INFORMATICA, 2017, Vol. 28, No. 3, 565–581

DOI: http://dx.doi.org/10.15388/Informatica.2017.145

A Closeness Index-Based TODIM Method for Hesitant Qualitative Group Decision Making

Xiaolu ZHANG
The Collaborative Innovation Center, Jiangxi University of Finance and Economics
Nanchang 330013, China
e-mail: xiaolu_jy@163.com

Received: June 2016; accepted: March 2017

Abstract. The purpose of this study is to develop a hesitant trapezoidal fuzzy TODIM (interactive and multi-criteria decision making) with a closeness index-based ranking method to handle hesitant qualitative group decision making problems. First, a novel closeness index-based ranking method is presented to compare the magnitude of hesitant trapezoidal fuzzy numbers (HTrFNs). Based on the developed ranking method, the dominance values of alternatives over others for each expert are calculated. Then, a nonlinear programming model is established to derive the dominance values of alternatives over others for the group and correspondingly the optimal ranking order of alternatives is obtained.
Key words: multi-criteria group decision making, the TODIM method, hesitant trapezoidal fuzzy numbers, comparative linguistic expressions, the closeness index.

1. Introduction

Multicriteria decision making (MCDM) is a usual human activity which helps making decisions mainly in terms of choosing, ranking or sorting the alternatives (Figueira *et al.*, 2005; Zhang, 2015; Zeng *et al.*, 2016a, 2016b, 2016c). In practical decision making process, it is convenient for the decision maker or expert to employ linguistic variables to express qualitative criteria values of alternatives. For example, when evaluating the cabin service of the service quality among airlines, the expert may utilize the linguistic terms like "*bad*" or "*good*" instead of numerical values to assess it. The linguistic fuzzy approach has been successfully applied in addressing qualitative MCDM problems (Zadeh, 1975). Several extended linguistic models, such as the linguistic 2-tuple model (Herrera and Martínez, 2000; Martínez and Herrera, 2012), the symbolic linguistic model (Rodriguez and Martínez, 2013; Xu, 2004; Yager, 1995), the linguistic model based on type-2 fuzzy set (Türkşen, 2002), the proportional 2-tuple model (Wang and Hao, 2006), the hesitant fuzzy linguistic term set (HFLTS) model (Rodriguez *et al.*, 2012), etc., have recently been developed to enrich linguistic fuzzy theory.

Among these previous linguistic models, the HFLTS model greatly increases the flexibility and capability of eliciting and representing linguistic information. The HFLTSs have been successfully applied to various decision making fields. For example, Rodriguez *et al*. (2012, 2013) proposed an envelope of HFLTS (i.e. linguistic intervals) to facilitate computing with words process (Herrera *et al*., 2009). Beg and Rashid (2013) developed an extended TOPSIS (*technique for order preference by similarity to ideal solution*) to handle MCDM problems with HFLTSs. Liu and Rodriguez (2014) proposed a trapezoidal fuzzy envelope of HFLTS to carry out computing with word processes. To effectively deal with group decision making problems with HFLTSs, Lee and Chen (2015) put forward a fuzzy group decision method based on the likelihood-based comparison relations of HFLTSs and a series of hesitant fuzzy linguistic aggregating operators. Chen and Hong (2014) also presented a new decision method based on the aggregation of linguistic terms represented by fuzzy numbers in HFLTSs to deal with hesitant fuzzy linguistic MCDM problems. What's more, Zhang *et al*. (2016) developed the concept of the hesitant trapezoidal fuzzy numbers (HTrFNs) to represent the semantic of the HFLTS. The HTrFNs benefited from the superiority of both trapezoidal fuzzy numbers (TrFNs) and hesitant fuzzy elements (HFEs) and it can be used to model effectively the imprecise and ambiguous information in real-world multicriteria group decision making (MCGDM) problems. In order to further solve the MCGDM problems in the environment of HFLTSs based on HTrFNs, it is necessary to develop the corresponding decision making methods.

The TODIM (interactive and multi-criteria decision making) developed by Gomes and Lima (1992) is a discrete MCDM approach based on prospect theory (Kahneman and Tversky, 1979), which can take the decision maker's psychological behaviour into account. Consider the fact that the relationships among criteria are interdependent; Gomes *et al*. (2013) developed a Choquet integral-based TODIM method to handle the MCDM problems with criteria interactions. The TODIM method has been successfully applied in various fields of decision making, such as the selection of the destination of natural gas (Gomes *et al*., 2009), the evaluation of residential properties (Gomes *et al*., 2013; Gomes and Rangel, 2009), the supplier selection problem (Tosun and Akyüz, 2015) and oil spill response (Passos *et al*., 2014), etc. Recently, the TODIM method has been extended into fuzzy environments because the crisp data is usually inadequate or insufficient to model the real-life complex decision problems (Zhang *et al*., 2015). For example, Krohling and de Souza (2012) developed a fuzzy extension of TODIM for handling MCDM problems with TrFNs. Fan *et al*. (2013) proposed a hybrid TODIM method to deal with the MCDM problems in which criteria values take the forms of crisp numbers, interval numbers and fuzzy numbers. Liu and Teng (2016) also extended the TODIM method to deal with MCDM problems in which the criteria values are in the form of 2-dimension uncertain linguistic variables. Lourenzutti and Krohling (2013) presented a generalization of the TODIM method which considers intuitionistic fuzzy information and an underlying random vector. Zhang and Xu (2014a) developed a hesitant fuzzy TODIM method for solving MCDM problems with HFEs. Wei *et al*. (2015) developed a hesitant fuzzy linguistic TODIM method for dealing with the MCDM problems with HFLTSs.

Despite their usefulness, these aforementioned TODIM-based methods fail to manage the HTrFN decision data which is collected by comparative linguistic expressions. To this end, in this study we develop a hesitant trapezoidal fuzzy TODIM approach with a closeness index-based ranking method for handling MCGDM problems with HTrFNs in which the weights of criteria are completely known and the weights of experts are completely unknown or partially known. First, we propose a novel closeness index for HTrFN and introduce a closeness index-based ranking method for HTrFNs. Next, we employ the closeness index-based ranking method to identify the gain and loss of each alternative relative to the others. Then, we calculate the dominance values of alternatives over others for each expert. Finally, we establish a nonlinear programming model to derive the dominance values of alternatives over others for the group and correspondingly we can obtain the optimal ranking order of alternatives. The rest of the paper is organized as follows: Section 2 reviews the basic concepts of HTrFNs. In Section 3, a hesitant trapezoidal fuzzy TODIM approach is proposed to solve the MCGDM problems with HTrFNs. In Section 4, a case study is presented. Section 5 presents our conclusions.

2. Preliminaries

In this section, we review some basic concepts of HTrFNs. Meanwhile, we define a novel closeness index of the HTrFN and introduce a closeness index-based ranking method for HTrFNs.

