INFORMATICA, 2017, Vol. 28, No. 4, 665–685

DOI: http://dx.doi.org/10.15388/Informatica.2017.151

An Extended VIKOR Method for Decision Making Problem with Interval-Valued Linguistic Intuitionistic Fuzzy Numbers Based on Entropy

Peide LIU[1]*, Xiyou QIN
School of Management Science and Engineering, Shandong University of Finance and Economics Jinan Shandong 250014, China
e-mail: peide.liu@gmail.com

Received: October 2016; accepted: August 2017

Abstract. In this paper, with respect to how to express the complex fuzzy information, we proposed the concept of interval-valued linguistic intuitionistic fuzzy numbers (IVLIFNs), whose membership and non-membership are represented by interval-valued linguistic terms, then the Hamming distance is defined, further, we also proposed the interval-valued linguistic intuitionistic fuzzy entropy. Considering that the VIKOR method can achieve the maximum "group utility" and minimum of "individual regret", we extended the VIKOR method to process the interval-valued linguistic intuitionistic fuzzy information (IVLIFI), and proposed an extended VIKOR method for the multiple attribute decision making (MADM) problems with IVLIFI. And an illustrative example shows the effectiveness of the proposed approach.

Key words: multiple attribute decision making, interval-valued linguistic intuitionistic fuzzy numbers, interval-valued linguistic intuitionistic fuzzy entropy, VIKOR method.

1. Introduction

Decision making has been widely used in economic, politics, military, management and other fields. But in real decision making, the decision information is often incomplete and fuzzy, it is difficult to obtain the values by the exact numbers. How to express this kind of information is a very worthwhile research issue. The theory of fuzzy sets (FSs) proposed by Zadeh (1965) is an important tool to describe fuzzy information. However, because the fuzzy set only has a membership function, sometimes, it is difficult to express some complex fuzzy information, such as the voting problem in which exists some of the opposition and abstain from voting. In order to solve the defects, Atanassov (1986, 1989a) proposed intuitionistic fuzzy set (IFS), which includes a membership function and a non-membership function. However, the membership function and non-membership function in IFS can only take the crisp numbers; sometimes it is still difficult to express the complex fuzzy information. Further, Atanassov (1989b), Atanassov and Gargov (1989) extended the membership degree and non-membership degree of IFS to interval numbers,

*Corresponding author.

and proposed the interval-valued intuitionistic fuzzy set (IVIFS), and defined some operational rules and relations of IVIFS. Now, research on IFS and IVIFS has been a hotspot, and a great number of research achievements about IFS and IVIFS are made (Liu, 2017; Liu and Chen, 2017; Liu *et al.*, 2017; Liu and Li, 2017).

Generally, on one hand, in a quantitative setting, we use the numerical values to express the information, and can get an effective result. On the other hand, when we present a decision problem in a qualitative setting, it is difficult to express the fuzzy information by the exact numerical value, and it is more feasible by linguistic terms rather than numerical values (Herrera *et al.*, 2000; Xu, 2004; Cabrerizo *et al.*, 2013; Dong and Herrera-Viedma, 2015; Massanet *et al.*, 2014; Liu *et al.*, 2016; Liu and Teng, 2016; Liu and Yu, 2014). In order to easily express the membership degree and non-membership degree of IFNs in a qualitative setting, one concept called linguistic intuitionistic fuzzy numbers (LIFNs) is proposed by Chen and Liu (2015), in which the membership degree and non-membership degree are expressed by linguistic variables based on the given linguistic term set. LIFNs combine the advantages of both linguistic term sets and IFNs, they can more effectively deal with the fuzzy information and have gotten more and more concerns in decision fields.

The entropy has become a hotspot of the research field. The entropy is originated from the Thermodynamics. Shannon introduced it into information theory to measure the uncertainty of information. Zadeh (1965) was first one to use the entropy to measure the fuzziness of a fuzzy set. Later, Burillo and Bustince (1996) defined the intuitionistic fuzzy entropy to measure the degree of hesitation in the intuitionistic fuzzy sets. Zhao and Xu (2016) proposed the entropy measures for interval-valued intuitionistic fuzzy information from a comparative perspective. Then, Guo (2004) and Liu *et al.* (2005) presented the axiomatic definition of interval-valued intuitionistic fuzzy entropy. Wang and Wei (2011) extended entropy of IFSs to IVIFSs. Subsequently, Gao and Wei (2012) defined a new entropy formula based on the improved Hamming distance for IVIFSs. However, these entropy definitions have some defects. For example, the constraint for the maximum values of entropy (Guo, 2004) considers only one aspect of uncertainty from fuzziness and neglects the other aspect of uncertainty from the lack of knowledge. Therefore, Xie and Lv (2016) improved the axiomatic definition of entropy for IVIFSs and proposed a new entropy formula which can consider both uncertainty from fuzziness and the lack of information, which can reflect the amount of information better.

In addition, the VIKOR method is an important decision tool to process the fuzzy MADM problems because it can consider the maximum "group utility" and minimum of "individual regret" and can consider two kinds of particular measures of "closeness" to the virtual ideal solution and the virtual negative ideal solution, simultaneously. Comparing with the other decision making methods, such as TOPSIS, ELECTRE, TODIM etc., the advantage of VIKOR can give one compromise optimal choice or a group of choices with no differences based on the maximum "group utility" and minimum of "individual regret", however, the other methods just can provide an optimal choice. Because the traditional VIKOR method can only deal with the crisp numbers, some new extensions of VIKOR for the different fuzzy information have been studied. Liu and Wang (2011) extended VIKOR to generalized interval-valued trapezoidal fuzzy numbers. Wu *et al.* (2016a) extended VIKOR to linguistic information. Liao *et al.* (2015) extended VIKOR to Hesitant fuzzy linguistic information. Keshavarz Ghorabaee *et al.* (2015) extended VIKOR to interval type-2 fuzzy sets. Zhang and Wei (2013) extended VIKOR to deal with HFS. Liu and Wu (2012) extended VIKOR to process the multi-granularity linguistic variables. Zhang *et al.* (2010) extended VIKOR to process the hybrid information, including linguistic variables, crisp numbers, interval numbers, triangular fuzzy numbers, trapezoid fuzzy numbers, and so on. Du and Liu (2011) extended VIKOR to deal with intuitionistic trapezoidal fuzzy numbers. Wu *et al.* (2016b) extended the VIKOR method to linguistic information and applied it to nuclear power industry. Gul *et al.* (2016) applied the VIKOR method to the state of art literature review. Kuo *et al.* (2015) extended VIKOR to develop a green supplier selection model. However, now it cannot process the IVLIFNs.

The IVIFNs are more convenient to express the complex fuzzy information than IFNs, however, their membership degree and non-membership degree are expressed by interval numbers. Similarly, in qualitative setting, it is easier to express the membership degree and non-membership degree by interval-valued linguistic variables than by interval numbers. So one of our goals in this paper is to propose the interval-valued linguistic intuitionistic fuzzy numbers (IVLIFNs), in which the membership degree and the non-membership degree are presented by interval-valued linguistic variables. Secondly, we also put forward the conception of interval-valued linguistic intuitionistic fuzzy entropy which can describe the uncertainty from fuzziness and the uncertainty from lack of knowledge of IVLIFNs better. Thirdly, we extend the VIKOR to IVLIFNs because the existing VIKOR didn't deal with IVLIFNs, and propose an extended VIKOR method to solve the MADM problems in which the attribute values take the form of IVLIFNs and the attribute weights are unknown. Here, interval-valued linguistic intuitionistic fuzzy entropy will be used to determine each attribute's weight.

In order to do that, the remainder of this paper is as follows. In Section 2, we briefly review some basic concepts of IVIFNs and IVIF Entropy, we also propose the notion of IVLIFNs and define the hamming distance of IVLIFNs, interval-valued linguistic intuitionistic fuzzy entropy. Further, the traditional VIKOR method was introduced. In Section 3, we extend the traditional VIKOR method to the IVLIF information, and a MADM approach is proposed. In Section 4, we give a numerical example to elaborate the effectiveness and feasibility of our approach. The comparison with other methods is conducted in Section 5. Concluding remark is made in Section 6.

2. Preliminaries

2.1. *Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs)*

Definition 1 (Atanassov and Gargov, 1989). Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite and non-empty universe of discourse. An interval-valued intuitionistic fuzzy set $\tilde{A}$ is given by:

$$\begin{aligned}\tilde{A} &= \big\{\langle x, \tilde{u}_{\tilde{A}}(x), \tilde{v}_{\tilde{A}}(x)\rangle \big| x \in X\big\}\\ &= \big\{\langle x, \big[u_A^-(x), u_A^+(x)\big], \big[v_A^-(x), v_A^+(x)\big]\rangle \big| x \in X\big\},\end{aligned} \tag{1}$$

where $u_A^-(x) \in [0,1]$, $u_A^+(x) \in [0,1]$ and $v_A^-(x) \in [0,1]$, $v_A^+(x) \in [0,1]$, the numbers $\tilde{u}_{\tilde{A}}(x)$ and $\tilde{v}_{\tilde{A}}(x)$ represent the membership degree and non-membership degree of the element x to the set $\tilde{A}$ respectively, and $u_A^-(x) \leqslant u_A^+(x)$, $v_A^-(x) \leqslant v_A^+(x)$, $u_A^+(x) + v_A^+(x) \leqslant 1$.

For a given $x \in X$, $\tilde{\pi}(x) = [1 - u_A^+(x) - v_A^+(x), 1 - u_A^-(x) - v_A^-(x)]$ is called the interval-valued intuitionistic fuzzy hesitation degree.

