Simultaneous Evaluation of Criteria and Alternatives (SECA) for Multi-Criteria Decision-Making

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Abstract. In the discrete form of multi-criteria decision-making (MCDM) problems, we are usually confronted with a decision-matrix formed from the information of some alternatives on some criteria. In this study, a new method is proposed for simultaneous evaluation of criteria and alternatives (SECA) in an MCDM problem. For making this type of evaluation, a multi-objective non-linear programming model is formulated. The model is based on maximization of the overall performance of alternatives with consideration of the variation information of decision-matrix within and between criteria. The standard deviation is used to measure the within-criterion, and the correlation is utilized to consider the between-criterion variation information. By solving the multi-objective model, we can determine the overall performance scores of alternatives and the objective weights of criteria simultaneously. To validate the proposed method, a numerical example is used, and three analyses are made. Firstly, we analyse the objective weights determined by the method, secondly, the stability of the performance scores and ranking results are examined, and finally, the ranking results of the proposed method are compared with those of some existing MCDM methods. The results of the analyses show that the proposed method is efficient to deal with MCDM problems.

Key words: multi-criteria decision-making (MCDM), criteria weight, performance evaluation, simultaneous evaluation of criteria and alternatives (SECA).

1. Introduction

Multi-criteria decision-making (MCDM) methods and techniques have been used and developed by many researchers and applied to many real-world problems. The MCDM

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methods and techniques are usually categorized into two classes: multi-attribute decision-making (MADM) and multi-objective decision-making (MODM). The MODM methods are used to deal with continuous MCDM problems, and the MADM methods are utilized to handle discrete MCDM problems (Rezaei, 2015). However, MCDM is a common term to describe the MADM (discrete MCDM) methods, and because the focus of this study is on this class, we also use this terminology. In this class, we are usually confronted with a decision-matrix which contains information about a number of alternatives with respect to a number of criteria or attributes. The aim of the MCDM methods is to evaluate the alternatives and/or criteria.

Several MCDM methods have been proposed during the past decades. Here, we briefly review some important methods which are used for criteria weight determination and alternative evaluation based on the information of the decision-matrix, and readers are referred to some recent review articles on MCDM methods and their applications for further details (Behzadian *et al.*, 2012; Abdullah and Adawiyah, 2014; Stefano *et al.*, 2015; Gul *et al.*, 2016; Mardani *et al.*, 2017; Keshavarz Ghorabaee *et al.*, 2017b). In the following, we present brief explanations of the SAW (Simple Additive Weighting), WASPAS (Weighted Aggregated Sum Product ASsessment), COPRAS (COmplex PRoportional ASsessment), TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution), VIKOR (from Serbian: VIseKriterijumska Optimizacija I Kompromisno Resenje) and EDAS (Evaluation based on Distance from Average Solution) methods which are used for the evaluation of alternatives. The range of the MCDM methods is wider than the mentioned methods. However, these methods are reviewed because we use them in the current study.

SAW is an old method which is likely the most common and very widely used method for the evaluation of alternatives. MacCrimmon (1968) is one of the first researchers that summarized this method. In this method, the performance of each alternative is determined by multiplying the rating of each alternative on each criterion by the weight assigned to the criterion and then summing these products over all criteria (Hwang and Yoon, 1981). This is a simple, easy to use and easy to understand method, and has been used in many studies (Chen and Hwang, 1992). The researches of Wang (2015), Wang *et al.* (2016a), Goodridge (2016), Kalibatas and Kovaitis (2017) and Muddineni *et al.* (2017), are some of the recent studies on the SAW method. Abdullah and Adawiyah (2014) presented a review of the applications of simple additive weighting and its fuzzy variants from 2003 to 2013.

Zavadskas *et al.* (2012) proposed the WASPAS method based on a combination of the SAW or weighted sum model (WSM) and weighted product model (WPM). This method has the advantages of both of them. Many researchers have utilized this efficient method in their studies. In some recent studies, Baušys and Juodagalvienė (2017), Karabasevic *et al.* (2017), Nie *et al.* (2017), Peng and Dai (2017) applied the WASPAS method to the MCDM problems. Mardani *et al.* (2017) presented a systematic review of applications and fuzzy developments of the WASPAS method.

The COPRAS method is an efficient MCDM method in which a ratio based on two measures (the summation of beneficial criteria performance and the summation of non-beneficial criteria performance) is used to evaluate alternatives (Zavadskas *et al.*, 1994).

We should define a set of beneficial and a set of non-beneficial criteria to use this method and determine the performance values of alternatives. This method has also been applied to many real-world MCDM problems. Wang *et al.* (2016b), Mulliner *et al.* (2016), Serrai *et al.* (2017), Rathi and Balamohan (2017), Turanoglu Bekar *et al.* (2016) and Nakhaei *et al.* (2016) have done some studies on this method recently. A review of this method and its applications was presented by Stefano *et al.* (2015).

TOPSIS is a value-based compensatory method introduced by Hwang and Yoon (1981). In this method, alternatives are ranked according to their distances from two reference points called ideal (positive-ideal) and nadir (negative-ideal) solutions. However, Opricovic and Tzeng (2004) noted that the relative importance of these distances is not considered in the TOPSIS method. This method is one of the popular MCDM methods among researchers. Some recent applications of the TOPSIS method can be found in the studies done by Mao *et al.* (2018), Cayir Ervural *et al.* (2018), Chen *et al.* (2018), Shen *et al.* (2018), Polat *et al.* (2017) and Walczak and Rutkowska (2017). A review of developments and applications of this method was made by Behzadian *et al.* (2012).