Definition 1. (Zadeh, 1975.) A fuzzy number $\tilde{\alpha} = T(a, b, c, d)$ is said to be a TrFN if its membership function is given as follows:

$$\mu_{\tilde{\alpha}}(x) = \begin{cases} (x-a)/(b-a), & (a \leqslant x < b), \\ 1, & (b \leqslant x \leqslant c), \\ (d-x)/(d-c), & (c < x \leqslant d), \\ 0, & \text{otherwise}, \end{cases} \tag{2.1}$$

where the closed interval $[b, c]$, a and d are the mode, low and upper limits of $\tilde{\alpha}$, respectively.

Remark 1. It is noted that a TrFN $\tilde{\alpha} = T(a, b, c, d)$ is reduced to a triangular fuzzy number if $b = c$. A TrFN $\tilde{\alpha} = T(a, b, c, d)$ is reduced to a real number if $a = b = c = d$. A TrFN $\tilde{\alpha} = T(a, b, c, d)$ is the normalized TrFN if $a \geqslant 0$ and $d \leqslant 1$. Thus, the TrFN $\tilde{1} = T(1, 1, 1, 1)$ is the maximal normalized TrFN which is also called the positive ideal TrFN, while the TrFN $\tilde{0} = T(0, 0, 0, 0)$ is the minimal normalized TrFN which is also called the negative ideal TrFN.

Definition 2. (See Zhang *et al.*, 2016.) Let X be a fixed set, a HTrFS $\mathscr{H}$ on X is defined as:

$$\mathscr{H} = \{\langle x, h_{\mathscr{H}}(x)\rangle \mid x \in X\} \tag{2.2}$$

where $h_{\mathscr{H}}(x)$ is a set of different normalized TrFNs, representing the possible membership degrees of the element $x \in X$ to $\mathscr{H}$.

For convenience, $h_{\mathscr{H}}(x)$ is called a HTrFN denoted by $h = \{\tilde{\alpha}^1, \tilde{\alpha}^2, \ldots, \tilde{\alpha}^{\#h}\}$ where the $\tilde{\alpha}^f = T(a^f, b^f, c^f, d^f)$ $(f = 1, 2, \ldots, \#h)$ is a normalized TrFN and $\#h$ is the number of all TrFNs in h. If $\#h = 1$, the HTrFN h is reduced a TrFN. If $b^f = c^f$ $(f = 1, 2, \ldots, \#h)$, the HTrFN h is reduced to a hesitant triangular fuzzy number (Zhao *et al.*, 2014).

Example 1. Let $X = \{x_1, x_2, x_3\}$, and let $h_{\mathscr{H}}(x_1) = \{T(0.2, 0.3, 0.4, 0.5), T(0.3, 0.4, 0.5, 0.6), T(0.35, 0.4, 0.45, 0.5)\}, h_{\mathscr{H}}(x_2) = \{T(0.1, 0.2, 0.3, 0.5), T(0.3, 0.4, 0.4, 0.6)\}$, and $h_{\mathscr{H}}(x_3) = \{T(0.2, 0.4, 0.5, 0.6), T(0.1, 0.3, 0.4, 0.6)\}$ be three HTrFNs of x_i $(i = 1, 2, 3)$ to a set $\mathscr{H}$. Thus, $\mathscr{H}$ can be called an HTrFS which is denoted as:

$$\mathscr{H} = \left\{ \begin{array}{l} \langle x_1, \{T(0.2, 0.3, 0.4, 0.5), T(0.3, 0.4, 0.5, 0.6), T(0.35, 0.4, 0.45, 0.5)\}\rangle, \\ \langle x_2, \{T(0.1, 0.2, 0.3, 0.5), T(0.3, 0.4, 0.4, 0.6)\}\rangle, \\ \langle x_3, \{T(0.2, 0.4, 0.5, 0.6), T(0.1, 0.3, 0.4, 0.6)\}\rangle \end{array} \right\}.$$

Remark 2. It is easy to see that the number of TrFNs in different HTrFNs is usually different. In such cases, it is necessary to extend the shorter one until both of them have the same length when we compare their magnitudes. To extend the shorter one, the best way is to add some TrFNs in it. Zhang *et al.* (2016) suggested that the optimist added the maximum TrFN because he/she often anticipated the desirable outcomes, while the pessimist added the minimum TrFN since he/she usually expected the unfavourable outcomes. The maximum TrFN or minimum TrFN in the shorter HTrFN can be identified by the sign distance-based ranking method develop by Abbasbandy and Asady (2006). Without loss of generality, in this study we assume that all possible TrFNs of HTrFNs are arranged in increasing order and the experts are pessimists.

Definition 3. Let $h_j = \{\tilde{\alpha}_j^1, \tilde{\alpha}_j^2, \ldots, \tilde{\alpha}_j^{\#h_j}\}$ $(j = 1, 2)$ be two HTrFNs, and $\#h_1 = \#h_2 = \#h$, then a nature quasi-ordering on HTrFNs is defined as follows:

$$h_1 \leqslant h_2 \text{ if and only if } \tilde{\alpha}_1^f \leqslant \tilde{\alpha}_2^f \quad (f = 1, 2, \ldots, \#h).$$

It is easily observed from Definition 3 that the HTrFN $h^+ = \{T(1, 1, 1, 1), \ldots, T(1, 1, 1, 1)\}$ is the biggest HTrFN and the HTrFN $h^- = \{T(0, 0, 0, 0), \ldots, T(0, 0, 0, 0)\}$ is the smallest HTrFN, respectively. We also call h^+ the positive ideal HTrFN and h^- the negative ideal HTrFN, respectively.

Zhang *et al.* (2016) developed hesitant trapezoidal Hamming distance for HTrFNs as below:

Definition 4. Given two HTrFNs $h_j = \{\tilde{\alpha}_j^1, \tilde{\alpha}_j^2, \ldots, \tilde{\alpha}_j^{\#h_j}\}$ $(j = 1, 2)$ with $\#h_1 = \#h_2 = \#h$, the hesitant trapezoidal Hamming distance between them is defined as follows:

$$d(h_1, h_2) = \frac{1}{6\#h} \sum_{f=1}^{\#h} \left(\left|a_1^f - a_2^f\right| + 2\left|b_1^f - b_2^f\right| + 2\left|c_1^f - c_2^f\right| + \left|d_1^f - d_2^f\right|\right). \tag{2.3}$$

Example 2. For two HTrFNs $h_1 = \{T(0.1, 0.2, 0.3, 0.5), T(0.3, 0.4, 0.4, 0.6)\}$ and $h_2 = \{T(0.1, 0.3, 0.4, 0.6), T(0.2, 0.4, 0.5, 0.6)\}$, the following result based on Definition 4 is obtained:

$$\begin{aligned} d(h_1, h_2) &= \frac{1}{6 \times 2}\begin{pmatrix} |0.1-0.1| + 2|0.2-0.3| + 2|0.3-0.4| + |0.5-0.6| + \\ |0.3-0.2| + 2|0.4-0.4| + 2|0.4-0.5| + |0.6-0.6| \end{pmatrix} \\ &= 0.0667. \end{aligned} \tag{2.4}$$

