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A New Procedure to Intuitionistic Uncertain Linguistic Group Decision Making

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Abstract. Intuitionistic uncertain linguistic variables (IULVs) are useful to express the qualitative and quantitative recognitions of decision makers. However, after reviewing the previous operational laws on IULVs, we find there are some limitations. To address these issues, we define several new operations on IULVs and give a new ranking method. To improve the utilization of IULVs, this paper defines two Choquet operators: the intuitionistic uncertain linguistic symmetrical Choquet averaging (IULSCA) operator and the intuitionistic uncertain linguistic symmetrical Choquet geometric mean (IULSCGM) operator, which can address the internal correlations among elements. To globally reflect the interactive characteristics of the importance of elements, two generalized Shapley intuitionistic uncertain linguistic symmetrical Choquet operators are presented. Subsequently, a new distance measure is defined, which is then used to build models to ascertain fuzzy measures on decision maker and criteria sets to address the case where the weighting information is partly known. After that, a new procedure to intuitionistic uncertain linguistic group decision making is developed. Finally, a specific example is offered to illustrate the practicality of the new procedure, and the comparison analysis is also made.

Key words: group decision making, intuitionistic uncertain linguistic variable, Choquet integral, generalized Shapley function.

1. Introduction

Group decision making (GDM) is one of critical researching topics in decision-making theory. How to express the judgments of decision makers (DMs) is a hot researching field. Because many fuzzy and uncertain factors usually exist during decision making, the criteria values cannot be expressed using concrete values. To address this situation, fuzzy sets (Zadeh, 1965) are more suitable tools. Since Zadeh (1965) first introduced fuzzy sets for us, many extending forms are developed, such as interval-valued fuzzy sets (Zadeh, 1973),

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intuitionistic fuzzy sets (Atanassov, 1983), and hesitant fuzzy sets (Torra, 2010). Meanwhile, many decision-making methods in fuzzy environment are proposed. For example, Meng *et al.* (2017a) introduced a decision making with interval reciprocal preference relations, and Meng (2018) discussed decision making with triangular fuzzy reciprocal preference relations. Kou *et al.* (2015) proposed a decision-making method with generalized fuzzy numbers (GFNs) based on the defined distance measures and built programming model. Li *et al.* (2016a, 2016b) introduced a GDM with integrating heterogeneous information based on the weighted-power average operator and consensus analysis. Meng *et al.* (2017b) researched multichoice games with trapezoidal fuzzy characteristic function and defined a fuzzy Shapley function. Xu *et al.* (2017) discussed PN equilibrium strategy for non-cooperative games by considering risk preference of DMs. Liu (2014) defined some interval-valued intuitionistic fuzzy Hamacher aggregation operators and studied their application in GDM. Liu (2016) reviewed researches about intuitionistic fuzzy decision making before 2016. Meng *et al.* (2017d) offered a method for group decision making with intuitionistic fuzzy preference relations. Meng *et al.* (2018a) studied interval-valued intuitionistic fuzzy GDM based on the built programming models. Liu and Peng (2017) used the defined geometric distance measure to study interval-valued intuitionistic fuzzy GDM. Stanujkic *et al.* (2017) evaluated the website quality in hotel industry based on triangular intuitionistic fuzzy numbers. Zhang *et al.* (2015), Zhang (2016) researched GDM with hesitant fuzzy preference relations and developed two GDM methods. Tang *et al.* (2017) also discussed GDM with hesitant fuzzy preference relations based on the consistency analysis. Additionally, in some cases, DMs may find that it is infeasible to assess precisely in a quantitative form Herrera and Herrera-Viedma (2000). To address this issue, Zadeh (1975) introduced linguistic variables (LVs) to express the qualitative recognitions of DMs. Herrera *et al.* (1996) extended the OWA operator to linguistic variables and researched linguistic fuzzy decision making based on the linguistic OWA operator. Cheng *et al.* (2017) defined some new interval-valued 2-tuple linguistic distance measures and studied their application. Park *et al.* (2011) defined some uncertain linguistic harmonic mean operators and discussed their application in GDM. Xu (2006) introduced uncertain multiplicative linguistic preference relations and studied their application based on the induced uncertain LOWG operator. Meng *et al.* (2017c) offered a GDM method with interval linguistic fuzzy preference relations based on consistency analysis. Tang and Meng (2017) studied GDM with hesitant fuzzy linguistic preference relations. Meng and Tang (2018) reviewed and analysed previous ranking methods for linguistic hesitant fuzzy sets and defined a new one. Herrera-Viedma and Lopez-Herrera (2010) reviewed information accessing systems based on linguistic modelling. To express the quantitative and qualitative judgments, uncertain linguistic hesitant fuzzy sets are proposed by Meng *et al.* (2018b), and intuitionistic hesitant fuzzy linguistic sets are introduced by Meng and Tan (2017). Meng *et al.* (2016) proposed a GDM method with intuitionistic linguistic preference relations based on consistency and consensus analysis. Liu and Qin (2017) studied GDM with interval-valued intuitionistic fuzzy information based on the geometric distance measure. Meanwhile, linguistic fuzzy decision making has been successfully applied in many fields, including performance appraisal (de Andrés *et al.*, 2010), medical treatment selection (Hu *et al.*, 2017), information retrieval system (Herrera-Viedma *et al.*, 2007), and engineering evaluation (Martínez *et al.*, 2007). Furthermore, Mardani *et al.* (2015) reviewed the techniques and applications of fuzzy decision making from 1994 to 2014, and Mardani *et al.* (2018) recalled decision-making methods based on fuzzy aggregation operators in the past three decades.

All of the above mentioned fuzzy sets only denote the qualitative or quantitative recognitions of DMs. However, none of them can express these two aspects simultaneously. Considering the situation, Wang and Li (2009) defined intuitionistic linguistic sets (ILSs) that are expressed using an LV and an intuitionistic fuzzy number (IFN). Note that this type of fuzzy sets applies an LV and an IFN to denote a qualitative and quantitative recognition of DMs. Following the original work of Wang and Li (2009), Liu (2013) introduced two aggregation operators on ILSs. Later, Liu and Jin (2012) introduced intuitionistic uncertain linguistic variables (IULVs), which are characterized using an IFN and an uncertain linguistic variable (ULV). Based on the operational laws on IULVs, Liu and Jin (2012) defined three operators on IULVs. Liu and Teng (2015) defined a Hamming distance on IULVs and then extended the TODIM into IULVs and proposed a TODIM based method for GDM with IULVs. Liu and Shi (2015) introduced the Einstein operations on IULVs and defined several Einstein operations. Using the defined aggregation operators, the authors proposed a group decision-making method. Furthermore, Liu *et al.* (2014a) defined several Heronian mean operators on IULVs, which are then used to compute the IULVs of alternatives. To reflect the interactions of importance, Chen and Li (2017) defined two Choquet operators on IULVs and showed their application in GDM.

After reviewing the above researches about decision making with IULVs, we find that three limitations exist: (i) These methods all use the operational laws in Liu and Jin (2012) that may lead to the unreasonable decisions; (ii) All of these decision-making methods assume that the independence of elements in a set is true, namely, the importance of elements is based on additive measures. As some scholars noted (Liu *et al.*, 2015; Xu, 2010), this assumption is not true. In this case, decision-making methods based on additive measures seem to be helpless. (iii) All of these methods are based on the assumption that the weighting information is exactly known. This is unrealistic because there are many factors, which may lead to the weighting information incompletely known. Following previous researches about decision making with IULVs, this paper continues to study this topic and develops a new procedure to GDM with IULVs that can address weighting information with correlations and that is partly known.

Section 2 reviews basic concepts, including ULVs, and IULVs. Then, it analyses the limitations of the previous operational laws and defines several new ones. Subsequently, the definitions of the Choquet integral and fuzzy measures (FMs) are reviewed. Section 3 proposes four new operators on IULVs, which overall define the weight of each combination and reflect their correlations. Section 4 constructs two models to ascertain FMs on decision maker and criteria sets, respectively. Then, a new procedure to GDM with IULVs is offered. Section 5 shows the efficiency of new results by an illustrative example and makes a comparison analysis. Conclusions are provided in the end.

2. Basic Concepts

This section includes three subsections. The first subsection recalls some concepts about IULVs and then analyses the issues in the previous operational laws. The second subsection introduces some new operations and defines a new ranking order that avoid the issues in the previous ones. The last subsection reviews the definitions of fuzzy measures and the Choquet integral.

2.1. *Intuitionistic Uncertain Linguistic Sets*

To denote the preferred and non-preferred membership degrees (P-NP-MDs) of a judgment, Atanassov (1983) introduced intuitionistic fuzzy sets (IFSs). To simplify, we let $X = \{x_1, x_2, \ldots, x_n\}$ denote the finite object set.

Definition 1 (*See* Atanassov, 1983). An IFS A on X is formulated as:

$$A = \{\langle x, u_A(x), v_A(x)\rangle | x \in X\}, \tag{1}$$

where $u_A(x) \in [0, 1]$ and $v_A(x) \in [0, 1]$ are the P-NP-MDs of $x \in X$ with $u_A(x) + v_A(x) \leqslant 1$, respectively. $\pi_A(x) = 1 - u_A(x) - v_A(x)$ is the hesitancy.

Different from quantitative fuzzy sets, Zadeh (1975) introduced linguistic variables (LVs) to express the qualitative recognitions of DMs. Considering the utilization of LVs, Herrera *et al.* (2000) offered linguistic term sets (LTSs).

Let $S = \{s_i | i = 1, 2, \ldots, t\}$ be an LTS with odd cardinality. Each element in S denotes a value for an LV, and elements in S own the properties (Herrera *et al.*, 2000): (i) if $i > j$, then $s_i > s_j$; (ii) if $s_i \geqslant s_j$, then $\max(s_i, s_j) = s_i$ and $\min(s_i, s_j) = s_j$.

For instance, an LTS S might be offered as: S={s_1: very slow, s_2: slow, s_3: a little slow, s_4: fair, s_5: a little fast, s_6: fast, s_7: very fast}.