The VIKOR method, which was proposed by Opricovic (1998), is a reference-based MCDM method that is able to solve decision problems with non-commensurable and conflicting criteria. The reference point of this method is an ideal solution, and alternatives are evaluated based on Manhattan and Chebyshev distances from this point. The VIKOR method is somewhat similar to the TOPSIS method, but there are some important differences that were explained by Opricovic and Tzeng (2004). This method has also been used in many real-world MCDM problems such as military airport location selection (Sennaroglu and Varlik Celebi, 2018), risk evaluation of construction projects (Wang *et al.*, 2018), maritime transportation (Soner *et al.*, 2017), energy management (Sakthivel *et al.*, 2017) and financial performance evaluation (Chang and Tsai, 2016). Gul *et al.* (2016) conducted a state-of-the-art literature review on VIKOR applications.

EDAS is an efficient and relatively new MCDM method introduced by Keshavarz Ghorabaee *et al.* (2015). The EDAS method measures the desirability of alternatives based on the distance from an average solution. Like some other MCDM methods, EDAS can be used to handle multi-criteria decision-making problems with non-commensurable and conflicting criteria. Despite the newness of the EDAS method, it has been applied to many practical MCDM problems like supplier selection (Keshavarz Ghorabaee *et al.*, 2016b; Stević *et al.*, 2017), life cycle sustainability assessment (Ren and Toniolo, 2018) and evaluation of architectural plans (Juodagalvienė *et al.*, 2017).

Besides the MCDM methods used for evaluation of alternatives, there are some other methods for determination of objective weights of criteria. The Standard Deviation (SD) and Entropy are two common methods which determine the objective weights based on the within-criterion variation information, and assign smaller weights to a criterion if it has similar values across alternatives. These methods have been used in many studies (Keshavarz Ghorabaee *et al.*, 2016a; Zhang *et al.*, 2011; Liu *et al.*, 2010). CRITIC (CRiteria Importance Through Inter-criteria Correlation) is another method which considers within- and between-criterion variation information (Diakoulaki *et al.*, 1995). The correlation between criteria is used to measure variations between criteria. In this

method, lower values of correlation have a positive effect on the weight of each criterion. Some researchers applied CRITIC to practical MCDM problems (Yalçın and Ünlü, 2017; Keshavarz Ghorabaee *et al.*, 2017a; Adalı and Işık, 2017; Rostamzadeh *et al.*, 2018).

In this paper, we introduce a new method that, unlike many other MCDM methods, can be used for simultaneous evaluation of criteria and alternatives (SECA) in a multi-criteria decision-making problem. A multi-objective non-linear mathematical model is developed to make the evaluation process. The model includes three objective functions. The first objective is related to maximization of the overall performance of alternatives, and the second and third objectives concern within- and between-criterion variation information. The within-criterion variation is measured by the standard deviation, and the between-criterion variation is considered using the correlation. By optimization of the developed mathematical model, the overall performance scores of alternatives and the objective weights of criteria can be determined simultaneously. A numerical example is used in this study for validation of the proposed method. Based on this example, the validity of objective criteria weights and the stability of the overall performance scores and ranking results are analysed. Moreover, a comparison is made between the ranking results of the proposed method with the results of the SAW, WASPAS, COPRAS, TOPSIS, VIKOR and EDAS methods.

The rest of this paper is organized as follows. Section 2 presents a new method for simultaneous evaluation of criteria and alternatives (SECA). In Section 3, we use a numerical example for validation of the proposed method by means of three analyses including "Analysis of criteria weights", "Analysis of the performance of alternatives", and "Comparative analysis". In Section 4, we present conclusions.

2. Simultaneous Evaluation of Criteria and Alternatives (SECA)

In this section, a new method is proposed to handle multi-criteria decision-making problems. The aim of this method is to determine the overall performance scores of alternatives and the weights of criteria simultaneously. A multi-objective non-linear mathematical model is formulated in this section to reach this aim. To formulate the mathematical model, two types of reference points are described for criteria weights. The first type is based on within-criterion variation information defined by the standard deviation and the second is related to between-criterion variation information determined based on the correlation measure. The multi-objective model seeks to maximize the overall performance of each alternative and minimize the deviation of criteria weights from the reference points. For maximization of the overall performance of each alternative, a weighted sum model is used as an objective. Also, we take advantage of the sum of squared deviations from the reference points to define the other objectives of the model.

Suppose that we have an MCDM problem with n alternatives and m criteria, and the weight of each criterion (w_j, $j \in \{1, 2, \ldots, m\}$) is unknown. We can define the decision-

matrix of this problem as follows:

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{im} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nj} & \cdots & x_{nm} \end{bmatrix}, \tag{1}$$

where x_{ij} denotes the performance value of ith ($i \in \{1, 2, \ldots, n\}$) alternative on jth ($j \in \{1, 2, \ldots, m\}$) criterion and $x_{ij} > 0$.

Based on Eq. (1), we can construct the normalized decision-matrix using the following equations

$$X^N = \begin{bmatrix} x_{11}^N & x_{12}^N & \cdots & x_{1j}^N & \cdots & x_{1m}^N \\ x_{21}^N & x_{22}^N & \cdots & x_{2j}^N & \cdots & x_{2m}^N \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1}^N & x_{i2}^N & \cdots & x_{ij}^N & \cdots & x_{im}^N \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{n1}^N & x_{n2}^N & \cdots & x_{nj}^N & \cdots & x_{nm}^N \end{bmatrix}, \tag{2}$$

where

$$x_{ij}^N = \begin{cases} \frac{x_{ij}}{\max_k x_{kj}} & \text{if } j \in BC, \\ \frac{\min_k x_{kj}}{x_{ij}} & \text{if } j \in NC, \end{cases} \tag{3}$$

and BC and NC are the sets of beneficial and non-beneficial criteria, respectively.

