INFORMATICA, 2018, Vol. 29, No. 2, 321–352

DOI: http://dx.doi.org/10.15388/Informatica.2018.170

Interval-Valued 2-Tuple Linguistic Induced Continuous Ordered Weighted Distance Measure and Its Application to Multiple Attribute Group Decision Making

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Received: January 2017; accepted: May 2018

Abstract. This paper aims to propose a new distance measure, the interval-valued 2-tuple linguistic induced continuous ordered weighted distance (IT-ICOWD) measure, which consists of the interval-valued 2-tuple linguistic induced continuous ordered weighted averaging (IT-ICOWA) operator and the ordered weighted distance (OWD) measure. In these operators, we consider the risk attitude of decision maker. Furthermore, we discuss some desired properties and various special cases of the IT-ICOWD measure. Additionally, a method of multiple attribute group decision making (MAGDM) in interval-valued 2-tuple linguistic environment is developed on the basis of the IT-ICOWD measure. Through this method, we obtain three simple and exact formulae to determine the order-inducing variables of the IT-ICOWD measure, the weighting vector of decision makers and the weighting vector of attributes, respectively. At last, a numerical example is presented to illustrate the practicability and feasibility of proposed method.

Key words: group decision making, distance measure, interval-valued 2-tuple linguistic information, *IOWA* operator, *COWA* operator.

1. Introduction

MAGDM is a crucial branch of decision theory. It is used to select the most highly preferred alternative(s) from a finite alternatives set. This process is accorded with experts' preference information, who are required to give their preferences based on multiple attributes. Bellman and Zadeh (1970) first studied the decision making under fuzzy environment: they took time pressure into consideration to have events uncertainty limited. After that, Zadeh (1965) proposed the traditional fuzzy environment, for instance, the interval-valued fuzzy set (Moore, 1966), the intuitionistic fuzzy set (Atanassov, 1986), the hesitant

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fuzzy set (Torra, 2010) and the type-2 fuzzy set (Mendel, 2007). These fuzzy sets have been widely studied and applied to decision making procedures (Zadeh, 1965; Torra, 2010; Mendel, 2007; Atanassov, 2012; Sengupta and Pal, 2009; Zhou *et al.*, 2014a). Aiming to express subjective evaluations of the decision makers, Zadeh (1975a, 1975b, 1975c) then introduced the concept of linguistic variable. Moreover, the linguistic variables have also been deeply studied and widely used to evaluate decision information, because the quantitative information is not feasible to all decision cases.

Afterwards, several linguistic representation models were proposed to fit different subjective situations. For example, 2-tuple linguistic representation model was developed by Herrera and Martinez (2000) to avoid information loss in the aggregation process of linguistic labels; the notion of linguistic intervals was introduced by Chen and Lee (2010) to describe linguistic information uncertainty; the concept of hesitant fuzzy linguistic term sets was proposed by Rodriguez *et al.* (2011) to express the hesitancy when linguistic labels are used; the unbalanced linguistic term set was put forward by Herrera *et al.* (2008), in which the linguistic labels around the centred linguistic label are not distributed symmetrically; an alternative form of uncertain linguistic variable was developed by Xu (2004) in order to demonstrate the uncertainty in linguistic information; multi-granular fuzzy linguistic modelling and fuzzy entropy methods were introduced by Morente-Molinera *et al.* (2017) to transform the training data in ways that represent their inner meaning more precisely; a linguistic computational model based on discrete fuzzy numbers whose support is a subset of consecutive natural numbers was presented by Massanet *et al.* (2014); a new method based on linguistic granular computing to solve group decision making problems defined in heterogeneous contexts was developed by Cabrerizo *et al.* (2013).

Decision making problem with interval-valued linguistic variable was utilized to deal with practical situation such as the health-care waste treatment technology evaluation and selection (Liu *et al.*, 2014a), the failure mode and effects analysis (Liu *et al.*, 2014b), searching for an optimal investment (Zhang, 2013), etc. Aggregation techniques are essential part in these applications. There are two ways to realize the aggregation: using the aggregation operators directly (Zhang, 2013; Zhang, 2012), and combining aggregation operators with information measures such as the weighted distance measure (Liu *et al.*, 2014b; Zhou *et al.*, 2013, 2014b; Liao *et al.*, 2014; Xu and Wang, 2011). Yager (1993) introduced the ordered weighted averaging (OWA) operator in 1993; since then, various aggregation operators were proposed and are combined with linguistic information (Meng *et al.*, 2016; Liu *et al.*, 2014a, 2014b; Zhang, 2013).

In addition, the distance measures are uniformly distributed on the corresponding interval variables, they are often utilized to deal with the aggregation information is denoted by exact numbers or defined by endpoints of intervals. Obviously, it varies among group decision making problems under uncertain environment. To solve this problem, Zhou *et al.* (2013, 2014b, 2016), developed the continuous intuitionistic fuzzy ordered weighted distance (*C-IFOWD*) measure, the continuous ordered weighted distance (*COWD*) measure and the linguistic continuous ordered weighted distance (*LCOWD*) measure, respectively. These three aggregation operators combine the *C-IFOWA* operator (or *COWA* operator/the *LCOWA* operator) with the ordered weighted distance (*OWD*) measure, considering the risk attribute of decision makers under interval variables environment.

Motivated by the work in Zhou *et al.* (2013), Zhou *et al.* (2014b), the aim of this paper is to develop a new distance measure named as interval-valued 2-tuples linguistic induced continuous ordered weighted distance (*IT-ICOWD*) measure, which is based on the *ICOWA* operator and the *OWD* measure with the interval-valued 2-tuples linguistic information. We also study some desirable properties and different families of the *IT-ICOWD* measure. Additionally, we extend the *IT-ICOWD* measure and obtain the quasi-*IT-ICOWD* measure. We also propose a new approach to *MAGDM* by using the *IT-ICOWD* measure. By this approach, we obtain promising formulae, which can determine the order-inducing variables in the *IT-ICOWD* measure, the weighting vector of decision makers and the weighting vector of attributes.

The rest of this paper is structured as follows. We briefly review some fundamental concepts about various relevant operators and measures in Section 2. In Section 3, we present the *IT-COWD* and the *IT-ICOWD* measure, and discuss some properties and families of the *IT-ICOWD* measure. We also develop some extensions of the *IT-ICOWD* measure. Section 4 provides an approach based on the *IT-ICOWD* measure for multiple attribute group decision making with interval-valued 2-tuple linguistic information and give a real-life example to illustrate the efficiency of the proposed method. At last, we give some further explanations in Section 5.

2. Preliminaries

In this section, we briefly review basic concepts about the 2-tuple linguistic, the OWA operator, the IOWA operator, the GOWA operator, the COWA operator, the ICOWA operator, the distance measure and the OWD measure.

2.1. *The 2-Tuple Fuzzy Linguistic Representation Model Processing*

Zadeh firstly introduced linguistic method in Zadeh (1975a, 1975b, 1975c) as an approximate technique representing qualitative information by means of linguistic labels.

Let $S = \{s_i \mid i = 0, 1, \dots, g\}$ be a linguistic term set with odd cardinality, each term s_i represents a possible value for a linguistic variable; for example, we can define S as follows:

$$S = \big\{s_0 = neither(N), s_1 = very\ \mathrm{low}(VL), s_2 = low(L), s_3 = medium(M), \\ s_4 = high(H), s_5 = very\ high(VH), s_6 = perfect(P)\big\},$$

where the mid-linguistic term s_3 represents "approximately 0.5" as an assessment, and the rest of the terms placed symmetrically around it. It should be clarified that term sets should satisfy the following characteristics:

1. Ordered set: $s_i > s_j \Leftrightarrow i > j$;

2. Negation operator: $Neg(s_i) = s_{g-i}$ ($g+1$ is the cardinality);
3. Minimum operator: $\min(s_i, s_j) = s_i \Leftrightarrow s_i \leqslant s_j$;
4. Maximum operator: $\max(s_i, s_j) = s_i \Leftrightarrow s_i \geqslant s_j$.

According to symbolic translation, Herrera and Martinez (2000) originally proposed the 2-tuple linguistic representation model for dealing with linguistic information. Being continuous in the domain is the primary advantage of this representation. A 2-tuple (s_i, α_i) is a 2-tuple linguistic representation model, where s_i is a linguistic label of predefined linguistic term set S and α_i is a numerical value representing the value of symbolic translation.

Definition 1 (*See* Herrera and Martinez, 2000). Let $S = \{s_i \mid i = 0, 1, \dots, g\}$ be a set of finite linguistic terms, and $\beta \in [0, g]$ is the numerical value demonstrating the result of a symbolic aggregation operation, then the function Δ denotes to obtain the 2-tuple linguistic information being equivalent to β, which can be defined as follows:

$$\Delta : [0, g] \to S \times [-0.5, 0.5), \tag{1}$$

$$\Delta(\beta) = (s_i, \alpha_i) \quad \text{with } \begin{cases} s_i, & i = round(\beta), \\ \alpha_i = \beta - i, & \alpha_i \in [-0.5, 0.5), \end{cases} \tag{2}$$

where $round(\beta)$ is the usual round operation, s_i has the closest index label of β and α_i is the value of the symbolic translation.

Definition 2 (*See* Herrera and Martinez, 2000). Let $S = \{s_i \mid i = 0, 1, \dots, g\}$ be a linguistic terms collection and (s_i, α_i) be a linguistic 2-tuple term. There is always a function Δ^{-1} such that the value which returns from 2-tuple is an equivalent numerical value $\beta \in [0, g]$, where

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, g], \tag{3}$$

$$\Delta^{-1}(s_i, \alpha_i) = i + \alpha_i = \beta, \tag{4}$$

and $\beta \in [0, g]$.

In the past few decades, multiple 2-tuple linguistic aggregation operators have been proposed in order to aggregate 2-tuple linguistic. However, these 2-tuple linguistic aggregation operators all simply focused on the usual 2-tuple. In another word, 2-tuple linguistic from different linguistic term set with different granularities cannot be aggregated directly. To overcome this obstacle, Chen and Tai (2005) put forward a generalized 2-tuple linguistic representation model and translation function.

Definition 3 (*See* Chen and Tai, 2005). Let $S = \{s_i \mid |i = 0, 1, \dots, g\}$ be an ordered linguistic term set, and crisp value $\beta \in [0, 1]$ can be transformed into one 2-tuple linguistic representation model through the following function:

$$\Delta : [0,1] \to S \times \left[-\frac{1}{2g}, \frac{1}{2g}\right), \tag{5}$$

$$\Delta(\beta) = (s_i, \alpha_i) \quad \text{with} \begin{cases} s_i, & i = round(\beta \cdot g); \\ \alpha_i = \beta - \frac{i}{g}, & \alpha_i \in [-\frac{1}{2g}, \frac{1}{2g}). \end{cases} \tag{6}$$

Conversely, the 2-tuple can be converted into a crisp $\beta \in [0, 1]$ as follows:

$$\Delta^{-1} : S \times \left[-\frac{1}{2g}, \frac{1}{2g}\right) \to [0,1], \tag{7}$$

$$\Delta^{-1}(s_i, \alpha_i) = \frac{i}{g} + \alpha_i = \beta, \tag{8}$$

where $\beta \in [0, 1]$ according to Definition 3. Therefore, in this method the 2-tuple linguistic representation model is standardized, so it is easier to compare 2-tuple linguistic terms with different multiple granularity linguistic term sets. In this paper, unless mentioned explicitly, the 2-tuple linguistic representation model is generalized 2-tuple representation model in Definition 3.

Considering merits of Definition 3, a new concept of the interval-valued 2-tuple linguistic representation model was introduced by Zhang (2012) with some aggregation operators with interval-valued 2-tuple linguistic information.