To avoid information losing, Xu (2004) proposed the continuous LTS $\bar{S} = \{s_\alpha | s_1 \leqslant s_\alpha \leqslant s_t,\ \alpha \in [1, t]\}$ by extending the discrete LTS S, whose elements satisfy all of the above characteristics too. To express the uncertain qualitative recognitions of DMs, Xu (2006) further proposed uncertain linguistic variables (ULVs):

Definition 2 (*See* Xu, 2006). Let $\tilde{s} = [s_\alpha, s_\beta]$, where $s_\alpha, s_\beta \in \bar{S}$ with $s_\alpha \leqslant s_\beta$. Then, $\tilde{s}$ is an ULV.

Following IFSs and ULVs, Liu and Jin (2012) presented intuitionistic uncertain linguistic sets (IULSs) to denote the qualitative and quantitative recognitions of DMs.

Definition 3 (*See* Liu and Jin, 2012). An IULS A on X is formulated as:

$$A = \{\langle x_i | [[s_{\theta(x_i)}, s_{\tau(x_i)}], (u_A(x_i), v_A(x_i))]\rangle | x_i \in X\}, \tag{2}$$

where $s_{\theta(x_i)}, s_{\tau(x_i)} \in \bar{S}$, $u_A(x_i)$ and $v_A(x_i)$ are the P-NP-MDs of $x \in X$ to $[s_{\theta(x_i)}, s_{\tau(x_i)}]$ with $\begin{cases} 0 \leqslant u_A(x_i) \wedge v_A(x_i) \leqslant 1, \\ u_A(x_i) + v_A(x_i) \leqslant 1, \end{cases}$ respectively.

When $s_{\theta(x_i)} = s_{\tau(x_i)}$, we get the intuitionistic linguistic set (ILS) A, denoted by $A = \{\langle x | s_{\theta(x_i)}, (u_A(x_i), v_A(x_i))\rangle | x_i \in X\}$ with $\begin{cases} 0 \leqslant u_A(x_i) \wedge v_A(x_i) \leqslant 1, \\ u_A(x_i) + v_A(x_i) \leqslant 1 \end{cases}$ (Liu, 2013). Considering the utilization of IULSs, Liu and Jin (2012) further introduced intuitionistic uncertain linguistic variables (IULVs) as follows:

Definition 4 (*See* Liu and Jin, 2012). An IULV $\tilde{\alpha}$ is formulated as $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$, where $u(\alpha)$ and $v(\alpha)$ are the P-NP-MDs to $[s_{\theta(\alpha)}, s_{\tau(\alpha)}]$ with $\begin{cases} 0 \leqslant u(\alpha) \wedge v(\alpha) \leqslant 1, \\ u(\alpha) + v(\alpha) \leqslant 1, \end{cases}$ respectively.

Furthermore, Liu and Jin (2012) defined the following operational laws on IULVs:

Definition 5 (*See* Liu and Jin, 2012). Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ and $\tilde{\beta} = [[s_{\theta(\beta)}, s_{\tau(\beta)}], (u(\beta), v(\beta))]$ be two IULVs. Then, some operations of $\tilde{\alpha}$ and $\tilde{\beta}$ are formulated as:

$$\tilde{\alpha} \oplus \tilde{\beta} = \big[[s_{\theta(\alpha)+\theta(\beta)}, s_{\tau(\alpha)+\tau(\beta)}], \big(1 - (1 - u(\alpha))(1 - u(\beta)), v(\alpha)v(\beta)\big)\big];$$
$$\tilde{\alpha} \otimes \tilde{\beta} = \big[[s_{\theta(\alpha)\theta(\beta)}, s_{\tau(\alpha)\tau(\beta)}], \big(u(\alpha)u(\beta), 1 - (1 - v(\alpha))(1 - v(\beta))\big)\big];$$
$$\lambda\tilde{\alpha} = \big[[s_{\lambda\theta(\alpha)}, s_{\lambda\tau(\alpha)}], \big(1 - (1 - u(\alpha))^{\lambda}, v(\alpha)^{\lambda}\big)\big], \quad \lambda \in [0, 1];$$
$$\tilde{\alpha}^{\lambda} = \big[[s_{\theta(\alpha)^{\lambda}}, s_{\tau(\alpha)^{\lambda}}], \big(u(\alpha)^{\lambda}, 1 - (1 - v(\alpha))^{\lambda}\big)\big], \quad \lambda \in [0, 1].$$

Considering the order relationship between IULVs, Liu and Jin (2012) defined the following concepts of the expect function and the accuracy function:

Definition 6 (*See* Liu and Jin, 2012). Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ be an IULV. The expected function $E(\tilde{\alpha})$ is formulated as:

$$E(\tilde{\alpha}) = s_{\frac{(\theta(\alpha)+\tau(\alpha))(u(\alpha)+1-v(\alpha))}{4}}, \tag{3}$$

and $H(\tilde{\alpha})$ is called the accuracy function with

$$H(\tilde{\alpha}) = s_{\frac{(\theta(\alpha)+\tau(\alpha))(u(\alpha)+v(\alpha))}{2}}. \tag{4}$$

Using the expected and accuracy functions, an order relationship for any two IULVs $\tilde{\alpha}$ and $\tilde{\beta}$ is offered (Liu and Jin, 2012):
If $E(\tilde{\alpha}) < E(\tilde{\beta})$, then $\tilde{\alpha} \prec \tilde{\beta}$.
If $E(\tilde{\alpha}) = E(\tilde{\beta})$, then $\begin{cases} H(\tilde{\alpha}) = H(\tilde{\beta}) \Rightarrow \tilde{\alpha} = \tilde{\beta}, \\ H(\tilde{\alpha}) < H(\tilde{\beta}) \Rightarrow \tilde{\alpha} \prec \tilde{\beta}. \end{cases}$

For the first and second operations, one can check that the non-membership degree $v(\alpha)v(\beta)$ of $\tilde{\alpha} \oplus \tilde{\beta}$ is zero when $v(\alpha) = 0 \vee v(\beta) = 0$, which is not influenced by the values of the other one. While the membership degree $1 - (1 - v(\alpha))(1 - v(\beta))$ of $\tilde{\alpha} \otimes \tilde{\beta}$ is one when $v(\alpha) = 1 \vee v(\beta) = 1$, which is also uninfluenced by the value of the other one. These conclusions seem to be unreasonable.

As Meng and Chen (2016) noted, the third and fourth operations cannot guarantee the ordered relationship between IULVs. For example, let $\tilde{\alpha} = [[s_3, s_4], (0.4, 0.3)]$ and $\tilde{\beta} = [[s_3, s_4], (0.5, 0.4)]$. According to formulae (3) and (4), we have $E(\tilde{\alpha}) = E(\tilde{\beta}) = 1.925$ and $H(\tilde{\alpha}) = 5.95 < 6.65 = H(\tilde{\beta})$. Thus, $\tilde{\alpha} \prec \tilde{\beta}$. Furthermore, let $\lambda = 0.7$. We get

$$\lambda\tilde{\alpha} = \big[[s_{2.1}, s_{2.8}], (0.3006, 0.4305)\big] \quad \text{and} \quad \lambda\tilde{\beta} = \big[[s_{2.1}, s_{2.8}], (0.3844, 0.5256)\big],$$

by which we derive $H(\lambda\tilde{\alpha}) = 1.0659 > 1.0509 = H(\lambda\tilde{\beta})$. Thus, $\tilde{\alpha} \succ \tilde{\beta}$.

Furthermore, let $\tilde{\alpha} = [[s_3, s_4], (0.25, 0.1)]$ and $\tilde{\beta} = [[s_3, s_4], (0.5, 0.4)]$. We derive $E(\tilde{\alpha}) = 2.0125 > E(\tilde{\beta}) = 1.925$ and $\tilde{\alpha} \succ \tilde{\beta}$. Let $\lambda = 0.2$. We obtain $\tilde{\alpha}^{\lambda} = [[s_{1.2457}, s_{1.3195}], (0.7579, 0.0209)]$ and $\tilde{\beta}^{\lambda} = [[s_{1,2457}, s_{1.3195}], (0.8706, 0.0912)]$. Using formula (3), we get $E(\tilde{\alpha}^{\lambda}) = 0.2580 < E(\tilde{\beta}^{\lambda}) = 0.2707$, by which we derive $\tilde{\alpha}^{\lambda} \prec \tilde{\beta}^{\lambda}$.

Different from the above operational laws, Liu and Teng (2015) further defined some Einstein operations on IULVs, which also exist the above listed issues. Taking the scalar multiplication, for example, for the IULVs $\tilde{\alpha} = [[s_3, s_4], (0.4, 0.3)]$, $\tilde{\beta} = [[s_3, s_4], (0.5, 0.4)]$ and $\lambda = 0.7$, we have

$$\lambda_E\tilde{\alpha} = \big[[s_{2.1}, s_{2.8}], (0.2882, 0.4579)\big] \quad \text{and} \quad \lambda_E\tilde{\beta} = \big[[s_{2.1}, s_{2.8}], (0.3666, 0.5496)\big]$$

by using formula (2.36) in Liu and Shi (2015). From formula (3), we have $E(\lambda_E\tilde{\alpha}) = 1.0171$ and $E(\lambda_E\tilde{\beta}) = 1.0008$. Thus, $\tilde{\alpha} \succ \tilde{\beta}$.

The above listed issues make the decisions obtained from methods in Liu and Jin (2012), Liu and Teng (2015), Liu and Shi (2015), Liu *et al.* (2014a) seem unreasonable.

2.2. *Several New Operations and a New Ranking Order*

To guarantee the decisions reasonably, it is necessary to define some new operations to address the issues listed in Section 2.1.