According to Definition 4, the distance between the HTrFN $h = \{\tilde{\alpha}^1, \tilde{\alpha}^2, \dots, \tilde{\alpha}^{\#h}\}$ and the positive ideal HTrFN h^+ can be calculated as follows:

$$\begin{aligned} d(h, h^+) &= \frac{1}{6\#h}\sum_{f=1}^{\#h}\left(1 - a^f + 2(1 - b^f) + 2(1 - c^f) + 1 - d^f\right) \\ &= \frac{1}{6\#h}\sum_{f=1}^{\#h}\left(6 - a^f - 2b^f - 2c^f - d^f\right) \end{aligned} \tag{2.5}$$

and the distance between the HTrFN h and the negative ideal HTrFN h^- can be computed as below:

$$d(h, h^-) = \frac{1}{6\#h}\sum_{f=1}^{\#h}\left(a^f + 2b^f + 2c^f + d^f\right). \tag{2.6}$$

In general, the smaller the distance $d(h, h^+)$ is, the bigger the HTrFN h is; and the larger the distance $d(h, h^-)$ is, the bigger the HTrFN h is. Motivated by the idea of TOPSIS method (Hwang and Yoon, 1981) we define a closeness index for the HTrFN as follows.

Definition 5. Let $h = \{\tilde{\alpha}^1, \tilde{\alpha}^2, \dots, \tilde{\alpha}^{\#h}\}$ be a HTrFN, h^+ be the positive ideal HTrFN and h^- be the negative ideal HTrFN; then the closeness index of h can be defined as follows:

$$\begin{aligned} \phi(h) &= \frac{d(h, h^-)}{d(h, h^-) + d(h, h^+)} \\ &= \frac{\sum_{f=1}^{\#h}(a^f + 2b^f + 2c^f + d^f)}{\sum_{f=1}^{\#h}(6 - a^f - 2b^f - 2c^f - d^f) + \sum_{f=1}^{\#h}(a^f + 2b^f + 2c^f + d^f)} \\ &= \frac{1}{6\#h}\sum_{f=1}^{\#h}\left(a^f + 2b^f + 2c^f + d^f\right). \end{aligned} \tag{2.7}$$

Obviously, if $h = h^-$, then $\phi(h) = 0$; while if $h = h^+$, then $\phi(h) = 1$.

Based on the closeness indices of HTrFNs, a comparison law for HTrFNs is introduced.

Definition 6. Given two HTrFNs $h_j = \{\tilde{\alpha}_j^1, \tilde{\alpha}_j^2, \ldots, \tilde{\alpha}_j^{\#h_j}\}$ $(j = 1, 2)$, $\phi(h_1)$ and $\phi(h_2)$ be the closeness indices of h_1 and h_2, respectively, then

1) if $\phi(h_1) < \phi(h_2)$, then $h_1 \prec h_2$;
2) if $\phi(h_1) > \phi(h_2)$, then $h_1 \succ h_2$;
3) if $\phi(h_1) = \phi(h_2)$, then $h_1 \sim h_2$.

Example 3. For two HTrFNs $h_1 = \{T(0.1, 0.2, 0.3, 0.5), T(0.3, 0.4, 0.4, 0.6)\}$ and $h_2 = \{T(0.1, 0.3, 0.4, 0.6), T(0.2, 0.4, 0.5, 0.6)\}$, the following result based on Definition 5 is obtained:

$$\phi(h_1) = \frac{1}{12}(0.1 + 0.4 + 0.6 + 0.5 + 0.3 + 0.8 + 0.8 + 0.6) = 0.2563,$$
$$\phi(h_2) = \frac{1}{12}(0.1 + 0.6 + 0.8 + 0.6 + 0.2 + 0.8 + 1.0 + 0.6) = 0.3912.$$

According to Definition 6, it is observed that $\phi(h_1) < \phi(h_2)$, i.e. $h_1 \prec h_2$.

3. Hesitant Trapezoidal Fuzzy TODIM Decision Analysis Method

Consider a decision environment based on HTrFNs for MCGDM problems in which the criteria values of alternatives take the form of comparative linguistic expressions. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of m $(m \geqslant 2)$ feasible alternatives, $\boldsymbol{C} = \{C_1, C_2, \ldots, C_n\}$ be a finite set of criteria, and $\boldsymbol{E} = \{E_1, E_2, \ldots, E_g\}$ be a group of experts. We also denote the weighting vector of criteria by $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$, where w_j is the weight of the criterion C_j, satisfying the normalization condition: $\sum_{j=1}^{n} w_j = 1$ and $w_j \geqslant 0$. Meanwhile, we denote the weighting vector of experts by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_g)^{\mathrm{T}}$, where λ_k is the weight of the expert E_k, satisfying the normalization condition: $\sum_{k=1}^{g} \lambda_k = 1$ and $\lambda_k \geqslant 0$. In this paper, the weights of criteria are completely known beforehand and the weights of experts are completely unknown or partially known. Let Δ be a set of the known weight information of experts, Δ can be constructed by the following five structure forms (Kim and Ahn, 1999; Wan and Li, 2013; Zhang and Xu, 2014b, 2015): (1) A weak ranking form: $\{\lambda_i \geqslant \lambda_j\}$ $(i \neq j)$; (2) A strict ranking form: $\{\lambda_i - \lambda_j \geqslant \alpha_i\}$ $(\alpha_i > 0)$; (3) A ranking of differences form: $\{\lambda_i - \lambda_j \geqslant \lambda_k - \lambda_l\}$ $(i \neq j \neq k \neq l)$; (4) A ranking with multiples form: $\{\lambda_i \geqslant \alpha_i \lambda_j\}$ $(i \neq j, 0 \leqslant \alpha_i \leqslant 1)$; (5) An interval form: $\{\alpha_i \leqslant \lambda_i \leqslant \alpha_i + \kappa_i\}$ $(0 \leqslant \alpha_i \leqslant \alpha_i + \kappa_i \leqslant 1)$.

The criteria value of the alternative $A_i \in \boldsymbol{A}$ with respect to the criterion $C_j \in \boldsymbol{C}$ provided by the expert $E_k \in \boldsymbol{E}$ can be represented by comparative linguistic expressions ll_{ij}^k. According to Zhang *et al.* (2016), the comparative linguistic expressions ll_{ij}^k can be normally converted into a HTrFN $h_{ij}^k = \{\tilde{\alpha}_{ij}^{k(1)}, \tilde{\alpha}_{ij}^{k(2)}, \ldots, \tilde{\alpha}_{ij}^{k(\#h_{ij}^k)}\}$. Thus the decision matrix $R^k = (ll_{ij}^k)_{m\times n}$ can be concisely expressed in the decision matrix format $R^k = (h_{ij}^k)_{m\times n}$ $(E_k \in \boldsymbol{E})$ as shown in Table 1.

Table 1
Hesitant trapezoidal fuzzy decision matrix.