Definition 4 (*See* Zhang, 2012). Let $S = \{s_i \mid i = 0, 1, \dots, g\}$ be a set of ordered linguistic terms. An interval-valued 2-tuple consists of two linguistic terms and two numbers, denoted as $[(s_i, \alpha_i), (s_j, \alpha_j)]$, where $i \leqslant j$, and $\alpha_i \leqslant \alpha_j$, $s_i, s_j \in S$, α_i, α_j are crisp numbers. The interval-valued 2-tuple that expresses the equivalent information to an interval-value $[\beta_1, \beta_2]$ $(\beta_1, \beta_2 \in [0, 1], \beta_1 \leqslant \beta_2)$ as follows:

$$\Delta([\beta_1, \beta_2]) = [(s_i, \alpha_i), (s_j, \alpha_j)] \quad \text{with} \begin{cases} s_i, & i = round(\beta_1 \cdot g); \\ s_j, & j = round(\beta_2 \cdot g); \\ \alpha_i = \beta_1 - \frac{i}{g}, & \alpha_i \in [-\frac{1}{2g}, \frac{1}{2g}); \\ \alpha_j = \beta_2 - \frac{j}{g}, & \alpha_j \in [-\frac{1}{2g}, \frac{1}{2g}). \end{cases} \tag{9}$$

There always exists the inverse function Δ^{-1} satisfying that for each interval-valued 2-tuple, it returns corresponding interval value $[\beta_1, \beta_2]$ $(\beta_1, \beta_2 \in [0, 1], \beta_1 \leqslant \beta_2)$ as follows:

$$\Delta^{-1}([(s_i, \alpha_i), (s_j, \alpha_j)]) = \left[\frac{i}{g} + \alpha_i, \frac{j}{g} + \alpha_j\right] = [\beta_1, \beta_2]. \tag{10}$$

Definition 5 (*See* Zhang, 2012). If $[(s_i, \alpha_i), (s_j, \alpha_j)]$ and $[(s_k, \alpha_k), (s_t, \alpha_t)]$ are any two interval-valued 2-tuple linguistic information, $l, l_1, l_2 \in [0, 1]$, then basic operational laws can be defined as follows:

1) $\big[(s_i,\alpha_i),(s_j,\alpha_j)\big]\oplus\big[(s_k,\alpha_k),(s_t,\alpha_t)\big]$
$=\big[\Delta(\min\{\Delta^{-1}(s_i,\alpha_i)+\Delta^{-1}(s_k,\alpha_k),1\}),$
$\Delta\big(\min\{\Delta^{-1}(s_j,\alpha_j)+\Delta^{-1}(s_t,\alpha_t),1\}\big)\big]$; 2)$l\otimes\big[(s_i,\alpha_i),(s_j,\alpha_j)\big]$
$=\big[\Delta(l\Delta^{-1}(s_i,\alpha_i)),\Delta(l\Delta^{-1}(s_j,\alpha_j))\big]$;
3) $\big(l_1\otimes[(s_i,\alpha_i),(s_j,\alpha_j)]\big)\oplus\big(l_2\otimes[(s_i,\alpha_i),(s_j,\alpha_j)]\big)$
$=(l_1+l_2)\otimes\big[(s_i,\alpha_i),(s_j,\alpha_j)\big]$;
4) $l\otimes\big(\big[(s_i,\alpha_i),(s_j,\alpha_j)\big]\oplus\big[(s_k,\alpha_k),(s_t,\alpha_t)\big]\big)$
$=\big(l\otimes\big[(s_i,\alpha_i),(s_j,\alpha_j)\big]\big)\oplus\big(l\otimes\big[(s_k,\alpha_k),(s_t,\alpha_t)\big]\big)$.

Aiming to compare two interval-valued 2-tuple linguistic terms, Zhang (2012) proposed the concept of the score and accuracy.

DEFINITION 6 (*See* Zhang, 2012). For an interval-valued 2-tuple $\tilde{A}=[(s_i,\alpha_i),(s_j,\alpha_j)]$, its score function can be defined as

$$(\tilde{A})=\frac{i+j}{2g}+\frac{\alpha_i+\alpha_j}{2}, \tag{11}$$

and the accuracy function can be defined as

$$H(\tilde{A})=\frac{j-i}{g}+\alpha_j-\alpha_i, \tag{12}$$

where $S=\{s_i\mid i=0,1,\dots,g\}$ is an ordered linguistic term set with $g+1$ linguistic labels. Obviously, $0\leqslant S(A)\leqslant 1$, and $0\leqslant H(A)\leqslant 1$.

DEFINITION 7 (*See* Zhang, 2012). Let $\tilde{A}=[(s_i,\alpha_i),(s_j,\alpha_j)]$ and $\tilde{B}=[(s_k,\alpha_k),(s_t,\alpha_t)]$ be two interval-valued linguistic 2-tuples. It follows that:

(1) If $S(\tilde{A})<S(\tilde{B})$, then $\tilde{A}<\tilde{B}$;
(2) If $S(\tilde{A})=S(\tilde{B})$, then:

$$\text{if } H(\tilde{A})<H(\tilde{B}),\quad \text{then } \tilde{A}>\tilde{B};$$
$$\text{if } H(\tilde{A})=H(\tilde{B}),\quad \text{then } \tilde{A}=\tilde{B}.$$

2.2. *The OWA Operator, the IOWA Operator and the GOWA Operator*

The *OWA* operator (Yager, 1988) provides a parameterized family of aggregation operators including the maximum, the minimum, and the average.

DEFINITION 8 (*See* Yager, 1988). The n-dimensional *OWA* operator is a mapping

$$OWA\colon R^n\to R,$$

where $W = (w_1, w_2, \ldots, w_n)$ is a weighting vector, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ satisfying

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{13}$$

where b_j is the j-th largest value among arguments $a_1, a_2, \ldots, a_n$.

The *IOWA* operator (Yager and Filev, 1999) can be regarded as extension of the *OWA* operator.

Definition 9 (*See* Yager and Filev, 1999). An n-dimensional *IOWA* operator is a mapping

$$IOWA\colon R^n \times R^n \to R,$$

where $W = (w_1, w_2, \ldots, w_n)$ is a weighting vector, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ satisfying

$$IOWA\big(\langle v_1, a_1\rangle, \langle v_2, a_2\rangle, \ldots, \langle v_n, a_n\rangle\big) = \sum_{j=1}^{n} w_j a_{\sigma(j)}, \tag{14}$$

where $a_{\sigma(j)}$ is the a_i value of the *IOWA* pair $\langle v_i, a_i\rangle$ having the j-th largest v_i.

Yager (2004a) proposed the generalized ordered weighted averaging (*GOWA*) operator.

Definition 10 (*See* Yager, 2004a). An n-dimensional *GOWA* operator is a mapping

$$GOWA\colon R^n \to R$$

where $W = (w_1, w_2, \ldots, w_n)$ is a weighting vector, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$ satisfying

$$GOWA(a_1, a_2, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j b_j^{\lambda}\right)^{1/\lambda}, \tag{15}$$

where $\lambda \in (-\infty, \infty)$, $\lambda \neq 0$, b_j is the j-th largest of the arguments among $a_1, a_2, \ldots, a_n$.

A group of special cases can be obtained through giving different values to parameter λ in the *GOWA* operator.

2.3. *The COWA Operator and the ICOWA Operator*

To deal with the case that the given argument is a continuous valued interval, Yager (2004b) introduced continuous ordered weighted averaging (*COWA*) operator.

Definition 11 (*See* Yager, 2004b). A *COWA* operator is a mapping $F:\Theta\to R^+$

$$F_Q(\tilde{a})=F_Q\big([a^L,a^U]\big)=\int_0^1\frac{dQ(y)}{dy}\big(a^U-y(a^U-a^L)\big)\,dy,\tag{16}$$

where Θ is the set of all nonnegative interval arguments and $\tilde{a}=[a^L,a^U]\in\Theta$, and Q is a basic unit monotonic (BUM) function.

Let $\lambda=\int_0^1 Q(y)\,dy$ be the attitudinal character of Q, then the general formulation of $F_Q(\tilde{a})$ is as follows:

$$F_Q(\tilde{a})=F_Q\big([a^L,a^U]\big)=\lambda a^U+(1-\lambda)a^L.\tag{17}$$

As we can see, the *COWA* operator $F_Q(\tilde{a})$ is the weighted arithmetical mean of end points according to the attitudinal character. The interval $\tilde{a}=[a^L,a^U]$ can be replaced by $F_Q(\tilde{a})$.

In Zhou *et al.* (2010) proposed induced continuous OWA (*ICOWA*) operator.

Definition 12 (*See* Zhou *et al.*, 2010). Let $[a^{L_1},a^{U_1}],[a^{L_2},a^{U_2}],\dots,[a^{L_n},a^{U_n}]$ be an interval number set. An *ICOWA* operator is a mapping *ICOWA*: $R^n\times\Theta^n\to R^+$ satisfying

$$\begin{aligned}&ICOWA\big(\langle u_1,[a^{L_1},a^{U_1}]\rangle,\langle u_2,[a^{L_2},a^{U_2}]\rangle,\dots,\langle u_n,[a^{L_n},a^{U_n}]\rangle\big)\\&\quad=ICOWA\big(\langle u_1,F_Q([a^{L_1},a^{U_1}])\rangle,\langle u_2,F_Q([a^{L_2},a^{U_2}])\rangle,\dots,\langle u_n,F_Q([a^{L_n},a^{U_n}])\rangle\big)\\&\quad=\sum_{j=1}^n w_jF_Q\big([a^{L_{\delta(j)}},a^{U_{\delta(j)}}]\big),\end{aligned}\tag{18}$$

where $W=(w_1,w_2,\dots,w_n)$ is an associated weighting vector with $\sum_{j=1}^n w_j=1$, $\delta(1),\delta(2),\dots,\delta(n)$ is any permutation of $(1,2,\dots,n)$, $(u_1,u_2,\dots,u_n)$ is a set of order inducing variables satisfying $u_{\delta(j-1)}\geqslant u_{\delta(j)}$, $j=2,3,\dots,n$, and $F_Q([a^{L_{\delta(j)}},a^{U_{\delta(j)}}]$ is the $F_Q([a^{L_i},a^{U_i}]$ value of the *ICOWA* pair $\langle u_i,[a^{L_i},a^{U_i}]\rangle$ having the j-th largest u_i, $F_Q([a^{L_i},a^{U_i}]$ is calculated by Eq. (17).

Similarly, we can obtain the interval-valued 2-tuple linguistic continuous OWA (*IT-COWA*) operator and the interval-valued 2-tuple linguistic induced continuous OWA (*IT-ICOWA*) operator. Let Ω be the set of all interval-valued 2-tuple linguistic information.

Definition 13. If $\tilde{A}=[(s_i,\alpha_i),(s_j,\alpha_j)]\in\Omega$, and $f_Q(\tilde{A})=f_Q([(s_i,\alpha_i),(s_j,\alpha_j)])=(s_k,\alpha_k)$, where

$$\begin{aligned}\Delta^{-1}(s_k,\alpha_k)&=F_Q\big([\Delta^{-1}(s_i,\alpha_i),\Delta^{-1}(s_j,\alpha_j)]\big)\\&=\int_0^1\frac{dQ(y)}{dy}\big(\Delta^{-1}(s_j,\alpha_j)-y(\Delta^{-1}(s_j,\alpha_j)-\Delta^{-1}(s_i,\alpha_i))\big)\,dy,\end{aligned}\tag{19}$$

then f is the interval-valued 2-tuples linguistic continuous OWA (*IT-COWA*) operator, where Q is the BUM function.

Especially, if $\lambda = \int_0^1 Q(y)\,dy$ is the attitudinal character of Q, then the *IT-COWA* operator can be rewritten as follows:

$$\begin{aligned} f_Q(\tilde{A}) &= \Delta\big(F_Q\big(\big[\Delta^{-1}(s_i,\alpha_i), \Delta^{-1}(s_j,\alpha_j)\big]\big)\big) \\ &= \Delta\big(\lambda\Delta^{-1}(s_j,\alpha_j) + (1-\lambda)\Delta^{-1}(s_i,\alpha_i)\big), \end{aligned} \tag{20}$$

Based on Eq. (20), we can see that the *IT-COWA* operator may be determined by the attitudinal character λ. For convenience, $f_\lambda(\tilde{A})$ denotes $f_Q(\tilde{A})$, i.e.

$$\begin{aligned} f_\lambda(\tilde{A}) &= \Delta\big(F_Q\big(\big[\Delta^{-1}(s_i,\alpha_i), \Delta^{-1}(s_j,\alpha_j)\big]\big)\big) \\ &= \Delta\big(\lambda\Delta^{-1}(s_j,\alpha_j) + (1-\lambda)\Delta^{-1}(s_i,\alpha_i)\big). \end{aligned} \tag{21}$$

Definition 14. An n-dimensional *IT-ICOWA* measure is a mapping *IT-ICOWA*

$$R^n \times \Omega^n \to \Omega,$$

which is associated weighting vector $W = (w_1, w_2, \ldots, w_n)$, $\sum_{j=1}^n w_j = 1$ and $w_j \in [0,1]$, satisfying

$$\begin{aligned} &\textit{IT-ICOWA}\big(\langle u_1, \big[(s_1,\alpha_1), (s_1',\alpha_1')\big]\rangle, \ldots, \langle u_n, \big[(s_n,\alpha_n), (s_n',\alpha_n')\big]\rangle\big) \\ &\quad = \textit{IT-ICOWA}\big(\langle u_1, f_\lambda\big(\big[(s_1,\alpha_1), (s_1',\alpha_1')\big]\big)\rangle, \ldots, \langle u_n, f_\lambda\big(\big[(s_n,\alpha_n), (s_n',\alpha_n')\big]\big)\rangle\big) \\ &\quad = \Delta\Bigg(\sum_{j=1}^n w_j \Delta^{-1}\big(f_\lambda\big(\big[(s_{\delta(j)},\alpha_{\delta(j)}), (s'_{\delta(j)},\alpha'_{\delta(j)})\big]\big)\big)\Bigg), \end{aligned} \tag{22}$$

where $\delta(1), \delta(2), \ldots, \delta(n)$ is any permutation of $(1,2,\ldots,n)$, $(u_1,u_2,\ldots,u_n)$ is a set of order inducing variables, such that $u_{\delta(j-1)} \geqslant u_{\delta(j)}$, $j = 2,3,\ldots,n$, and $f_\lambda([(s_{\delta(j)},\alpha_{\delta(j)}), (s'_{\delta(j)},\alpha'_{\delta(j)})])$ is the $f_\lambda([(s_i,\alpha_i), (s_i',\alpha_i')])$ value of the *IT-ICOWA* pair $\langle u_i, (s_i,\alpha_i), (s_i',\alpha_i')\rangle$ with the j-th largest u_i, where $f_\lambda([(s_i,\alpha_i), (s_i',\alpha_i')])$ can be determined by Eq. (21).