For convenience, we use $IVIF(X)$ to express the set of all IVIFS.

Definition 2 (Atanassov and Gargov, 1989). If $\tilde{A} = \{\langle x, [u_A^-(x), u_A^+(x)], [v_A^-(x), v_A^+(x)]\rangle | x \in X\}$, $\tilde{B} = \{\langle x, [u_B^-(x), u_B^+(x)], [v_B^-(x), v_B^+(x)]\rangle | x \in X\}$ are two IVIFSs, the basic operations can be defined as follows.

$$\text{(1)}\quad \tilde{A} \subseteq \tilde{B} \text{ if and only if } \begin{cases} u_A^-(x) \leqslant u_B^-(x), & u_A^+(x) \leqslant u_B^+(x), \\ v_A^-(x) \geqslant v_B^-(x), & v_A^+(x) \geqslant v_B^+(x); \end{cases} \tag{2}$$

$$\text{(2)}\quad \tilde{A} = \tilde{B} \text{ if and only if } \tilde{A} \subseteq \tilde{B},\ \tilde{A} \supseteq \tilde{B}; \tag{3}$$

$$\text{(3)}\quad \tilde{A}^c = \{\langle x, [v_A^-(x), v_A^+(x)], [u_A^-(x), u_A^+(x)]\rangle | x \in X\}. \tag{4}$$

Definition 3 (Delgado *et al.*, 1998). If $\tilde{A} = \{\langle x_i, [u_A^-(x_i), u_A^+(x_i)], [v_A^-(x_i), v_A^+(x_i)]\rangle | x_i \in X\}$, $\tilde{B} = \{\langle x_i, [u_B^-(x_i), u_B^+(x_i)], [v_B^-(x_i), v_B^+(x_i)]\rangle | x_i \in X\}$ are two IVIFSs, then Hamming distance between $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$d(\tilde{A}, \tilde{B}) = \frac{1}{4n} \sum_{i=1}^{n} \big[|u_A^-(x_i) - u_B^-(x_i)| + |u_A^+(x_i) - u_B^+(x_i)| + |v_A^-(x_i) - v_B^-(x_i)| + |v_A^+(x_i) - v_B^+(x_i)| + |\pi_A^-(x_i) - \pi_B^-(x_i)| + |\pi_A^+(x_i) - \pi_B^+(x_i)| \big]. \tag{5}$$

2.2. *Interval-Valued Intuitionistic Fuzzy Entropy*

In this section, we will introduce the interval-valued intuitionistic fuzzy entropy which can consider both uncertainty from fuzziness and from the lack of information, the fuzzier the information, and the more information is missing, the greater the entropy value.

Definition 4 (Xie and Lv, 2016). Let $\forall \tilde{A} \in IVIF(X)$, the mapping E: $IVIF(X) \to [0,1]$ is called entropy if E satisfies the following conditions:

Condition 1. $E(\tilde{A}) = 0$ if and only if $\tilde{A}$ is a crisp set, the crisp set includes $\tilde{A} = \{\langle x_i, (1,1), (0,0)\rangle | x_i \in X\}$ and $\tilde{A} = \{\langle x_i, (0,0), (1,1)\rangle | x_i \in X\}$;

Condition 2. $E(\tilde{A}) = 1$ if and only if $[v_A^-(x_i), v_A^+(x_i)] = [u_A^-(x_i), u_A^+(x_i)] = [0,0]$ for every $x_i \in X$;

Condition 3. $E(\tilde{A}) = E(\tilde{A}^C)$ for every $\tilde{A} \in IVIFS(X)$;

Condition 4. For any $\tilde{B} \in IVIFS(X)$ if $\tilde{A} \subseteq \tilde{B}$ when $u_B^-(x_i) \leqslant v_B^-(x_i)$, $u_B^+(x_i) \leqslant v_B^+(x_i)$ for every $x_i \in X$, or $\tilde{A} \supseteq \tilde{B}$ when $u_B^-(x_i) \geqslant v_B^-(x_i)$, $u_B^+(x_i) \geqslant v_B^+(x_i)$ for every $x_i \in X$, then $E(\tilde{A}) \leqslant E(\tilde{B})$.

Theorem 1. *Let* $X = \{x_1, x_2, \dots, x_n\}$ *be a universe,* $\tilde{A} = \{\langle x_i, [u_A^-(x_i), u_A^+(x_i)], [v_A^-(x_i), v_A^+(x_i)]\rangle | x_i \in X\}$*, the formula of the entropy is as follows*:

$$E(\tilde{A}) = \frac{1}{n}\sum_{i=1}^{n} \frac{2 - |u_A^+(x_i) - v_A^+(x_i)|^2 - |u_A^-(x_i) - v_A^-(x_i)|^2 + (\pi_A^-(x_i))^2 + (\pi_A^+(x_i))^2}{4}. \tag{6}$$

2.3. *VIKOR Method*

The VIKOR is a good MADM method which can consider both the group utility and individual regret. The decision making problem can be expressed as follows.

Suppose there are m alternatives which are presented as $X_1, X_2, \dots, X_m$, and there are n attributes which are presented as $A_1, A_2, \dots, A_n$, the evaluation value of alternative X_i with respect to attribute A_j is expressed by x_{ij}, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$. We suppose the x_j^* expresses the virtual positive ideal value and the x_j^- expresses virtual negative ideal value under the attribute X_j. $w = (w_1, w_2, \dots, w_n)^T$ is the attribute weight vector which satisfies $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$. The compromise ranking by VIKOR method is begun with the form of L_p-metric [21].

$$L_{pi} = \left\{ \sum_{j=1}^{n} \left[\frac{(f_j^* - f_{ij})}{(f_j^* - f_j^-)} \right]^p \right\}^{1/p} \quad 1 \leqslant p \leqslant \infty;\ i = 1, 2, \dots, m. \tag{7}$$

In the VIKOR method, the maximum group utility can be presented by min S_i and minimum individual regret can be presented by min R_i, where $S_i = L_{1,i}$, and $R_i = L_{\infty,i}$.

The steps of the VIKOR method can be described as follows:

Step 1: Normalize the decision matrix.

Step 2: Compute the virtual positive ideal x_j^* and the virtual negative ideal x_j^- values under the attribute A_j, we have

$$x_j^* = \max_i x_{ij}, \qquad x_j^- = \min_i x_{ij}. \tag{8}$$

Step 3: Computing the group utility value and individual regret value R_i; $i = 1, 2, \dots, m$, as follows:

$$S_i = \sum_{j=1}^{n} w_j \big(x_j^* - x_{ij}\big) / \big(x_j^* - x_j^-\big), \tag{9}$$

$$R_i = \max_j w_j \big(x_j^* - x_{ij}\big) / \big(x_j^* - x_j^-\big). \tag{10}$$

Step 4: Compute the values: $i = 1, 2, \dots, m$, according to the following formulas:

$$Q_i = \frac{v(S_i - S^*)}{(S^- - S^*)} + \frac{(1-v)(R_i - R^*)}{(R^- - R^*)}, \tag{11}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, v is the balance parameter of decision strategy which can balance the factors between group utility and individual regret. Then, it explains that considering "the maximum group utility" is more and considering "the minimum individual regret" is less, then, it explains that considering "the minimum individual regret" is more and considering "the maximum group utility" is less. (In our research, we suppose that the "minimum individual regret" and "the maximum group utility" are both important.)

Step 5: Rank all the alternatives. According to the values S and Q, we will get three ranking results, and then we can obtain a set of the compromise solutions.

Step 6: Obtain a compromise solution $X^{(1)}$, which is in the first position of all ranking alternatives produced by the value Q (i.e. the alternative is with minimum value Q) if it meets the following two conditions:

Condition 1. Acceptable advantage: $Q(X^{(2)}) - Q(X^{(1)}) \geqslant \frac{1}{m-1}$, where $Q(X^{(2)})$ is the Q value in the second position of all ranking alternatives produced by the value Q, and m is the number of alternatives;

Condition 2. Acceptable stability. Alternative $X^{(1)}$ must also be in the first position of all ranking alternatives produced by the value by S and R.

If one of above two conditions is not met, we will get a collection of compromise alternatives and not one compromise solution.

(1) If condition 2 is not met, then we can get that alternatives $X^{(1)}$ and $X^{(2)}$ should be compromise solutions.
(2) If condition 1 is not met, then the maximum M can be gotten by the formula $Q(X^{(M)}) - Q(X^{(1)}) < MQ$, $MQ = \frac{1}{m-1}$, and we can get the alternatives, $X^{(1)}, X^{(2)}, \dots, X^{(M)}$ are compromise solutions.

Based on the above analysis, we know that the best solution is the one with the minimum Q value when the conditions 1 and 2 are met, and when one of two conditions is not met, we may have more than one compromise solution.

The VIKOR method is a useful tool for solving the MADM problems, and it can get a collection of compromise solutions or one compromise solution according to some conditions based on the maximum "group utility" and minimum "individual regret".

3. Distance and Entropy for Interval-Valued Linguistic Intuitionistic Fuzzy Sets (IVLIFSs)

3.1. *Interval-Valued Linguistic Intuitionistic Fuzzy Sets*

In order to easily understand the IVLIFSs, firstly, we introduce the basic concept about the linguistic variables as follows.

In real decision making, there is a lot of qualitative information, which is easily represented by means of linguistic variables (Delgado *et al.*, 1998; Herrera and Herrera-Viedma, 1997; Xu, 2004; Zadeh, 1975).