Definition 7. Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ and $\tilde{\beta} = [[s_{\theta(\beta)}, s_{\tau(\beta)}], (u(\beta), v(\beta))]$ be any two IULVs. Two symmetrical operations are formulated as:

(i) $\lambda_1\tilde{\alpha} \oplus \lambda_2\tilde{\beta} = [[s_{\lambda_1\theta(\alpha)+\lambda_2\theta(\beta)}, s_{\lambda_1\tau(\alpha)+\lambda_2\tau(\beta)}], (\lambda_1 u(\alpha) + \lambda_2 u(\beta), \lambda_1 v(\alpha) + \lambda_2 v(\beta))]$, $\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leqslant 1$;

(ii) $\tilde{\alpha}^{\lambda_1} \otimes \tilde{\beta}^{\lambda_2} = [[s_{\theta(\alpha)^{\lambda_1}\theta(\beta)^{\lambda_2}}, s_{\tau(\alpha)^{\lambda_1}\tau(\beta)^{\lambda_2}}], (u(\alpha)^{\lambda_1} u(\beta)^{\lambda_2}, v(\alpha)^{\lambda_1} v(\beta)^{\lambda_2})]$, $\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leqslant 1$.

From the operational laws (i) and (ii), one can easily obtain:

a. $\lambda\tilde{\alpha} = \big[[s_{\lambda\theta(\alpha)}, s_{\lambda\tau(\alpha)}], (\lambda u(\alpha), \lambda v(\alpha))\big], \quad \lambda \in [0, 1];$

b. $\tilde{\alpha}^{\lambda} = \big[[s_{\theta(\alpha)^{\lambda}}, s_{\tau(\alpha)^{\lambda}}], (u(\alpha)^{\lambda}, v(\alpha)^{\lambda})\big], \quad \lambda \in [0, 1].$

Property 1. Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ and $\tilde{\beta} = [[s_{\theta(\beta)}, s_{\tau(\beta)}], (u(\beta), v(\beta))]$ be any two IULVs. Then,

(i) $\lambda(\tilde{\alpha} \oplus \tilde{\beta}) = \lambda\tilde{\alpha} \oplus \lambda\tilde{\beta}$, $\lambda \in [0, 1]$;
(ii) $(\lambda_1 + \lambda_2)\tilde{\alpha} = \lambda_1\tilde{\alpha} \oplus \lambda_2\tilde{\alpha}$, $\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leqslant 1$;
(iii) $\lambda(\tilde{\alpha} \otimes \tilde{\beta}) = \lambda\tilde{\alpha} \otimes \lambda\tilde{\beta}$, $\lambda \in [0, 1]$;
(iv) $\tilde{\alpha}^{\lambda_1+\lambda_2} = \tilde{\alpha}^{\lambda_1} \otimes \tilde{\alpha}^{\lambda_2}$, $\lambda_1, \lambda_2 \in [0, 1] \wedge \lambda_1 + \lambda_2 \leqslant 1$.

Proof. From Definition 7, the conclusions are easily derived. □

Because we do not use $\tilde{\alpha} \oplus \tilde{\beta}$ and $\tilde{\alpha} \otimes \tilde{\beta}$ in this paper, Definition 7 does not consider these two operations. To rank IULVs, we offer the following new ranking method:

Definition 8. Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ be an IULV. Then, the new score function is defined as:

$$NS(\tilde{\alpha}) = s_{\frac{(\theta(\alpha)+\tau(\alpha))(u(\alpha)-v(\alpha))}{2}} \tag{5}$$

and the new accuracy function is given as:

$$NA(\tilde{\alpha}) = s_{\frac{(\theta(\alpha)+\tau(\alpha))(u(\alpha)+v(\alpha))}{2}}. \tag{6}$$

Using the new score and accuracy functions in Definition 8, we offer the following order relationship between IULVs $\tilde{\alpha}$ and $\tilde{\beta}$:
If $NS(\tilde{\alpha}) < NS(\tilde{\beta})$, then $\tilde{\alpha} \prec \tilde{\beta}$.
If $NS(\tilde{\alpha}) = NS(\tilde{\beta})$, then $\begin{cases} NA(\tilde{\alpha}) = NA(\tilde{\beta}) \Rightarrow \tilde{\alpha} = \tilde{\beta}, \\ NA(\tilde{\alpha}) < NA(\tilde{\beta}) \Rightarrow \tilde{\alpha} \prec \tilde{\beta}. \end{cases}$

Property 2. Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ and $\tilde{\beta} = [[s_{\theta(\beta)}, s_{\tau(\beta)}], (u(\beta), v(\beta))]$ be any two IULVs, and let $\lambda \in (0, 1)$. Then,

(i) $NS(\tilde{\alpha}) \leqslant NS(\tilde{\beta})$ if and only if $NS(\lambda\tilde{\alpha}) \leqslant NS(\lambda\tilde{\beta})$,
(ii) $NA(\tilde{\alpha}) \leqslant NA(\tilde{\beta})$ if and only if $NS(\tilde{\alpha}^{\lambda}) \leqslant NS(\tilde{\beta}^{\lambda})$.

Proof. Following $NS(\tilde{\alpha}) \leqslant NS(\tilde{\beta})$, we derive

$$\frac{(\theta(\alpha) + \tau(\alpha))(u(\alpha) - \theta(\alpha))}{2} \leqslant \frac{(\theta(\beta) + \tau(\beta))(u(\beta) - v(\beta))}{2},$$

by which we get

$$\frac{(\lambda\theta(\alpha)+\lambda\tau(\alpha))(\lambda u(\alpha)-\lambda\theta(\alpha))}{2}=\lambda^2\frac{(\theta(\alpha)+\tau(\alpha))(u(\alpha)-\theta(\alpha))}{2}$$
$$\leqslant\lambda^2\frac{(\theta(\beta)+\tau(\beta))(u(\beta)-v(\beta))}{2}=\frac{(\lambda\theta(\beta)+\lambda\tau(\beta))(\lambda u(\beta)-\lambda\theta(\beta))}{2}.$$

Thus, $NS(\lambda\tilde{\alpha})\leqslant NS(\lambda\tilde{\beta})$.

Similarly, one can prove the second conclusion in Property 2. □

Without special explanation, this paper always adopts the operational laws shown in Definition 7.

2.3. *Fuzzy Measures and the Choquet Integral*

FM (Sugeno, 1974) is powerful to measure the importance of elements with correlations that is researched by many scholars.

Definition 9 (*See* Sugeno, 1974). Let $N=\{1,2,\dots,n\}$ be a finite set. A FM μ on N is a set function $\mu: P(N)\to[0,1]$ with conditions:

(i) $\mu(N)=1$ and $\mu(\emptyset)=0$;
(ii) $\mu(A)\leqslant\mu(B)$ for all $A\subseteq B\subseteq N$, where $P(N)$ is the power set of N.

$\mu(A)$ can be regarded as the weight of the criteria subset A in multi-criteria decision making. Thus, weights for all combinations of criteria are considered.

Fuzzy integrals are important aggregation tools with respect to FMs, and Choquet integral (Grabisch, 1997) is the most widely used one.

Definition 10 (*See* Grabisch, 1997). Let μ be an FM on $N=\{1,2,\dots,n\}$, and let f be a positive real-valued function on $X=\{x_1,x_2,\dots,x_n\}$. The discrete Choquet integral of f for μ is formulated as:

$$C_\mu\big(f(x_{(1)}),f(x_{(2)}),\dots,f(x_{(n)})\big)=\sum_{i=1}^{n}f(x_{(i)})\big(\mu(A_{(i)})-\mu(A_{(i+1)})\big),\tag{7}$$

where $(\cdot)$ is a permutation on N such that $f(x_{(1)})\leqslant f(x_{(2)})\leqslant\dots\leqslant f(x_{(n)})$, and $A_{(i)}=\{i,\dots,n\}$ with $A_{(n+1)}=\emptyset$.

From Definition 10, one can check that when there are no interactive characteristics, the FM reduces to an additive measure (AM), and Choquet integral degenerates to the ordered weighted average (OWA) operator. Due to the advantages of Choquet integral for addressing decision making with interactions, many Choquet integral-based decision-making methods with different types of fuzzy sets are introduced, such as the intuitionistic fuzzy probabilistic Choquet aggregation operator (Sirbiladze and Badagadze, 2017), the triangular intuitionistic fuzzy Choquet aggregation operator (Liu *et al.*, 2015), and the intuitionistic fuzzy Einstein Choquet integral operator (Xu *et al.*, 2014). Meanwhile, the utilization of Choquet integral-based decision making are studied in many fields, including supplier selection (Nia *et al.*, 2016), assessing payment instrument alternatives (Ferreira *et al.*, 2017), and evaluating emerging technology enterprises (Wei *et al.*, 2014).

3. Several Intuitionistic Uncertain Linguistic Symmetrical Choquet Aggregation Operators

There is usually more than one criterion in current decision-making problems. This needs us to aggregate the alternatives' criteria values into the comprehensive ones. To do this, the aggregation operator is one of efficient tools. This section focuses on the aggregation operators on IULVs and defines two types of intuitionistic uncertain linguistic symmetrical Choquet aggregation (IULSCA) operators.

3.1. *Intuitionistic Uncertain Linguistic Symmetrical Choquet Aggregation Operators*

To address the situation that correlations between elements in a set exist, this subsection defines two IULSCA operators: the intuitionistic uncertain linguistic symmetrical Choquet averaging (IULSCA) operator and the intuitionistic uncertain linguistic symmetrical Choquet geometric mean (IULSCGM) operator.

Definition 11. Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \dots, n\}$. The IULSCA operator is formulated as:

$$\mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigoplus_{i=1}^{n} \big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)\tilde{\alpha}_{(i)}, \tag{8}$$

where $(\cdot)$ is a permutation on N with $\tilde{\alpha}_{(1)} \preceq \alpha_{(2)} \preceq \cdots \preceq \alpha_{(n)}$, and $A_{(i)} = \{\tilde{\alpha}_{(i)}, \dots, \tilde{\alpha}_{(n)}\}$ with $A_{(n+1)} = \emptyset$.