Experts	Alternatives	Criteria			
		C_1	C_2	$\dots$	C_n
E_1	A_1	h_{11}^1	h_{12}^1	$\dots$	h_{1n}^1
	A_2	h_{21}^1	h_{22}^1	$\dots$	h_{2n}^1
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
	A_m	h_{m1}^1	h_{m2}^1	$\dots$	h_{mn}^1
$\dots$	A_1	h_{11}^k	h_{12}^k	$\dots$	h_{1n}^k
	A_2	h_{21}^k	h_{22}^k	$\dots$	h_{2n}^k
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
	A_m	h_{m1}^k	h_{m2}^k	$\dots$	h_{mn}^k
E_g	A_1	h_{11}^g	h_{12}^g	$\dots$	h_{1n}^g
	A_2	h_{21}^g	h_{22}^g	$\dots$	h_{2n}^g
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
	A_m	h_{m1}^g	h_{m2}^g	$\dots$	h_{mn}^g

In what follows, we develop a new technique to deal effectively with the above MCGDM problem. The focus of the proposed method is to measure the dominance degree of each alternative over the others by constructing the prospect value function based on prospect theory. For this purpose, we need to identify the reference criterion and calculate the relative weight of each criterion to the reference criterion. According to the idea of the classical TODIM (Gomes and Lima, 1992), the criterion with the highest weight is usually regarded as the reference criterion C_r, namely,

$$C_r = \left\{ C_j : \max_{j=1}^{n} w_j \right\}. \tag{3.1}$$

Then, the relative weight w_{jr} of the criterion $C_j \in \boldsymbol{C}$ to the reference criterion C_r can be obtained by the following equation:

$$w_{jr} = w_j / w_r, \quad r, j \in \{1, 2, \dots, n\} \tag{3.2}$$

where w_r is the weight of the reference criterion C_r.

Furthermore, by employing the closeness index-based ranking method of HTrFNs we compare with the magnitude of the criteria values of alternatives with respect to each criterion. Then, for the expert $E_k \in \boldsymbol{E}$, the dominance value of the alternative $A_\xi \in \boldsymbol{A}$ over the alternative $A_\zeta \in \boldsymbol{A}$ concerning the criterion $C_j \in \boldsymbol{C}$ can be calculated by using the following expression:

$$Q_j^k(A_\xi, A_\zeta) = \begin{cases} \sqrt{\frac{w_{jr}(\phi(h_{\xi j}^k) - \phi(h_{\zeta j}^k))}{\sum_{j=1}^n w_{jr}}}, & \text{if } \phi(h_{\xi j}^k) - \phi(h_{\zeta j}^k) > 0, \\ 0, & \text{if } \phi(h_{\xi j}^k) - \phi(h_{\zeta j}^k) = 0, \\ -\frac{1}{\theta}\sqrt{\frac{(\sum_{j=1}^n w_{jr})(\phi(h_{\zeta j}^k) - \phi(h_{\xi j}^k))}{w_{jr}}}, & \text{if } \phi(h_{\xi j}^k) - \phi(h_{\zeta j}^k) < 0 \end{cases} \tag{3.3}$$

Table 2
The overall dominance values of alternatives for each expert.

Alternatives	Experts			
	E_1	E_2	...	E_g
A_1	$Q^1(A_1)$	$Q^2(A_1)$	...	$Q^g(A_1)$
A_2	$Q^1(A_2)$	$Q^2(A_2)$	...	$Q^g(A_2)$
...	...	...	...	...
A_m	$Q^1(A_m)$	$Q^2(A_m)$	...	$Q^g(A_m)$

where $\phi(h^k_{\xi j})$ and $\phi(h^k_{\zeta j})$ are respectively the closeness indices of the criteria values $h^k_{\xi j}$ and $h^k_{\zeta j}$, and the parameter $\theta \in [1, 10]$ represents the attenuation factor of the losses.

From Eq. (3.3), it is easily observed that: (1) if $\phi(h^k_{\xi j}) - \phi(h^k_{\zeta j}) > 0$, then $Q^k_j(A_\xi, A_\zeta)$ represents a gain; (2) if $\phi(h^k_{\xi j}) - \phi(h^k_{\zeta j}) = 0$, then $Q^k_j(A_\xi, A_\zeta)$ represents a nil; (3) if $\phi(h^k_{\xi j}) - \phi(h^k_{\zeta j}) < 0$, then $Q^k_j(A_\xi, A_\zeta)$ represents a loss.

For the expert $E_k \in \boldsymbol{E}$, the weighted dominance value of the alternative $A_\xi \in \boldsymbol{A}$ over the alternative $A_\zeta \in \boldsymbol{A}$ can be obtained as follows:

$$Q^k(A_\xi, A_\zeta) = \sum_{j=1}^{n} Q^k_j(A_\xi, A_\zeta). \tag{3.4}$$

The overall dominance value of the alternative $A_\xi \in \boldsymbol{A}$ for the expert $E_k \in \boldsymbol{E}$ can be obtained by the following equation and is listed in Table 2:

$$Q^k(A_\xi) = \frac{\sum_{\zeta=1}^{m} Q^k(A_\xi, A_\zeta) - \min_{\xi=1}^{m}\{\sum_{\zeta=1}^{m} Q^k(A_\xi, A_\zeta)\}}{\max_{\xi=1}^{m}\{\sum_{\zeta=1}^{m} Q^k(A_\xi, A_\zeta)\} - \min_{\xi=1}^{m}\{\sum_{\zeta=1}^{m} Q^k(A_\xi, A_\zeta)\}}, \quad \xi \in \{1, 2, \dots, m\}. \tag{3.5}$$

After obtaining the overall dominance value $Q^k(A_\xi)$ of the alternative A_ξ ($\xi \in \{1, 2, \dots, m\}$) for the expert E_k ($k \in \{1, 2, \dots, g\}$), we need to determine the overall dominance value for the group which is represented by $Q^*(A_\xi)$ ($\xi \in \{1, 2, \dots, m\}$). Based on the decision data in Table 2, we further establish a nonlinear programming model to calculate $Q^*(A_\xi)$ ($\xi \in \{1, 2, \dots, m\}$) for the group as follows:

$$\begin{cases} \min \quad Z = \sum_{\xi=1}^{m}\sum_{k=1}^{g} \lambda_k \left| Q^k(A_\xi) - Q^*(A_\xi) \right| \\ \text{s.t.} \quad \sum_{k=1}^{g} \lambda_k = 1,\ \lambda_k \geqslant 0,\ k \in \{1, 2, \dots, g\}. \end{cases} \tag{MOD-1}$$

To solve the model (MOD-1), let

$$\eta^k_\xi = \frac{1}{2}\left(\left|Q^k(A_\xi) - Q^*(A_\xi)\right| + \left(Q^k(A_\xi) - Q^*(A_\xi)\right)\right) \tag{3.6}$$

and

$$\rho_\xi^k = \frac{1}{2}\left(\left|Q^k(A_\xi) - Q^*(A_\xi)\right| - \left(Q^k(A_\xi) - Q^*(A_\xi)\right)\right). \tag{3.7}$$