Suppose that $S = \{s_i \,|\, i = 1, 2, \dots, t\}$ is a linguistic term set with odd cardinality, where t is a positive integer, s_i represents a possible value for a linguistic variable. For example, a set of nine linguistic terms S could be given as follows (see Chen and Liu, 2015):

$$S = \{s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{slightly poor}, s_4 = \text{fair}, \\ s_5 = \text{slightly good}, s_6 = \text{good}, s_7 = \text{very good}, s_8 = \text{extremely good}\}.$$

Next, we will extend the IVIFSs to the interval-valued linguistic intuitionistic fuzzy sets (IVLIFSs). The concept and related properties of IVLIFSs are shown as follows:

Definition 5. Let $R = \{\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_n\}$ be a finite and non-empty universe of discourse, $S_{[0,t]}$ is a linguistic term universe of discourse with odd cardinality and $s_{\alpha_i^-}, s_{\alpha_i^+}, s_{\beta_i^-}, s_{\beta_i^+} \in S_{[0,t]}$. An interval-valued linguistic intuitionistic fuzzy number can be expressed by $\tilde{r}_i = (s_{\tilde{\alpha}_i}, s_{\tilde{\beta}_i}) = ([s_{\alpha_i^-}, s_{\alpha_i^+}], [s_{\beta_i^-}, s_{\beta_i^+}])$, $i = 1, 2, \dots, n$. Then an interval-valued linguistic intuitionistic fuzzy set $\tilde{M}$ can be given by:

$$\tilde{M} = \left\{(s_{\tilde{\alpha}_{\tilde{M}}(\tilde{r}_i)}, s_{\tilde{\beta}_{\tilde{M}}(\tilde{r}_i)}) \middle| \tilde{r} \in R\right\} = \left\{\left([s_{\alpha^-_{\tilde{M}}(\tilde{r}_i)}, s_{\alpha^+_{\tilde{M}}(\tilde{r}_i)}], [s_{\beta^-_{\tilde{M}}(\tilde{r}_i)}, s_{\beta^+_{\tilde{M}}(\tilde{r}_i)}]\right) \middle| \tilde{r}_i \in R\right\} \quad (12)$$

where $\alpha^-_{\tilde{M}}(\tilde{r}_i) \in [0, t]$, $\alpha^+_{\tilde{M}}(\tilde{r}_i) \in [0, t]$ and $\beta^-_{\tilde{M}}(\tilde{r}_i) \in [0, t]$, $\beta^+_{\tilde{M}}(\tilde{r}_i) \in [0, t]$, the numbers $s_{\tilde{\alpha}_{\tilde{M}}(\tilde{r}_i)}$ and $s_{\tilde{\beta}_{\tilde{M}}(\tilde{r}_i)}$ represent the membership degree and non-membership degree to the number $\tilde{r}_i$ of set $\tilde{M}$, respectively, and $\alpha^-_{\tilde{M}}(\tilde{r}_i) \leqslant \alpha^+_{\tilde{M}}(\tilde{r}_i)$, $\beta^-_{\tilde{M}}(\tilde{r}_i) \leqslant \beta^+_{\tilde{M}}(\tilde{r}_i)$, $\alpha^+_{\tilde{M}}(\tilde{r}_i) + \beta^+_{\tilde{M}}(\tilde{r}_i) \leqslant t$.

Moreover, $s_{\tilde{\pi}_{\tilde{M}}(\tilde{r}_i)}$ is called the interval-valued linguistic intuitionistic fuzzy hesitation degree, and

$$\tilde{\pi}_{\tilde{M}}(\tilde{r}_i) = \left[\pi^-_{\tilde{M}}(\tilde{r}_i), \pi^+_{\tilde{M}}(\tilde{r}_i)\right] = \left[1 - \alpha^+_{\tilde{M}}(\tilde{r}_i) - \beta^+_{\tilde{M}}(\tilde{r}_i), 1 - \alpha^-_{\tilde{M}}(\tilde{r}_i) - \beta^-_{\tilde{M}}(\tilde{r}_i)\right].$$

Definition 6. Let $R_{(0,t)}$ be the set of all IVLIFSs based on $S_{(0,t)}$ and $\tilde{M} = \{([s_{\alpha^-_{\tilde{M}}(\tilde{r}_i)}, s_{\alpha^+_{\tilde{M}}(\tilde{r}_i)}], [s_{\beta^-_{\tilde{M}}(\tilde{r}_i)}, s_{\beta^+_{\tilde{M}}(\tilde{r}_i)}]) | \tilde{r}_i \in R\}$, $\tilde{N} = \{([s_{\alpha^-_{\tilde{N}}(\tilde{r}_i)}, s_{\alpha^+_{\tilde{N}}(\tilde{r}_i)}], [s_{\beta^-_{\tilde{N}}(\tilde{r}_i)}, s_{\beta^+_{\tilde{N}}(\tilde{r}_i)}]) | \tilde{r}_i \in R\}$, the following basic operations can be defined:

$$(1) \quad \tilde{M} \subseteq \tilde{N} \text{ if and only if } \begin{cases} \alpha^-_{\tilde{M}}(\tilde{r}_i) \leqslant \alpha^-_{\tilde{N}}(\tilde{r}_i), & \alpha^+_{\tilde{M}}(\tilde{r}_i) \leqslant \alpha^+_{\tilde{N}}(\tilde{r}_i), \\ \beta^-_{\tilde{M}}(\tilde{r}_i) \geqslant \beta^-_{\tilde{N}}(\tilde{r}_i), & \beta^+_{\tilde{M}}(\tilde{r}_i) \geqslant \beta^+_{\tilde{N}}(\tilde{r}_i), \end{cases} \quad (13)$$

$$(2) \quad \tilde{M} = \tilde{N} \text{ if and only if } \tilde{M} \subseteq \tilde{N} \text{ and } \tilde{M} \supseteq \tilde{N}, \quad (14)$$

$$(3) \quad \tilde{M}^c = \left\{\left([s_{\beta^-_{\tilde{M}}(\tilde{r}_i)}, s_{\beta^+_{\tilde{M}}(\tilde{r}_i)}], [s_{\alpha^-_{\tilde{M}}(\tilde{r}_i)}, s_{\alpha^+_{\tilde{M}}(\tilde{r}_i)}]\right) \middle| \tilde{r}_i \in R\right\}. \quad (15)$$

3.2. *The Distance Between Two IVLIFSs*

Definition 7. Let $R_{(0,t)}$ be the set of all IVLIFSs based on $S_{(0,t)}$ and $\tilde{M}=\{([s_{\alpha^-_{\tilde{M}}(\tilde{r}_i)}, s_{\alpha^+_{\tilde{M}}(\tilde{r}_i)}], [s_{\beta^-_{\tilde{M}}(\tilde{r}_i)}, s_{\beta^+_{\tilde{M}}(\tilde{r}_i)}]).|\tilde{r}_i\in R\}$, $\tilde{N}=\{([s_{\alpha^-_{\tilde{N}}(\tilde{r}_i)}, s_{\alpha^+_{\tilde{N}}(\tilde{r}_i)}], [s_{\beta^-_{\tilde{N}}(\tilde{r}_i)}, s_{\beta^+_{\tilde{N}}(\tilde{r}_i)}])|\tilde{r}_i\in R\}$, then Hamming distance measure between $\tilde{M}$ and $\tilde{N}$ is defined as follows:

$$d(\tilde{M},\tilde{N})=\frac{1}{4tn}\sum_{i=1}^{n}\left|\alpha^-_{\tilde{M}}(r_i)-\alpha^-_{\tilde{N}}(r_i)\right|+\left|\alpha^+_{\tilde{M}}(r_i)-\alpha^+_{\tilde{N}}(r_i)\right|+\left|\beta^-_{\tilde{M}}(r_i)-\beta^-_{\tilde{N}}(r_i)\right| \\ +\left|\beta^+_{\tilde{M}}(r_i)-\beta^+_{\tilde{N}}(r_i)\right|+\left|\pi^-_{\tilde{M}}(r_i)-\pi^-_{\tilde{N}}(r_i)\right|+\left|\pi^+_{\tilde{M}}(r_i)-\pi^+_{\tilde{N}}(r_i)\right|. \quad (16)$$

Theorem 2. *Let* $\tilde{M}_1=\{([s_{\alpha^-_{\tilde{M}_1}(r_i)}, s_{\alpha^+_{\tilde{M}_1}(r_i)}], [s_{\beta^-_{\tilde{M}_1}(r_i)}, s_{\beta^+_{\tilde{M}_1}(r_i)}])|r_i\in R\}$, $\tilde{M}_2=\{([s_{\alpha^-_{\tilde{M}_2}(r_i)}, s_{\alpha^+_{\tilde{M}_2}(r_i)}], \dots, [s_{\beta^-_{\tilde{M}_2}(r_i)}, s_{\beta^+_{\tilde{M}_2}(r_i)}])|r_i\in R\}$, $\tilde{M}_3=\{([s_{\alpha^-_{\tilde{M}_3}(r_i)}, s_{\alpha^+_{\tilde{M}_3}(r_i)}], [s_{\beta^-_{\tilde{M}_3}(r_i)}, s_{\beta^+_{\tilde{M}_3}(r_i)}])|r_i\in R\}$, *are any three sets of IVLIFSs, then the Hamming distance satisfies the following conditions*:

$$(1)\quad 0\leqslant d(\tilde{M}_1,\tilde{M}_2)\leqslant 1, \quad (17)$$

$$(2)\quad \textit{If } d(\tilde{M}_1,\tilde{M}_2)=0, \quad \textit{then } \tilde{M}_1=\tilde{M}_2, \quad (18)$$

$$(3)\quad d(\tilde{M}_1,\tilde{M}_2)=d(\tilde{M}_2,\tilde{M}_1), \quad (19)$$

$$(4)\quad d(\tilde{M}_1,\tilde{M}_2)+d(\tilde{M}_2,\tilde{M}_3)\geqslant d(\tilde{M}_1,\tilde{M}_3). \quad (20)$$

