functional unit;
- as a fault detection circuit or any transmitting unit in the chain fail, there appear an unchangeble signal at the output of the chain independently of the input signals of the fault detection circuit.

Let us introduce the following parameters, which specify the reliability of the i-th functional unit:

λ_{ui} is the failure rate of the unchecked part of the i-th functional unit;

λ_{ci} is the failure rate of the checked part of the i-th functional unit;

λ_{di} is the failure rate of the built-in fault detection circuit of the i-th functional unit;

λ_{rji} is the failure rate of the j-th transmitting unit in the chain from the i-th functional unit up to the display device of the checking results;

Δt is the time interval between the neighbouring checking procedures.

2. Determination of units reliability. A probability of reliable functioning of the i-th unit can be expressed in such a form:

$$P_i(t) = P_{ci}(t)P_{ui}(t), \tag{2.1}$$

where $P_{ci}(t)$ is the probability of reliable operation of the checked part of the i-th unit, $P_{ui}(t)$ is probability of reliable operation of the unchecked part of the same unit.

When the time t is over from the very beginning of the system operation or its complete restoration, the probability of reliable functioning of the unchecked part of the i-th unit can be written in such a form:

$$P_{ui}(t) = \exp[-(\lambda_{ui} + \lambda_{di})t], \tag{2.2}$$

since the fault detection circuit belong to the unchecked part of the i-th unit.

The reliable operation probability of the checked part of the i-th unit immediately after the checking procedure is equal to 1 since the unit is completely restored after a failure was detected.

At a moment before the checking procedure the reliable operation probability of the checked part of the i-th unit has a minimal value equal to:

$$P^*_{ci}(\Delta t) = \exp[-\lambda_{ci}\Delta t]. \tag{2.3}$$

Now let us define a mean value of the reliable operation probability of the checked part of the i-th unit during the time interval Δt. It is equal to:

$$\overline{P}_{ci}(\Delta t) = (1/\lambda_{ci}\Delta t)[1 - \exp(-\lambda_{ci}\Delta t)]. \tag{2.4}$$

If

$$\lambda_{ci}\Delta t \ll 1, \tag{2.5}$$

then $P_{ci}(t)$ can be replaced by $\overline{P}_{ci}(\Delta t)$ with preset precision, and we obtain

$$P_i(t) = \exp[-(\lambda_{ui} + \lambda_{di})t](1/\lambda_{ci}\Delta t)[1 - \exp(-\lambda_{ci}\Delta t)]. \tag{2.6}$$

3. Determination of a false failure probability. According to Sotskov (1970) a false failure probability of the system can be determinated by a well-known formula for the series units connection (in the sense of reliability):

$$\alpha(t) = 1 - \prod_{i=1}^{n}[1 - q_{wi}(t)], \tag{3.1}$$

where $q_{wi}(t)$ is the false failure probability of the i-th unit, n is the number of units in the system.

The signals from the built-in fault detection circuits are often transmitted to the place of their processing and display devices through a chain of transmitting units which, in turn, may also fail. In the case of a failure the transmitting unit gives a signal either "i-th unit is correct" or "i-th unit is incorrect" independently of the signal at its input. The same situation is observed as the built-in fault detection circuits fail. Therefore, the false failure probability of the i-th unit can be determined by the expression

$$q_{wi}(t) = P_{ci}(t)\Big[a_{di}q_{di}(t)p_{mi}(t) + \sum_{j=1}^{m} a_{ji}q_{ji}(t)p_{ji}(t)\Big], \tag{3.2}$$

where $P_{ci}(t)$ is the reliable operation probability of the checked part of the i-th unit; a_{di} is the probability of the appearance of the signal "i-th unit is incorrect" at the output of the i-th fault detection circuit when it fails; $q_{di}(t)$ is the failure probability of the fault detection circuit of the i-th unit; $p_{mi}(t)$ is the reliable operation probability of the whole chain of transmitting units; a_{ji} is a probability of the appearance of the signal "i-th unit is incorrect" at the output of the j-th transmitting unit as it fails; $q_{ji}(t)$ is a failure probability of the j-th transmitting unit; $p_{ji}(t)$ is the reliable operation probability of the part of transmitting units in the range from the j-th to the m-th unit.