Remark 1. If no interactions among elements in A exist, the IULSCA operator reduces to the intuitionistic uncertain linguistic symmetrical weighted averaging (IULSWA) operator:

$$\mathrm{IULSWA}_{w}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigoplus_{i=1}^{n} w_i \tilde{\alpha}_{(i)}, \tag{9}$$

where $w = (w_1, w_2, \dots, w_3)$ is a weighting vector defined on A under conditions $\begin{cases} \sum_{i=1}^{n} w_i = 1, \\ w_i \geqslant 0, \quad i = 1, 2, \dots, n, \end{cases}$ and the other notations as shown in Definition 11.

Furthermore, if each ULV in $\tilde{\alpha}_i$ for all $i = 1, 2, \dots, n$ degenerates to an LV, then the intuitionistic linguistic symmetrical Choquet averaging (ILSCA) operator is obtained:

$$\mathrm{ILSCA}_\mu(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigoplus_{i=1}^{n} \big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)\tilde{\alpha}_{(i)}, \tag{10}$$

where $s_{\theta(\alpha_i)} = s_{\tau(\alpha_i)}$ for all $i = 1, 2, \dots, n$, and the other notations as shown in the IUL-SWA operator.

Theorem 1. *Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i\in N}$ with $N = \{1, 2, \dots, n\}$. Their aggregation value using the IULSCA operator is still an IULV, where*

$$\begin{aligned}&\mathrm{IULSCA}_\mu(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)\\ &\quad = \Bigg[[s_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i+1)}))\theta(\alpha_i)}, s_{\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i+1)}))\tau(\alpha_i)}],\\ &\qquad \Bigg(\sum_{i=1}^{n}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)u(\alpha_i), \sum_{i=1}^{n}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)v(\alpha_i)\Bigg)\Bigg],\end{aligned} \tag{11}$$

and the other notations are as shown in Definition 11.

Proof. Following Definition 4, the first conclusion is easily obtained. Next, formula (11) is proved by using mathematical induction for the value of n.

(i) Let $n = 2$, we have

$$\begin{aligned}&\big(\mu(A_{(1)}) - \mu(A_{(2)})\big)(\tilde{\alpha}_{(1)})\\ &\quad = \big[\big[s_{(\mu(A_{(1)})-\mu(A_{(2)}))\theta(\alpha_{(1)})}, s_{(\mu(A_{(1)})-\mu(A_{(2)}))\tau(\alpha_{(1)})}\big],\\ &\qquad \big(\big(\mu(A_{(1)}) - \mu(A_{(2)})\big)u(\alpha_{(1)}), \big(\mu(A_{(1)}) - \mu(A_{(2)})\big)v(\alpha_{(1)})\big)\big]\end{aligned}$$

and

$$\begin{aligned}&\big(\mu(A_{(2)}) - \mu(A_{(3)})\big)(\tilde{\alpha}_{(2)}) = \big(\mu(A_{(2)}) - \mu(\emptyset)\big)(\tilde{\alpha}_{(2)})\\ &\quad = \big[\big[s_{(\mu(A_{(2)})-\mu(\emptyset))\theta(\alpha_{(2)})}, s_{(\mu(A_{(2)})-\mu(\emptyset))\tau(\alpha_{(2)})}\big],\\ &\qquad \big(\big(\mu(A_{(2)}) - \mu(\emptyset)\big)u(\alpha_{(2)}), \big(\mu(A_{(2)}) - \mu(\emptyset)\big)v(\alpha_{(2)})\big)\big].\end{aligned}$$

Because $(\mu(A_{(1)}) - \mu(A_{(2)})), (\mu(A_{(2)}) - \mu(A_{(3)})) \in [0, 1]$ and $(\mu(A_{(1)}) - \mu(A_{(2)})) + (\mu(A_{(2)}) - \mu(A_{(3)})) = \mu(A_{(1)}) = 1$, we obtain

$$\begin{aligned}&\mathrm{IULSCA}_\mu(\tilde{\alpha}_1, \tilde{\alpha}_2)\\ &\quad = \big(\mu(A_{(1)}) - \mu(A_{(2)})\big)(\tilde{\alpha}_{(1)}) \oplus \big(\mu(A_{(2)}) - \mu(\emptyset)\big)(\tilde{\alpha}_{(2)})\\ &\quad = \Bigg[\big[s_{\sum_{i=1}^{2}(\mu(A_{(i)})-\mu(A_{(i+1)}))\theta(\alpha_{(i)})}, s_{\sum_{i=1}^{2}(\mu(A_{(i)})-\mu(A_{(i+1)}))\tau(\alpha_{(i)})}\big],\\ &\qquad \Bigg(\sum_{i=1}^{2}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)u(\alpha_{(i)}), \sum_{i=1}^{2}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)v(\alpha_{(i)})\Bigg)\Bigg].\end{aligned}$$

(ii) Let formula (11) hold for $n = k$ $(k \geqslant 2)$, then

$$
\begin{aligned}
&\mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_k)\\
&= \Bigg[\big[s_{\sum_{i=1}^{k}(\mu(A_{(i)})-\mu(A_{(i+1)}))\theta(\alpha_i)}, s_{\sum_{i=1}^{k}(\mu(A_{(i)})-\mu(A_{(i+1)}))\tau(\alpha_i)}\big],\\
&\quad\Bigg(\sum_{i=1}^{k}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)u(\alpha_i), \sum_{i=1}^{k}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)v(\alpha_i)\Bigg)\Bigg].
\end{aligned}
$$

When $n = k + 1$, from (ii) we get

$$
\begin{aligned}
&\mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_{k+1})\\
&= \Bigg[\Bigg[\big[s_{\sum_{i=1}^{k}(\mu(A_{(i)})-\mu(A_{(i+1)}))\theta(\alpha_i)}, s_{\sum_{i=1}^{k}(\mu(A_{(i)})-\mu(A_{(i+1)}))\tau(\alpha_i)}\big],\\
&\quad\Bigg(\sum_{i=1}^{k}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)u(\alpha_i), \sum_{i=1}^{k}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)v(\alpha_i)\Bigg)\Bigg]\Bigg]\\
&\quad\oplus\big[\big[s_{(\mu(A_{(k+1)})-\mu(A_{(k+2)}))\theta(\alpha_{(k+1)})}, s_{(\mu(A_{(k+1)})-\mu(A_{(k+2)}))\tau(\alpha_{(k+1)})}\big],\\
&\quad\big(\big(\mu(A_{(k+1)}) - \mu(A_{(k+2)})\big)u(\alpha_{(k+1)}), \big(\mu(A_{(k+1)}) - \mu(A_{(k+2)})\big)v(\alpha_{(k+1)})\big)\big]\\
&= \Bigg[\Bigg[\big[s_{\sum_{i=1}^{k+1}(\mu(A_{(i)})-\mu(A_{(i+1)}))\theta(\alpha_i)}, s_{\sum_{i=1}^{k+1}(\mu(A_{(i)})-\mu(A_{(i+1)}))\tau(\alpha_i)}\big],\\
&\quad\Bigg(\sum_{i=1}^{k+1}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)u(\alpha_i), \sum_{i=1}^{k+1}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)v(\alpha_i)\Bigg)\Bigg]\Bigg].
\end{aligned}
$$

Thus, formula (11) still holds for $n = k + 1$. □

The IULSCA operator is in fact an utilization of the mathematical weighted averaging operator. Next, we define the IULSCGM operator from the view of the geometric mean.

Definition 12. Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \ldots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \ldots, n\}$. The IULSCGM operator is formulated as:

$$
\mathrm{IULSCGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \bigotimes_{i=1}^{n} \tilde{\alpha}_{(i)}^{\mu(A_{(i)})-\mu(A_{(i+1)})}, \tag{12}
$$

where the notations are as shown in Definition 11.

Remark 2. If no interactions among elements in A exist, the IULSCGM operator reduces to the intuitionistic uncertain linguistic symmetrical weighted geometric mean (IULSWGM) operator:

$$
\mathrm{IULSWGM}_{w}(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) = \bigotimes_{i=1}^{n} \tilde{\alpha}_{(i)}^{w_i}, \tag{13}
$$

where the notations are as shown in the IULSWA operator.

Furthermore, if each ULV in $\tilde{\alpha}_i$ for all $i = 1, 2, \dots, n$ reduces to an LV, it reduces to the intuitionistic linguistic symmetrical Choquet geometric mean (ILSCGM) operator:

$$\mathrm{IULSWGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigotimes_{i=1}^{n} \tilde{\alpha}_{(i)}^{\mu(A_{(i)}) - \mu(A_{(i+1)})}, \tag{14}$$

where the notations are as shown in the ILSCA operator.

Theorem 2. *Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \dots, n\}$. Their aggregation value using the IULSCGM operator is still an IULV, where*

$$\begin{aligned} &\mathrm{IULSCGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \\ &= \Bigg[\Big[s_{\prod_{i=1}^{n} \theta(\alpha_i)^{\mu(A_{(i)}) - \mu(A_{(i+1)})}}, s_{\prod_{i=1}^{n} \tau(\alpha_i)^{\mu(A_{(i)}) - \mu(A_{(i+1)})}} \Big], \\ &\quad \Bigg(\prod_{i=1}^{n} u(\alpha_i)^{\mu(A_{(i)}) - \mu(A_{(i+1)})}, \prod_{i=1}^{n} v(\alpha_i)^{\mu(A_{(i)}) - \mu(A_{(i+1)})} \Bigg) \Bigg], \end{aligned} \tag{15}$$

and the notations are as shown in Definition 11.

Proof. Following Theorem 1, Theorem 2 can be easily proved. □

To show the rationality of the IULSCA and IULSCGM operators, let us briefly consider the following desirable properties.

PROPERTY 3. Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \dots, n\}$.