2.4. *Distance Measure*

Definition 15 (*See* Zhou *et al.*, 2013, 2014b). Let A_1, A_2, A_3 be elements of a set. A distance measure D should satisfy properties as follows:

- Nonnegativity: $D(A_1,A_2) \geqslant 0$;
- Commutativity: $D(A_1,A_2) = D(A_2,A_1)$;
- Reflexivity: $D(A_1,A_1) = 0$;
- Triangle inequality: $D(A_1,A_2) + D(A_1,A_3) \geqslant D(A_2,A_3)$.

Note that different D can derive different types of distance measures.

On the basis of Liu *et al.* (2014b), Zhou *et al.* (2014b), Li *et al.* (2014), Xu (2005), now we present several interval-valued linguistic distance measures including the interval-valued linguistic distance measure, the continuous interval-valued linguistic distance measure and the interval-valued 2-tuple linguistic distance measure.

– Interval-valued linguistic distance measure:

$$d(A_1, A_2) = d\big([s_i, s_j], [s_k, s_t]\big) = s_{\frac{1}{g}(\frac{|i-k|+|j-t|}{2})}, \tag{23}$$

where $[s_i, s_j]$ and $[s_k, s_t]$ are two interval-valued linguistic variables, respectively.

– Continuous interval-valued linguistic distance measure:

$$\begin{aligned} d'(A_1, A_2) &= d'\big([s_i, s_j], [s_k, s_t]\big) = s_{\frac{|F_Q([i,j])-F_Q([k,t])|}{g}} \\ &= s_{\frac{|\lambda j+(1-\lambda)i-(\lambda t+(1-\lambda)k)}{|}g}, \end{aligned} \tag{24}$$

where Q is the BUM function, $\lambda = \int_0^1 Q(y)\,dy$.

– Interval-valued 2-tuple linguistic distance measure:

$$\begin{aligned} & d''(\tilde{A}_1, \tilde{A}_2) \\ &= d''\big([(s_i, \alpha_i), (s_j, \alpha_j)], [(s_k, \alpha_k), (s_t, \alpha_t)]\big) \\ &= \Delta\Big(\frac{|\Delta^{-1}(s_i, \alpha_i) - \Delta^{-1}(s_k, \alpha_k)| + |\Delta^{-1}(s_j, \alpha_j) - \Delta^{-1}(s_t, \alpha_t)|}{2}\Big), \end{aligned} \tag{25}$$

where $[(s_i, \alpha_i), (s_j, \alpha_j)]$ and $[(s_k, \alpha_k), (s_t, \alpha_t)]$ are two interval-valued 2-tuple linguistic information, respectively.

2.5. *The Ordered Weighted Distance Measure*

Xu and Chen (2008) developed the ordered weighted distance (*OWD*) measure.

DEFINITION 16 (*See* Xu and Chen, 2008). An n-dimensional *OWD* measure is a mapping *OWD*: $R^{+n} \times R^{+n} \to R^+$ satisfying:

$$OWD(\alpha, \beta) = \Bigg(\sum_{j=1}^{n} w_j \big(d(a_{\sigma(j)}, b_{\sigma(j)})\big)^{\lambda}\Bigg)^{1/\lambda}, \tag{26}$$

where $W = (w_1, w_2, \dots, w_n)$ is associated with weighting vector, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$, $\sigma(1), \sigma(2), \dots, \sigma(n)$ is any permutation of $(1, 2, \dots, n)$, $d(a_{\sigma(j-1)}, b_{\sigma(j-1)}) \geqslant d(a_{\sigma(j)}, b_{\sigma(j)})$, $j = 1, 2, \dots, n$, $d(a_j, b_j) = |a_j - b_j|$ is the distance of a_j and b_j. $\alpha = (a_1, a_2, \dots, a_n)$ and $\beta = (b_1, b_2, \dots, b_n)$ are two collections of arguments, and $\lambda > 0$.

The *OWD* measure is monotonic, commutative, idempotent and bounded. We can obtain a group of special cases when considering as many as values of the parameter λ in the *OWD* measure. For instance, the ordered weighted Hamming distance (*OWHD*) measure, the ordered weighted Euclidean distance (*OWED*) measure and the ordered weighted Geometric (*OWGD*) measure can be determined in the following way:

– The *OWHD* measure is obtained if $\lambda = 1$.
– The *OWED* measure is obtained if $\lambda = 2$.
– The *OWGD* measure is obtained if $\lambda \to 0$.

3. The Interval-Valued 2-Tuples Linguistic Induced Continuous Ordered Weighted Distance Measure

In this section, we introduce the interval-valued 2-tuples linguistic continuous ordered weighted distance (*IT-COWD*) measure and the interval-valued 2-tuples linguistic induced continuous ordered weighted distance (*IT-ICOWD*) measure.

3.1. *The IT-COWD Measure and the IT-ICOWD Measure*

Definition 17. Let $\tilde{A}_1 = [(s_1, \alpha_1), (s_1', \alpha_1')] \in \Omega$ and $\tilde{A}_2 = [(s_2, \alpha_2), (s_2', \alpha_2')] \in \Omega$. If

$$d_\lambda(\tilde{A}_1, \tilde{A}_2) = \Delta\big(\big|\Delta^{-1}\big(f_\lambda(\tilde{A}_1)\big) - \Delta^{-1}\big(f_\lambda(\tilde{A}_2)\big)\big|\big), \tag{27}$$

then $d_\lambda(\tilde{A}_1, \tilde{A}_2)$ is called the distance between $\tilde{A}_1$ and $\tilde{A}_2$ based on the *IT-COWA* operator, where $f_\lambda(\tilde{A}_1)$ and $f_\lambda(\tilde{A}_2)$ can be calculated by Eq. (21).

According to Eq. (27), $d_\lambda(\tilde{A}_1, \tilde{A}_2)$ can be defined as follows:

$$\begin{aligned} d_\lambda(\tilde{A}_1, \tilde{A}_2) &= \Delta\big(\big|\Delta^{-1}\big(f_\lambda(\tilde{A}_1)\big) - \Delta^{-1}\big(f_\lambda(\tilde{A}_2)\big)\big|\big) \\ &= \Delta\big(\big|\lambda\Delta^{-1}(s_1', \alpha_1') + (1-\lambda)\Delta^{-1}(s_1, \alpha_1) - (\lambda\Delta^{-1}(s_2', \alpha_2') \\ &\quad + (1-\lambda)\Delta^{-1}(s_2, \alpha_2))\big|\big) \end{aligned} \tag{28}$$

where $\lambda = \int_0^1 Q(y)\,dy$ is the attitudinal character of Q.

Example 1. Let $S^7 = \{s_i^7 \mid i = 0, 1, \dots, 6\}$ be a linguistic term set, $A_1 = [s_2^7, s_5^7]$ and $A_2 = [s_3^7, s_4^7]$ be two interval-valued linguistic variables, and $Q(y) = y^3$, so $\lambda = \int_0^1 Q(y)\,dy = \frac{1}{4}$. Therefore, A_1, A_2 can be rewritten as interval-valued 2-tuple linguistic information $\tilde{A}_1 = [(s_2^7, 0), (s_5^7, 0)]$ and $\tilde{A}_2 = [(s_3^7, 0), (s_4^7, 0)]$. By Eq. (23)–(25), (28), we have

(1) $d(A_1, A_2) = d\big([s_2^7, s_5^7], [s_3^7, s_4^7]\big) = s_{\frac{1}{6}(\frac{|2-3|+|5-4|}{2})} = s_{\frac{1}{6}}$,
(2) $d'(A_1, A_2) = d'\big([s_2^7, s_5^7], [s_3^7, s_4^7]\big) = s_{\frac{|\frac{5}{4}+(1-\frac{1}{4})2-\frac{4}{4}-(1-\frac{1}{4})3)|}{6}} = s_{\frac{1}{12}}$,

(3) $d''(\tilde{A}_1, \tilde{A}_2) = d''\big([(s_2^7, 0), (s_5^7, 0)], [(s_3^7, 0), (s_4^7, 0)]\big)$
$= \Delta\Big(\frac{|\Delta^{-1}(s_2^7, 0) - \Delta^{-1}(s_3^7, 0)| + |\Delta^{-1}(s_5^7, 0) - \Delta^{-1}(s_4^7, 0)|}{2}\Big) = (s_1^7, 0),$

(4) $d_\lambda(\tilde{A}_1, \tilde{A}_2) = d_\lambda\big([(s_2^7, 0), (s_5^7, 0)], [(s_3^7, 0), (s_4^7, 0)]\big)$
$= \Delta\Big(\Big|\frac{1}{4}\Delta^{-1}(s_5^7, 0) + \Big(1 - \frac{1}{4}\Big)\Delta^{-1}(s_2^7, 0) - \frac{1}{4}\Delta^{-1}(s_4^7, 0)$
$-\Big(1 - \frac{1}{4}\Big)\Delta^{-1}(s_3^7, 0))\Big|\Big)$
$= (s_1^7, -\frac{1}{12}).$

As we can see, the results do not match any of the initial linguistic terms in the aggregation process of the distance measure $d(A_1, A_2)$ and $d'(A_1, A_2)$, but these two distance measures can express the results in the initial expression domain. Moreover, they are able to deal with the situations where the input arguments are represented with interval-valued 2-tuples linguistic information.

From Definition 17, we can get the following theorems:

Theorem 1. *If* $\tilde{A}_1 = [(s_1, \alpha_1), (s_1', \alpha_1')] \in \Omega$*,* $\tilde{A}_2 = [(s_2, \alpha_2), (s_2', \alpha_2')] \in \Omega$ *and* $\tilde{A}_3 = [(s_3, \alpha_3), (s_3', \alpha_3')] \in \Omega$*, then*

(1) *Nonnegativity:* $d_\lambda(\tilde{A}_1, \tilde{A}_2) \geqslant 0$*;*
(2) *Commutativity:* $d_\lambda(\tilde{A}_1, \tilde{A}_2) = d_\lambda(\tilde{A}_2, \tilde{A}_1)$*;*
(3) *Reflexivity:* $d_\lambda(\tilde{A}_1, \tilde{A}_1) = 0$*;*
(4) *Triangle inequality:* $d_\lambda(\tilde{A}_1, \tilde{A}_2) + d_\lambda(\tilde{A}_1, \tilde{A}_3) \geqslant d_\lambda(\tilde{A}_2, \tilde{A}_3)$.

Theorem 2. *If* $\tilde{A}_1 = [(s_1, \alpha_1), (s_1', \alpha_1')] \in \Omega$*,* $\tilde{A}_2 = [(s_2, \alpha_2), (s_2', \alpha_2')] \in \Omega$*, then* $\Delta^{-1}(d_\lambda(\tilde{A}_1, \tilde{A}_2)) \leqslant 1$.

The proofs of theorems are straightforward, thus omitted.

Suppose that $\tilde{\mathrm{A}} = (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \in \Omega^n$ and $\tilde{\mathrm{B}} = (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) \in \Omega^n$, we can define the interval-valued 2-tuple linguistic continuous ordered weighted distance (*IT-COWD*) measure as follows:

Definition 18. An n-dimensional *IT-COWD* measure is a mapping

$$IT\text{-}COWD\colon \Omega^n \times \Omega^n \to \Omega,$$

satisfying:

$$IT\text{-}COWD(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = \Delta\Bigg(\bigg(\sum_{j=1}^{n} w_j\big[\Delta^{-1}\big(d_\lambda(\tilde{A}_{\sigma(j)}, \tilde{B}_{\sigma(j)})\big)\big]^\tau\bigg)^{1/\tau}\Bigg), \tag{29}$$

where $\sigma(1), \sigma(2), \dots, \sigma(n)$ is any permutation of $(1, 2, \dots, n)$ such that $d_\lambda(\tilde{A}_{\sigma(j-1)}, \tilde{B}_{\sigma(j-1)}) \geqslant d_\lambda(\tilde{A}_{\sigma(j)}, \tilde{B}_{\sigma(j)})$, $j = 2, 3, \dots, n$, and $d_\lambda(\tilde{A}_j, \tilde{B}_j)$ is the distance between $\tilde{A}_j$

Table 1
Aggregation result.