Let $\mathbf{V}_j = [x_{ij}^N]_{n \times 1}$ denotes the vector of jth ($j \in \{1, 2, \ldots, m\}$) criterion. The standard deviation of the elements of each vector (σ_j) can get the within-criterion variation information. To capture the between-criterion variation information from the decision-matrix, we need to calculate the correlation between each pair of vectors of criteria. Let us denote by r_{jl} the correlation between jth and lth vectors (j and $l \in \{1, 2, \ldots, m\}$). Then the following summation (π_j) can reflect the degree of conflict between jth criterion and the other criteria (Diakoulaki *et al.*, 1995):

$$\pi_j = \sum_{l=1}^{m} (1 - r_{jl}). \tag{4}$$

An increase in the variation within the vector of a criterion (σ_j), as well as an increase in the degree of conflict between a criterion and the other criteria (π_j), intensifies the objective importance of that criterion. Accordingly, we define the normalized values of

σ_j and π_j as the reference points for the weights of criteria. These values can be calculated as follows:

$$\sigma_j^N = \frac{\sigma_j}{\sum_{l=1}^{m} \sigma_l}, \tag{5}$$

$$\pi_j^N = \frac{\pi_j}{\sum_{l=1}^{m} \pi_l}. \tag{6}$$

Based on the above description, a multi-objective non-linear programming model is formulated as follows:

$$\max \ S_i = \sum_{j=1}^{m} w_j x_{ij}^N, \quad \forall i \in \{1, 2, \ldots, n\}, \tag{7.1}$$

$$\min \ \lambda_b = \sum_{j=1}^{m} \left(w_j - \sigma_j^N\right)^2, \tag{7.2}$$

$$\min \ \lambda_c = \sum_{j=1}^{m} \left(w_j - \pi_j^N\right)^2, \tag{7.3}$$

$$\text{s.t.} \ \sum_{j=1}^{m} w_j = 1, \tag{7.4}$$

$$w_j \leqslant 1, \quad \forall j \in \{1, 2, \ldots, m\}, \tag{7.5}$$

$$w_j \geqslant \varepsilon, \quad \forall j \in \{1, 2, \ldots, m\}. \tag{7.6}$$

In Model (7), the first equation maximizes the overall performance of each alternative, and the second and third equations minimize the deviation of criteria weights from the reference points for each criterion. Eq. (7.4) guarantees that the sum of weights is equal to 1. Eqs. (7.5) and (7.6) set the weights of criteria to some values in the interval $[\varepsilon, 1]$. It should be noted that ε is a small positive parameter considered as a lower bound for criteria weights. In this study, this parameter is set to 10^{-3}.

To optimize Model (7), we use some techniques of the multi-objective optimization and transform the model to Model (8) as follows:

$$\max \ Z = \lambda_a - \beta(\lambda_b + \lambda_c), \tag{8.1}$$

$$\text{s.t.} \ \lambda_a \leqslant S_i, \quad \forall i \in \{1, 2, \ldots, n\}, \tag{8.2}$$

$$S_i = \sum_{j=1}^{m} w_j x_{ij}^N, \quad \forall i \in \{1, 2, \ldots, n\}, \tag{8.3}$$

$$\lambda_b = \sum_{j=1}^{m} \left(w_j - \sigma_j^N\right)^2, \tag{8.4}$$

Table 1
The decision-matrix of the numerical example.

	$C_1 \in BC$	$C_2 \in BC$	$C_3 \in BC$	$C_4 \in NC$	$C_5 \in NC$	$C_6 \in NC$	$C_7 \in NC$
A_1	23	264	2.37	0.05	167	8900	8.71
A_2	20	220	2.2	0.04	171	9100	8.23
A_3	17	231	1.98	0.15	192	10800	9.91
A_4	12	210	1.73	0.2	195	12300	10.21
A_5	15	243	2	0.14	187	12600	9.34
A_6	14	222	1.89	0.13	180	13200	9.22
A_7	21	262	2.43	0.06	160	10300	8.93
A_8	20	256	2.6	0.07	163	11400	8.44
A_9	19	266	2.1	0.06	157	11200	9.04
A_{10}	8	218	1.94	0.11	190	13400	10.11

$$\lambda_c = \sum_{j=1}^{m} \left(w_j - \sigma_j^N \right)^2, \tag{8.5}$$

$$\sum_{j=1}^{m} w_j = 1, \tag{8.6}$$

$$w_j \leqslant 1, \quad \forall j \in \{1, 2, \ldots, m\}, \tag{8.7}$$

$$w_j \geqslant \varepsilon, \quad \forall j \in \{1, 2, \ldots, m\}. \tag{8.8}$$

According to the objective function of Model (8), the minimum of the overall performance score of alternatives (λ_a) is maximized. Because the deviations from reference points should be minimized, they are subtracted from the objective function with a coefficient β ($\beta \geqslant 0$). This coefficient affects the importance of reaching the reference points of criteria weights.

The overall performance score of each alternative (S_i) and the objective weight of each criterion (w_j) are determined by solving Model (8).