3.3. *Interval-Valued Linguistic Intuitionistic Fuzzy Entropy*

Definition 8. Let $IVLIF(R)$ be the set of all IVLIFSs, $\forall\tilde{M}\in IVLIF(R)$, the mapping $E: IVLIF(R)\to[0,t]$ is called entropy if E satisfies the following conditions:

Condition 1. $E(\tilde{M})=0$ if and only if $\tilde{M}$ is a crisp set, the crisp set includes $\tilde{M}=\{\langle r_i,(s_t,s_t),(s_0,s_0)\rangle|r_i\in R\}$ and $\tilde{M}=\{\langle r_i,(s_0,s_0),(s_t,s_t)\rangle|r_i\in R\}$;

Condition 2. $E(\tilde{M})=1$ if and only if $[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}]=[s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^+_{\tilde{M}}(r_i)}]=[s_0,s_0]$ for every $r_i\in R$;

Condition 3. $E(\tilde{M})=E(\tilde{M}^C)$ for every $\tilde{M}\in IVLIF(R)$;

Condition 4. For any $\tilde{N}\in IVLIF(R)$, if $\tilde{M}\subseteq\tilde{N}$ when $\alpha^-_{\tilde{N}}(r_i)\leqslant\beta^-_{\tilde{N}}(r_i)$, $\alpha^+_{\tilde{N}}(r_i)\leqslant\beta^+_{\tilde{N}}(r_i)$ for every $r_i\in R$, or $\tilde{M}\supseteq\tilde{N}$ when $\alpha^-_{\tilde{N}}(r_i)\geqslant\beta^-_{\tilde{N}}(r_i)$, $\alpha^+_{\tilde{N}}(r_i)\geqslant\beta^+_{\tilde{N}}(r_i)$ for every $r_i\in R$, then $E(\tilde{M})\leqslant E(\tilde{N})$.

Theorem 3. *Let* $R=\{r_1,r_2,\dots,r_n\}$ *be a universe,* $\tilde{M}=\{([s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}], [s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^+_{\tilde{M}}(r_i)}])|r_i\in R\}$, *the formula of the entropy is as follows:*

$$E(\tilde{M}) = \frac{1}{n}\sum_{i=1}^{n}\frac{2t^2 - |\alpha^+_{\tilde{M}}(r_i) - \beta^+_{\tilde{M}}(r_i)|^2 - |\alpha^-_{\tilde{M}}(r_i) - \beta^-_{\tilde{M}}(r_i)|^2 + (\pi^-_{\tilde{M}}(r_i))^2 + (\pi^+_{\tilde{M}}(r_i))^2}{4t^2}. \tag{21}$$

Proof.

Condition 1. $\tilde{M}$ is a crisp set, namely

$$\left[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}\right] = [s_0, s_0], \qquad \left[s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^-_{\tilde{M}}(r_i)}\right] = [s_t, s_t] \quad \text{or}$$

$$\left[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}\right] = [s_t, s_t], \qquad \left[s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^-_{\tilde{M}}(r_i)}\right] = [s_0, s_0],$$

then $E(\tilde{M}) = 0$.

If $E(\tilde{M}) = 0$, since

$$2t^2 + \left(\pi^-_{\tilde{M}}(r_i)\right)^2 + \left(\pi^+_{\tilde{M}}(r_i)\right)^2 \geqslant 2t^2,$$

$$\left|\alpha^+_{\tilde{M}}(r_i) - \beta^+_{\tilde{M}}(r_i)\right|^2 + \left|\alpha^-_{\tilde{M}}(r_i) - \beta^-_{\tilde{M}}(r_i)\right|^2 \leqslant 2t^2,$$

so $\left[s_{\pi^-_{\tilde{M}}(r_i)}, s_{\pi^+_{\tilde{M}}(r_i)}\right] = [s_0, s_0]$ and

$$\left[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}\right] = [s_0, s_0], \qquad \left[s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^+_{\tilde{M}}(r_i)}\right] = [s_t, s_t]$$

or $\left[s_{\pi^-_{\tilde{M}}(r_i)}, s_{\pi^+_{\tilde{M}}(r_i)}\right] = [s_0, s_0]$ and

$$\left[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}\right] = [s_t, s_t], \qquad \left[s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^+_{\tilde{M}}(r_i)}\right] = [s_0, s_0],$$

namely, $\tilde{M}$ is a crisp set.

Condition 2. If $[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}] = [s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^+_{\tilde{M}}(r_i)}] = [s_0, s_0]$, it's obvious that $E(\tilde{M}) = 1$.

If $E(\tilde{M}) = 1$, since

$$2t^2 + \left(\pi^-_{\tilde{M}}(r_i)\right)^2 + \left(\pi^+_{\tilde{M}}(r_i)\right)^2 \leqslant 4t^2,$$

$$\left|\alpha^+_{\tilde{M}}(r_i) - \beta^+_{\tilde{M}}(r_i)\right|^2 + \left|\alpha^-_{\tilde{M}}(r_i) - \beta^-_{\tilde{M}}(r_i)\right|^2 \geqslant 0,$$

so $\left[s_{\pi^-_{\tilde{M}}(r_i)}, s_{\pi^+_{\tilde{M}}(r_i)}\right] = [s_t, s_t]$ and $\left[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^-_{\tilde{M}}(r_i)}\right] = \left[s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^-_{\tilde{M}}(r_i)}\right] = [s_0, s_0]$.

Condition 3. For the two IVLIFSs $\tilde{M}$ and $\tilde{M}^c$, $[s_{\pi^-_{\tilde{M}}(r_i)}, s_{\pi^+_{\tilde{M}}(r_i)}] = [s_{\pi^-_{\tilde{M}^c}(r_i)}, s_{\pi^+_{\tilde{M}^c}(r_i)}]$, so it's obvious that the condition 3 is right.

Condition 4.

$$E(\tilde{M}) = \frac{1}{n}\sum_{i=1}^{n}\frac{2t^2 - |\alpha^+_{\tilde{M}}(r_i) - \beta^+_{\tilde{M}}(r_i)|^2 - |\alpha^-_{\tilde{M}}(r_i) - \beta^-_{\tilde{M}}(r_i)|^2 + (\pi^-_{\tilde{M}}(r_i))^2 + (\pi^+_{\tilde{M}}(r_i))^2}{4t^2}$$

$$= \frac{1}{n}\sum_{i=1}^{n}\frac{2t^2 + \alpha^+_{\tilde{M}}(r_i)(\beta^+_{\tilde{M}}(r_i) - t) + \beta^+_{\tilde{M}}(r_i)(\alpha^+_{\tilde{M}}(r_i) - t) + \alpha^-_{\tilde{M}}(r_i)(\beta^-_{\tilde{M}}(r_i) - t) + \beta^-_{\tilde{M}}(r_i)(\alpha^-_{\tilde{M}}(r_i) - t)}{2t^2}$$

and if $\alpha^-_{\tilde{N}}(r_i) \leqslant \beta^-_{\tilde{N}}(r_i), \alpha^+_{\tilde{N}}(r_i) \leqslant \beta^+_{\tilde{N}}(r_i)$ and $\tilde{M} \subseteq \tilde{N}$ for every $r_i \in R$ then $\beta^-_{\tilde{M}}(r_i) \geqslant \beta^-_{\tilde{N}}(r_i) \geqslant \alpha^-_{\tilde{N}}(r_i) \geqslant \alpha^-_{\tilde{M}}(r_i)$ and $\beta^+_{\tilde{M}}(r_i) \geqslant \beta^+_{\tilde{N}}(r_i) \geqslant \alpha^+_{\tilde{N}}(r_i) \geqslant \alpha^+_{\tilde{M}}(r_i)$, so

$$\alpha^+_{\tilde{M}}(r_i)\big(\beta^+_{\tilde{M}}(r_i) - t\big) \leqslant \alpha^+_{\tilde{N}}(r_i)\big(\beta^+_{\tilde{N}}(r_i) - t\big),$$
$$\beta^+_{\tilde{M}}(r_i)\big(\alpha^+_{\tilde{M}}(r_i) - t\big) \leqslant \beta^+_{\tilde{N}}(r_i)\big(\alpha^+_{\tilde{N}}(r_i) - t\big),$$
$$\alpha^-_{\tilde{M}}(r_i)\big(\beta^-_{\tilde{M}}(r_i) - t\big) \leqslant \alpha^-_{\tilde{N}}(r_i)\big(\beta^-_{\tilde{N}}(r_i) - t\big),$$
$$\beta^-_{\tilde{M}}(r_i)\big(\alpha^-_{\tilde{M}}(r_i) - t\big) \leqslant \beta^-_{\tilde{N}}(r_i)\big(\alpha^-_{\tilde{N}}(r_i) - t\big),$$

then $E(\tilde{M}) \leqslant E(\tilde{N})$.

As the above method, when $\alpha^-_{\tilde{N}}(r_i) \geqslant \beta^-_{\tilde{N}}(r_i), \alpha^+_{\tilde{N}}(r_i) \geqslant \beta^+_{\tilde{N}}(r_i)$ and $\tilde{M} \supseteq \tilde{N}$ for every $r_i \in R$, we can conclude that $E(\tilde{M}) \leqslant E(\tilde{N})$.

So, the Condition 4 is right.