Then, the optimal model (MOD-1) is transformed into the following optimal model:

$$\begin{cases} \min \quad Z = \sum_{\xi=1}^{m}\sum_{k=1}^{g} \lambda_k\left(\eta_\xi^k + \rho_\xi^k\right) \\ \text{s.t.} \\ \quad Q^k(A_\xi) - Q^*(A_\xi) - \eta_\xi^k + \rho_\xi^k = 0;\ \xi \in \{1,2,\ldots,m\},\ k \in \{1,2,\ldots,g\}, \\ \quad \eta_\xi^k \geqslant 0,\ \rho_\xi^k \geqslant 0,\ \eta_\xi^k \rho_\xi^k = 0;\ \xi \in \{1,2,\ldots,m\},\ k \in \{1,2,\ldots,g\}, \\ \quad \sum_{k=1}^{g} \lambda_k = 1,\ \lambda_k \geqslant 0,\ k \in \{1,2,\ldots,g\}. \end{cases} \tag{MOD-2}$$

It is observed that the model (MOD-2) is a linear programming model and can be easily executed by using the MATLAB 7.4.0 or LINGO 11.0. By solving this model, we get the optimal solutions $Q^*(A_\xi)$ ($\xi \in \{1,2,\ldots,m\}$) and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_g)^T$.

In addition, there are real-world situations that the weights of experts are not completely unknown but partially known. For these cases, based on the set of the known weight information of experts Δ, we construct the following optimization model to get the optimal solutions $Q^*(A_\xi)$ ($\xi \in \{1,2,\ldots,m\}$) and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_g)^T$:

$$\begin{cases} \min \quad Z = \sum_{\xi=1}^{m}\sum_{k=1}^{g} \lambda_k(\eta_\xi^k + \rho_\xi^k) \\ \text{s.t.} \\ \quad Q^k(A_\xi) - Q^*(A_\xi) - \eta_\xi^k + \rho_\xi^k = 0;\ \xi \in \{1,2,\ldots,m\},\ k \in \{1,2,\ldots,g\}, \\ \quad \eta_\xi^k \geqslant 0,\ \rho_\xi^k \geqslant 0,\ \eta_\xi^k \rho_\xi^k = 0;\ \xi \in \{1,2,\ldots,m\},\ k \in \{1,2,\ldots,g\}, \\ \quad (\lambda_1, \lambda_2, \ldots, \lambda_g) \in \Delta \end{cases} \tag{MOD-3}$$

where Δ is also a set of constraint conditions that the expert weight λ_k ($k \in \{1,2,\ldots,g\}$) should satisfy according to the requirements in real-world situations.

Obviously, the greater the value of $Q^*(A_\xi)$ ($\xi \in \{1,2,\ldots,m\}$) is, the better the alternative A_ξ will be. Therefore, we can determine the ranking order of alternatives according to the increasing order of the $Q^*(A_\xi)$ ($\xi \in \{1,2,\ldots,m\}$), and select the best alternative from the alternative set $\{A_1, A_2, \ldots, A_m\}$. According to the above analysis, the steps of the proposed method are summarized as follows:

Step 1. Identify the criteria values of alternatives on criteria under each expert and the weights of criteria, respectively;

Step 2. For each expert, we employ Eq. (3.3) to calculate the dominance value of the alternative $A_\xi \in \boldsymbol{A}$ over the alternative $A_\zeta \in \boldsymbol{A}$ concerning the criterion $C_j \in \boldsymbol{C}$;

Step 3. For each expert, we use Eq. (3.4) to compute the weighted dominance value of the alternative $A_\xi \in \boldsymbol{A}$ over the alternative $A_\zeta \in \boldsymbol{A}$;

Step 4. We utilize Eq. (3.5) to determine the overall dominance value of the alternative $A_\xi \in \boldsymbol{A}$ for the expert $E_k \in \boldsymbol{E}$;

Step 5. If the weights of experts are completely unknown, according to the model (MOD-2) we construct a linear programming model to determine the overall dominance value of the alternative $A_\xi \in \boldsymbol{A}$ for the group; if the weights of experts are partially known, based on the model (MOD-3) we construct an optimal model to determine the overall dominance value of the alternative $A_\xi \in \boldsymbol{A}$ for the group;

Step 6. Rank the alternatives by comparing the magnitude of the overall dominance value of the alternative $A_\xi \in \boldsymbol{A}$ for the group.

4. Case Illustration

In this section, we consider an evaluation problem of the service quality among airlines discussed in Liou *et al.* (2011), Zhang and Xu (2014a) to demonstrate the decision process and the applicability of the proposed approach.

Due to the development of high-speed railroad, airline marketing has faced a powerful challenge. More and more airlines have attempted to attract customers by reducing price. Unfortunately, they soon found that there was a no-win situation and only service quality is the critical and fundamental element to survive in this highly competitive domestic market. In order to improve the service quality of Taiwan airline, the civil aviation administration (CAA) wants to know which airline is the best one and then calls for the others to learn from it. So the CAA invites a committee including three experts (E_1, E_2, E_3) to investigate four major Taiwan airlines, which are UNI Air (A_1), Transasia (A_2), Mandarin (A_3) and Daily Air (A_4), according to the following four qualitative criteria: Booking and ticketing service (C_1), Check-in and boarding process (C_2), Cabin service (C_3), and Responsiveness (C_4). The weight vector of the criteria is $\boldsymbol{w} = (0.2, 0.25, 0.35, 0.2)^{\mathrm{T}}$. The weight vector of the experts is given as follows:

$$\Delta = \left\{ \begin{array}{l} \lambda_3 \geqslant \lambda_1,\ 0.15 \leqslant \lambda_2 - \lambda_1 \leqslant 0.25,\ \lambda_1 + \lambda_3 \geqslant \lambda_2,\ 0.2 \leqslant \lambda_2 \leqslant 0.35, \\ \lambda_1 + \lambda_2 + \lambda_3 = 1,\ \lambda_1 \geqslant 0,\ \lambda_2 \geqslant 0,\ \lambda_3 \geqslant 0 \end{array} \right\}.$$

All experts employ linguistic terms or comparison linguistic expressions to provide the assessment values of alternatives with respect to each criterion as shown in Table 3.

The top-left cell "*Between P and MP*" in Table 3 indicates that the degree to which the alternative A_1 (UNI Air) satisfies the criterion C_1 (booking and ticketing service) is *between Poor and Medium Poor*. The others in Table 3 have the similar meanings.

Usually, the uncertainty and vagueness of the linguistic assessments can be captured and represented by TrFNs. The corresponding relation between TrFNs and linguistic variables with seven-point rating scales is expressed in Table 4.

Table 3
The linguistic criteria values of alternatives for each expert.