3. Computational Analysis

In this section, a computational analysis is made to validate the results of the proposed method. This analysis is presented in the three following sub-sections. Firstly, we compare the objective weights resulted from the proposed method with the results of some other methods, secondly the overall performance scores and ranks of alternatives are analysed and finally we make a comparative analysis based on some existing MCDM methods. It should be noted that the model is solved using the LINGO 11 software, and the code and instructions can be found in Keshavarz Ghorabaee (2018).

To make the computational analysis, a numerical example is borrowed from the study of Keshavarz Ghorabaee *et al.* (2015). The decision-matrix of the example along with the type of each criterion is shown in Table 1, and the normalized decision-matrix, which is constructed using Eqs. (2) and (3), is represented in Table 2. As can be seen in these

Table 2
The normalized decision-matrix of the numerical example.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	1	0.9925	0.9115	0.8	0.9401	1	0.9449
A_2	0.8696	0.8271	0.8462	1	0.9181	0.9780	1
A_3	0.7391	0.8684	0.7615	0.2667	0.8177	0.8241	0.8305
A_4	0.5217	0.7895	0.6654	0.2000	0.8051	0.7236	0.8061
A_5	0.6522	0.9135	0.7692	0.2857	0.8396	0.7063	0.8812
A_6	0.6087	0.8346	0.7269	0.3077	0.8722	0.6742	0.8926
A_7	0.9130	0.9850	0.9346	0.6667	0.9813	0.8641	0.9216
A_8	0.8696	0.9624	1	0.5714	0.9632	0.7807	0.9751
A_9	0.8261	1	0.8077	0.6667	1	0.7946	0.9104
A_{10}	0.3478	0.8195	0.7462	0.3636	0.8263	0.6642	0.8140

Table 3
The sets of criteria weights determined by changing β.

	β									
	0.1	0.2	0.3	0.4	0.5	1	2	3	4	5
w_1	0.001	0.001	0.035	0.068	0.081	0.133	0.148	0.153	0.156	0.158
w_2	0.284	0.253	0.233	0.221	0.213	0.175	0.157	0.151	0.148	0.146
w_3	0.001	0.083	0.115	0.131	0.142	0.129	0.126	0.126	0.125	0.125
w_4	0.001	0.001	0.001	0.001	0.010	0.104	0.159	0.178	0.187	0.193
w_5	0.289	0.239	0.212	0.196	0.186	0.145	0.125	0.118	0.115	0.113
w_6	0.134	0.185	0.192	0.189	0.184	0.172	0.162	0.159	0.157	0.156
w_7	0.289	0.238	0.211	0.194	0.183	0.143	0.122	0.116	0.112	0.110

tables, there are ten alternatives (A_1 to A_{10}) which need to be evaluated with respect to seven criteria (C_1 to C_7).

3.1. *Analysis of Criteria Weights*

In this sub-section, we solve Model (8) with the data provided in Table 2 and different values for the parameter β ($\beta = 0.1, 0.2, 0.3, 0.4, 0.5, 1, 2, 3, 4$ and 5). By solving the model, 10 sets of criteria weights are determined. The weights of criteria related to changing the values of β are presented in Table 3, and the variation of the weights is shown in Fig. 1.

According to Fig. 1, we can see that the weights of criteria are more stable when the values of the β parameter are greater than 3 ($\beta \geqslant 3$). Now, to validate the results, the weights of criteria are determined using some other methods. Here three methods including the SD, CRITIC and Entropy methods are chosen for the comparison analysis (Zardari *et al.*, 2014). It should be noted that the normalization step in the CRITIC method is skipped because we use the normalized decision-matrix (Table 2) for computations. The weights of criteria determined by these methods are presented in Table 4.

To make a comparison, the correlations between the results of the proposed method in different values of β and the results of the other methods (SD, CRITIC and Entropy) are computed. If the value of correlation is greater than 0.6, we can say that there is a
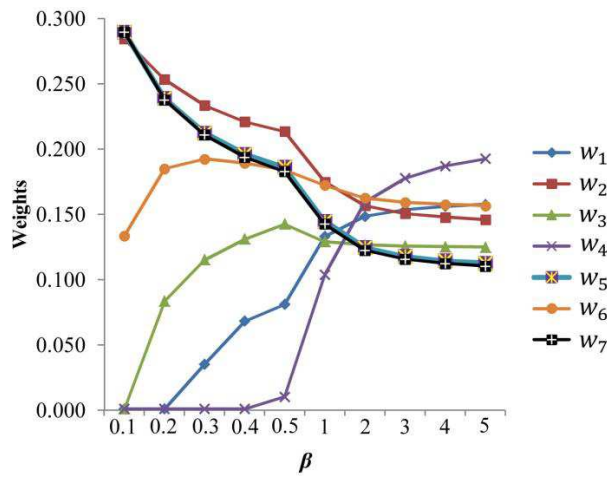
Fig. 1. The variation of the criteria weights related to β.

Table 4
The objective weights of criteria for comparison.

	SD	CRITIC	Entropy
w_1	0.2206	0.1706	0.1991
w_2	0.0888	0.1212	0.0199
w_3	0.1148	0.1104	0.0397
w_4	0.2934	0.2891	0.6596
w_5	0.0801	0.0748	0.0162
w_6	0.1295	0.1647	0.0521
w_7	0.0727	0.0693	0.0134

Table 5
The correlation between the results of criteria weight determination.