(i) *Commutativity*: Let $\{\tilde{\alpha}'_1, \tilde{\alpha}'_2, \dots, \tilde{\alpha}'_n\}$ be a permutation of $\{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\}$, then

$$\mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \mathrm{IULSCA}_{\mu}(\tilde{\alpha}'_1, \tilde{\alpha}'_2, \dots, \tilde{\alpha}'_n), \tag{16}$$

$$\mathrm{IULSCGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \mathrm{IULSCGM}_{\mu}(\tilde{\alpha}'_1, \tilde{\alpha}'_2, \dots, \tilde{\alpha}'_n); \tag{17}$$

(ii) *Idempotency*: When the IULVs $\tilde{\alpha}_i$ for all $i = 1, 2, \dots, n$ equal, namely, $\tilde{\alpha}_i = \tilde{\alpha}$ for any i, then

$$\mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \tilde{\alpha}, \tag{18}$$

$$\mathrm{IULSCGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \tilde{\alpha}; \tag{19}$$

(iii) *Comonotonicity*: Let $\tilde{\beta}_i = [[s_{\theta(\beta_i)}, s_{\tau(\beta_i)}], (u(\beta_i), v(\beta_i))]$ for all $i = 1, 2, \dots, n$ be another set of IULVs. If

$$\tilde{\alpha}_{(1)} \preceq \alpha_{(2)} \preceq \dots \preceq \alpha_{(n)} \quad \text{if and only if} \quad \tilde{\beta}_{(1)} \preceq \beta_{(2)} \preceq \dots \preceq \beta_{(n)} \tag{20}$$

for some permutation $(\cdot)$, then

$$\mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \preceq \mathrm{IULSCA}_{\mu}(\tilde{\beta}_1, \tilde{\beta}_2, \dots, \tilde{\beta}_n), \tag{21}$$

$$\mathrm{IULSCGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \preceq \mathrm{IULSCGM}_{\mu}(\tilde{\beta}_1, \tilde{\beta}_2, \dots, \tilde{\beta}_n); \tag{22}$$

(iv) *Boundary*: We have:

$$\begin{aligned} \min\{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\} &\preceq \mathrm{IULSCA}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \\ &\preceq \max\{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\}, \end{aligned} \tag{23}$$

$$\begin{aligned} \min\{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\} &\preceq \mathrm{IULSCGM}_{\mu}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \\ &\preceq \max\{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n\}. \end{aligned} \tag{24}$$

From Theorems 1 and 2 as well as Definitions 11 and 12, one can easily get the proofs and therefore we omit them.

3.2. *Generalized Shapley Choquet Operators*

Following Choquet integral, this subsection defines two intuitionistic uncertain linguistic operators. From Definitions 11 and 12, we know when there exist correlations between the importance of elements, the IULSCA and IULSCGM operators only consider the interactions between two adjoining coalitions and, where $i = 1, 2, \dots, n$, it seems to be unreasonable. To overall reflect the interactive characteristics between elements, this subsection further studies the intuitionistic uncertain linguistic operators with respect to the generalized Shapley function and Choquet integral.

The generalized Shapley function (Marichal, 2000) is formulated as:

$$\Phi_S(\mu, N) = \sum_{T \subseteq N \setminus S} \frac{(n-t-s)!t!}{(n-s+1)!} \big(\mu(S \cup T) - \mu(T)\big), \quad \forall S \subseteq N, \tag{25}$$

where n, s and t are the numbers of elements in N, S and T, respectively.

Following formula (25), when only one element exists in S, the Shapley function is derived as Shapley (1953):

$$\Phi_i(\mu, N) = \sum_{T \subseteq N \setminus i} \frac{(n-t-1)!t!}{n!} \big(\mu(i \cup T) - \mu(T)\big), \quad \forall i \in N. \tag{26}$$

Next, the generalized Shapley intuitionistic uncertain linguistic symmetrical Choquet averaging (GSIULSCA) operator is offered as:

Definition 13. Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \dots, n\}$. The GSIULSCA operator of $\tilde{\alpha}_i$ is formulated as:

$$\mathrm{GSIULSCA}_{\Phi}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigoplus_{i=1}^{n} \big(\Phi_{A_{(i)}}(\mu, A) - \Phi_{A_{(i+1)}}(\mu, A)\big)\tilde{\alpha}_{(i)}, \tag{27}$$

as shown in formula (25).

Theorem 3. *Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \dots, n\}$. Their aggregation value using the GSIULSCA operator is still an IULV, where*

$$\begin{aligned}
&\mathrm{GSIULSCA}_{\Phi}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) \\
&\quad = \Bigg[\Big[s_{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,A)-\Phi_{A_{(i+1)}}(\mu,A))\theta(\alpha_i)}, s_{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,A)-\Phi_{A_{(i+1)}}(\mu,A))\tau(\alpha_i)}\Big], \\
&\qquad \Bigg(\sum_{i=1}^{n} \big(\Phi_{A_{(i)}}(\mu, A) - \Phi_{A_{(i+1)}}(\mu, A)\big)u(\alpha_i), \\
&\qquad \sum_{i=1}^{n} \big(\Phi_{A_{(i)}}(\mu, A) - \Phi_{A_{(i+1)}}(\mu, A)\big)v(\alpha_i)\Bigg)\Bigg],
\end{aligned} \tag{28}$$

and the notations are as shown in Definition 11.

Proof. From formula (25), it is not difficult to know that $\sum_{i=1}^{n} \Phi_{A_{(i)}}(\mu, A) = 1$ and $\Phi_{A_{(i)}}(\mu, A) \geqslant 0$ for all $A_{(i)} \subseteq A$. According to Theorem 1, one can easily get the conclusion. □

Similarly, the generalized Shapley intuitionistic uncertain linguistic symmetrical Choquet geometric mean (GSIULSCGM) operator can be derived.

Definition 14. Let $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ for all $i = 1, 2, \dots, n$ be a set of IULVs, and let μ be an FM on $A = \{\tilde{\alpha}_i\}_{i \in N}$ with $N = \{1, 2, \dots, n\}$. The GSIULSCGM operator is formulated as:

$$\mathrm{GSIULSCGM}_{\Phi}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \bigotimes_{i=1}^{n} \tilde{\alpha}_{(i)}^{\Phi_{A_{(i)}}(\mu,A)-\Phi_{A_{(i+1)}}(\mu,A)}, \tag{29}$$

where Φ is as shown in formula (25).

Theorem 4. *Let* $\tilde{\alpha}_i = [[s_{\theta(\alpha_i)}, s_{\tau(\alpha_i)}], (u(\alpha_i), v(\alpha_i))]$ *for all* $i = 1, 2, \ldots, n$ *be a set of IULVs, and let* μ *be an FM on* $A = \{\tilde{\alpha}_i\}_{i \in N}$ *with* $N = \{1, 2, \ldots, n\}$. *Then, their aggregation value using the GSIULSCGM operator is an IULV, where*

$$\begin{aligned} &\mathrm{GSIULSCGM}_\Phi(\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n) \\ &= \Bigg[\Big[s_{\prod_{i=1}^n \theta(\alpha_i)^{\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)}}, s_{\prod_{i=1}^n \tau(\alpha_i)^{\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)}}\Big], \\ &\quad \left(\prod_{i=1}^n u(\alpha_i)^{\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)}, \prod_{i=1}^n v(\alpha_i)^{\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)}\right)\Bigg], \end{aligned} \tag{30}$$

and the notations are as shown in Definition 11.

Proof. Following Theorems 2 and 3, the results are easily derived. □

Similarly to the IULSCA and IULSCGM operators, the GSIULSCA and GSIULSCGM operators satisfy the properties: commutativity, idempotency, comonotonicity and boundary.

4. A New Procedure to GDM

Considering a multi-criteria GDM problem, where the importance of DMs and criteria might be correlative, respectively. Let $E = \{e_1, e_2, \ldots, e_q\}$ be the collection of DMs, let $C = \{c_1, c_2, \ldots, c_n\}$ be the collection of criteria, and let $A = \{a_1, a_2, \ldots, a_m\}$ be the collection of alternatives. We use $\tilde{A}^k = (\tilde{a}_{ij}^k)_{m \times n}$ to denote the IULV matrix given by e_k, and $\tilde{a}_{ij}^k = [[s_{\theta(a_{ij}^k)}, s_{\tau(a_{ij}^k)}], (u(a_{ij}^k), v(a_{ij}^k))]$ is the IULV for $a_i \in A$ with respect to $c_j \in C$.

When the numerical weighting vectors on DM and criteria sets are completely known, one can use the defined intuitionistic uncertain linguistic operators to develop a method for multi-criteria GDM with IULVs. However, the weighting information may be incompletely known because of various reasons.

4.1. *Models for Ascertaining Fuzzy Measures*

To build models for determining the weights of DMs and criteria, we define the following distance measure between any two IULVs:

Definition 15. Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$ and $\tilde{\beta} = [[s_{\theta(\beta)}, s_{\tau(\beta)}], (u(\beta), v(\beta))]$ be any two IULVs, the distance between $\tilde{\alpha}$ and $\tilde{\beta}$ is defined as:

$$D(\tilde{\alpha}, \tilde{\beta}) = \frac{(|\theta(\alpha) - \theta(\beta)| + |\tau(\alpha) - \tau(\beta)|)/t + |u(\alpha) - u(\beta)| + |v(\alpha) - v(\beta)|}{4}. \tag{31}$$

PROPERTY 4. Let $\tilde{\alpha} = [[s_{\theta(\alpha)}, s_{\tau(\alpha)}], (u(\alpha), v(\alpha))]$, $\tilde{\beta} = [[s_{\theta(\beta)}, s_{\tau(\beta)}], (u(\beta), v(\beta))]$ and $\tilde{\gamma} = [[s_{\theta(\gamma)}, s_{\tau(\gamma)}], (u(\gamma), v(\gamma))]$ be any three IULVs. Then, their distance measure listed in formula (31) satisfies:

(i) $D(\tilde{\alpha}, \tilde{\beta}) = D(\tilde{\beta}, \tilde{\alpha})$;

(ii) $D(\tilde{\alpha}, \tilde{\beta}) = 0$ if and only if $\begin{cases} \theta(\alpha) = \theta(\beta), \\ \tau(\alpha) = \tau(\beta), \end{cases}$ and $\begin{cases} u(\alpha) = u(\beta), \\ v(\alpha) = v(\beta); \end{cases}$

(iii) $D(\tilde{\alpha}, \tilde{\beta}) + D(\tilde{\beta}, \tilde{\gamma}) \geqslant D(\tilde{\alpha}, \tilde{\gamma})$.