Experts	Alternatives	Criteria			
		C_1	C_2	C_3	C_4
E_1	A_1	Between MG and G	MG	Between MG and G	At most MP
	A_2	MG	Between MP and F	MP	Between F and MG
	A_3	Between MP and F	Between P and MP	At least MG	MG
	A_4	MG	F	G	Between P and MP
E_2	A_1	Between F and MG	MP	P	G
	A_2	Between G and VG	At least MG	F	F
	A_3	G	MG	MP	MP
	A_4	MP	G	F	Between P and MP
E_3	A_1	At least MG	F	At most MP	MG
	A_2	F	MG	At least G	P
	A_3	Between F and G	Between MP and F	Between P and MP	At least G
	A_4	MG	G	Between F and G	G

Note: VP: Very poor; P: Poor; MP: Medium poor; F: Fair; MG: Medium good; G: Good; VG: Very good.

Table 4
Linguistic terms and their corresponding TrFNs.

Rating	Abbreviation	TrFNs
s_0: Very poor	VP	$T(0.0, 0.0, 0.1, 0.2)$
s_1: Poor	P	$T(0.1, 0.2, 0.2, 0.3)$
s_2: Medium poor	MP	$T(0.2, 0.3, 0.4, 0.5)$
s_3: Fair	F	$T(0.4, 0.5, 0.5, 0.6)$
s_4: Medium good	MG	$T(0.5, 0.6, 0.7, 0.8)$
s_5: Good	G	$T(0.7, 0.8, 0.8, 0.9)$
s_6: Very good	VG	$T(0.8, 0.9, 1.0, 1.0)$

Then, according to Zhang *et al.* (2016), all comparative linguistic expressions are transformed into HTrFNs which are listed in Table 5.

In what follows, we employ the proposed hesitant trapezoidal fuzzy TODIM method to solve the above decision problem. Based on the closeness index-based ranking method of HTrFNs, we first compare with the magnitude of the criteria values and obtain the superior-inferior table as in Table 6.

The top-left cell "$_{1/2}\mathrm{S}_1$" in Table 6 indicates that for the expert E_1 and under the criterion C_1 the alternative A_1 is superior to the alternative A_2 because of $\phi(h_{11}^1) = 0.725 > \phi(h_{21}^1) = 0.65$. Similar logic is used to determine the remaining entries in Table 6. Then, we notice that the weight of the criterion C_3 is the biggest one among these four criteria. According to the proposed method (i.e. Eq. (3.2)), the criterion C_3 is regarded as the reference criterion and thus the weight of the reference criterion $w_r = w_{(C_3)}$ is 0.35. Furthermore, using Eq. (3.2) the relative weight of the criterion C_1 is $w_{1r} = w_{(C_1)}/w_r = 0.2/0.35 = 0.57$. Analogously, the other relative weights can be obtained. Without loss of generality, we take the value of the parameter θ as 3 and the dominance value of the alternative $A_\xi \in \boldsymbol{A}$ over the alternative $A_\zeta \in \boldsymbol{A}$ under the criterion $C_j \in \boldsymbol{C}$ can be calculated by using Eq. (3.3). The calculated results are listed in Tables 7–10. Furthermore, by aggregating the gains and losses of the alternative $A_\xi \in \boldsymbol{A}$ over the alternative $A_\zeta \in \boldsymbol{A}$

Table 5
The hesitant criteria values of alternatives.

Experts	Alternatives	Criteria			
		C1	C2	C3	C4
E_1	A_1	$\{T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9)\}$	$T(0.5, 0.6, 0.7, 0.8)$	$\{T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9)\}$	$\{T(0.0, 0.0, 0.1, 0.2), T(0.1, 0.2, 0.2, 0.3), T(0.2, 0.3, 0.4, 0.5)\}$
	A_2	$T(0.5, 0.6, 0.7, 0.8)$	$\{T(0.2, 0.3, 0.4, 0.5), T(0.4, 0.5, 0.5, 0.6)\}$	$T(0.7, 0.8, 0.8, 0.9)$	$\{T(0.4, 0.5, 0.5, 0.6), T(0.5, 0.6, 0.7, 0.8)\}$
	A_3	$\{T(0.2, 0.3, 0.4, 0.5), T(0.4, 0.5, 0.5, 0.6)\}$	$\{T(0.1, 0.2, 0.2, 0.3), T(0.2, 0.3, 0.4, 0.5)\}$	$\{T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9), T(0.8, 0.9, 1.0, 1.0)\}$	$T(0.5, 0.6, 0.7, 0.8)$
	A_4	$T(0.5, 0.6, 0.7, 0.8)$	$T(0.4, 0.5, 0.5, 0.6)$	$T(0.7, 0.8, 0.8, 0.9)$	$\{T(0.1, 0.2, 0.2, 0.3), T(0.2, 0.3, 0.4, 0.5)\}$
E_2	A_1	$\{T(0.4, 0.5, 0.5, 0.6), T(0.5, 0.6, 0.7, 0.8)\}$	$T(0.2, 0.3, 0.4, 0.5)$	$T(0.1, 0.2, 0.2, 0.3)$	$T(0.7, 0.8, 0.8, 0.9)$
	A_2	$\{T(0.7, 0.8, 0.8, 0.9), T(0.8, 0.9, 1.0, 1.0)\}$	$\{T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9), T(0.8, 0.9, 1.0, 1.0)\}$	$T(0.4, 0.5, 0.5, 0.6)$	$T(0.4, 0.5, 0.5, 0.6)$
	A_3	$T(0.7, 0.8, 0.8, 0.9)$	$T(0.5, 0.6, 0.7, 0.8)$	$T(0.2, 0.3, 0.4, 0.5)$	$T(0.2, 0.3, 0.4, 0.5)$
	A_4	$T(0.2, 0.3, 0.4, 0.5)$	$T(0.7, 0.8, 0.8, 0.9)$	$T(0.4, 0.5, 0.5, 0.6)$	$\{T(0.4, 0.5, 0.5, 0.6), T(0.5, 0.6, 0.7, 0.8)\}$
E_3	A_1	$\{T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9), T(0.8, 0.9, 1.0, 1.0)\}$	$T(0.4, 0.5, 0.5, 0.6)$	$\{T(0.0, 0.0, 0.1, 0.2), T(0.1, 0.2, 0.2, 0.3), T(0.2, 0.3, 0.4, 0.5)\}$	$T(0.5, 0.6, 0.7, 0.8)$
	A_2	$T(0.4, 0.5, 0.5, 0.6)$	$T(0.5, 0.6, 0.7, 0.8)$	$\{T(0.7, 0.8, 0.8, 0.9), T(0.8, 0.9, 1.0, 1.0)\}$	$T(0.1, 0.2, 0.2, 0.3)$
	A_3	$\{T(0.4, 0.5, 0.5, 0.6), T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9)\}$	$\{T(0.2, 0.3, 0.4, 0.5), T(0.4, 0.5, 0.5, 0.6)\}$	$\{T(0.4, 0.5, 0.5, 0.6), T(0.5, 0.6, 0.7, 0.8)\}$	$\{T(0.7, 0.8, 0.8, 0.9), T(0.8, 0.9, 1.0, 1.0)\}$
	A_4	$T(0.5, 0.6, 0.7, 0.8)$	$T(0.7, 0.8, 0.8, 0.9)$	$\{T(0.4, 0.5, 0.5, 0.6), T(0.5, 0.6, 0.7, 0.8), T(0.7, 0.8, 0.8, 0.9)\}$	$T(0.7, 0.8, 0.8, 0.9)$

Table 6
A superior-inferior table over alternatives with criteria for the expert E_1.