β	SD	CRITIC	Entropy
0.1	−0.7745	−0.6788	−0.5920
0.2	−0.8896	−0.7404	−0.7312
0.3	−0.9345	−0.7887	−0.8245
0.4	−0.9534	−0.8296	−0.8965
0.5	−0.9577	−0.8404	−0.9215
1	−0.6839	−0.5063	−0.7397
2	0.5552	0.7391	0.4719
3	**0.8011**	**0.9238**	**0.7368**
4	**0.8604**	**0.9597**	**0.8040**
5	**0.8849**	**0.9725**	**0.8326**

strong relationship between the results (Walters, 2009). In Table 5, we present the values of correlations between the results.

As it can be seen it Table 5, the values of correlations are greater than 0.6 for $\beta \geqslant 3$ (the values are bold in the table). Accordingly, $\beta = 3$ can be a good threshold value for computations in the proposed method.

Table 6
The overall performance scores of alternatives in different values of β.

| | β | | | | | | | | | |
	0.1	0.2	0.3	0.4	0.5	1	2	3	4	5
S_1	0.9643	0.9631	0.9635	0.9641	0.9625	0.9500	0.9416	0.9388	0.9374	0.9365
S_2	0.9239	0.9196	0.9157	0.9125	0.9113	0.9169	0.9203	0.9215	0.9220	0.9224
S_3	0.8360	0.8295	0.8237	0.8193	0.8120	0.7547	0.7218	0.7108	0.7053	0.7020
S_4	0.7890	0.7738	0.7593	0.7482	0.7378	0.6700	0.6335	0.6213	0.6152	0.6116
S_5	0.8540	0.8370	0.8247	0.8162	0.8075	0.7441	0.7098	0.6983	0.6926	0.6892
S_6	0.8400	0.8180	0.8031	0.7928	0..7836	0.7221	0.6893	0.6784	0.6729	0.6697
S_7	0.9490	0.9421	0.9389	0.9373	0.9342	0.9055	0.8894	0.8840	0.8814	0.8798
S_8	0.9416	0.9347	0.9310	0.9289	0.9252	0.8850	0.8633	0.8560	0.8524	0.8503
S_9	0.9459	0.9242	0.9130	0.9064	0.9009	0.8694	0.8525	0.8469	0.8441	0.8424
S_{10}	0.7982	0.7841	0.7644	0.7482	0.7378	0.6735	0.6426	0.6324	0.6272	0.6241

Table 7
The ranks of alternatives in different values of β.

| | | β | | | | | | | | | |
		0.1	0.2	0.3	0.4	0.5	1	2	3	4	5
Rank	A_1	1	1	1	1	1	1	1	1	1	1
	A_2	5	5	4	4	4	2	2	2	2	2
	A_3	8	7	7	6	6	6	6	6	6	6
	A_4	10	10	10	10	10	10	10	10	10	10
	A_5	6	6	6	7	7	7	7	7	7	7
	A_6	7	8	8	8	8	8	8	8	8	8
	A_7	2	2	2	2	2	3	3	3	3	3
	A_8	4	3	3	3	3	4	4	4	4	4
	A_9	3	4	5	5	5	5	5	5	5	5
	A_{10}	9	9	9	10	10	9	9	9	9	9

3.2. *Analysis of the Performance of Alternatives*

In this sub-section, we obtain the overall performance score of each alternative (S_i) by solving Model (8) based on the data provided in Table 2 and the same values of β as we have used for the analysis of criteria weights. The overall performance scores obtained are presented in Table 6 and the corresponding ranks of alternatives are shown in Table 7. Also, the graphical view of performance scores is represented in Fig. 2.

As we can see in Fig. 2 (and Table 6), the performance of alternatives are more distinguishable and more stable when the values of β are greater than 3 ($\beta \geqslant 3$).

To analyse the stability of the ranks of criteria in different values of β, the Spearman's rank correlation coefficients (r_s) between each column of Table 7 are computed. The results are presented in Table 8.

As can be seen in Table 7, we have a completely stable rank when the values of β are greater than 1. Generally, we can say that $\beta = 3$ is also a good threshold for determination of the overall performance scores and ranks of alternatives.
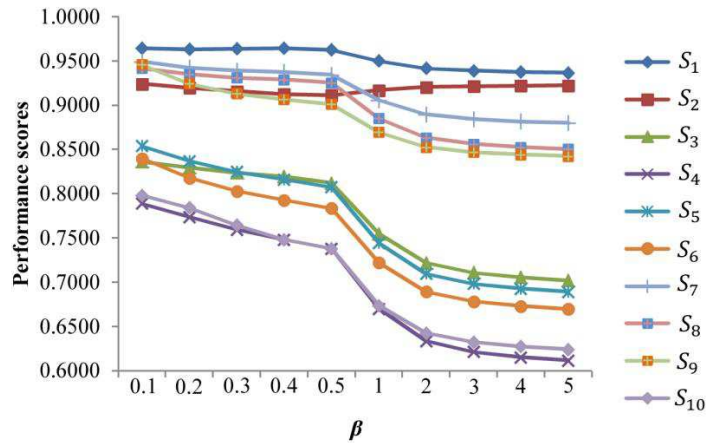
Fig. 2. Variations in the overall performance scores related to β.

Table 8
The correlation (r_s) between the ranking results with different values of β.