Proof. Following formula (30), the conclusions can be easily derived. □

Note that the Hamming distance measure in Liu and Teng (2015) does not satisfy the condition (ii) in Property 4. Similar to models for determining the optimal fuzzy measures provided by Zhang *et al.* (2018) and Meng *et al.* (2016), we build the following programming models for ascertaining the optimal fuzzy measures on the DM set and the criteria set.

When the weights of DMs are incompletely known, the individual IULV matrices are used to establish model for ascertaining the FM μ^j on E for c_j, $j = 1, 2, \ldots, n$, where

$$\varphi^* = \min \sum_{k=1}^{q} \sum_{l=1}^{q} D(\tilde{A}_j^k, \tilde{A}_j^l) \phi_{e_k}(\mu^j, E),$$
$$\text{s.t.} \begin{cases} \mu^j(E) = 1, \\ \mu^j(S) \leqslant \mu^j(T), & \forall S, T \subseteq E, S \subseteq T, \\ \mu^j(e_k) \in W_{e_k}^j, \ \mu^j(e_k) \geqslant 0, & k = 1, 2, \ldots, q, \end{cases} \tag{32}$$

$D(\tilde{A}_j^k, \tilde{A}_j^l) = \sum_{i=1}^{m} D(\tilde{a}_{ij}^k, \tilde{a}_{ij}^l)$ with $\tilde{A}_j^k$ being the jth column of the individual IULV matrix $\tilde{A}^k$, $\phi_{e_k}(\mu^j, E)$ is the e_k's Shapley value, and $W_{e_k}^j$ is the given weighting information.

The FM μ^j has the following desirable characteristics: the closer the DM's evaluation value for other DMs' values is, the larger his/her FM value will be. This can decrease the influence of the unduly high or low values induced by DMs' limited expertise.

When the weights of criteria are incompletely known, the TOPSIS method (Negi, 1989) is adopted to construct model for the FM on C.

We let $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$ denote the collective IULV matrix, where $\tilde{a}_{ij} = [[s_{\theta(a_{ij})}, s_{\tau(a_{ij})}], (u(a_{ij}), v(a_{ij}))]$. Furthermore, we define $\tilde{h}^+ = \{\tilde{h}_1^+, \tilde{h}_2^+, \ldots, \tilde{h}_n^+\}$ and $\tilde{h}^- = \{\tilde{h}_1^-, \tilde{h}_2^-, \ldots, \tilde{h}_n^-\}$ with

$$\tilde{h}_j^+ = \Big([s_{\max_{1 \leqslant i \leqslant m} \theta(a_{ij})}, s_{\max_{1 \leqslant i \leqslant m} \tau(a_{ij})}], \Big(\max_{1 \leqslant i \leqslant m} u(a_{ij}), \min_{1 \leqslant i \leqslant m} v(a_{ij})\Big)\Big), \tag{33}$$

$$\tilde{h}_j^- = \Big([s_{\min_{1 \leqslant i \leqslant m} \theta(a_{ij})}, s_{\min_{1 \leqslant i \leqslant m} \tau(a_{ij})}], \Big(\min_{1 \leqslant i \leqslant m} u(a_{ij}), \max_{1 \leqslant i \leqslant m} v(a_{ij})\Big)\Big), \tag{34}$$

where $j = 1, 2, \ldots, n$.

Define

$$D_{ij} = \frac{D(\tilde{a}_{ij}, \tilde{h}_j^+)}{D(\tilde{a}_{ij}, \tilde{h}_j^+) + D(\tilde{a}_{ij}, \tilde{h}_j^-)}. \tag{35}$$

Then, model for the FM v on C is established:

$$\phi^* = \min \sum_{i=1}^{m} \sum_{j=1}^{n} D_{ij} \phi_{c_j}(v, C),$$

$$\text{s.t.} \begin{cases} v(C) = 1, & \\ v(S) \leqslant v(T), & \forall S, T \subseteq C, S \subseteq T, \\ v(c_j) \in W_{c_j},\ v(c_j) \geqslant 0, & j = 1, 2, \ldots, n, \end{cases} \tag{36}$$

where $\phi_{c_j}(v, C)$ is the c_j's Shapley value, and W_{c_j} is the given weighting information.

Remark 3. In models (32) and (36), we use the Shapley values of DMs and criteria to denote the weights, which overall reflect their interactions. If there are no interactions, models for additive weighting vectors are obtained.

4.2. *A New Algorithm*

Following the defined operators and built models for FMs, we present the following algorithm to multi-criteria GDM with IULVs:

Step 1: Assuming that the judgment of a_i for c_j offered by e_k is an IULV $\tilde{a}_{ij}^k = [[s_{\theta(a_{ij}^k)}, s_{\tau(a_{ij}^k)}], (u(a_{ij}^k), v(a_{ij}^k))]$, where $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, q$. Let $\tilde{A}^k = (\tilde{a}_{ij}^k)_{m \times n}$ denote the IULV matrix;

Step 2: Model (32) is used to derive the FM μ^j on E for the criterion c_j, where $j = 1, 2, \ldots, n$;

Step 3: The IULSCA or IULSCGM operator for the FMs FM μ^j for all $j = 1, 2, \ldots, n$ is adopted to compute the comprehensive IULV matrix $\tilde{A} = (\tilde{a}_{ij})_{m \times n}$, where $\tilde{a}_{ij} = \text{IULSCA}_{\mu^j}(\tilde{a}_{ij}^1, \tilde{a}_{ij}^2, \ldots, \tilde{a}_{ij}^q)$ or $\tilde{a}_{ij} = \text{IULSCGM}_{\mu^j}(\tilde{a}_{ij}^1, \tilde{a}_{ij}^2, \ldots, \tilde{a}_{ij}^q)$ for each pair of (i, j);

Step 4: For the comprehensive IULV matrix $\tilde{A}$, we adopt model (36) to obtain the FM on C;

Step 5: The IULSCA or IULSCGM operator is utilized to compute the comprehensive IULV $\tilde{a}_i = [[s_{\theta(a_i)}, s_{\tau(a_i)}], (u(a_i), v(a_i))]$ of a_i, where $\tilde{a}_i = \text{IULSCA}_\mu(\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{in})$ and $\tilde{a}_i = \text{IULSCGM}_\mu(\tilde{a}_{i1}, \tilde{a}_{i2}, \ldots, \tilde{a}_{in})$ for all $i = 1, 2, \ldots, m$;

Step 6: NS and NE are used to compute the score and accuracy of each comprehensive IULV;

Step 7: According to $NS(\tilde{a}_i)$ and $NA(\tilde{a}_i)$ for all $i = 1, 2, \ldots, m$, the ranking of alternatives as well as the best option(s) are derived;

Step 8: End.

In this procedure, we only apply the IULSCA or IULSCGM operator to make decisions. Similarly, we can adopt the GSIULSCA or GSIULSCGM operator to give the decision-making procedure.

5. Case Study and Comparison Analysis

5.1. *A Case Study*

To illustrate the utilization of the new procedure, this section provides an example. Meanwhile, comparison analysis is also made.

An investment company plans to invest a sum of money for deriving the best return (Liu and Jin, 2012). Four companies are selected as possible alternatives: a car company a_1; a computer company a_2; a TV company a_3; a food company a_4. Following four criteria: the risk index c_1; the growth index c_2; the social-political impact index c_3; the environmental impact index c_4, the investment company needs to make the best choice. Now, three DMs $E = \{e_1, e_2, e_3\}$ are invited to offer their judgments using IULVs obtained from the LTS $S = \{s_0$: very bad, s_1: bad, s_2: slightly bad, s_3: fair, s_4: slightly good, s_5: good, s_6: very good$\}$ following these four criteria. The IULV matrices are listed as shown in Tables 1–3.

Assume that the given weighting information of DMs is $w^1 = ([0.3, 0.5], [0.4, 0.5], [0.1, 0.2], [0.15, 0.3])$, $w^2 = ([0.2, 0.3], [0.2, 0.3], [0.3, 0.4], [0.35, 0.5])$, $w^3 = ([0.25, 0.4], [0.2, 0.3], [0.25, 0.3], [0.3, 0.45])$, and the given weighting information of criteria is

Table 1
Individual IULV matrix $\tilde{A}^1$.

	c_1	c_2	c_3	c_4
a_1	$[[s_5, s_5], (0.2, 0.7)]$	$[[s_2, s_3], (0.4, 0.6)]$	$[[s_5, s_6], (0.5, 0.5)]$	$[[s_3, s_4], (0.2, 0.6)]$
a_2	$[[s_4, s_5], (0.4, 0.6)]$	$[[s_5, s_5], (0.4, 0.5)]$	$[[s_3, s_4], (0.1, 0.8)]$	$[[s_4, s_4], (0.5, 0.5)]$
a_3	$[[s_3, s_4], (0.2, 0.7)]$	$[[s_4, s_4], (0.2, 0.7)]$	$[[s_4, s_5], (0.3, 0.7)]$	$[[s_4, s_5], (0.2, 0.7)]$
a_4	$[[s_6, s_6], (0.5, 0.4)]$	$[[s_2, s_3], (0.2, 0.8)]$	$[[s_3, s_4], (0.2, 0.6)]$	$[[s_3, s_3], (0.3, 0.6)]$

Table 2
Individual IULV matrix $\tilde{A}^2$.

	c_1	c_2	c_3	c_4
a_1	$[[s_4, s_4], (0.1, 0.7)]$	$[[s_3, s_4], (0.2, 0.7)]$	$[[s_3, s_4], (0.2, 0.8)]$	$[[s_6, s_6], (0.4, 0.5)]$
a_2	$[[s_5, s_6], (0.4, 0.5)]$	$[[s_3, s_4], (0.3, 0.6)]$	$[[s_4, s_5], (0.2, 0.6)]$	$[[s_3, s_4], (0.2, 0.7)]$
a_3	$[[s_4, s_5], (0.2, 0.6)]$	$[[s_4, s_4], (0.2, 0.7)]$	$[[s_2, s_3], (0.4, 0.6)]$	$[[s_3, s_4], (0.3, 0.7)]$
a_4	$[[s_5, s_5], (0.3, 0.6)]$	$[[s_4, s_5], (0.4, 0.5)]$	$[[s_2, s_3], (0.3, 0.6)]$	$[[s_4, s_4], (0.2, 0.6)]$