τ	$\to 0+$	1	2
$IT\text{-}COWD(\tilde{A},\tilde{B})$	$(s_0^7, 0.0252)$	$(s_0^7, 0.0418)$	$(s_0^7, 0.0543)$
τ	**3**	**4**	**5**
$IT\text{-}COWD(\tilde{A},\tilde{B})$	$(s_0^7, 0.0617)$	$(s_0^7, 0.0664)$	$(s_0^7, 0.0697)$
τ	**6**	**7**	**8**
$IT\text{-}COWD(\tilde{A},\tilde{B})$	$(s_0^7, 0.0682)$	$(s_0^7, 0.0739)$	$(s_0^7, 0.0754)$
τ	**9**	**10**	**15**
$IT\text{-}COWD(\tilde{A},\tilde{B})$	$(s_0^7, 0.0766)$	$(s_0^7, 0.0776)$	$(s_0^7, 0.0808)$

and $\tilde{B}_j$ based on the *IT-COWA* operator and the parameter $\tau > 0$, $W = (w_1, w_2, \dots, w_n)$ is associated with vector, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0, 1]$.

Here we present a simple numerical example showing how to use the *IT-COWD* measure in an aggregation process.

EXAMPLE 2. Let $\tilde{\text{A}} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4) = ([(s_3^7, 0.05), (s_5^7, 0.01)], [(s_2^7, 0.02), (s_4^7, -0.06)], [(s_4^7, -0.04), (s_5^7, -0.07)], [(s_1^7, -0.02), (s_2^7, 0.03)])$ and $\tilde{\text{B}} = (\tilde{B}_1, \tilde{B}_2, \tilde{B}_3, \tilde{B}_4) = ([(s_4^7, 0.02), (s_5^7, -0.06)], [(s_2^7, 0.04), (s_3^7, 0.03)], [(s_4^7, -0.02), (s_6^7, -0.08)], [(s_1^7, -0.05), (s_2^7, 0.01)])$ be two collections of interval-valued 2-tuples linguistic information, and $Q(y) = y^3$, so $\lambda = \int_0^1 Q(y)\,dy = \frac{1}{4}$. From Eq. (28), we have

$$\begin{aligned} d_\lambda(\tilde{A}_1, \tilde{B}_1) &= \Delta\left(\left|\frac{1}{4}\Delta^{-1}(s_5^7, 0.01) + \left(1 - \frac{1}{4}\right)\Delta^{-1}(s_3^7, 0.05)\right.\right. \\ &\qquad \left.\left. - \left(\frac{1}{4}\Delta^{-1}(s_5^7, -0.06) + \left(1 - \frac{1}{4}\right)\Delta^{-1}(s_4^7, 0.02)\right)\right|\right) \\ &= \Delta(0.0875) = (s_1^7, -0.0792), \\ d_\lambda(\tilde{A}_2, \tilde{B}_2) &= \Delta(0.005) = (s_0^7, 0.005), \\ d_\lambda(\tilde{A}_3, \tilde{B}_3) &= \Delta(0.055) = (s_0^7, 0.055), \\ d_\lambda(\tilde{A}_4, \tilde{B}_4) &= \Delta(0.01) = (s_0^7, 0.01). \end{aligned}$$

So,

$$\begin{aligned} &d_l(\tilde{A}_{s(1)}, \tilde{B}_{s(1)}) = (s_1^7, -0.0792), \qquad d_l(\tilde{A}_{s(2)}, \tilde{B}_{s(2)}) = (s_0^7, 0.055), \\ &d_l(\tilde{A}_{s(3)}, \tilde{B}_{s(3)}) = (s_0^7, 0.01), \qquad d_l(\tilde{A}_{s(4)}, \tilde{B}_{s(4)}) = (s_0^7, 0.005). \end{aligned}$$

Let $W = (0.3, 0.2, 0.4, 0.1)$. By Eq. (29), we determine the distances of τ, which are shown in Table 1.

From Table 1, it is demonstrated that the aggregation result $IT\text{-}COWD(\tilde{A}, \tilde{B})$ increases as the parameter τ steadily increases.

We define the interval-valued 2-tuples linguistic induced continuous ordered weighted distance (*IT-ICOWD*) measure:

Definition 19. An n-dimensional *IT-ICOWD* measure is a mapping

$$IT-ICOWD\colon R^n \times \Omega^n \times \Omega^n \to \Omega,$$

which is associated with the weighting vector $W = (w_1, w_2, \dots, w_n)$, $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, satisfying:

$$\begin{aligned} &\textit{IT-ICOWD}\big(\langle u_1, \tilde{A}_1, \tilde{B}_1\rangle, \langle u_2, \tilde{A}_2, \tilde{B}_2\rangle, \dots, \langle u_n, \tilde{A}_n, \tilde{B}_n\rangle\big) \\ &\quad = \Delta\left(\left[\sum_{j=1}^{n} w_j \big[\Delta^{-1}\big(d_\lambda(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)})\big)\big]^{\tau}\right]^{1/\tau}\right), \end{aligned} \tag{30}$$

where $\delta(1), \delta(2), \dots, \delta(n)$is any permutation of $(1, 2, \dots, n)$, $(u_1, u_2, \dots, u_n)$ is a set of order inducing variables, such that $u_{\delta(j-1)} \geqslant u_{\delta(j)}$, $j = 2, 3, \dots, n$, and $d_\lambda(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)})$ is the $d_\lambda(\tilde{A}_i, \tilde{B}_i)$ value of the *IT-ICOWD* pair $\langle u_i, \tilde{A}_i, \tilde{B}_i\rangle$ having the jth largest u_i, $d_\lambda(\tilde{A}_j, \tilde{B}_j)$ can be calculated by Eq. (28) and the parameter $\tau > 0$.

In the following, we present a simple numerical example showing how to use the *IT-ICOWD* measure in an aggregation process.

Example 3. Let $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \tilde{A}_3, \tilde{A}_4) = ([(s_3^7, 0.05), (s_5^7, 0.01)], [(s_2^7, 0.02), (s_4^7, -0.06)], [(s_4^7, -0.04), (s_5^7, -0.07)], [(s_1^7, -0.02), (s_2^7, 0.03)])$ and $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \tilde{B}_3, \tilde{B}_4) = ([(s_4^7, 0.02), (s_5^7, -0.06)], [(s_2^7, 0.04), (s_3^7, 0.03)], [(s_4^7, -0.02), (s_6^7, -0.08)], [(s_1^7, -0.05), (s_2^7, 0.01)])$ be two interval-valued 2-tuples linguistic information collections, and $Q(y) = y^2$, so $\lambda = \int_0^1 Q(y)\,dy = \frac{1}{3}$. From Eq. (28), we have

$$\begin{aligned} d_\lambda(\tilde{A}_1, \tilde{B}_1) &= \Delta\left(\left|\frac{1}{3}\Delta^{-1}\big(s_5^7, 0.01\big) + \left(1 - \frac{1}{3}\right)\Delta^{-1}\big(s_3^7, 0.05\big)\right.\right. \\ &\qquad \left.\left. - \left(\frac{1}{3}\Delta^{-1}\big(s_5^7, -0.06\big) + \left(1 - \frac{1}{3}\right)\Delta^{-1}\big(s_4^7, 0.02\big)\right)\right|\right) \\ &= \Delta(0.07) = \big(s_0^7, 0.07\big), \\ d_\lambda(\tilde{A}_2, \tilde{B}_2) &= \big(s_0^7, 0.0133\big), \qquad d_\lambda(\tilde{A}_3, \tilde{B}_3) = \big(s_0^7, 0.0667\big), \\ d_\lambda(\tilde{A}_4, \tilde{B}_4) &= \big(s_0^7, 0.0067\big). \end{aligned}$$

Let both sets be of the same order inducing variables: $(u_1, u_2, u_3, u_4) = (5, 1, 7, 3)$. Thus,

$$\begin{aligned} &d_\lambda(\tilde{A}_{\delta(1)}, \tilde{B}_{\delta(1)}) = \big(s_0^7, 0.0667\big), \qquad d_\lambda(\tilde{A}_{\delta(2)}, \tilde{B}_{\delta(2)}) = \big(s_0^7, 0.07\big), \\ &d_\lambda(\tilde{A}_{\delta(3)}, \tilde{B}_{\delta(3)}) = \big(s_0^7, 0.0067\big), \qquad d_\lambda(\tilde{A}_{\delta(4)}, \tilde{B}_{\delta(4)}) = \big(s_0^7, 0.0133\big). \end{aligned}$$

Table 2
Aggregation result.

τ	$\to 0$	1	2
$IT\text{-}ICOWD(\tilde{A}, \tilde{B})$	$(s_0^7, 0.0228)$	$(s_0^7, 0.038)$	$(s_0^7, 0.0485)$
$IT\text{-}IOWD(\tilde{A}, \tilde{B})$	$(s_0^7, 0.048)$	$(s_0^7, 0.061)$	$(s_0^7, 0.071)$
τ	**3**	**4**	**5**
$IT\text{-}ICOWD(\tilde{A}, \tilde{B})$	$(s_0^7, 0.054)$	$(s_0^7, 0.0572)$	$(s_0^7, 0.0593)$
$IT\text{-}IOWD(\tilde{A}, \tilde{B})$	$(s_0^7, 0.0775)$	$(s_0^7, 0.0818)$	$(s_1^7, -0.082)$
τ	**6**	**7**	**8**
$IT\text{-}ICOWD(\tilde{A}, \tilde{B})$	$(s_0^7, 0.0607)$	$(s_0^7, 0.0617)$	$(s, 0.0625)$
$IT\text{-}IOWD(\tilde{A}, \tilde{B})$	$(s_1^7, -0.0798)$	$(s_1^7, -0.0782)$	$(s_1^7, -0.0768)$
τ	**9**	**10**	**15**
$IT\text{-}ICOWD(\tilde{A}, \tilde{B})$	$(s_0^7, 0.0631)$	$(s_0^7, 0.0636)$	$(s_0^7, 0.0652)$
$IT\text{-}IOWD(\tilde{A}, \tilde{B})$	$(s_1^7, -0.0757)$	$(s_1^7, -0.0748)$	$(s_1^7, -0.0715)$

Let $W = (0.3, 0.2, 0.4, 0.1)$. By Eq. (30), we have the distances of parameter τ, which are shown in Table 2.

From Table 2, the aggregation result $IT\text{-}ICOWD(\tilde{A}, \tilde{B})$ increases as the parameter τ steadily increases.

3.2. *Properties of the IT-ICOWD Measure*

The *IT-ICOWD* measure is of desirable properties. Now we discuss them through following theorems.

Theorem 3 (Monotonicity-distance measure). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) \in \Omega^n$, $C = (\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_n) \in \Omega^n$, *and* $d_\lambda(\tilde{A}_j, \tilde{B}_j) \leqslant d_\lambda(\tilde{A}_j, \tilde{C}_j)$ *for all* j, *then*

$$IT\text{-}ICOWD(\tilde{A}, \tilde{B}) \leqslant IT\text{-}ICOWD(\tilde{A}, \tilde{C}). \tag{31}$$

Theorem 4 (Monotonicity-parameter τ). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) \in \Omega^n$, *and* $\tau_1 \leqslant \tau_2$, *then*

$$IT\text{-}ICOWD_{\tau_1}(\tilde{A}, \tilde{B}) \leqslant IT\text{-}ICOWD_{\tau_2}(\tilde{A}, \tilde{B}). \tag{32}$$

Theorem 5 (Idempotency). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) \in \Omega^n$, *and* $d_\lambda(\tilde{A}_j, \tilde{B}_j) = d$ *for all* j, *then*

$$IT\text{-}ICOWD(\tilde{A}, \tilde{B}) = d. \tag{33}$$

Theorem 6 (Boundedness). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \dots, \tilde{B}_n) \in \Omega^n$, *and* $\max_j d_\lambda(\tilde{A}_j, \tilde{B}_j) = d_{\max}$, $\min_j d_\lambda(\tilde{A}_j, \tilde{B}_j) = d_{\min}$, *then*

$$d_{\min} \leqslant IT\text{-}ICOWD(\tilde{A}, \tilde{B}) \leqslant d_{\max}. \tag{34}$$

Theorem 7 (Commutativity-*GOWA* aggregation). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_n) \in \Omega^n$, *and* $((\hat{A}_1, \hat{B}_1), (\hat{A}_2, \hat{B}_2), \ldots, (\hat{A}_n, \hat{B}_n))$ *is a permutation of* $((\tilde{A}_1, \tilde{B}_1), (\tilde{A}_2, \tilde{B}_2), \ldots, (\tilde{A}_n, \tilde{B}_n))$, *then*

$$IT\text{-}ICOWD(\hat{A}, \hat{B}) = IT\text{-}ICOWD(\tilde{A}, \tilde{B}), \tag{35}$$

where $\hat{A} = (\hat{A}_1, \hat{A}_2, \ldots, \hat{A}_n) \in \Omega^n$, $\hat{B} = (\hat{B}_1, \hat{B}_2, \ldots, \hat{B}_n) \in \Omega^n$.