						β					
		0.1	0.2	0.3	0.4	0.5	1	2	3	4	5
β	0.1	1	0.9758	0.9515	0.9240	0.9240	0.8788	0.8788	0.8788	0.8788	0.8788
	0.2	0.9758	1	0.9879	0.9726	0.9726	0.9152	0.9152	0.9152	0.9152	0.9152
	0.3	0.9515	0.9879	1	0.9848	0.9848	0.9515	0.9515	0.9515	0.9515	0.9515
	0.4	0.9240	0.9726	0.9848	1	1	0.9605	0.9605	0.9605	0.9605	0.9605
	0.5	0.9240	0.9726	0.9848	1	1	0.9605	0.9605	0.9605	0.9605	0.9605
	1	0.8788	0.9152	0.9515	0.9605	0.9605	1	1	1	1	1
	2	0.8788	0.9152	0.9515	0.9605	0.9605	1	1	1	1	1
	3	0.8788	0.9152	0.9515	0.9605	0.9605	1	1	1	1	1
	4	0.8788	0.9152	0.9515	0.9605	0.9605	1	1	1	1	1
	5	0.8788	0.9152	0.9515	0.9605	0.9605	1	1	1	1	1

3.3. *Comparative Analysis*

In this sub-section, the results of the proposed method are compared with those of some existing MCDM methods. We use six MCDM methods including SAW, WASPAS, CO-PRAS, TOPSIS, VIKOR and EDAS to make this comparison. Based on the results of the previous sections, the value of β is set to 3 for computations.

The methods which are used for comparative analysis need a set of weights to rank the alternatives. In the comparative analysis, we use the set of weights which has been determined by the proposed method with $\beta = 3$ ($w_1 = 0.153$, $w_2 = 0.151$, $w_3 = 0.126$, $w_4 = 0.178$, $w_5 = 0.118$, $w_6 = 0.159$ and $w_7 = 0.116$). Using the decision-matrix and this set of weights for criteria, the rank of alternatives is determined by the considered MCDM methods. The ranking results of the proposed method and the other methods are presented in Table 9. In the last row of this table, we can see the Spearman's rank correlation coefficients (r_s) between the ranking result of SECA and the other methods.

Table 9
The ranking results of different MCDM methods and the correlation values.

	SAW	WASPAS	COPRAS	TOPSIS	VIKOR	EDAS	SECA
A_1	1	1	1	1	1	1	1
A_2	2	2	2	2	5	2	2
A_3	6	6	6	9	6	6	6
A_4	10	10	10	10	10	10	10
A_5	7	7	7	8	7	7	7
A_6	8	8	8	7	8	8	8
A_7	3	3	3	3	2	3	3
A_8	4	4	4	5	3	4	4
A_9	5	5	5	4	4	5	5
A_{10}	9	9	9	6	9	9	9
r_s	**1**	**1**	**1**	**0.867**	**0.927**	**1**	**–**

According to Table 9, all of the correlation values are more than 0.8. Thus it can be said that there is a very strong relationship between the result of SECA and the MCDM methods considered for the comparative analysis (Walters, 2009). Based on these results we can conclude that the results of the proposed method are valid, and SECA can be considered as an efficient method to deal with the multi-criteria decision-making problems.

For using the proposed method, the suggested value of β is $\beta = 3$. However, this parameter can be set according to the preferences of decision-makers and characteristics of the decision-making problem. The appropriate value of β can be estimated by making a sensitivity analysis based on the decision-matrix of the problem.

It should be noted that the elements of the decision-matrix should be greater than zero $(x_{ij} > 0)$ to use SECA. If there are some elements with negative and/or zero values, they should be transformed.

4. Conclusions

Many MCDM methods have been developed in the past decades. Most of these methods were designed to evaluate some alternatives with respect to a given set of criteria weights. There have also been some methods for determination of the subjective and objective weights of criteria. In this study, we have proposed a new method for simultaneous evaluation of criteria and alternatives (SECA). A multi-objective non-linear mathematical model has been formulated for the proposed method. The objectives of the model have been defined to maximize the overall performance of alternatives considering the within- and between-criterion variation information of decision-matrix. By using the proposed method, we can determine the overall performance scores of alternatives and objective weights of criteria simultaneously. For validation of SECA, the objective weights of criteria and the overall performance of alternatives determined by the method have been analysed. The results show that by setting the parameter of the method (β) in an appropriate value, we can get stable weights for criteria and performance scores for alternatives. A comparison has also been made between the results of SECA and those of some existing

methods. This comparison also shows the validity and efficiency of the proposed method. Future research can extend SECA for MCDM problems in fuzzy and other uncertain environments.

References

Abdullah, L., Adawiyah, C.R. (2014). Simple additive weighting methods of multi criteria decision making and applications: a decade review. *International Journal of Information Processing and Management*, 5(1), 39–49.

Adalı, E.A., Işık, A.T. (2017). CRITIC and MAUT methods for the contract manufacturer selection problem. *European Journal of Multidisciplinary Studies*, 5(1), 93–101.

Baušys, R., Juodagalvienė, B. (2017). Garage location selection for residential house by WASPAS-SVNS method. *Journal of Civil Engineering and Management*, 23(3), 421–429.

Behzadian, M., Otaghsara, S.K., Yazdani, M., Ignatius, J. (2012). A state-of the-art survey of TOPSIS applications. *Expert Systems with Applications*, 39(17), 13051–13069.

Cayir Ervural, B., Zaim, S., Demirel, O.F., Aydin, Z., Delen, D. (2018). An ANP and fuzzy TOPSIS-based SWOT analysis for Turkey's energy planning. *Renewable and Sustainable Energy Reviews*, 82, 1538–1550.

Chang, S.-C., Tsai, P.-H. (2016). A hybrid financial performance evaluations model for wealth management banks following the global financial crisis. *Technological and Economic Development of Economy*, 22(1), 21–46.