From above conditions, we can see that the interval-valued linguistic intuitionistic fuzzy entropy can consider both uncertainty from fuzziness and from the lack of information based on IVLIFS, when IVLIFS is a crisp set, the entropy value is smallest; when IVLIFS present the form of $[s_{\alpha^-_{\tilde{M}}(r_i)}, s_{\alpha^+_{\tilde{M}}(r_i)}] = [s_{\beta^-_{\tilde{M}}(r_i)}, s_{\beta^+_{\tilde{M}}(r_i)}] = [s_0, s_0]$, the information of IVLIFS is fuzziest and lack all the information, the entropy value is largest; for given a IVLIFSs, the fuzzier the information, and the more information is missing, the greater the entropy value.

4. An Extended VIKOR Method for Interval-Valued Linguistic Intuitionistic Fuzzy Numbers Based on Entropy

In this paper, we will extend the VIKOR method to solve MADM problem with the interval-valued linguistic intuitionistic fuzzy information (IVLIFI).

In order to do this, we describe the decision making problem firstly.

For a multiple attribute decision making problem, let $X = \{x_1, x_2, \ldots, x_m\}$ be a group of alternatives, $C = \{c_1, c_2, \ldots, c_n\}$ be a group of attributes, and the attribute weights are unknown. Suppose that $\tilde{r}_{ij} = ([s_{\alpha^-(\tilde{r}_{ij})}, s_{\alpha^+(\tilde{r}_{ij})}], [s_{\beta^-(\tilde{r}_{ij})}, s_{\beta^+(\tilde{r}_{ij})}])$ is the evaluation value of the alternative with respect to the attributes C_j which is expressed by the IVLIFI, where $[s_{\alpha^-(\tilde{r}_{ij})}, s_{\alpha^+(\tilde{r}_{ij})}], [s_{\beta^-(\tilde{r}_{ij})}, s_{\beta^+(\tilde{r}_{ij})}]$ represent the membership degree and non-membership degree of IVLIFNs, and $s_{\alpha^-(\tilde{r}_{ij})}, s_{\alpha^+(\tilde{r}_{ij})}, s_{\beta^-(\tilde{r}_{ij})}, s_{\beta^+(\tilde{r}_{ij})} \in S_{[0,t]}$. The decision matrix denoted by IVLIFNs is listed in Table 1, and the goal of this MADM problem is to rank the alternatives.

In this study, we think the weight information is unknown, and we use the interval-valued linguistic intuitionistic fuzzy entropy to calculate the weight.

The procedures of the proposed method are shown as follows:

Table 1
Decision making matrix with the interval-valued linguistic intuitionistic fuzzy information.

	c_1	c_2	...	c_n
x_1	$\tilde{r}_{11}$	$\tilde{r}_{12}$	...	$\tilde{r}_{1n}$
x_2	$\tilde{r}_{21}$	$\tilde{r}_{22}$	...	$\tilde{r}_{2n}$
...	...	...	...	...
x_m	$\tilde{r}_{m1}$	$\tilde{r}_{m2}$	...	$\tilde{r}_{mn}$

Step 1. Normalize the decision matrix.

Since there are different types of attributes, we should convert different type to the same type.

In general, we can transform the cost attribute values to benefit type, and the transformed decision matrix is expressed by $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$ $(i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n)$, where

$$\tilde{r}_{ij} = \begin{cases} ([s_{\alpha^-(\tilde{r}_{ij})}, s_{\alpha^+(\tilde{r}_{ij})}], [s_{\beta^-(\tilde{r}_{ij})}, s_{\beta^+(\tilde{r}_{ij})}]) & \text{for benefit attribute } C_j, \\ ([s_{\beta^-(\tilde{r}_{ij})}, s_{\beta^+(\tilde{r}_{ij})}], [s_{\alpha^-(\tilde{r}_{ij})}, s_{\alpha^+(\tilde{r}_{ij})}]) & \text{for cost attribute } C_j. \end{cases} \tag{22}$$

Step 2. Obtain the virtual positive ideal solution and the virtual negative ideal solution.

According to the partial order relation, we have the virtual positive ideal solution (PIS):

$$X^* = \{\tilde{r}_1^*, \tilde{r}_2^*, \ldots, \tilde{r}_n^*\} \tag{23}$$

where

$$\begin{aligned} \tilde{r}_j^* = \{&[s_{\max\{\alpha_{1j}^-,\alpha_{2j}^-,\ldots,\alpha_{mj}^-\}}, s_{\max\{\alpha_{1j}^+,\alpha_{2j}^+,\ldots,\alpha_{mj}^+\}}], \\ &[s_{\min\{\beta_{1j}^-,\beta_{2j}^-,\ldots,\beta_{mj}^-\}}, s_{\min\{\beta_{1j}^+,\beta_{2j}^+,\ldots,\beta_{mj}^+\}}]\}, \quad j = 1, 2, \ldots, n, \end{aligned} \tag{24}$$

the virtual negative ideal solution (NIS)

$$X^- = \{\tilde{r}_1^-, \tilde{r}_2^-, \ldots, \tilde{r}_n^-\} \tag{25}$$

where

$$\begin{aligned} \tilde{r}_j^- = \{&[s_{\min\{\alpha_{1j}^-,\alpha_{2j}^-,\ldots,\alpha_{mj}^-\}}, s_{\min\{\alpha_{1j}^+,\alpha_{2j}^+,\ldots,\alpha_{mj}^+\}}], \\ &[s_{\max\{\beta_{1j}^-,\beta_{2j}^-,\ldots,\beta_{mj}^-\}}, s_{\max\{\beta_{1j}^+,\beta_{2j}^+,\ldots,\beta_{mj}^+\}}]\}, \quad j = 1, 2, \ldots, n. \end{aligned} \tag{26}$$

Step 3. Calculate the entropy of every attribute $E_j = \sum_{i=1}^m e_{ij}$ by formula (16), where $j = 1, 2, \ldots, n$.

Step 4. Calculate the weight value of each attribute by using the model as follows Wang *et al.* (2012):

$$w_j = \frac{E_j^{-1}}{\sum_{i=1}^n E_j^{-1}}, \quad j = 1, 2, \ldots, n. \tag{27}$$

Step 5. Compute S_i and R_i, and we have

$$S_i = \frac{\sum_{j=1}^{n} w_j \|\tilde{r}_j^* - \tilde{r}_{ij}\|}{\|\tilde{r}_j^* - \tilde{r}_j^-\|}, \quad i = 1, 2, \dots, m, \tag{28}$$

$$R_i = \frac{\max w_j \|\tilde{r}_j^* - \tilde{r}_{ij}\|}{\|\tilde{r}_j^* - \tilde{r}_j^-\|}, \quad i = 1, 2, \dots, m \tag{29}$$

where $|\tilde{r}_1 - \tilde{r}_2|$ is the distance between two interval-valued linguistic intuitionistic fuzzy numbers $\tilde{r}_1$ and $\tilde{r}_2$, which is defined by Eq. (11).

Step 6. Compute the value Q_i, and we have

$$Q_i = \frac{v(S_i - S^*)}{(S^- - S^*)} + \frac{(1 - v)(R_i - R^*)}{(R^- - R^*)} \tag{30}$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, v is the balance parameter which can balance the group utility and individual regret, here suppose that $v = 0.5$, it shows that the "minimum individual regret" and the "maximum group utility" are both important.

Step 7. Same as the step 5 of Section 2.

Step 8. Same as the step 6 of Section 2.

5. Illustrative Example

5.1. *Description of the Example*

In this part, we will give an example to explain the proposed method. Suppose there is a problem of selecting the sites of subsidiary company which is described as follows.

A manufacturing company hopes to build a new subsidiary company. Suppose that $X = \{x_1, x_2, x_3, x_4\}$ is a group of four potential sites, they are the alternatives and $C = \{c_1, c_2, c_3, c_4, c_5\}$ is a group of attributes, where $(c_1, c_2, c_3, c_4, c_5)$ stand for "the price of land", "the distance of sale market", "the distance of discourse", "the labour market", "local economical", respectively, and the weight vector of attributes is unknown. The alternatives x_i $(1, \dots, 4)$ are to be evaluated with respect to the five attributes by the IVLIFNs based on the linguistic term set:

$$S = \{s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{slightly poor}, s_4 = \text{fair}, s_5 = \text{slightly good}, s_6 = \text{good}, s_7 = \text{very good}, s_8 = \text{extremely good}\}.$$

Then we can build the decision matrix $R = (\gamma_{ij})_{4\times 5}$ shown in Table 2.

The goal of this decision making is to select the best site of subsidiary company.

Table 2
Decision matrix R of IVLIFNs.

	c_1	c_2	c_3	c_4	c_5
x_1	$([s_6,s_7],[s_1,s_1])$	$([s_5,s_6],[s_1,s_2])$	$([s_4,s_5],[s_1,s_3])$	$([s_6,s_7],[s_1,s_1])$	$([s_5,s_6],[s_1,s_2])$
x_2	$([s_5,s_6],[s_1,s_2])$	$([s_5,s_7],[s_1,s_1])$	$([s_5,s_6],[s_1,s_2])$	$([s_5,s_6],[s_1,s_2])$	$([s_6,s_7],[s_1,s_1])$
x_3	$([s_5,s_6],[s_1,s_2])$	$([s_4,s_5],[s_2,s_3])$	$([s_6,s_7],[s_1,s_1])$	$([s_5,s_6],[s_1,s_2])$	$([s_3,s_4],[s_3,s_4])$
x_4	$([s_4,s_5],[s_2,s_3])$	$([s_6,s_7],[s_1,s_1])$	$([s_4,s_5],[s_2,s_3])$	$([s_4,s_6],[s_1,s_2])$	$([s_3,s_4],[s_3,s_4])$

5.2. *Steps of Decision Making for this Example*

Next, we present the procedure of decision making based on the VIKOR method and interval-valued linguistic intuitionistic fuzzy entropy.