Theorem 8 (Commutativity-distance measure). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_n) \in \Omega^n$, *then*

$$IT\text{-}ICOWD(\tilde{A}, \tilde{B}) = IT\text{-}ICOWD(\tilde{B}, \tilde{A}). \tag{36}$$

Theorem 9 (Nonnegativity). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) \in \Omega^n$, $\tilde{B} = (\tilde{B}_1, \tilde{B}_2, \ldots, \tilde{B}_n) \in \Omega^n$, *then*

$$IT\text{-}ICOWD(\tilde{A}, \tilde{B}) \geqslant 0. \tag{37}$$

Theorem 10 (Reflexivity). *If* $\tilde{A} = (\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) \in \Omega^n$, *then*

$$IT\text{-}ICOWD(\tilde{A}, \tilde{A}) = 0. \tag{38}$$

The proofs of theorems above are straightforward, thus omitted.

3.3. *Families of the IT-ICOWD Measure*

Now we discuss families of the *IT-ICOWD* measure by using different τ and weighting vectors to get different types of distance measure.

REMARK 1. If $\tau = 1$, the *IT-ICOWD* measure reduces to the *IT-ICOWHD* measure:

$$IT\text{-}ICOWHD(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = \Delta\left(\sum_{j=1}^{n} w_j \left[\Delta^{-1}\left(d_\lambda(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)})\right)\right]\right). \tag{39}$$

If $\tau = 2$, the *IT-ICOWD* measure reduces to the *IT-ICOWED* measure:

$$IT\text{-}ICOWED(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = \Delta\left(\left(\sum_{j=1}^{n} w_j \left[\Delta^{-1}\left(d_\lambda(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)})\right)\right]^2\right)^{1/2}\right). \tag{40}$$

If $\tau \to 0^+$, we obtain *IT-ICOWGD* measure:

$$IT\text{-}ICOWGD(\tilde{\mathrm{A}}, \tilde{\mathrm{B}}) = \Delta\left(\prod_{j=1}^{n} \left[\Delta^{-1}\left(d_\lambda(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)})\right)\right]^{w_j}\right). \tag{41}$$

REMARK 2. Some other special measures can be obtained as follows:

- The *IT-ICMAXD* measure: $w_l = 1$ and $w_j = 0$ for all $j \neq l$, and $d_\lambda(\tilde{A}_{\delta(l)}, \tilde{B}_{\delta(l)}) = \max\{d_\lambda(\tilde{A}_i, \tilde{B}_i)\}$, $i = 1, 2, \ldots, n$.
- The *IT-ICMIND* measure: $w_l = 1$ and $w_j = 0$ for all $j \neq l$, and $d_\lambda(\tilde{A}_{\delta(l)}, \tilde{B}_{\delta(l)}) = \min\{d_\lambda(\tilde{A}_i, \tilde{B}_i)\}$, $i = 1, 2, \ldots, n$.
- Step-*IT-ICOWD* measure: $w_k = 1$ and $w_j = 0$ for all $j \neq k$.
- The *IT-ICND* measure: $w_j = 1/n$ for all j; specially, if $\tau = 1$, we get the *IT-ICNHD* measure. If $\tau = 2$, we get the *IT-ICNED* measure. If $\tau \to 0^+$, we get the *IT-ICNGD* measure.
- Median *IT-ICOWD* measure: $w_{(n+1)/2} = 1$ and $w_j = 0$ for all $j \neq (n+1)/2$, n is odd; or $w_{n/2} = w_{(n/2)+1} = \frac{1}{2}$ and $w_j = 0$ for all $j \neq n/2, (n/2)+1$, n is even.

REMARK 3. Based on Yager (1993), we can get families of *IT-ICOWD* measure. For instance:

- The Olympic *IT-ICOWD* measure: $w_1 = w_n = 0$ and $w_j = 1/(n-2)$ for all $j \neq 1, \ldots, n$.
- The general Olympic *IT-ICOWD* measure: $w_j = 0$ for all $j = 1, 2, \ldots, k, n, n-1, \ldots, n-k+1\sqrt{a^2+b^2}$ and for all others $w_j = 1/(n-2k)$, where $k < n/2$.
- The Window *IT-ICOWD* measure: $w_j = 1/m$ for $k \leqslant j \leqslant k+m-1$, or $w_j = 0$ for $j \geqslant k+m$ and $j < k$.
- The generalized S-*IT-ICOWD* measure: $w_k = (1-(\alpha+\beta)/n)+\alpha$, $w_t = (1-(\alpha+\beta)/n)+\beta$ and $w_j = 1-(\alpha+\beta)/n$ for all $j \neq k, t$, where $\tilde{A}_k = \max_i\{\tilde{A}_i\}$, $\tilde{A}_t = \min_i\{\tilde{A}_i\}$, and $\alpha+\beta \leqslant 1$ with $\alpha, \beta \in [0, 1]$.

3.4. *Extensions of the IT-ICOWD Measure*

We can develop the extension of the *IT-COWD* measure through the quasi-arithmetic means, which can be named as *Quasi-IT-COWD* measure. The primary merit of this measure is providing a more complete generalization including a lot of particular cases that are not included in the *IT-COWD* measure.

DEFINITION 20. An n-dimensional *Quasi-IT-COWD* measure is a mapping

$$\textit{Quasi-IT-COWD}\colon \Omega^n \times \Omega^n \to \Omega,$$

which is associated with the weighting vector $W = (w_1, w_2, \ldots, w_n)$, $\sum_{j=1}^n w_j = 1$ and $w_j \in [0, 1]$, satisfying:

$$\textit{Quasi-IT-COWD}(\tilde{A}, \tilde{B}) = \Delta\left(g^{-1}\left(\sum_{j=1}^{n} w_j g\left[\Delta^{-1}\left(d_\lambda(\tilde{A}_{\sigma(j)}, \tilde{B}_{\sigma(j)})\right)\right]\right)\right), \tag{42}$$

where g is a strictly continuous monotonic function, $\sigma(1), \sigma(2), \ldots, \sigma(n)$ is any permutation of $(1, 2, \ldots, n)$ such that $d_\lambda(\tilde{A}_{\sigma(j-1)}, \tilde{B}_{\sigma(j-1)}) \geqslant d_\lambda(\tilde{A}_{\sigma(j)}, \tilde{B}_{\sigma(j)})$, $j = 2, 3, \ldots, n$, and $d_\lambda(\tilde{A}_j, \tilde{B}_j)$ is the distance of $\tilde{A}_j$ and $\tilde{B}_j$ based on the *IT-COWA* operator and the parameter.

Note that the *IT-COWD* measure is a particular case of the *Quasi-IT-COWD* measure when $g(x) = x^\tau$.

Similarly, we can obtain the *Quasi-IT-ICOWD* measure.

Definition 21. An n-dimensional *Quasi-IT-ICOWD* measure is a mapping

$$\textit{Quasi-IT-ICOWD}: \Omega^n \times \Omega^n \to \Omega,$$

which is associated with the weighting vector $W = (w_1, w_2, \dots, w_n)$, $\sum_{j=1}^n w_j = 1$ and $w_j \in [0, 1]$, satisfying

$$\begin{aligned} &\textit{Quasi-IT-ICOWD}\big(\langle u_1, \tilde{A}_1, \tilde{B}_1\rangle, \langle u_2, \tilde{A}_2, \tilde{B}_2\rangle, \dots, \langle u_n, \tilde{A}_n, \tilde{B}_n\rangle\big) \\ &\quad = \Delta\left(g^{-1}\left[\sum_{j=1}^n w_j g\big(\Delta^{-1}\big(d_\lambda\big(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)}\big)\big)\big)\right]\right), \end{aligned} \tag{43}$$

where g is a strictly continuous monotonic function, $\delta(1), \delta(2), \dots, \delta(n)$ is any permutation of $(1, 2, \dots, n)$, $(u_1, u_2, \dots, u_n)$ is a set of order inducing variables such that $u_{\delta(j-1)} \geqslant u_{\delta(j)}$, $j = 2, 3, \dots, n$, and $d_\lambda(\tilde{A}_{\delta(j)}, \tilde{B}_{\delta(j)})$ is the $d_\lambda(\tilde{A}_i, \tilde{B}_i)$ value of the *IT-ICOWD* pair $\langle u_i, \tilde{A}_i, \tilde{B}_i\rangle$ having the j-th largest u_i, $d_\lambda(\tilde{A}_j, \tilde{B}_j)$ is calculated by Eq. (28), $\lambda > 0$.

4. An Approach to 2-Tuple Linguistic Multiple Attribute Group Decision Making

4.1. *The Process of MAGDM Based on the IT-ICOWD Measure*

The *IT-ICOWD* measure is of high feasibility in a broad range of situations, especially in solving multiple attribute group decision making where the attribute assessment values are represented by interval-valued 2-tuple linguistic information.

Here we propose an approach to MAGDM with interval-valued 2-tuple linguistic information by the *IT-ICOWD* measure. Moreover, we can determine the order-inducing variables of the *IT-ICOWD* measure, the weighting vector of decision makers and the weighting vector of attributes.

Let $X = \{X_1, X_2, \dots, X_m\}$ be a discrete set of alternatives, $C = \{C_1, C_2, \dots, C_n\}$ be an attributes set, and $w = (w_1, w_2, \dots, w_n)^T$ be the weighting vector satisfying $w_j \in [0, 1]$, associated with weight of C_j, and $\sum_{j=1}^n w_j = 1$. Let $E = \{e_1, e_2, \dots, e_t\}$ be a decision maker collection, and $\omega = (\omega_1, \omega_2, \dots, \omega_t)^T$ be the weighting vector of decision makers, where $\sum_{k=1}^t \omega_k = 1$, $\omega_k \in [0, 1]$. The decision makers e_k $(k = 1, 2, \dots, t)$ are required to give his/her assessment values of alternative X_i with respect to attribute C_j in linguistic term sets S^{T_k} (S^{T_k} may have different granularity), therefore, the decision matrix $\tilde{R}^k$ can be built as $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m\times n} = ([r_{ij}^k, r_{ij}'^k])_{m\times n}$, where $\tilde{r}_{ij}^k$ is linguistic variable

Table 3
Ideal alternative.

	C_1	C_2	...	C_j	...	C_n
$\tilde{\varphi}^k$	$\tilde{\varphi}_1^k$	$\tilde{\varphi}_2^k$	...	$\tilde{\varphi}_j^k$	...	$\tilde{\varphi}_n^k$

$s_{ij}^k, s_{ij}^k \in S^{T_k}$, $S^{T_k} = \{s_i^{T_k} \mid i \in \{0, 1, \dots, T_k - 1\}\}$. The decision makers also establish the ideal alternative by giving the ideal levels of each characteristic, which is shown in Table 3, where $\tilde{\varphi}^k$ is the ideal alternative and $\tilde{\varphi}_j^k = [\varphi_j^k, \varphi_j'^k]$ is the j-th ideal characteristic of $\tilde{\varphi}^k$.

The process with the *IT-ICOWD* measure in MAGDM involves the following steps.

Step 1. Transform the decision matrix $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m\times n}$ $(k = 1, 2, \dots, t)$ and the ideal alternative $\tilde{\phi}^k$ into interval-valued 2-tuple linguistic decision matrix $\hat{R}^k = (\hat{r}_{ij}^k)_{m\times n} = ([(r_{ij}^k, 0), (r_{ij}'^k, 0)])_{m\times n}$ $(k = 1, 2, \dots, t)$ and interval-valued 2-tuple linguistic the ideal alternative $\hat{\phi}^k = (\hat{\phi}_j^k)_{1\times n} = ([(\phi_1^k, 0), (\phi_1'^k, 0)], [(\phi_2^k, 0), (\phi_2'^k, 0)], \dots, [(\phi_n^k, 0), (\phi_n'^k, 0)])$ respectively.

Step 2. Calculate the distance between each assessment value $\hat{r}_{ij}^k$ provided by the decision maker e_k and his/her ideal assessment value $\hat{\phi}_j^k$ by Eq. (28):

$$d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k) = \Delta\Big(\Big|\big(\lambda\Delta^{-1}(r_{ij}'^k, 0) + (1-\lambda)\Delta^{-1}(r_{ij}^k, 0)\big) - \big(\lambda\Delta^{-1}(\phi_j'^k, 0) + (1-\lambda)\Delta^{-1}(\phi_j^k, 0)\big)\Big|\Big) \tag{44}$$

where $k = 1, 2, \dots, t$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, $\lambda = \int_0^1 Q(y)\,dy$ is the attitudinal character of Q.