Table 3
Individual IULV matrix $\tilde{A}^3$.

	c_1	c_2	c_3	c_4
a_1	$[[s_5, s_5], (0.2, 0.6)]$	$[[s_3, s_4], (0.3, 0.7)]$	$[[s_4, s_5], (0.4, 0.5)]$	$[[s_4, s_4], (0.2, 0.7)]$
a_2	$[[s_4, s_5], (0.3, 0.7)]$	$[[s_5, s_5], (0.3, 0.6)]$	$[[s_2, s_3], (0.1, 0.8)]$	$[[s_3, s_4], (0.4, 0.6)]$
a_3	$[[s_4, s_4], (0.2, 0.7)]$	$[[s_5, s_5], (0.3, 0.6)]$	$[[s_1, s_3], (0.1, 0.8)]$	$[[s_4, s_4], (0.2, 0.7)]$
a_4	$[[s_3, s_4], (0.2, 0.7)]$	$[[s_3, s_4], (0.1, 0.7)]$	$[[s_4, s_5], (0.3, 0.6)]$	$[[s_5, s_5], (0.4, 0.5)]$

Table 4
Individual IULV matrix $\tilde{A}$.

	c_1	c_2	c_3	c_4
a_1	$[[s_{5.0000}, s_{5.0000}], (0.2000, 0.6750)]$	$[[s_{2.6000}, s_{3.6000}], (0.3400, 0.6600)]$	$[[s_{3.5000}, s_{4.5000}], (0.2800, 0.7100)]$	$[[s_{4.7000}, s_{4.7000}], (0.2700, 0.6300)]$
a_2	$[[s_{4.2000}, s_{5.2000}], (0.3500, 0.6300)]$	$[[s_{5.0000}, s_{5.0000}], (0.3400, 0.5600)]$	$[[s_{2.7000}, s_{3.7000}], (0.1000, 0.8000)]$	$[[s_{3.3000}, s_{4.0000}], (0.4300, 0.5700)]$
a_3	$[[s_{3.5000}, s_{4.0000}], (0.2000, 0.7000)]$	$[[s_{4.3000}, s_{4.3000}], (0.2300, 0.6700)]$	$[[s_{3.4000}, s_{4.4000}], (0.3300, 0.6700)]$	$[[s_{3.6500}, s_{4.5500}], (0.2350, 0.7000)]$
a_4	$[[s_{4.5000}, s_{5.000}], (0.3500, 0.5500)]$	$[[s_{3.0300}, s_{4.0000}], (0.1800, 0.6800)]$	$[[s_{2.7000}, s_{3.7000}], (0.2300, 0.60000)]$	$[[s_{3.9000}, s_{3.9000}], (0.3450, 0.5550)]$

$w = ([0.3, 0.4], [0.15, 0.25], [0.2, 0.25], [0.25, 0.3])$. To derive the best option, the procedure is offered as:

Step 1: According to individual IULV matrices and model (32), model for the FM μ^1 on the DN set E for the criterion c_1 is built:

$$
\begin{aligned}
\varphi^* = \min & -0.0139\big(\mu^1(e_1) - \mu^1(e_2, e_3)\big) + 0.0153\big(\mu^1(e_2) \\
& - \mu^1(e_1, e_3)\big) - 0.0014\big(\mu^1(e_3) - \mu^1(e_1, e_2)\big) + 0.5055, \\
\text{s.t. } & \begin{cases} \mu^1(e_1, e_2, e_3) = 1 \\ \mu^1(S) \leqslant \mu^1(T), \quad \forall S, T \subseteq \{e_1, e_2, e_3\}, S \subseteq T, \\ \mu^1(e_1) \in [0.3, 0.5], \quad \mu^1(e_2) \in [0.2, 0.3], \quad \mu^1(e_3) \in [0.25, 0.4]. \end{cases}
\end{aligned}
\tag{37}
$$

Solving model (37) using Matlab, we obtain:

$$
\begin{aligned}
& \mu^1(e_1) = \mu^1(e_1, e_2) = 0.5, \qquad \mu^1(e_2) = 0.2, \qquad \mu^1(e_3) = \mu^1(e_2, e_3) = 0.25, \\
& \mu^1(e_1, e_3) = \mu^1(e_1, e_2, e_3) = 1.
\end{aligned}
$$

Similarly, other FMs are:

$$
\begin{aligned}
& \mu^2(e_1) = \mu^2(e_1, e_2) = 0.4, \quad \mu^2(e_2) = 0.2, \qquad \mu^2(e_3) = 0.3, \\
& \mu^2(e_1, e_3) = \mu^2(e_2, e_3) = \mu^2(e_1, e_2, e_3) = 1; \\
& \mu^3(e_1) = 0.2, \qquad \mu^3(e_2) = \mu^3(e_3) = \mu^3(e_1, e_2) = \mu^3(e_2, e_3) = 0.3, \\
& \mu^3(e_1, e_3) = \mu^3(e_1, e_2, e_3) = 1; \\
& \mu^4(e_1) = 0.3, \qquad \mu^4(e_2) = \mu^4(e_1, e_2) = 0.35, \qquad \mu^4(e_3) = \mu^4(e_2, e_3) = 0.45, \\
& \mu^4(e_1, e_3) = \mu^4(e_1, e_2, e_3) = 1.
\end{aligned}
$$

Step 2: Using the IULSCA operator, collective IULV matrix $\tilde{A}$ is obtained shown in Table 4.

Step 3: Following comprehensive IULV matrix $\tilde{A}$, model for the FM v on the criteria set C is constructed:

$$
\begin{aligned}
\phi^* = \ & \min -0.00045\big(v(c_1) - v(c_2, c_3, c_4)\big) + 0.00054\big(v(c_2) - v(c_1, c_3, c_4)\big) \\
& -0.00027\big(v(c_3) - v(c_1, c_2, c_4)\big) + 0.00018\big(v(c_4) - v(c_1, c_2, c_3)\big) \\
& +0.00005\big(v(c_1, c_2) - v(c_3, c_4)\big) - 0.00036\big(v(c_1, c_3) - v(c_2, c_4)\big) \\
& -0.000013\big(v(c_1, c_4) - v(c_2, c_3)\big) + 0.00228
\end{aligned}
$$

$$\text{s.t.}\begin{cases} v(C)=1, \\ v(S)\leqslant v(T), \quad \forall S,T\subseteq C \text{ s.t. } S\subseteq T, \\ v(c_1)\in[0.3,0.4], \quad v(c_2)\in[0.15,0.25], \\ v(c_3)\in[0.2,0.25], \quad v(c_4)\in[0.25,0.3]. \end{cases} \tag{38}$$

Solving model (38) using Matlab, the following optimal fuzzy measure is obtained:

$$v(c_1)=v(c_1,c_2)=v(c_1,c_4)=v(c_1,c_2,c_4)=0.4, \quad v(c_2)=0.15,$$
$$v(c_3)=v(c_4)=v(c_2,c_3)=v(c_2,c_4)=v(c_3,c_4)=v(c_2,c_3,c_4)=0.25,$$
$$v(c_1,c_3)=v(c_1,c_2,c_3)=v(c_1,c_3,c_4)=v(c_1,c_2,c_3,c_4)=1.$$

Step 4: Following the FM v and the comprehensive IULV matrix $\tilde{A}$, the IULSCA operator is utilized to compute the comprehensive IULVs, where

$$\tilde{a}_1=\big[[s_{4.8600},s_{5.0100}],(0.2380,0.7020)\big],$$
$$\tilde{a}_2=\big[[s_{3.0750},s_{4.0000}],(0.2200,0.7170)\big],$$
$$\tilde{a}_3=\big[[s_{3.4750},s_{4.1000}],(0.2325,0.6925)\big],$$
$$\tilde{a}_4=\big[[s_{3.2700},s_{3.9450}],(0.2768,0.5813)\big].$$

Step 5: For comprehensive IULVs, the scores are:

$$NS(\tilde{a}_1)=-2.2898, \qquad NS(\tilde{a}_2)=-1.7581,$$
$$NS(\tilde{a}_3)=-1.7423, \qquad NS(\tilde{a}_4)=-1.0985.$$

Step 6: Following the scores of comprehensive IULVs, we derive $\tilde{a}_4\succ\tilde{a}_3\succ\tilde{a}_2\succ\tilde{a}_1$. Thus, the food company a_4 is the best option.

When the IULCM operator is adopted in Example 1, the comprehensive IULVs are:

$$\tilde{a}_1=\big[[s_{4.8590},s_{5.1093}],(0.2044,0.6529)\big],$$
$$\tilde{a}_2=\big[[s_{2.9953},s_{3.9490}],(0.1733,0.7077)\big],$$
$$\tilde{a}_3=\big[[s_{3.4090},s_{4.0705}],(0.2262,0.6920)\big],$$
$$\tilde{a}_4=\big[[s_{3.1114},s_{3.8592}],(0.2634,0.5768)\big].$$

From formula (5), we have

$$NS(\tilde{a}_1)=-2.22352, \qquad NS(\tilde{a}_2)=-1.8555,$$
$$NS(\tilde{a}_3)=-1.7419, \qquad NS(\tilde{a}_4)=-1.0924.$$

and $\tilde{a}_4\succ\tilde{a}_3\succ\tilde{a}_2\succ\tilde{a}_1$, by which the best option is the food company a_4.