Step 1. We suppose the decision matrix R_1 has been normalized.

Step 2. Obtain the virtual positive ideal solution and the virtual negative ideal solution by Eqs. (23)–(26), and we can get

$$\begin{aligned} X^* &= \{\tilde{r}_1^*, \tilde{r}_2^*, \tilde{r}_3^*, \tilde{r}_4^*, \tilde{r}_5^*\} \\ &= \{([s_6,s_7],[s_1,s_1]), ([s_6,s_7],[s_1,s_1]), ([s_6,s_7],[s_1,s_1]), ([s_6,s_7],[s_1,s_1]), \\ &\quad ([s_6,s_7],[s_1,s_1])\}, \\ X^- &= \{\tilde{r}_1^-, \tilde{r}_2^-, \tilde{r}_3^-, \tilde{r}_4^-, \tilde{r}_5^-\} \\ &= \{([s_4,s_5],[s_2,s_3]), ([s_4,s_5],[s_2,s_3]), ([s_4,s_5],[s_2,s_3]), ([s_4,s_6],[s_1,s_2]), \\ &\quad ([s_3,s_4],[s_3,s_4])\}. \end{aligned}$$

Step 3. Calculate the entropy of every attribute $E_j = \frac{1}{m}\sum_{i=1}^{m} e_{ij}$ by formula (16), where $j = 1, 2, \dots, n$. We can get

$$E_1 = 0.3828, \quad E_2 = 0.3633, \quad E_3 = 0.4063, \quad E_4 = 0.3711, \quad E_5 = 0.4219.$$

Step 4. Calculate the weight value of each attribute by Eq. (27), we can get

$$\begin{aligned} &w_1 = 0.202632, \quad w_2 = 0.213526, \quad w_3 = 0.190942, \\ &w_4 = 0.209031, \quad w_5 = 0.183870. \end{aligned}$$

Step 5. Compute S_i, R_i, and we have

$$\begin{aligned} S_1 &= \frac{w_1\|r_1^* - r_{11}\|}{\|r_1^* - r_1^-\|} + \frac{w_2\|r_2^* - r_{12}\|}{\|r_2^* - r_2^-\|} + \frac{w_3\|r_3^* - r_{13}\|}{\|r_3^* - r_3^-\|} + \frac{w_4\|r_4^* - r_{14}\|}{\|r_4^* - r_4^-\|} \\ &\quad + \frac{w_5\|r_5^* - r_{15}\|}{\|r_5^* - r_5^-\|} = 0.358995, \end{aligned}$$

$$S_2 = \frac{w_1\|r_1^* - r_{21}\|}{\|r_1^* - r_1^-\|} + \frac{w_2\|r_2^* - r_{22}\|}{\|r_2^* - r_2^-\|} + \frac{w_3\|r_3^* - r_{23}\|}{\|r_3^* - r_3^-\|} + \frac{w_4\|r_4^* - r_{24}\|}{\|r_4^* - r_4^-\|} + \frac{w_5\|r_5^* - r_{25}\|}{\|r_5^* - r_5^-\|} = 0.389522,$$

$$S_3 = \frac{w_1\|r_1^* - r_{31}\|}{\|r_1^* - r_1^-\|} + \frac{w_2\|r_2^* - r_{32}\|}{\|r_2^* - r_2^-\|} + \frac{w_3\|r_3^* - r_{33}\|}{\|r_3^* - r_3^-\|} + \frac{w_4\|r_4^* - r_{34}\|}{\|r_4^* - r_4^-\|} + \frac{w_5\|r_5^* - r_{35}\|}{\|r_5^* - r_5^-\|} = 0.638066,$$

$$S_4 = \frac{w_1\|r_1^* - r_{41}\|}{\|r_1^* - r_1^-\|} + \frac{w_2\|r_2^* - r_{42}\|}{\|r_2^* - r_2^-\|} + \frac{w_3\|r_3^* - r_{43}\|}{\|r_3^* - r_3^-\|} + \frac{w_4\|r_4^* - r_{44}\|}{\|r_4^* - r_4^-\|} + \frac{w_5\|r_5^* - r_{45}\|}{\|r_5^* - r_5^-\|} = 0.786474,$$

$$R_1 = \max_5 \left\{ \frac{w_1\|r_1^* - r_{11}\|}{\|r_1^* - r_1^-\|}, \frac{w_2\|r_2^* - r_{12}\|}{\|r_2^* - r_2^-\|}, \frac{w_3\|r_3^* - r_{13}\|}{\|r_3^* - r_3^-\|}, \frac{w_4\|r_4^* - r_{14}\|}{\|r_4^* - r_4^-\|}, \frac{w_5\|r_5^* - r_{15}\|}{\|r_5^* - r_5^-\|} \right\} = 0.190942,$$

$$R_2 = \max_5 \left\{ \frac{w_1\|r_1^* - r_{21}\|}{\|r_1^* - r_1^-\|}, \frac{w_2\|r_2^* - r_{22}\|}{\|r_2^* - r_2^-\|}, \frac{w_3\|r_3^* - r_{23}\|}{\|r_3^* - r_3^-\|}, \frac{w_4\|r_4^* - r_{24}\|}{\|r_4^* - r_4^-\|}, \frac{w_5\|r_5^* - r_{25}\|}{\|r_5^* - r_5^-\|} \right\} = 0.139354,$$

$$R_3 = \max_5 \left\{ \frac{w_1\|r_1^* - r_{31}\|}{\|r_1^* - r_1^-\|}, \frac{w_2\|r_2^* - r_{32}\|}{\|r_2^* - r_2^-\|}, \frac{w_3\|r_3^* - r_{33}\|}{\|r_3^* - r_3^-\|}, \frac{w_4\|r_4^* - r_{34}\|}{\|r_4^* - r_4^-\|}, \frac{w_5\|r_5^* - r_{35}\|}{\|r_5^* - r_5^-\|} \right\} = 0.213526,$$

$$R_4 = \max_5 \left\{ \frac{w_1\|r_1^* - r_{41}\|}{\|r_1^* - r_1^-\|}, \frac{w_2\|r_2^* - r_{42}\|}{\|r_2^* - r_2^-\|}, \frac{w_3\|r_3^* - r_{43}\|}{\|r_3^* - r_3^-\|}, \frac{w_4\|r_4^* - r_{44}\|}{\|r_4^* - r_4^-\|}, \frac{w_5\|r_5^* - r_{45}\|}{\|r_5^* - r_5^-\|} \right\} = 0.209031.$$

Step 6. Compute the values Q_i $(i = 1, 2, 3, 4)$ by Eq. (30) (suppose $v = 0.5$), we have

$$Q_1 = 0.652241, \quad Q_2 = 0.964294, \quad Q_3 = 0.173585, \quad Q_4 = 0.030301.$$

Step 7. Rank the alternatives.

We firstly gave the ranking results by the values, and the smaller the values are, the better the alternatives are. The results are listed in Table 3.

Table 3
The ranking and the best alternative by S, Q and R.

	x_1	x_2	x_3	x_4	Ranking	The best alternative
S	0.358995	0.389522	0.638066	0.786474	$x_1 \succ x_2 \succ x_3 \succ x_4$	x_1
R	0.190942	0.139354	0.213526	0.209031	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2
Q $(v = 0.5)$	0.347759	0.035706	0.826415	0.969699	$x_2 \succ x_1 \succ x_3 \succ x_4$	x_2

Step 8. Obtain the compromise ranking results. Firstly, we rank the alternatives by Q in increasing order, the alternative with first position is x_2 with $Q(x_2) = 0.035706$, and alternative x_2 is not the best ranked by, which does not satisfy the condition 2. x_1 is the alternative with second position with $Q(x_1) = 0.347759$. So x_2, x_1 are both the compromise solutions.

As $\mathrm{MQ} = 1/(m-1) = 1/(4-1) = 0.333333$, so

$$Q(x_2) - Q(x_1) = 0.312053,$$

which does not satisfy the condition 1, but we can get

$$Q(x_3) - Q(x_1) = 0.790709.$$

So, x_2, x_1, x_3 are all compromise solutions.

5.3. *Comparison Analysis*

Since there is no decision making method based on interval-valued linguistic intuitionistic fuzzy information, in order to verify the effectiveness of the method proposed in this paper, we use the TOPSIS method to solve this example again, and compare the ranking result and select the best alternative. Furthermore, since the LIFN is a special case of the IVLIFN, we can extend LIFNs to IVLIFNs, so we can use this method to solve the example in Chen and Liu (2015), compare the ranking results and the best alternative, too.

5.3.1. *Compared with TOPSIS Method*

In Hu and Xu (2007), they proposed TOPSIS method for MADM with interval-valued intuitionistic fuzzy information. We extend this method to IVLIFNs, and our example above will be solved again by TOPSIS method, the procedures are shown as follows:

Steps 1–4. Similar to steps 1–4 of Section 4.2.