Step 3. Let $\bar{d}_{ij} = \Delta(\frac{1}{t}\sum_{k=1}^t \Delta^{-1}(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k)))$, i.e. $(\bar{d}_{ij})_{m\times n}$ is the mean distance matrix of $d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k)$, $k = 1, 2, \dots, t$, and $(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}))_{m\times n} = (\Delta(|\Delta^{-1}(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k)) - \Delta^{-1}(\bar{d})|))_{m\times n}$ is the absolute distance matrix between $d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k)$ and $\bar{d}_{ij}$. Then, the similarity measure can be defined as follows:

$$Sim_k = 1 - \frac{\sum_{i=1}^m \sum_{j=1}^n \Delta^{-1}(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}_{ij}))}{\sum_{k=1}^t \sum_{i=1}^m \sum_{j=1}^n \Delta^{-1}(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}_{ij}))}. \tag{45}$$

The closer Sim_k is to 1, the more representative and reliable the information provided by the k-th expert is. That is the absolute distance matrix with the more similarity measure should be more important. Thus, we can use the similarity measure Sim_k as the order-inducing variables of the assessment values to be aggregated in the process of group decision making. Thus, the weighting vector $\omega = (\omega_1, \omega_2, \dots, \omega_t)^T$ can be determined by the following formula:

$$\omega_k = \frac{Sim_k}{\sum_{k=1}^t Sim_k}, \quad k = 1, 2, \dots, t. \tag{46}$$

Moreover, according to the principle that the closer a preference value is to the mid one(s), the more the weight, the weighting vector $w = (w_1, w_2, \dots, w_n)^T$ can be determined by the following formula:

$$\begin{aligned} w_j &= \frac{Sim'_j}{\sum_{j=1}^{n} Sim'_j} \\ &= \left(1 - \frac{\sum_{k=1}^{t}\sum_{i=1}^{m} \Delta^{-1}(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}_{ij}))}{\sum_{k=1}^{t}\sum_{i=1}^{m}\sum_{j=1}^{n} \Delta^{-1}(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}_{ij}))}\right) \Big/ \\ &\quad \sum_{j=1}^{n}\left(1 - \frac{\sum_{k=1}^{t}\sum_{i=1}^{m} \Delta^{-1}(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}_{ij}))}{\sum_{k=1}^{t}\sum_{i=1}^{m}\sum_{j=1}^{n} \Delta^{-1}(\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d}_{ij}))}\right). \end{aligned} \tag{47}$$

Step 4. Utilize the *IT-ICOWD* measure

$$\begin{aligned} \tilde{r}_{ij} &= (r_{ij}, a_{ij}) \\ &= IT\text{-}ICOWD\begin{pmatrix} \langle Sim_1, [(r_{ij}^1, 0), (r_{ij}'^1, 0)], [(\phi_j^1, 0), (\phi_j'^1, 0)]\rangle, \\ \cdots, \langle Sim_n, [(r_{ij}^t, 0), (r_{ij}'^t, 0)], [(\phi_j^t, 0), (\phi_j'^t, 0)]\rangle \end{pmatrix} \\ &= \Delta\left(\left[\sum_{k=1}^{t} \omega_k \left(\Delta^{-1}\left[d_\lambda(\hat{r}_{\delta(j)}^k, \hat{\phi}_{\delta(j)}^k)\right]\right)^\tau\right]^{1/\tau}\right) \end{aligned} \tag{48}$$

to aggregate all the 2-tuple linguistic distance matrices into the collective 2-tuple linguistic distance matrix $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = ((r_{ij}, a_{ij}))_{m\times n}$, where $\omega = (\omega_1, \omega_2, \dots, \omega_t)^T$ is the weighting vector of decision makers. Here, it should be mentioned that $r_{ij} \in S^{T_k}$ and $a_{ij} \in [-\frac{1}{2T_k}, \frac{1}{2T_k})$.

Step 5. Utilize the *T-GOWA* operator (Liu *et al.*, 2011)

$$\begin{aligned} \tilde{r}_i &= (r_i, a_i) = T\text{-}GOWA(\tilde{r}_{i1}, \tilde{r}_{i2}, \dots, \tilde{r}_{in}) \\ &= T\text{-}GOWA\big((r_{i1}, a_{i1}), (r_{i2}, a_{i2}), \dots, (r_{in}, a_{in})\big) \end{aligned} \tag{49}$$

to aggregate all of the preference values $\tilde{r}_{ij}$ $(j = 1, 2, \dots, n)$ in the i-th line of $\tilde{R}$, and then derive the collective overall preference values $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, \dots, m)$ of the alternative X_i $(i = 1, 2, \dots, m)$, where $w = (w_1, w_2, \dots, w_n)^T$ is the weighting vector of attribute.

Step 6. According to the comparison law, rank the $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, \dots, m)$ in descending order.

Step 7. Rank all of the alternatives X_i $(i = 1, 2, \dots, m)$, and then select the best one(s) in accordance with the collective overall preference values $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, \dots, m)$. The best choice is the one with the smallest distance.

Step 8. End.

4.2. *Illustrative Example*

In this section, we employ a practical MAGDM problem to illustrate the efficiency of the proposed method in dealing with problems of interval-valued 2-tuple linguistic information. Suppose that an investment company wants to find an optimal investment. There are four possible alternatives to invest the money:

X_1: car industry; X_2: food company; X_3: computer companyX_4: arms industry.

The investment company must make a decision according to the following four attributes:

C_1: risk analysis; C_2: growth analysis; C_3: social-political impact analysis; C_4: environment impact analysis.

In order to eliminate influence among them, three decision makers are invited to provide their preferences for each possible alternative on each attributes in anonymity and in different linguistic term sets respectively, which are seven terms: $S^7 = \{s_0^7, s_1^7, s_2^7, s_3^7, s_4^7, s_5^7, s_6^7\}$, five terms: $S^5 = \{s_0^5, s_1^5, s_2^5, s_3^5, s_4^5\}$ and nine terms: $S^9 = \{s_0^9, s_1^9, s_2^9, s_3^9, s_4^9, s_5^9, s_6^9, s_7^9, s_8^9\}$.

The linguistic decision matrices $\tilde{R}^k = (\tilde{r}_{ij}^k)_{4\times 4}$ $(k = 1, 2, 3)$ and the ideal alternative $\tilde{\phi}^k$ are provided as follows:

Linguistic decision matrix $\tilde{R}^1$ provided by D_1

$$\tilde{R}^1 = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} [s_1^7, s_3^7] & [s_1^7, s_2^7] & [s_1^7, s_2^7] & [s_3^7, s_5^7] \\ [s_3^7, s_5^7] & [s_1^7, s_3^7] & [s_0^7, s_2^7] & [s_1^7, s_3^7] \\ [s_4^7, s_5^7] & [s_1^7, s_4^7] & [s_1^7, s_3^7] & [s_1^7, s_2^7] \\ [s_0^7, s_1^7] & [s_2^7, s_5^7] & [s_2^7, s_4^7] & [s_2^7, s_3^7] \end{pmatrix} \end{array}.$$

Linguistic decision matrix $\tilde{R}^2$ provided by D_2

$$\tilde{R}^2 = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} [s_1^5, s_2^5] & [s_1^5, s_3^5] & [s_0^5, s_1^5] & [s_1^5, s_3^5] \\ [s_2^5, s_3^5] & [s_0^5, s_2^5] & [s_1^5, s_3^5] & [s_1^5, s_3^5] \\ [s_1^5, s_3^5] & [s_1^5, s_2^5] & [s_0^5, s_3^5] & [s_2^5, s_3^5] \\ [s_0^5, s_1^5] & [s_2^5, s_3^5] & [s_1^5, s_3^5] & [s_0^5, s_2^5] \end{pmatrix} \end{array}.$$

Linguistic decision matrix $\tilde{R}^3$ provided by D_3

$$\tilde{R}^3 = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} [s_1^9, s_4^9] & [s_4^9, s_7^9] & [s_3^9, s_4^9] & [s_1^9, s_2^9] \\ [s_1^9, s_2^9] & [s_1^9, s_4^9] & [s_4^9, s_6^9] & [s_2^9, s_5^9] \\ [s_5^9, s_7^9] & [s_5^9, s_6^9] & [s_3^9, s_4^9] & [s_4^9, s_6^9] \\ [s_4^9, s_6^9] & [s_3^9, s_5^9] & [s_1^9, s_2^9] & [s_1^9, s_4^9] \end{pmatrix} \end{array}.$$

Table 4
Ideal alternative.

	C_1	C_2	C_3	C_4
$\tilde{\phi}^1$	(s_4^7, s_6^7)	(s_4^7, s_5^7)	(s_3^7, s_4^7)	(s_4^7, s_5^7)
$\tilde{\phi}^2$	(s_3^5, s_4^5)	(s_2^5, s_4^5)	(s_2^5, s_3^5)	(s_3^5, s_4^5)
$\tilde{\phi}^3$	(s_6^9, s_8^9)	(s_5^9, s_7^9)	(s_6^9, s_7^9)	(s_5^9, s_6^9)

Then, we utilize the method developed to obtain the best alternative(s).

Step 1. Transform the decision matrix $\tilde{R}^k = (\tilde{r}_{ij}^k)_{4\times 4}$, $k = 1, 2, 3$ and the ideal alternative $\tilde{\phi}^k$ into interval-valued 2-tuple linguistic decision matrix $\hat{R}^k = (\hat{r}_{ij}^k)_{4\times 4} = ([(r_{ij}^k, 0), (r_{ij}'^k, 0)])_{4\times 4}$ and interval-valued 2-tuple linguistic ideal alternative $\hat{\phi}^k = (\hat{\phi}_j^k)_{1\times 4} = ([(\phi_1^k, 0), (\phi_1'^k, 0)], [(\phi_2^k, 0), (\phi_2'^k, 0)], \dots, [(\phi_4^k, 0), (\phi_4'^k, 0)])$, shown as follows:

Interval-valued 2-tuple linguistic decision matrix $\hat{R}^1$ provided by D_1

$$\hat{R}^1 = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} [(s_1^7, 0), (s_3^7, 0)] & [(s_1^7, 0), (s_2^7, 0)] & [(s_1^7, 0), (s_2^7, 0)] & [(s_3^7, 0), (s_5^7, 0)] \\ [(s_3^7, 0), (s_5^7, 0)] & [(s_1^7, 0), (s_3^7, 0)] & [(s_0^7, 0), (s_2^7, 0)] & [(s_1^7, 0), (s_3^7, 0)] \\ [(s_4^7, 0), (s_5^7, 0)] & [(s_1^7, 0), (s_4^7, 0)] & [(s_1^7, 0), (s_3^7, 0)] & [(s_1^7, 0), (s_2^7, 0)] \\ [(s_0^7, 0), (s_1^7, 0)] & [(s_2^7, 0), (s_5^7, 0)] & [(s_2^7, 0), (s_4^7, 0)] & [(s_2^7, 0), (s_3^7, 0)] \end{pmatrix} \end{array}.$$

Interval-valued 2-tuple linguistic decision matrix $\hat{R}^2$ provided by D_2

$$\hat{R}^2 = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} [(s_1^5, 0), (s_2^5, 0)] & [(s_1^5, 0), (s_3^5, 0)] & [(s_0^5, 0), (s_1^5, 0)] & [(s_1^5, 0), (s_3^5, 0)] \\ [(s_2^5, 0), (s_3^5, 0)] & [(s_0^5, 0), (s_2^5, 0)] & [(s_1^5, 0), (s_3^5, 0)] & [(s_1^5, 0), (s_3^5, 0)] \\ [(s_1^5, 0), (s_3^5, 0)] & [(s_1^5, 0), (s_2^5, 0)] & [(s_0^5, 0), (s_3^5, 0)] & [(s_2^5, 0), (s_3^5, 0)] \\ [(s_0^5, 0), (s_1^5, 0)] & [(s_2^5, 0), (s_3^5, 0)] & [(s_1^5, 0), (s_3^5, 0)] & [(s_0^5, 0), (s_2^5, 0)] \end{pmatrix} \end{array}.$$