Furthermore, if the GSIULSCA operator is used to compute the comprehensive IULVs, we derive:

$$\tilde{a}_1 = \big[[s_{4.1206}, s_{4.4595}], (0.2547, 0.6389)\big],$$
$$\tilde{a}_2 = \big[[s_{3.62997}, s_{4.2339}], (0.3512, 0.6025)\big],$$
$$\tilde{a}_3 = \big[[s_{3.7604}, s_{4.2892}], (0.2194, 0.6903)\big],$$
$$\tilde{a}_4 = \big[[s_{4.1006}, s_{4.2964}], (0.3239, 0.5615)\big].$$

From formula (5), we have

$$NS(\tilde{a}_1) = -1.6480, \qquad NS(\tilde{a}_2) = -0.9880,$$
$$NS(\tilde{a}_3) = -1.8952, \qquad NS(\tilde{a}_4) = -0.9977$$

and $\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$, which shows that the computer company a_2 is the best option.

Moreover, if the GSIULSCGM operator is used to compute the comprehensive IULVs, we obtain:

$$\tilde{a}_1 = \big[[s_{3.9942}, s_{4.4010}], (0.2394, 0.6343)\big],$$
$$\tilde{a}_2 = \big[[s_{3.5252}, s_{4.1967}], (0.3207, 0.5994)\big],$$
$$\tilde{a}_3 = \big[[s_{3.6854}, s_{4.2472}], (0.2145, 0.6911)\big],$$
$$\tilde{a}_4 = \big[[s_{3.9335}, s_{4.1805}], (0.3033, 0.5539)\big].$$

From formula (5), we have

$$NS(\tilde{a}_1) = -1.6575, \qquad NS(\tilde{a}_2) = -1.0760,$$
$$NS(\tilde{a}_3) = -1.8905, \qquad NS(\tilde{a}_4) = -1.0165$$

and $\tilde{a}_4 \succ \tilde{a}_2 \succ \tilde{a}_1 \succ \tilde{a}_3$. Thus, the food company a_4 is still the best option.

Following the IULSCA, IULCM, and GSIULSCGM operators, the same best option is derived, which is different from the best option that is obtained from the GSIULSCA operator. However, the difference between the scores of the comprehensive IULVs and derived from the GSIULSCA operator is small, which is less than 1%.

With respect to different methods as well as different aggregation operators, the final values and orders are shown in Table 5.

5.2. *Comparison Analysis*

The above example shows that different best options might be derived following different operators. Thus, when DMs make decisions, they should first choose the adopted aggregation operator. When we cannot make sure that no interaction exists between the importance

Table 5
Final values and orders.

Methods	Final values of a_1	Final values of a_2	Final values of a_4	Final values of a_4	Orders
The first method in Liu and Jin (2012)	$s_{1.2100}$	$s_{1.3900}$	$s_{1.0400}$	$s_{1.2600}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
The second method in Liu and Jin (2012)	$s_{1.2160}$	$s_{1.4840}$	$s_{1.0440}$	$s_{1.3360}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
The method in Chen and Li (2017) using the IULCWA operator	$s_{1.7427}$	$s_{1.5160}$	$s_{1.1814}$	$s_{1.7295}$	$\tilde{a}_1 \succ \tilde{a}_4 \succ \tilde{a}_2 \succ \tilde{a}_3$
The method in Chen and Li (2017) using the IULCGM operator	$s_{1.5622}$	$s_{1.3579}$	$s_{1.0842}$	$s_{1.5397}$	$\tilde{a}_1 \succ \tilde{a}_4 \succ \tilde{a}_2 \succ \tilde{a}_3$
The TODIM method in Liu and Teng (2015) with $= 1$	$\xi_1 = 0.8415$	$\xi_2 = 1$	$\xi_3 = 0$	$\xi_4 = 0.9606$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
The method in Liu and Shi (2015) using the LULFPEWA operator	$s_{2.2140}$	$s_{2.4010}$	$s_{1.9940}$	$s_{2.2510}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
The method in Liu and Shi (2015) using the LULFPEWG operator	$s_{0.7660}$	$s_{0.9540}$	$s_{0.6750}$	$s_{0.8310}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
The method in Liu *et al.* (2014a) using the IULWAHM operator with $p = q = 1$	$s_{1.3600}$	$s_{1.5500}$	$s_{1.1100}$	$s_{1.4400}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
The method in Liu *et al.* (2014a) using the IULWGHM operator with $p = q = 1$	$s_{1.3100}$	$s_{1.4800}$	$s_{1.1100}$	$s_{1.3700}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
New method using the IULSCA operator	$s_{-2.2898}$	$s_{-1.7581}$	$s_{-1.7423}$	$s_{-1.0985}$	$\tilde{a}_4 \succ \tilde{a}_3 \succ \tilde{a}_2 \succ \tilde{a}_1$
New method using the IULCM operator	$s_{2.2352}$	$ss_{1.8555}$	$s_{1.7419}$	$s_{1.0924}$	$\tilde{a}_4 \succ \tilde{a}_3 \succ \tilde{a}_2 \succ \tilde{a}_1$
New method using the GSIULSCA operator	$s_{1.6480}$	$s_{0.9880}$	$s_{1.8952}$	$s_{0.9977}$	$\tilde{a}_2 \succ \tilde{a}_4 \succ \tilde{a}_1 \succ \tilde{a}_3$
New method using the GSIULSCGM operator	$s_{1.6575}$	$s_{1.0760}$	$s_{1.8905}$	$s_{1.0165}$	$\tilde{a}_4 \succ \tilde{a}_3 \succ \tilde{a}_1 \succ \tilde{a}_2$

Note: ξ_i is the collective overall dominance of the alternative a_i (see formula (33) in Liu and Teng, 2015), where $i = 1, 2, 3, 4$. Furthermore, with the different values of θ, the same ranking order is derived (see Table 21 in Liu and Teng, 2015). The LULFPEWA and LULFPEWG operators are the abbreviation of the intuitionistic uncertain linguistic fuzzy powered Einstein weighted operator and the intuitionistic uncertain linguistic fuzzy Einstein weighted geometric operator, respectively (Liu and Shi, 2015). The IULCWA and IULCGM operators are respective of the abbreviation of the intuitionistic uncertain linguistic Choquet weighted averaging operator and the intuitionistic uncertain linguistic Choquet geometric mean operator (Chen and Li, 2016). The IULWAHM and IULWGHM operators are respective of the abbreviation of the intuitionistic uncertain linguistic weighted arithmetic Heronian mean operator and the intuitionistic uncertain linguistic weighted geometric Heronian mean operator (Liu *et al.*, 2014a). For different values of p and q, different best choices are obtained (see Table 6 in Liu *et al.*, 2014a). Methods in Liu and Jin (2012), Liu and Shi (2015), Liu *et al.* (2014a) calculate the ranking values using formula (3), while the new method adopts formula (5).

of DMs and that of criteria, we suggest DMs to adopt the IULSCA or IULSCM operator. To globally reflect the interactive characteristics, we recommend DMs to apply the GSIULSCA or GSIULSCGM operator. Note that no matter which method is chosen, we suggest DMs to apply the new operational laws and new ranking method. Thus, two differences exist between new operators and previous ones listed in Table 5: (i) the adopted operational laws; and (ii) reflecting the interactive characteristics among weights of elements.

The differences between new method and previous ones (Chen and Li, 2017; Liu and Jin, 2012; Liu and Teng, 2015; Liu and Shi, 2015; Liu *et al.*, 2014a) include:

(i) The new method uses symmetrical operations listed in Definition 7, while previous methods adopt the operations offered in Definition 5 that there are undesirable properties;
(ii) The new method is based on the new operators that overall reflect the interactions among weights of elements, while previous methods cannot;
(iii) The new method can address the situations where the weighting information is partly known or completely unknown, while previous methods are based on the assumption that the weighting information is completely known.

Note that all of reviewed methods listed in Table 5 can address GDM with IULVs. Following the above analysis, we suggest DMs to apply the new method to avoid the limitations in previous ones.

When there is no interaction, then the algorithm in subsection 4.2 reduces to a method for GDM with IULVs that uses additive measures. Because the adopted operations in methods (Liu and Jin, 2012; Liu and Teng, 2015; Liu and Shi, 2015; Liu *et al.*, 2014a) have the limitations listed in Section 2.1, we suggest DMs to apply the new operations given in Definition 7. Table 5 shows that influences of the listed issues in introduction for the final ranking of alternatives.

6. A New Procedure to GDM

Due to the advantages of IULVs to express the judgments of DMs, many GDM methods with intuitionistic uncertain linguistic information are developed, such as Liu and Jin (2012) introduced a GDM method based on the intuitionistic uncertain linguistic hybrid geometric operator; Liu (2014) presented a GDM method based on the intuitionistic uncertain linguistic Hamacher aggregation operator; Liu and Teng (2015) developed a TODIM method for intuitionistic uncertain linguistic decision making; Liu *et al.* (2014a) provided a GDM method based on the intuitionistic uncertain linguistic Heronian mean operator, and GDM methods based on the intuitionistic uncertain linguistic partitioned Bonferroni mean operator were studied by Liu *et al.* (2014b) and Liu and Liu (2017). Furthermore, Liu and Shi (2015) proposed a GDM method based on the intuitionistic uncertain linguistic powered Einstein aggregation operator. Due to the used operations on IULVs, unreasonable rankings of alternatives as well as decisions may be derived following these methods because these methods are based on the aggregation operators using the scalar multiplication. Furthermore, all of these methods are based on the assumptions that the weighting information is completely known and independent. Nevertheless, in some situations, the weighting information is incompletely known and interdependent. All of these issues restrict the application of IULVs. Thus, this paper continued to study decision making with intuitionistic uncertain linguistic information and offered a new GDM method. Compared with these previous methods, there are three main contributions of the new method: (i) it is based on the new operations on IULVs that avoid issues in previous operations; (ii) it can address the situation where the weighting information is incompletely known; (iii) it can cope with the situation where the weighting information is interdependent.

From the given example, we can find that the different best options may be derived using different methods. Thus, it is important to select the appropriate method following the needs of decision making. Considering the issues in previous methods, we suggest the DMs to apply the new one. This paper only considered the utilization of the new method in the investment problem, and it could also be used in some other fields, such as clustering analysis, human resource management, pattern recognition, and expert system. Additionally, we will continue to research the theory of decision making with IULVs.

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