Step 5. Calculate the distances between each alternative and the virtual positive ideal solution and the virtual negative ideal solution, the results are shown as follows:

(1) The distance between each alternative and the virtual positive ideal solution is shown as follows:

$$d(x_1, X^*) = 0.0974, \qquad d(x_2, X^*) = 0.0887,$$
$$d(x_3, X^*) = 0.1738, \qquad d(x_4, X^*) = 0.2065.$$

(2) The distance between each alternative and the virtual positive ideal solution is shown as follows:

$$d(x_1, X^-) = 0.1979, \qquad d(x_2, X^-) = 0.1832,$$
$$d(x_3, X^-) = 0.1436, \qquad d(x_4, X^-) = 0.1466.$$

Step 6. Calculate the relative closeness between each alternative and the ideal solution, we can get:

$$C_1 = 0.3299, \qquad C_2 = 0.3262, \qquad C_3 = 0.5476, \qquad C_4 = 0.5848.$$

Step 7. Rank the alternatives according to the proximities and select the best one, we can get:

$$x_2 \succ x_1 \succ x_3 \succ x_4,$$

and the best one is x_2.

Clearly, the ranking result is the same as that by Q in Table 3. The advantage of the extended VIKOR method is that it can provide the compromise alternative set by the maximum "group utility" and minimum "individual regret". According to the real decision problem, we can adjust the values of the balance parameter v to balance the factors between group utility and individual regret, and get different alternative rankings; it is more feasible and scientific to solve the MADM problems in the real world. In this example, we can think the alternatives x_2, x_1, x_3 are the same according to the new method. But the best alternatives are only x_2 calculated by Hu and Xu's method, obviously, it can't get multiple compromise alternatives. The advantage of Hu and Xu's method is that it is simple in ranking the alternatives, while our proposed method is a little complex. In addition, because the ranking principle is different, it is reasonable not to get the completely same ranking results. In this example, these two methods produced the same best and worst alternatives, and this can show the validity of the proposed method.

5.3.2. *Compared with Another Aggregation Operator*

In Chen and Liu (2015), $X = \{x_1, x_2, x_3, x_4\}$ is a set of four potential global suppliers under consideration and $A = \{a_1, a_2, a_3, a_4, a_5\}$ is a set of attributes, where a_i ($i = 1, 2, \ldots, 5$) stands for "overall cost of the product", "quality of the product", "service performance of supplier", "supplier's profile", "risk factor", respectively. The weight vector of attributes is $\omega = (\omega_1, \omega_2, \omega_3, \omega_4, \omega_5)^T = (0.25, 0.2, 0.15, 0.18, 0.22)^T$ for decision makers d_k ($k = 1, 2, \ldots, 4$) under the above five attributes, and construct the linguistic intuitionistic fuzzy decision matrices, the maker weight vector of attributes is $w = (w_1, w_2, w_3, w_4)^T = (0.25, 0.2, 0.3, 0.25)^T$.

Chen and Liu (2015) utilized the LIFWA operator to aggregate all attributes of each alternative and get a comprehensive decision making matrix, the result is presented in Table 4.

Table 4
The synthesis of decision making matrix aggregated by Chen and Liu (2015).

	d_1	d_2	d_3	d_4
x_1	$(s_{6.199}, s_{1.578})$	$(s_{5.458}, s_{2.363})$	$(s_{5.598}, s_{1.647})$	$(s_{5.129}, s_{2.023})$
x_2	$(s_{6.138}, s_{1.444})$	$(s_{5.715}, s_{1.433})$	$(s_{6.343}, s_{1.301})$	$(s_{6.127}, s_{1.133})$
x_3	$(s_{5.428}, s_{1.690})$	$(s_{5.501}, s_{1.966})$	$(s_{4.673}, s_{1.842})$	$(s_{4.572}, s_{2.471})$
x_4	$(s_{5.510}, s_{1.902})$	$(s_{5.644}, s_{2.093})$	$(s_{5.846}, s_{1.712})$	$(s_{4.871}, s_{1.927})$

Table 5
The decision making matrix R_2 of IVLIFNs extended by Table 4.

	d_1	d_2	d_3	d_4
x_1	$([s_{6.199}, s_{6.199}], [s_{1.578}, s_{1.578}])$	$([s_{5.458}, s_{5.458}], [s_{2.363}, s_{2.363}])$	$([s_{5.598}, s_{5.598}], [s_{1.647}, s_{1.647}])$	$([s_{5.129}, s_{5.129}], [s_{2.023}, s_{2.023}])$
x_2	$([s_{6.138}, s_{6.138}], [s_{1.444}, s_{1.444}])$	$([s_{5.715}, s_{5.715}], [s_{1.433}, s_{1.433}])$	$([s_{6.343}, s_{6.343}], [s_{1.301}, s_{1.301}])$	$([s_{6.127}, s_{6.127}], [s_{1.133}, s_{1.133}])$
x_3	$([s_{5.129}, s_{5.129}], [s_{1.690}, s_{1.690}])$	$([s_{5.501}, s_{5.501}], [s_{1.966}, s_{1.966}])$	$([s_{4.673}, s_{4.673}], [s_{1.842}, s_{1.842}])$	$([s_{4.572}, s_{4.572}], [s_{2.471}, s_{2.471}])$
x_4	$([s_{5.510}, s_{5.510}], [s_{1.902}, s_{1.902}])$	$([s_{5.644}, s_{5.644}], [s_{2.093}, s_{2.093}])$	$([s_{5.846}, s_{5.846}], [s_{1.712}, s_{1.712}])$	$([s_{4.871}, s_{4.871}], [s_{1.927}, s_{1.927}])$

Firstly, we extend the LIFNs which are shown in Table 4 to IVLIFNs, for example, the linguistic intuitionistic fuzzy number $(s_{6.199}, s_{1.578})$ can be extended to IVLIFNs $([s_{6.199}, s_{6.199}], [s_{1.578}, s_{1.578}]) \in \Gamma_{[0,8]}$, and the decision matrix of LIFNs had been normalized obviously, the results are shown in Table 5.

Next, we use our method to rank the alternatives based on the decision making matrix of Table 5, and obtain the compromise ranking results. The procedures are presented as follows:

Step 1. Since decision matrix R_2 has been normalized, we start the next step directly.

Step 2. Obtain the virtual positive ideal solution and the virtual negative ideal solution.

Step 3. Compute S_i and R_i. (Since the weight of each expert has been given, we don't need to calculate the entropy anymore.)

Step 4. Compute the values Q_i $(i = 1, 2, 3, 4)$ (suppose $v = 0.5$).

Step 5. Rank the alternatives. The results are shown in Table 6.

Step 6. Obtain the compromise ranking results. According to the rules of VIKOR method, firstly, we rank the alternatives by Q in increasing order, the alternative with first position is x_2, we can get the x_2 as the only compromise solution. In addition, the alternative x_2 is also best ranked by S and R, which satisfies the condition 2.

As $\mathrm{MQ} = 1/(m-1) = 1/(4-1) = 0.333333$, so

$$Q(x_2) - Q(x_1) = 0.529596,$$

Table 6
The ranking and the best alternative by S, Q and R.

	x_1	x_2	x_3	x_4	Ranking	The best alternative
S	0.562322	0.012383	0.965570	0.623102	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2
R	0.202598	0.012383	0.406821	0.254974	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2
$Q(v = 0.5)$	0.529596	0	1	0.627870	$x_2 \succ x_1 \succ x_4 \succ x_3$	x_2

it also satisfies the condition 1, so we can get the alternative x_2 as the only compromise solution.

It is easy to see that the ranking results obtained by the method proposed in this paper and by the method in Chen and Liu (2015) are completely the same, and the best alternative of two methods is always x_2. So it can prove the method in this paper is effective. Our proposed method can select the compromise alternative by the maximum "group utility" and minimum "individual regret" and Chen's method does not consider these factors, so the advantage of our method is that it is more flexible and effective to solve the MADM problems in the real world.

6. Conclusions

The intuitionistic fuzzy sets have become more and more important tools to deal with imprecise and uncertain information. In this paper, we propose the concept of interval-valued linguistic intuitionistic fuzzy numbers, in which the membership degree and the non-membership degree are presented by interval-valued linguistic variables. The Hamming distance between two IVLIFNs is defined. We also define the interval-valued linguistic intuitionistic fuzzy entropy, which can reflect fuzziness and the degree of lack of information. Further, the traditional VIKOR method is extended to process the IVLIFNs and used to solve the MADM problems, which are based on the maximum "group utility" and minimum "individual regret". Finally, an example is used to demonstrate their practicality and effectiveness. By comparing it to the other MADM methods, the advantages of the proposed method are that it can simultaneously consider the "group utility' and "individual regret" and can give a compromise best solution or a set of compromise solutions. In further research, it is necessary and meaningful to use the proposed method to real decision making problems such as environment evaluation (Wu, 2016; Zhang, 2016; Jain, 2016; He *et al.*, 2015), etc. Some other methods based on IVLIFNs should also be developed to neutrosophic set (Li *et al.*, 2016), interval neutrosophic hesitant fuzzy sets (Ye, 2016). In addition, we can extend our analysis to some other problems as consensus or management of incomplete information (Cabrerizo *et al.*, 2015; Ureńa *et al.*, 2015), and so on.

Acknowledgements. This paper is supported by the National Natural Science Foundation of China (Nos. 71771140, 71471172 and 71271124), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), Shandong Provincial Social

Science Planning Project (Nos. 16CGLJ31 and 16CKJJ27), the Natural Science Foundation of Shandong Province (No. ZR2017MG007), the Teaching Reform Research Project of Undergraduate Colleges and Universities in Shandong Province (No. 2015Z057), and Key research and development program of Shandong Province (No. 2016GNC110016).

References

Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87–96.

Atanassov, K.T. (1989a). More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 33, 37–46.

Atanassov, K.T. (1989b). Operators over interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 64, 159–174.

Atanassov, K.T., Gargov, G. (1989). Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 3, 343–349.

Burillo, P., Bustince, H. (1996). Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets. *Fuzzy Sets Systems*, 78(3), 305–316.