Interval-valued 2-tuple linguistic decision matrix $\hat{R}^3$ provided by D_3

$$\hat{R}^3 = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} [(s_1^9, 0), (s_4^9, 0)] & [(s_4^9, 0), (s_7^9, 0)] & [(s_3^9, 0), (s_4^9, 0)] & [(s_1^9, 0), (s_2^9, 0)] \\ [(s_1^9, 0), (s_2^9, 0)] & [(s_1^9, 0), (s_4^9, 0)] & [(s_4^9, 0), (s_6^9, 0)] & [(s_2^9, 0), (s_5^9, 0)] \\ [(s_5^9, 0), (s_7^9, 0)] & [(s_5^9, 0), (s_6^9, 0)] & [(s_3^9, 0), (s_4^9, 0)] & [(s_4^9, 0), (s_6^9, 0)] \\ [(s_4^9, 0), (s_6^9, 0)] & [(s_3^9, 0), (s_5^9, 0)] & [(s_1^9, 0), (s_2^9, 0)] & [(s_1^9, 0), (s_4^9, 0)] \end{pmatrix} \end{array}.$$

Interval-valued 2-tuple linguistic ideal alternative $\hat{\phi}^k$ $(k = 1, 2, 3)$

$$\begin{array}{ccccc} & C_1 & C_2 & C_3 & C_4 \\ \hat{\phi}^1 & [(s_4^7, 0), (s_6^7, 0)] & [(s_4^7, 0), (s_5^7, 0)] & [(s_3^7, 0), (s_4^7, 0)] & [(s_4^7, 0), (s_5^7, 0)] \\ \hat{\phi}^2 & [(s_3^5, 0), (s_4^5, 0)] & [(s_2^5, 0), (s_4^5, 0)] & [(s_2^5, 0), (s_3^5, 0)] & [(s_3^5, 0), (s_4^5, 0)] \\ \hat{\phi}^3 & [(s_6^9, 0), (s_8^9, 0)] & [(s_5^9, 0), (s_7^9, 0)] & [(s_6^9, 0), (s_7^9, 0)] & [(s_5^9, 0), (s_6^9, 0)] \end{array}.$$

Step 2. Calculate the distance of each assessment value $\hat{r}_{ij}^k$ provided by the decision maker e_k and his/her ideal assessment value $\hat{\phi}_j^k$ by Eq. (44), where $Q(y) = y^2$, $\lambda = \frac{1}{3}$. The results are listed as follows:

Distance matrix of decision maker D_1

$$\begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_3^7, 0) & (s_3^7, 0) & (s_2^7, 0) & (s_1^7, -0.0553) \\ (s_1^7, 0) & (s_3^7, -0.0557) & (s_3^7, -0.0553) & (s_3^7, -0.0557) \\ (s_0^7, 0.0557) & (s_2^7, 0.0553) & (s_2^7, -0.0557) & (s_3^7, 0) \\ (s_4^7, 0.0557) & (s_1^7, 0.056) & (s_1^7, -0.0553) & (s_2^7, 0.0003) \end{pmatrix} \end{array}.$$

Distance matrix of decision maker D_2

$$\begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_2^5, 0) & (s_1^5, 0) & (s_2^5, 0) & (s_2^5, -0.0833) \\ (s_1^5, 0) & (s_2^5, 0) & (s_1^5, -0.0833) & (s_2^5, -0.0833) \\ (s_2^5, -0.0833) & (s_1^5, 0.0833) & (s_1^5, 0.0833) & (s_1^5, 0) \\ (s_3^5, 0) & (s_0^5, 0.0833) & (s_1^5, -0.0833) & (s_3^5, -0.0833) \end{pmatrix} \end{array}.$$

Distance matrix of decision maker D_3

$$\begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_5^9, -0.0417) & (s_1^9, -0.0417) & (s_3^9, 0) & (s_4^9, 0) \\ (s_5^9, 0.0417) & (s_4^9, -0.0417) & (s_2^9, -0.0417) & (s_2^9, 0.0417) \\ (s_1^9, 0) & (s_0^9, 0.0417) & (s_3^9, 0) & (s_1^9, -0.0417) \\ (s_2^9, 0) & (s_2^9, 0) & (s_5^9, 0) & (s_3^9, 0.0417) \end{pmatrix} \end{array}.$$

Step 3. Calculate the mean distance matrix $\bar{d}$ and the absolute distance matrix $\Lambda(d_\lambda(\hat{r}_{ij}^k, \hat{\phi}_j^k), \bar{d})$, $k = 1, 2, 3$, where $\bar{d}$ chooses the linguistic term sets S^5. The results are shown as follows:

The mean distance matrix $\bar{d}$

$$(\bar{d}_{ij})_{4\times 4} = \begin{pmatrix} (s_2^5, 0.0278) & (s_1^5, 0.0278) & (s_2^5, -0.0972) & (s_1^5, 0.0927) \\ (s_1^5, 0.1112) & (s_2^5, -0.0324) & (s_1^5, 0.0232) & (s_2^5, -0.1158) \\ (s_1^5, -0.0509) & (s_1^5, 0.0046) & (s_1^5, 0.0787) & (s_1^5, 0.0278) \\ (s_2^5, 0.0741) & (s_1^5, -0.0647) & (s_1^5, 0.051) & (s_2^5, -0.0277) \end{pmatrix}.$$

The absolute distance matrix of decision maker D_1

$$(\Lambda(d_\lambda(\hat{r}_{ij}^1, \hat{\phi}_j^1), \bar{d}_{ij}))_{4\times 4} = \begin{pmatrix} (s_0^7, 0.0278) & (s_1^7, 0.0556) & (s_1^7, -0.0694) & (s_1^7, 0.0647) \\ (s_1^7, 0.0276) & (s_0^7, 0.0232) & (s_0^7, 0.0117) & (s_0^7, 0.0601) \\ (s_1^7, -0.0232) & (s_1^7, -0.0326) & (s_1^7, -0.0323) & (s_1^7, 0.0556) \\ (s_1^7, -0.0184) & (s_0^7, 0.0373) & (s_2^7, 0.0161) & (s_1^7, -0.028) \end{pmatrix}.$$

The absolute distance matrix of decision maker D_2

$$\left(\Lambda(d_\lambda(\hat{r}_{ij}^2, \hat{\phi}_j^2), \bar{d}_{ij})\right)_{4\times 4} = \begin{pmatrix} (s_0^5, 0.0278) & (s_0^5, 0.0278) & (s_0^5, 0.0972) & (s_0^5, 0.074) \\ (s_0^5, 0.1112) & (s_0^5, 0.0324) & (s_0^5, 0.1066) & (s_0^5, 0.0324) \\ (s_1^5, -0.0324) & (s_0^5, 0.0788) & (s_0^5, 0.0047) & (s_0^5, 0.0278) \\ (s_1^5, -0.0741) & (s_0^5, 0.102) & (s_1^5, -0.1157) & (s_1^5, -0.0557) \end{pmatrix}.$$

The absolute distance matrix of decision maker D_3

$$(\Lambda(d_\lambda(\hat{r}_{ij}^3, \hat{\phi}_j^3), \bar{d}_{ij}))_{4\times 4} = \begin{pmatrix} (s_0^9, 0.0556) & (s_2^9, -0.0556) & (s_0^9, 0.0556) & (s_1^9, 0.0323) \\ (s_2^9, 0.0554) & (s_0^9, 0.0092) & (s_2^9, -0.0254) & (s_1^9, -0.0324) \\ (s_1^9, -0.0509) & (s_2^9, -0.0317) & (s_0^9, 0.037) & (s_2^9, -0.0556) \\ (s_3^9, -0.0509) & (s_1^9, -0.0603) & (s_1^9, 0.0393) & (s_0^9, 0.0557) \end{pmatrix}.$$

Then, we can get the similarity measure by Eq. (45) and the weighting vector ω by Eq. (46):

$$Sim_1 = 0.6386, \quad Sim_2 = 0.7471, \quad Sim_3 = 0.6143,$$
$$\omega_1 = 0.3193, \quad \omega_2 = 0.3736, \quad \omega_3 = 0.3071.$$

Moreover, we can obtain the weighting vector w by Eq. (47):

$$w_1 = 0.228, \quad w_2 = 0.2669, \quad w_3 = 0.2582, \quad w_4 = 0.2469.$$

Step 4. Utilize the *IT-ICOWD* measure to aggregate 2-tuple linguistic distance matrix into the collective 2-tuple linguistic distance matrix $\tilde{R} = (\tilde{r}_{ij})_{4\times 4} = ((r_{ij}, a_{ij}))_{4\times 4}$, where $\tau = 3$. We can use the similarity measure Sim_k as the order-inducing variables. Note that $r_{ij} \in S^{T_k}$ and $a_{ij} \in [-\frac{1}{2T_k}, \frac{1}{2T_k})$.

The collective 2-tuple linguistic distance matrix $\tilde{R}$

$$\tilde{R} = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_2^5, 0.0284) & (s_1^5, 0.1229) & (s_2^5, -0.088) & (s_2^5, -0.1042) \\ (s_2^5, -0.0394) & (s_2^5, -0.0324) & (s_2^5, 0.0835) & (s_2^5, -0.1011) \\ (s_1^5, 0.0375) & (s_1^5, 0.0733) & (s_1^5, 0.0803) & (s_1^5, 0.1229) \\ (s_3^5, -0.0956) & (s_1^5, -0.0412) & (s_2^5, -0.0746) & (s_2^5, 0.0075) \end{pmatrix} \end{array}.$$

Step 5. Utilize the *T-GOWA* operator to derive the collective overall preference value $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, 3, 4)$ of the alternative X_i $(i = 1, 2, 3, 4)$:

$$\tilde{r}_1 = \left(s_2^5, -0.0671\right), \qquad \tilde{r}_2 = \left(s_2^5, -0.0807\right),$$
$$\tilde{r}_3 = \left(s_1^5, 0.0821\right), \qquad \tilde{r}_4 = \left(s_2^5, -0.0055\right).$$

Assume that the parameter τ in the *T-GOWA* operator is equal to the parameter τ in the *IT-ICOWD* measure.

Step 6. According to the comparison law, rank the $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, 3, 4)$ in descending order:

$$\tilde{r}_4 > \tilde{r}_1 > \tilde{r}_2 > \tilde{r}_3.$$

Step 7. Rank all of the alternatives X_i $(i = 1, 2, 3, 4)$ as follows:

$$X_3 \succ X_2 \succ X_1 \succ X_4,$$

and the best alternative is thus X_3. i.e. the best alternative is the computer company.

Furthermore, it is possible to analyse how the different particular cases of the *IT-ICOWD* measure influence for the aggregation results. Here we consider the *IT-ICOWHD* measure, the *IT-ICOWED* measure, the *IT-ICOWGD* measure, the *IT-ICMAXD* measure, the *IT-ICMIND* measure, the Step-*IT-ICOWD* measure ($k = 2$), the *IT-ICND* measure, the *IT-ICNHD* measure, the *IT-ICNED* measure, the *IT-ICNGD* measure, the Median *IT-ICOWD* measure and the Olympic *IT-ICOWD* measure. The results are clearly demonstrated in Table 5. Now, we are able to propose the order of the companies for each case. The results are clearly demonstrated in Table 6. Note that the best and the most optimal investment is the one possessing the lowest distance.

As we can see, the company order varies with category of *IT-ICOWD* measures.

Moreover, we can also analyse how different parameter τ affects the aggregation results. Considering different values of parameter $\tau \in (0, 20)$ provided by the decision makers, here we take $\lambda = \frac{1}{3}$. The results are shown in Fig. 1.

Similarly, the company order varies with parameter τ. From Fig. 1, we can conclude that

(1) when $\tau \in (0, 0.398]$, alternative's rank is $X_3 \succ X_4 \succ X_1 \succ X_2$, and the best alternative is X_3;

Table 5
Aggregated results.

	IT-ICOWHD	*IT-ICOWED*	*IT-ICOWGD*	*IT-ICMAXD*
X_1	$(s_2^5, -0.1167)$	$(s_2^5, -0.0864)$	$(s_1^5, 0.0839)$	$(s_2^5, 0.0214)$
X_2	$(s_1^5, 0.118)$	$(s_2^5, -0.1042)$	$(s_1^5, 0.0864)$	$(s_2^5, 0.0287)$
X_3	$(s_1^5, 0.0179)$	$(s_1^5, 0.0572)$	$(s_1^5, -0.0431)$	$(s_2^5, -0.072)$
X_4	$(s_2^5, -0.122)$	$(s_2^5, -0.0558)$	$(s_1^5, 0.0519)$	$(s_2^5, 0.1233)$
	IT-ICMIND	Step-*IT-ICOWD*	*IT-ICND*	*IT-ICNHD*
X_1	$(s_1^5, 0.0888)$	$(s_2^5, -0.0977)$	$(s_2^5, -0.0657)$	$(s_2^5, -0.1151)$
X_2	$(s_1^5, 0.0565)$	$(s_1^5, 0.1106)$	$(s_2^5, -0.0792)$	$(s_1^5, 0.1179)$
X_3	$(s_1^5, -0.0779)$	$(s_1^5, 0.033)$	$(s_1^5, 0.0783)$	$(s_1^5, 0.014)$
X_4	$(s_1^5, 0.021)$	$(s_2^5, -0.0204)$	$(s_2^5, -0.0044)$	$(s_2^5, -0.12)$
	IT-ICNED	*IT-ICNGD*	Median *IT-ICOWD*	Olympic *IT-ICOWD*
X_1	$(s_2^5, -0.0849)$	$(s_1^5, -0.0853)$	$(s_2^5, -0.0887)$	$(s_2^5, -0.0887)$
X_2	$(s_2^5, -0.1036)$	$(s_1^5, 0.086)$	$(s_2^5, -0.0933)$	$(s_2^5, -0.0933)$
X_3	$(s_1^5, 0.0532)$	$(s_1^5, -0.0465)$	$(s_2^5, -0.1233)$	$(s_2^5, -0.1233)$
X_4	$(s_2^5, -0.054)$	$(s_1^5, 0.0544)$	$(s_2^5, -0.375)$	$(s_2^5, -0.0375)$

Table 6
Ordering of the companies.