Chen, S.-J., Hwang, C.-L. (1992). *Fuzzy Multiple Attribute Decision Making: Methods and Applications*. Springer, Berlin.

Chen, W., Shen, Y., Wang, Y. (2018). Evaluation of economic transformation and upgrading of resource-based cities in Shaanxi province based on an improved TOPSIS method. *Sustainable Cities and Society*, 37, 232–240.

Diakoulaki, D., Mavrotas, G., Papayannakis, L. (1995). Determining objective weights in multiple criteria problems: the critic method. *Computers & Operations Research*, 22(7), 763–770.

Goodridge, W.S. (2016). Sensitivity analysis using simple additive weighting method. *International Journal of Intelligent Systems and Applications*, 8(5), 27–33.

Gul, M., Celik, E., Aydin, N., Taskin Gumus, A., Guneri, A.F. (2016). A state of the art literature review of VIKOR and its fuzzy extensions on applications. *Applied Soft Computing*, 46, 60–89.

Hwang, C.L., Yoon, K.P. (1981). *Multiple Attribute Decision Making Methods and Applications: A State-of-the Art Survey*. Springer, London.

Juodagalvienė, B., Turskis, Z., Šaparauskas, J., Endriukaitytė, A. (2017). Integrated multi-criteria evaluation of house's plan shape based on the EDAS and SWARA methods. *Engineering Structures and Technologies*, 9(3), 117–125.

Kalibatas, D., Kovaitis, V. (2017). Selecting the most effective alternative of waterproofing membranes for multifunctional inverted flat roofs. *Journal of Civil Engineering and Management*, 23(5), 650–660.

Karabasevic, D., Stanujkic, D., Maksimovic, M. (2017). An approach to personnel selection in the tourism industry based on the SWARA and the WASPAS methods. *Economic Computation and Economic Cybernetics Studies and Research*, 51(1), 75–88.

Keshavarz Ghorabaee, M. (2018). The LINGO code and instructions for SECA, figshare. https://doi.org/10.6084/m9.figshare.5830623.

Keshavarz Ghorabaee, M., Zavadskas, E.K., Olfat, L., Turskis, Z. (2015). Multi-criteria inventory classification using a new method of evaluation based on distance from average solution (EDAS). *Informatica*, 26(3), 435–451.

Keshavarz Ghorabaee, M., Zavadskas, E.K., Amiri, M., Antucheviciene, J. (2016a). A new method of assessment based on fuzzy ranking and aggregated weights (AFRAW) for MCDM problems under type-2 fuzzy environment. *Economic Computation and Economic Cybernetics Studies and Research*, 50(1), 39–68.

Keshavarz Ghorabaee, M., Zavadskas, E.K., Amiri, M., Turskis, Z. (2016b). Extended EDAS method for fuzzy multi-criteria decision-making: an application to supplier selection. *International Journal of Computers Communications & Control*, 11(3), 358–371.

Keshavarz Ghorabaee, M., Amiri, M., Zavadskas, E.K., Antucheviciene, J. (2017a). Assessment of third-party logistics providers using a CRITIC–WASPAS approach with interval type-2 fuzzy sets. *Transport*, 32(1), 66–78.

Keshavarz Ghorabaee, M., Amiri, M., Zavadskas, E.K., Antucheviciene, J. (2017b). Supplier evaluation and selection in fuzzy environments: a review of MADM approaches. *Economic Research-Ekonomska Istraži-vanja*, 30(1), 1073–1118.

Liu, L., Zhou, J., An, X., Zhang, Y., Yang, L. (2010). Using fuzzy theory and information entropy for water quality assessment in Three Gorges region, China. *Expert Systems with Applications*, 37(3), 2517–2521.

MacCrimmon, K.R. (1968). *Decision Making among Multiple-attribute Alternatives: A Survey and Consolidated Approach*. RAND Corporation, Santa Monica.

Mao, N., Song, M., Pan, D., Deng, S. (2018). Comparative studies on using RSM and TOPSIS methods to optimize residential air conditioning systems. *Energy*, 144, 98–109.

Mardani, A., Nilashi, M., Zakuan, N., Loganathan, N., Soheilirad, S., Saman, M.Z.M., Ibrahim, O. (2017). A systematic review and meta-Analysis of SWARA and WASPAS methods: Theory and applications with recent fuzzy developments. *Applied Soft Computing*, 57, 265–292.

Muddineni, V.P., Sandepudi, S.R., Bonala, A.K. (2017). Improved weighting factor selection for predictive torque control of induction motor drive based on a simple additive weighting method. *Electric Power Components and Systems*, 45(13), 1450–1462.

Mulliner, E., Malys, N., Maliene, V. (2016). Comparative analysis of MCDM methods for the assessment of sustainable housing affordability. *Omega*, 59(Part B), 146–156.

Nakhaei, J., Lale Arefi, S., Bitarafan, M., Kildienė, S. (2016). Evaluation of light supply in the public underground safe spaces by using of COPRAS-SWARA methods. *International Journal of Strategic Property Management*, 20(2), 198–206.

Nie, R.-X., Wang, J.-Q., Zhang, H.-Y. (2017). Solving solar-wind power station location problem using an extended weighted aggregated sum product assessment (WASPAS) technique with interval neutrosophic sets. *Symmetry*, 9(7), 106.

Opricovic, S. (1998). *Multicriteria Optimization of Civil Engineering Systems*. Faculty of Civil Engineering, Belgrade (in Serbian).