Cabrerizo, F.J., Herrera-Viedma, E., Pedrycz, W. (2013). A method based on PSO and granular computing of linguistic information to solve group decision making problems defined in heterogeneous contexts. *European Journal of Operational Research*, 230(3), 624–633.

Cabrerizo, F.J., Chiclana, F., Al-Hmouz, R., Morfeq, A., Balamash, A.S., Herrera-Viedma, E. (2015). Fuzzy decision making and consensus: challenges. *Journal of Intelligent & Fuzzy Systems*, 29(3), 1109–1118.

Chen, Z.C., Liu, P.H. (2015). An approach to multiple attribute group decision making based on linguistic intuitionistic fuzzy numbers. *International Journal of Computational Intelligence Systems*, 8(4), 747–760.

Delgado, M., Herrera, F., Herrera-Viedma, E., Martínez, L. (1998). Combining numerical and linguistic information in group decision making. *Information Sciences*, 107, 177–194.

Dong, Y., Herrera-Viedma, E. (2015). Consistency-driven automatic methodology to set interval numerical scales of 2-tuple linguistic term sets and its use in the linguistic GDM with preference relation. *IEEE Transactions on Cybernetics*, 45(4), 780–792.

Du, Y., Liu, P.D. (2011). Extended fuzzy VIKOR method with intuitionistic trapezoidal fuzzy numbers. *Information – An International Interdisciplinary Journal*, 14(8), 2575–2584.

Gao, Z.H., Wei, C.P. (2012). Formula of interval-valued intuitionistic fuzzy entropy and its application. *Comput. Eng. Appl*, 48(2), 53–55.

Gul, M., Celik, E., Aydin, N., Gumus, A.T., Guneri, A.F. (2016). A state of the art literature review of VIKOR and its fuzzy extensions on applications. *Applied Soft Computing*, 46, 60–89.

Guo, X. (2004). *The Discussion an Expansion on Measurement of Fuzzy Uncertainty*. Northwest University, Xian, pp. 43–45.

He, J.K., Teng, F., Qi, Y. (2015). Towards a new climate economics: research areas and prospects. *Chinese Journal of Population, Resources and Environment*, 13(1), 1–9.

Herrera, F., Herrera-Viedma, E. (1997). Aggregation operators for linguistic weighted information. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 27(5), 646–656.

Herrera, F., Herrera-Viedma, E., Martínez, L. (2000). A fusion approach for managing multi-granularity linguistic term sets in decision-making. *Fuzzy Sets and Systems*, 114(1), 43–58.

Hu, H., Xu, Z.S. (2007). TOPSIS method for multiple attribute decision making with interval-valued intuitionistic fuzzy information. *Fuzzy Systems and Mathematics*, 21(5), 108–112.

Jain, P. (2016). Population and development: impacts on environmental performance. *Chinese Journal of Population, Resources and Environment*, 14(3), 208–214.

Keshavarz Ghorabaee, M., Amiri, M., Zavadskas, E.K. (2015). Multi-criteria project selection using an extended VIKOR method with interval type-2 fuzzy sets. *International Journal of Information Technology & Decision Making*, 14(5), 993–1016.

Kuo, T.C., Hsu, C.W., Li, J.Y. (2015). Developing a green supplier selection model by using the DANP with VIKOR. *Sustainability*, 7(2), 1661–1689.

Li, Y.H., Liu, P.D., Chen, Y.B. (2016). Some single valued neutrosophic number heronian mean operators and their application in multiple attribute group decision making. *Informatica*, 27(1), 85–110.

Liao, H.C., Xu, Z.S., Zeng, X.J. (2015). Hesitant fuzzy linguistic VIKOR method and its application in qualitative multiple criteria decision making. *IEEE Transactions on Fuzzy Systems*, 23(5), 1343–1355.

Liu, P. (2017). Multiple attribute group decision making method based on interval-valued intuitionistic fuzzy power Heronian aggregation operators. *Computers & Industrial Engineering*, 108, 199–212.
Liu, P., Wang, M. (2011). An extended VIKOR method for multiple attribute group decision making based on generalized interval-valued trapezoidal fuzzy numbers. *Scientific Research and Essays*, 6(4), 766–776.
Liu, P., Wu, X. (2012). A competency evaluation method of human resources managers based on multi-granularity linguistic variables and VIKOR method. *Technological and Economic Development of Economy*, 18(4), 696–710.
Liu, P., Yu, X. (2014). 2-dimension uncertain linguistic power generalized weighted aggregation operator and its application for multiple attribute group decision making. *Knowledge-Based Systems*, 57(1), 69–80.
Liu, P., Teng, F. (2016). An extended TODIM method for multiple attribute group decision-making based on 2-dimension uncertain linguistic variable. *Complexity*, 21(5), 20–30.
Liu, P., Chen, S.M. (2017). Group decision making based on Heronian aggregation operators of intuitionistic fuzzy numbers. *IEEE Transactions on Cybernetics*, 47(9), 2514–2530.
Liu, P., Li, H. (2017). Interval-valued intuitionistic fuzzy power Bonferroni aggregation operators and their application to group decision making. *Cognitive Computation*, 9(4), 494–512.
Liu, X., Zheng, S., Xiong, F. (2005). Entropy and subsethood for general interval-valued intuitionistic fuzzy sets. In: *International Conference on Fuzzy Systems and Knowledge Discovery*. Springer, Berlin, Heidelberg, pp. 42–52.
Liu, P., He, L., Yu, X.C. (2016). Generalized hybrid aggregation operators based on the 2-dimension uncertain linguistic information for multiple attribute group decision making. *Group Decision and Negotiation*, 25(1), 103–126.
Liu, P., Chen, S.M., Liu, J. (2017). Some intuitionistic fuzzy interaction partitioned Bonferroni mean operators and their application to multi-attribute group decision making. *Information Sciences*, 411, 98–121.
Massanet, S., Riera, J.V., Torrens, J., Herrera-Viedma, E. (2014). A new linguistic computational model based on discrete fuzzy numbers for computing with words. *Information Sciences*, 258, 277–290.
Ureńa, M.R., Chiclana, F., Morente-Molinera, J.A., Herrera-Viedma, E. (2015). Managing incomplete preference relations in decision making: a review and future trends. *Information Sciences*, 302(1), 14–32.
Wang, P., Wei, C. (2011). Constructing method of interval-valued intuitionistic fuzzy entropy. *Computer Engineering and Applications*, 47(2), 43–45.
Wang, C.C., Yao, D.B., Mao, J.J. (2012). Intuitionistic fuzzy multiple attributes decision making method based on entropy and correlation coefficient. *Journal of Computer Applications*, 32, 3002–3004.
Wu, H.X. (2016). The impact of climate changes on mass events in China. *Chinese Journal of Population, Resources and Environment*, 14(1), 11–15.
Wu, Y.N., Chen, K.F., Zeng, B.X., Xu, H., Yang, Y.S. (2016a). Supplier selection in nuclear power industry with extended VIKOR method under linguistic information. *Applied Soft Computing*, 48, 444–457.
Wu, Z., Ahmad, J., Xu, J. (2016b). A group decision making framework based on fuzzy VIKOR approach for machine tool selection with linguistic information. *Applied Soft Computing*, 42, 314–324.
Xie, X.J., Lv, X.X. (2016). Improved interval-valued intuitionistic fuzzy entropy and its applications in multi-attribute decision making problems. *Fuzzy Systems & Operations Research and Management*. Springer International Publishing, pp. 201–211.
Xu, Z.S. (2004). Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment. *Information Sciences*, 168, 171–184.
Ye, J. (2016). Correlation coefficients of interval neutrosophic hesitant fuzzy sets and its application in a multiple attribute decision making method. *Informatica*, 27(1), 179–202.
Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8, 338–356.
Zadeh, L.A. (1975). The concept of a linguistic variable and its applications to approximate reasoning-1. *Information Sciences*, 8(3), 199–249.
Zhang, X. (2016). A new approach to natural capital sustainable development. *Chinese Journal of Population, Resources and Environment*, 14(2), 105–111.
Zhang, N., Wei, G.W. (2013). Extension of VIKOR method for decision making problem based on hesitant fuzzy set. *Applied Mathematical Modelling*, 37(7), 4938–4947.
Zhang X., Du, H.T., Liu, P.D. (2010). The research of enterprise credit risk evaluation based on hybrid decision-making index and VIKOR method. *ICIC-EL*, 4(2), 527–532.
Zhao, N., Xu, Z.S. (2016). Entropy measures for interval-valued intuitionistic fuzzy information from a comparative perspective and their application to decision making. *Informatica*, 27(1), 203–229.

P. Liu received the BS and MS degrees in signal and information processing from Southeast University, Nanjing, China, in 1988 and 1991, respectively, and the PhD degree in information management from Beijng Jiaotong University, Beijing, China, in 2010.

He is currently a professor with the School of Management Science and Engineering, Shandong University of Finance and Economics, Shandong, China. He is an associate editor of the *Journal of Intelligent and Fuzzy Systems*, a member of the editorial board of the journal *Technological and Economic Development of Economy*, and a member of editorial boards of other 12 journals. He has authored or co-authored more than 170 publications. His research interests include aggregation operators, fuzzy logic, fuzzy decision making, and their applications.

X. Qin received the BS degree in computer sciences, Shandong Jianzhu University, Jinan, China, in 2015. Now he is studying for master's degree in management science and engineering, Shandong University of Finance and Economics, Shandong, China.

He has authored or co-authored more than 4 publications. His research interests include aggregation operators, fuzzy logic, fuzzy decision making, and their applications.