	A_1/A_2	A_1/A_3	A_1/A_4	A_2/A_3	A_2/A_4	A_3/A_4
C_1	$_{1/2}\mathrm{S}_1$	$_{1/3}\mathrm{S}_1$	$_{1/4}\mathrm{S}_1$	$_{2/3}\mathrm{S}_1$	$_{2/4}\mathrm{E}_1$	$_{3/4}\mathrm{I}_1$
C_2	$_{1/2}\mathrm{S}_2$	$_{1/3}\mathrm{S}_2$	$_{1/4}\mathrm{S}_2$	$_{2/3}\mathrm{S}_2$	$_{2/4}\mathrm{I}_2$	$_{3/4}\mathrm{I}_2$
C_3	$_{1/2}\mathrm{I}_3$	$_{1/3}\mathrm{I}_3$	$_{1/4}\mathrm{I}_3$	$_{2/3}\mathrm{S}_3$	$_{2/4}\mathrm{E}_3$	$_{3/4}\mathrm{I}_3$
C_4	$_{1/2}\mathrm{I}_4$	$_{1/3}\mathrm{I}_4$	$_{1/4}\mathrm{I}_4$	$_{2/3}\mathrm{I}_4$	$_{2/4}\mathrm{S}_4$	$_{3/4}\mathrm{S}_4$

Note: 'S' denotes "superior to", 'I' denotes "inferior to", "E" denotes "equal to".

Table 7
Gains and losses of alternatives over the others for the criterion C_1 and the expert E_1.

	A_1	A_2	A_3	A_4
A_1	0	0.1225	0.2449	0.1225
A_2	−0.2041	0	0.2121	0
A_3	−0.4082	−0.3536	0	−0.3536
A_4	−0.2041	0	0.2121	0

under the criterion $C_j \in \boldsymbol{C}$ using Eq. (3.4), we can obtain the weighted dominance value of each alternative over the others, listed in Table 11.

Using Eq. (3.5), the overall dominance values of alternatives for the expert E_1 are obtained as follows:

Table 8
Gains and losses of alternatives over the others for the criterion C_2 and the expert E_1.

	A_1	A_2	A_3	A_4
A_1	0	0.2372	0.3062	0.1936
A_2	−0.3162	0	0.1936	−0.1826
A_3	−0.4082	−0.2582	0	−0.3162
A_4	−0.2582	0.1369	0.2372	0

Table 9
Gains and losses of alternatives over the others for the criterion C_3 and the expert E_1.

	A_1	A_2	A_3	A_4
A_1	0	−0.1543	−0.1484	−0.1543
A_2	0.1620	0	0.0443	0
A_3	0.1559	−0.0422	0	−0.0422
A_4	0.1620	0	0.0443	0

Table 10
Gains and losses of alternatives over the others for the criterion C_4 and the expert E_1.

	A_1	A_2	A_3	A_4
A_1	0	−0.4530	−0.4969	−0.1964
A_2	0.2718	0	−0.2041	0.2449
A_3	0.2981	0.1225	0	0.2739
A_4	0.1178	−0.4082	−0.4564	0

Table 11
Weighted dominance values of alternatives over the others for the expert E_1.

	A_1	A_2	A_3	A_4
A_1	0	−0.2476	−0.0942	−0.0346
A_2	−0.0865	0	0.2459	0.0623
A_3	−0.3624	−0.5315	0	−0.4381
A_4	−0.1825	−0.2713	0.0372	0

$$Q^1(A_1) = 0.6150, \quad Q^1(A_2) = 1.0, \quad Q^1(A_3) = 0.0, \quad Q^1(A_4) = 0.5892.$$

Analogously, we can also calculate the overall dominance values of alternatives for the expert E_2 as:

$$Q^2(A_1) = 0.0, \quad Q^2(A_2) = 1.0, \quad Q^2(A_3) = 0.0218, \quad Q^2(A_4) = 0.3991,$$

and the overall dominance values of alternatives for the expert E_3 as follows:

$$Q^3(A_1) = 0.0210, \quad Q^3(A_2) = 0.0, \quad Q^3(A_3) = 0.3669, \quad Q^3(A_4) = 1.0.$$

According to the model (MOD-3), we construct the following optimal model:

$$\begin{cases} \min \quad Z = \sum_{i=1}^{4} \sum_{k=1}^{3} \lambda_k(\eta_i^k + \rho_i^k) \\ \text{s.t.} \\ \quad 0.615 - Q^*(A_1) - \eta_1^1 + \rho_1^1 = 0; \quad 0.0 - Q^*(A_1) - \eta_1^2 + \rho_1^2 = 0; \\ \quad 0.021 - Q^*(A_1) - \eta_1^3 + \rho_1^3 = 0; \quad 1.0 - Q^*(A_2) - \eta_2^1 + \rho_2^1 = 0; \\ \quad 1.0 - Q^*(A_2) - \eta_2^2 + \rho_2^2 = 0; \quad 0.0 - Q^*(A_2) - \eta_2^3 + \rho_2^3 = 0; \\ \quad 0.0 - Q^*(A_3) - \eta_3^1 + \rho_3^1 = 0; \quad 0.0218 - Q^*(A_3) - \eta_3^2 + \rho_3^2 = 0; \\ \quad 0.3669 - Q^*(A_3) - \eta_3^3 + \rho_3^3 = 0; \quad 0.5892 - Q^*(A_4) - \eta_4^1 + \rho_4^1 = 0; \\ \quad 0.3991 - Q^*(A_4) - \eta_4^2 + \rho_4^2 = 0; \quad 1.0 - Q^*(A_4) - \eta_4^3 + \rho_4^3 = 0; \\ \quad \lambda_3 \geqslant \lambda_1,\ 0.15 \leqslant \lambda_2 - \lambda_1 \leqslant 0.25,\ \lambda_1 + \lambda_3 \geqslant \lambda_2,\ 0.2 \leqslant \lambda_2 \leqslant 0.35, \\ \quad \sum_{k=1}^{3} \lambda_k = 1,\ \lambda_k \geqslant 0,\ k \in \{1,2,3\}, \\ \quad \eta_i^k \geqslant 0,\ \rho_i^k \geqslant 0,\ \eta_i^k \rho_i^k = 0;\ i \in \{1,2,3,4\},\ k \in \{1,2,3\}. \end{cases} \tag{MOD-4}$$

By solving the above model (MOD-4), the overall dominance values of alternatives for the group can be obtained as follows:

$$\lambda_1 = 0.2, \quad \lambda_2 = 0.35, \quad \lambda_3 = 0.45, \quad Q^*(A_1) = 0.0210,$$
$$Q^*(A_2) = 1.0, \quad Q^*(A_3) = 0.0218, \quad Q^*(A_4) = 0.5892.$$

Apparently, the ranking order of alternatives is obtained as $A_2 \succ A_4 \succ A_3 \succ A_1$, and the best alternative is A_2.