	Ordering		Ordering
IT-ICOWHD	$X_3 \succ X_2 \succ X_4 \succ X_1$	*IT-ICND*	$X_3 \succ X_2 \succ X_1 \succ X_4$
IT-ICOWED	$X_3 \succ X_2 \succ X_1 \succ X_4$	*IT-ICNHD*	$X_3 \succ X_2 \succ X_4 \succ X_1$
IT-ICOWGD	$X_3 \succ X_4 \succ X_1 \succ X_2$	*IT-ICNED*	$X_3 \succ X_2 \succ X_1 \succ X_4$
IT-ICMAXD	$X_3 \succ X_1 \succ X_2 \succ X_4$	*IT-ICNGD*	$X_3 \succ X_4 \succ X_1 \succ X_2$
IT-ICMIND	$X_3 \succ X_4 \succ X_2 \succ X_1$	Median *IT-ICOWD*	$X_3 \succ X_2 \succ X_1 \succ X_4$
Step-*IT-ICOWD*	$X_3 \succ X_2 \succ X_1 \succ X_4$	Olympic *IT-ICOWD*	$X_3 \succ X_2 \succ X_1 \succ X_4$

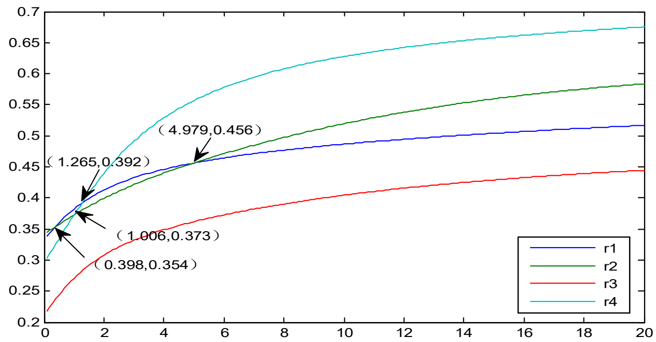

Fig. 1. Variations of the aggregation results with parameter τ.

(2) when $\tau \in (0.398, 1.006]$, alternative's rank is $X_3 \succ X_4 \succ X_2 \succ X_1$, and the best alternative is X_3;

(3) when $\tau \in (1.006, 1.265]$, alternative's rank is $X_3 \succ X_2 \succ X_4 \succ X_1$, and the best alternative is X_3.

(4) when $\tau \in (1.265, 4.979]$, alternative's rank is $X_3 \succ X_2 \succ X_1 \succ X_4$, and the best alternative is X_3;
(5) when $\tau \in (4.979, 20]$, alternative's rank is $X_3 \succ X_1 \succ X_2 \succ X_4$, and the best alternative is X_3.

4.3. *Discussion of Comparative Analysis*

In order to evaluate the performance and effectiveness of the proposed distance measure with the existing one, this paper conducts a comparative study in this subsection, including the comparison with well-applied distance measures and algorithm-based aggregation tools.

(1) Comparison with the existing distance measure.

Compared with the previous distance measure, we can conclude that the *IT-ICOWD* measure is very useful to deal with deviated problem in aggregation on continuous valued interval 2-tuple linguistic information. The prominent characteristic of the *IT-ICOWD* measure is that it combines the *GOWA* operator with the distance measure and the *IT-ICOWA* operator in the same formula. The decision maker is able to consider the MAGDM problem more clearly according to his/her risk attitude in aggregation process because the parameter λ, which lies in the interval [0, 1], can be considered as the measure of the decision maker's attitudinal character.

(2) Comparison with the aggregation tool based algorithm.

In Liu *et al.* (2014b), Liu *et al.* proposed the interval 2-tuple hybrid weighted distance (*IT-HWD*) measure to aggregate the interval-valued 2-tuple linguistic information. To facilitate a comparison with the proposed approach, we adopt the algorithm proposed by Liu *et al.* (2014b) and solve the same illustrative example described above. The steps are as follows:

Step 1. Interval-valued 2-tuple linguistic decision matrix $\hat{R}^k = (\hat{r}^k_{ij})_{4\times4} = ([(r^k_{ij}, 0), (r'^k_{ij}, 0)])_{4\times4}$ $(k = 1, 2, 3)$ and interval-valued 2-tuple linguistic ideal alternative $\hat{\phi}^k = (\hat{\phi}^k_j)_{1\times4} = ([(\phi^k_1, 0), (\phi'^k_1, 0)], [(\phi^k_2, 0), (\phi'^k_2, 0)], \dots, [(\phi^k_4, 0), (\phi'^k_4, 0)])$ are listed as shown in Section 4.2.

Step 2. According to the interval 2-tuple linguistic distance (Liu *et al.*, 2014b) between two interval 2-tuple linguistic information, the distance of each assessment value $\hat{r}^k_{ij}$ provided by the decision maker e_k and his/her ideal assessment value $\hat{\phi}^k_j$ are calculated. The results are listed as follows:

Distance matrix of decision maker D_1

$$\begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} \quad C_1 \quad & \quad C_2 \quad & \quad C_3 \quad & \quad C_4 \quad \end{array} \\ \begin{pmatrix} (s^7_3, 0) & (s^7_3, 0) & (s^7_2, 0.0002) & (s^7_1, -0.0486) \\ (s^7_1, 0.0003) & (s^7_3, -0.0752) & (s^7_3, -0.0748) & (s^7_3, -0.0752) \\ (s^7_1, -0.0486) & (s^7_2, 0.0392) & (s^7_2, -0.0699) & (s^7_3, 0) \\ (s^7_5, -0.0788) & (s^7_1, 0.0695) & (s^7_1, -0.0486) & (s^7_2, 0.0002) \end{pmatrix} \end{array}.$$

Distance matrix of decision maker D_2

$$\begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_2^5, 0) & (s_1^5, 0) & (s_2^5, 0) & (s_2^5, -0.1047) \\ (s_1^5, 0) & (s_2^5, 0) & (s_1^5, -0.0732) & (s_2^5, -0.1047) \\ (s_2^5, -0.1047) & (s_2^5, -0.1047) & (s_1^5, 0.1036) & (s_1^5, 0) \\ (s_3^5, 0) & (s_1^5, -0.0732) & (s_1^5, -0.0732) & (s_3^5, -0.1126) \end{pmatrix} \end{array}.$$

Distance matrix of decision maker D_3

$$\begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_5^9, -0.059) & (s_1^9, -0.0366) & (s_3^9, 0) & (s_4^9, 0) \\ (s_6^9, -0.0597) & (s_4^9, -0.0581) & (s_2^9, -0.0524) & (s_2^9, 0.0295) \\ (s_1^9, 0) & (s_1^9, -0.0366) & (s_3^9, 0) & (s_1^9, -0.0366) \\ (s_2^9, 0) & (s_2^9, 0) & (s_5^9, 0) & (s_3^9, 0.0203) \end{pmatrix} \end{array}.$$

Step 3. Calculate the collective 2-tuple linguistic distance matrix $\tilde{R} = (\tilde{r}_{ij})_{4\times4} = ((r_{ij}, a_{ij}))_{4\times4}$ by using the *IT-HWD* measure, which is proposed by Liu's algorithm. Here, in order to eliminate the unnecessary impacts, we also suppose the objective weight vector $\omega = (0.3193, 0.3736, 0.3071)^T$, the subjective weight vector $W = (0.3, 0.4, 0.3)^T$ and the parameter $\lambda = 3$.

The collective 2-tuple linguistic distance matrix $\tilde{R}$

$$\tilde{R} = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array} \\ \begin{pmatrix} (s_2^5, 0.027) & (s_1^5, 0.0901) & (s_2^5, -0.0492) & (s_2^5, -0.0909) \\ (s_2^5, -0.0557) & (s_2^5, -0.0214) & (s_1^5, 0.0352) & (s_2^5, -0.1089) \\ (s_1^5, 0.0773) & (s_1^5, 0.1142) & (s_1^5, 0.0999) & (s_1^5, 0.0901) \\ (s_3^5, -0.0431) & (s_1^5, -0.0334) & (s_2^5, -0.1069) & (s_2^5, 0.0518) \end{pmatrix} \end{array}.$$

It is noted that the balancing coefficient is $n = 3$ and $r_{ij} \in S^5$.

Step 4. Utilize the *T-GOWA* operator to derive the collective overall preference value $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, 3, 4)$ of the alternative X_i $(i = 1, 2, 3, 4)$:

$$\tilde{r}_1 = \left(s_2^5, -0.0613\right), \qquad \tilde{r}_2 = \left(s_2^5, -0.0878\right),$$
$$\tilde{r}_3 = \left(s_1^5, 0.0967\right), \qquad \tilde{r}_4 = \left(s_2^5, 0.0198\right).$$

Assume that the parameter λ in the *T-GOWA* operator is equal to the parameter λ in the *IT-HWD* measure.

Step 5. According to the comparison law, rank the $\tilde{r}_i = (r_i, a_i)$ $(i = 1, 2, 3, 4)$ in descending order:

$$\tilde{r}_4 > \tilde{r}_1 > \tilde{r}_2 > \tilde{r}_3.$$

Step 6. Rank all of the alternatives X_i $(i = 1, 2, 3, 4)$ as follows:

$$X_3 \succ X_2 \succ X_1 \succ X_4.$$

Therefore, the best alternative is X_3, i.e. the best alternative is the computer company.

From the comparison with interval 2-tuple linguistic distance of Liu *et al.* (2014b), the newly proposed approach and Liu's have their own merits. For one thing, the interval 2-tuple linguistic distance (Liu *et al.*, 2014b) is defined by endpoints of interval-valued 2-tuple linguistic information. This measure varies with the uncertain linguistic environment in GDM and makes it more flexible in diverse circumstances. For another, the approach proposed in this paper is able to provide more decision-related information such as order-inducing variables in the *IT-ICOWD* measure, the weighting vector of decision makers and the weighting vector of attributes. This enables decision-making process more evidential and reliable, and these intermediate results can be applied for multiple times when necessary. However, this benefit requires more efforts in computation, opposed to Liu's approach. Therefore, decision makers who request a deterministic answer as well as reasonable and solid evidence would prefer the novel *IT-ICOWD* measure regardless of computational complexity.

5. Conclusion

In this paper, we introduced the interval-valued 2-tuple linguistic induced continuous ordered weighted distance (*IT-ICOWD*) measure. Comparing with existing methods of aggregating interval-valued linguistic variables, we firstly demonstrated that the *IT-ICOWD* measure is of high practicality to uncertain cases when the decision maker is only able to express preference information in interval-valued 2-tuples linguistic terms. Furthermore, we discussed several desirable properties and different families of the *IT-ICOWD* measure. At last, feasibility and practicability of proposed approach were illustrated by a numerical example.

In future research, we look forward to applying the *IT-ICOWD* measure to different decision making application, such as dynamic decision making (Pérez *et al.*, 2010), consensus reaching process (Dong *et al.*, 2016), social media (Dong *et al.*, 2017), heterogeneous information merging process (Liu *et al.*, 2017). Simultaneously, we are going to develop further extensions of the *IT-ICOWD* measure to other types of distance measure and decision information.

Acknowledgements. The authors would like to thank the editor and the anonymous referees for their valuable comments and suggestions for improving the paper. The work was supported by Anhui Provincial Natural Science Foundation (No. 1808085QG211), Statistics and Science Research Foundation of China (No. 2017LZ11), Doctoral Research Start-up Funds Projects of Hefei Normal University (No. 2017rcjj03), Open Project of School of Mathematical Sciences, Anhui University, Provincial Natural Science Research Project of Anhui Colleges (No. KJ2015A379), Provincial Natural Science Research

Project of Anhui Colleges (No.KJ2017A026), National Natural Science Foundation of China (Nos. 71301001, 71371011, 11426033, 11501005), Project of Anhui Province for Excellent Young Talents.

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