Opricovic, S., Tzeng, G.-H. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455.

Peng, X., Dai, J. (2017). Hesitant fuzzy soft decision making methods based on WASPAS, MABAC and CO-PRAS with combined weights. *Journal of Intelligent & Fuzzy Systems*, 33(2), 1313–1325.

Polat, G., Eray, E., Bingol, B. N. (2017). An integrated fuzzy MCGDM approach for supplier selection problem. *Journal of Civil Engineering and Management*, 23(7), 926–942.

Rathi, K., Balamohan, S. (2017). A mathematical model for subjective evaluation of alternatives in fuzzy multi-criteria group decision making using COPRAS method. *International Journal of Fuzzy Systems*, 19(5), 1290–1299.

Ren, J., Toniolo, S. (2018). Life cycle sustainability decision-support framework for ranking of hydrogen production pathways under uncertainties: an interval multi-criteria decision making approach. *Journal of Cleaner Production*, 175, 222–236.

Rezaei, J. (2015). Best-worst multi-criteria decision-making method. *Omega*, 53, 49–57.

Rostamzadeh, R., Keshavarz Ghorabaee, M., Govindan, K., Esmaeili, A., Nobar, H.B.K. (2018). Evaluation of sustainable supply chain risk management using an integrated fuzzy TOPSIS- CRITIC approach. *Journal of Cleaner Production*, 175, 651–669.

Sakthivel, G., Sivakumar, R., Saravanan, N., Ikua, B.W. (2017). A decision support system to evaluate the optimum fuel blend in an IC engine to enhance the energy efficiency and energy management. *Energy*, 140, 566–583.

Sennaroglu, B., Varlik Celebi, G. (2018). A military airport location selection by AHP integrated PROMETHEE and VIKOR methods. *Transportation Research Part D: Transport and Environment*, 59, 160–173.

Serrai, W., Abdelli, A., Mokdad, L., Hammal, Y. (2017). Towards an efficient and a more accurate web service selection using MCDM methods. *Journal of Computational Science*, 22, 253–267.

Shen, F., Ma, X., Li, Z., Xu, Z., Cai, D. (2018). An extended intuitionistic fuzzy TOPSIS method based on a new distance measure with an application to credit risk evaluation. *Information Sciences*, 428, 105–119.

Soner, O., Celik, E., Akyuz, E. (2017). Application of AHP and VIKOR methods under interval type 2 fuzzy environment in maritime transportation. *Ocean Engineering*, 129, 107–116.

Stefano, N.M., Filho, N.C., Vergara, L.G.L., Rocha, R.U.G.D. (2015). COPRAS (complex proportional assessment): state of the art research and its applications. *IEEE Latin America Transactions*, 13(12), 3899–3906.

Stević, Ž., Pamučar, D., Vasiljević, M., Stojić, G., Korica, S. (2017). Novel integrated multi-criteria model for supplier selection: case study construction company. *Symmetry*, 9(11), 279.

Turanoglu Bekar, E., Cakmakci, M., Kahraman, C. (2016). Fuzzy COPRAS method for performance measurement in total productive maintenance: a comparative analysis. *Journal of Business Economics and Management*, 17(5), 663–684.

Walczak, D., Rutkowska, A. (2017). Project rankings for participatory budget based on the fuzzy TOPSIS method. *European Journal of Operational Research*, 260(2), 706–714.

Walters, S.J. (2009). *Quality of Life Outcomes in Clinical Trials and Health-Care Evaluation: A Practical Guide to Analysis and Interpretation*. Wiley, New York.

Wang, Y.-J. (2015). A fuzzy multi-criteria decision-making model based on simple additive weighting method and relative preference relation. *Applied Soft Computing*, 30, 412–420.

Wang, L.-E., Liu, H.-C., Quan, M.-Y. (2016a). Evaluating the risk of failure modes with a hybrid MCDM model under interval-valued intuitionistic fuzzy environments. *Computers & Industrial Engineering*, 102, 175–185.

Wang, P., Zhu, Z., Wang, Y. (2016b). A novel hybrid MCDM model combining the SAW, TOPSIS and GRA methods based on experimental design. *Information Sciences*, 345, 27–45.

Wang, L., Zhang, H.-Y., Wang, J.-Q., Li, L. (2018). Picture fuzzy normalized projection-based VIKOR method for the risk evaluation of construction project. *Applied Soft Computing*, 64, 216–226.

Yalçın, N., Ünlü, U. (2017). A multi-criteria performance analysis of Initial Public Offering (IPO) firms using CRITIC and VIKOR methods. *Technological and Economic Development of Economy*, 24(2), 534–560.

Zardari, N.H., Ahmed, K., Shirazi, S.M., Yusop, Z.B. (2014). *Weighting Methods and their Effects on Multi-Criteria Decision Making Model Outcomes in Water Resources Management*. Springer International Publishing.

Zavadskas, E.K., Kaklauskas, A., Sarka, V. (1994). The new method of multicriteria complex proportional assessment of projects. *Technological and Economic Development of Economy*, 1(3), 131–139.

Zavadskas, E.K., Turskis, Z., Antucheviciene, J., Zakarevicius, A. (2012). Optimization of weighted aggregated sum product assessment. *Elektronika ir Elektrotechnika*. 122(6), 3–6.

Zhang, H., Gu, C.-L., Gu, L.-W., Zhang, Y. (2011). The evaluation of tourism destination competitiveness by TOPSIS & information entropy – a case in the Yangtze River Delta of China. *Tourism Management*, 32(2), 443–451.

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