The quantity $P_{ci}(t)$ is determined by formula (2.4). Concerning quantities a_{di} and a_{ji} there is assumed that for a particular unit they don't depend on time and may be defined experimentally by using statistical or reference data. The quantities $q_{di}(t)$ and $q_{ji}(t)$ are defined by the expressions:

$$q_{di}(t) = 1 - \exp(-\lambda_{di}t), \tag{3.3}$$

$$q_{ji}(t) = 1 - \exp(-\lambda_{ji}t). \tag{3.4}$$

The quantities $p_{mi}(t)$ and $p_{ji}(t)$ are defined by the following expressions:

$$p_{mi}(t) = \prod_{j=1}^{m}[1 - q_{ji}(t)] = \exp\left[-\sum_{j=1}^{m}\lambda_{rji}t\right], \tag{3.5}$$

$$p_{ji}(t) = \prod_{s=j}^{m}[1 - q_{si}(t)] = \exp\left[-\sum_{s=j}^{m}\lambda_{rsi}t\right], \tag{3.6}$$

where λ_{rji} is the failure rate of the j-th transmitting unit being in the i-th chain. It is necessary to notice that in (3.5) and (3.6) there is assumed all transmitting units to be without fault detecting circuits.

By substituting (2.4), (3.3) - (3.6) into (3.2) we obtain the expression for determination of the false failure probability of the i-th functional unit:

$$\begin{aligned} q_{wi}(t) = &(1/\lambda_{ci}\Delta t)[1 - \exp(-\lambda_{ci}\Delta t)] \\ &\times \left\{a_{di}[1 - \exp(-\lambda_{di}t)]\exp\left(-\sum_{j=1}^{m}\lambda_{rji}t\right)\right. \\ &\left.+\sum_{j=1}^{m}a_{ij}[1 - \exp(-\lambda_{rji}t)]\exp\left(-\sum_{s=j}^{m}\lambda_{rsi}t\right)\right\}. \end{aligned} \tag{3.7}$$

4. Determination of an undetected failure probability. According to Sotskov (1970) an undetected failure probability can be determined by the following formula

$$\beta(t) = 1 - \prod_{i=1}^{n}[1 - q_{ui}(t)], \tag{4.1}$$

where $q_{ui}(t)$ is the undetected failure probability of the i-th unit of the system.

The probability $q_{ui}(t)$ consists of two parts:

a) the failure probability of the unchecked part of the i-th unit provided that there is no false failure of its checked part, and

b) the failure probability of the checked part of the i-th unit provided that there is a false failure signal "i-th unit is correct". Therefore, it is determined by the expression:

$$q_{ui}(t) = [1 - P_{ui}(t)][1 - q_{wi}(t)] + [1 - P_{ci}(t)]q_{ri}(t), \tag{4.2}$$

where $P_{ui}(t)$, $q_{wi}(t)$ and $P_{ci}(t)$ are determined by (2.2), (3.2) and (2.4). The quantity $q_{ri}(t)$ is the probability of the false signal "i-th unit is correct" at the output of the chain of transmitting units, including the built-in fault detection circuit of the i-th unit. These remarks lead up to:

$$\begin{aligned} q_{ri}(t) =& [1 - a_{di}]q_{di}(t)p_{mi}(t) \\ &+ \sum_{j=1}^{n}[1 - a_{ji}]q_{ji}(t)p_{ji}(t), \end{aligned} \tag{4.3}$$

where a_{di} and a_{ji} are described together with the formulas (3.2) explanation; $q_{di}(t)$, $p_{mi}(t)$, $q_{ji}(t)$ and $p_{ji}(t)$ are determined by (3.3), (3.5), (3.4) and (3.6).

Conclusions. The explicit expressions for the quantities included into formula for the checking fidelity of a system with built-in fault detection circuits and information transmission facilities are obtained in this paper.

The expressions (3.7) and (4.2) allow us to evaluate the checking fidelity of systems more exactly, taking into account the fact that failed checking or transmitting facilities of information on the failures of functional units yield unchangeble signals on their outputs, not depending on the real state of correspondent functional units.

REFERENCES

Kudrickij, V.D., M.A.Sinica and P.J.Chinayev (1977). *Checking Automation of Radioelectronic Equipment.* Soviet Radio, Moscow (in Russian).

Sotskov, B.S. (1970). *The fundamentals of the Theory and Calculations of Reliability of the Elements and Equipment of Automation and Hardware.* Vyschaja Shkola, Moscow (in Russian).

Received March 1992

K. Žukauskas received the Degree of the Candidate of Physical-mathematical Sciences at Vilnius University in 1964. At present he is head of the laboratory at the Institute of Mathematics and Informatics, Lithuania. The field of his scientific interests concerns the reliability theory and reliability checking of large systems.

K. Serapinas received the Degree of the Candidate of Technical Sciences at Kaunas Polytechnical Institute in 1972. At present he is a senior scientist researcher of the Institute of Mathematics and Informatics, Lithuania. The field of his scientific interests concerns the reliability theory, equipment redundancy methods.