5. Conclusions

In this paper, we have developed a hesitant trapezoidal fuzzy TODIM approach with a closeness index-based ranking method to handle MCGDM problems in which decision data is expressed as comparative linguistic expressions based on HTrFNs. The key contribution of this paper is fivefold: (1) a novel closeness index for HTrFN has been introduced; (2) an effective ranking method for HTrFNs has been proposed; (3) the dominance values of alternatives over the others have been defined; (4) the overall dominance values of alternatives over the others for each expert has been calculated; (5) a nonlinear programming model has been established to determine the overall dominance values of alternatives over the others for the group. The biggest limitation to the proposed approach is the degree of computational complexity corresponding to the large-scale group decision making problems. In terms of future research, one decision support system based on the proposed method is developed to help practitioners solve the real-world large-scale group decision making problems in which the decision information takes the form of comparative linguistic expressions based on HTrFNs. Additionally, the potential of combining the proposed approach with other useful decision making techniques within the environment of HTrFNs will also be taken into consideration in the future.

Acknowledgements. The author is very grateful to the anonymous reviewers and the editor for their insightful and constructive comments and suggestions that have led to an improved version of this paper. The work was supported by the National Natural Science Foundation of China (No. 71661010), the Natural Science Foundation of Jiangxi Province of China (No. 20161BAB211020) and the Technology Project of Education Department of Jiangxi Province of China (No. GJJ160455).

References

Abbasbandy, S., Asady, B. (2006). Ranking of fuzzy numbers by sign distance. *Information Sciences*, 176, 2405–2416.

Beg, I., Rashid, T. (2013). TOPSIS for hesitant fuzzy linguistic term sets. *International Journal of Intelligent Systems*, 28, 1162–1171.

Chen, S.-M., Hong, J.-A. (2014). Multicriteria linguistic decision making based on hesitant fuzzy linguistic term sets and the aggregation of fuzzy sets. *Information Sciences*, 286, 63–74.

Fan, Z.-P., Zhang, X., Chen, F.-D., Liu, Y. (2013). Extended TODIM method for hybrid multiple attribute decision making problems. *Knowledge-Based Systems*, 42, 40-48.

Figueira, J., Greco, S., Ehrgott, M. (2005). *Multiple Criteria Decision Analysis: State of the Art Surveys*, Vol. 78. Springer Science & Business Media.

Gomes, L.F.A.M., Lima, M.M.P.P. (1992). TODIM: Basics and application to multicriteria ranking of projects with environmental impacts. *Foundations of Computing and Decision Sciences*, 16, 113–127.

Gomes, L.F.A.M., Rangel, D. (2009). An application of the TODIM method to the multicriteria rental evaluation of residential properties. *European Journal of Operational Research*, 193, 204–211.

Gomes, L.F.A.M., Rangel, L.A.D., Maranhäo, F.J.C. (2009). Multicriteria analysis of natural gas destination in Brazil: an application of the TODIM method. *Mathematical and Computer Modelling*, 50, 92–100.

Gomes, L.F.A.M., Machado, M.A.S., Rangel, L.A.D. (2013). Behavioral multi-criteria decision analysis: the TODIM method with criteria interactions. *Annals of Operations Research*, 211, 531–548.

Herrera, F., Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 8, 746–752.

Herrera, F., Alonso, S., Chiclana, F., Herrera-Viedma, E. (2009). Computing with words in decision making: foundations, trends and prospects. *Fuzzy Optimization and Decision Making*, 8, 337–364.

Hwang, C.L., Yoon, K.S. (1981). *Multiple Attribute Decision Methods and Applications*. Springer, Berlin, Heidelberg.

Kahneman, D., Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47, 263–291.

Kim, S.H., Ahn, B.S. (1999). Interactive group decision making procedure under incomplete information. *European Journal of Operational Research*, 116, 498–507.

Krohling, R.A., de Souza, T.T.M. (2012). Combining prospect theory and fuzzy numbers to multi-criteria decision making. *Expert Systems with Applications*, 39, 11487–11493.

Lee, L.-W., Chen, S.-M. (2015). Fuzzy decision making based on likelihood-based comparison relations of hesitant fuzzy linguistic term sets and hesitant fuzzy linguistic operators. *Information Sciences*, 294, 513–529.

Liou, J.J., Tsai, C.-Y., Lin, R.-H., Tzeng, G.-H. (2011). A modified VIKOR multiple-criteria decision method for improving domestic airlines service quality. *Journal of Air Transport Management*, 17, 57–61.

Liu, H.B., Rodriguez, R.M. (2014). A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. *Information Sciences*, 258, 220–238.

Liu, P. D., Teng, F. (2016). An extended TODIM method for multiple attribute group decision-making based on 2-dimension uncertain linguistic Variable. *Complexity*, 21(5), 20–30.

Lourenzutti, R., Krohling, R.A. (2013). A study of TODIM in a intuitionistic fuzzy and random environment. *Expert Systems with Applications*, 40, 6459–6468.

Martínez, L., Herrera, F. (2012). An overview on the 2-tuple linguistic model for computing with words in decision making: extensions, applications and challenges. *Information Sciences*, 207, 1–18.

Passos, A.C., Teixeira, M.G., Garcia, K.C., Cardoso, A.M., Gomes, L.F.A.M. (2014). Using the TODIM-FSE method as a decision-making support methodology for oil spill response. *Computers & Operations Research*, 42, 40–48.

Rodriguez, R.M., Martínez, L. (2013). An analysis of symbolic linguistic computing models in decision making. *International Journal of General Systems*, 42, 121–136.

Rodriguez, R. M., Martínez, L., Herrera, F. (2012). Hesitant fuzzy linguistic term sets for decision making. *IEEE Transactions on Fuzzy Systems*, 20, 109–119.

Rodriguez, R.M., Martínez, L., Herrera, F. (2013). A group decision making model dealing with comparative linguistic expressions based on hesitant fuzzy linguistic term sets. *Information Sciences*, 241, 28–42.

Türkşen, I.B. (2002). Type 2 representation and reasoning for CWW. *Fuzzy Sets and Systems*, 127, 17–36.

Tosun, Ö., Akyüz, G. (2015). A fuzzy TODIM approach for the supplier selection problem. *International Journal of Computational Intelligence Systems*, 8, 317–329.

Wan, S.-P., Li, D.-F. (2013). Fuzzy LINMAP approach to heterogeneous MADM considering comparisons of alternatives with hesitation degrees. *Omega*, 41, 925–940.

Wang, J.-H., Hao, J.J. (2006). A new version of 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 14, 435–445.

Wei, C., Ren, Z., Rodríguez, R.M. (2015). A hesitant fuzzy linguistic TODIM method based on a score function. *International Journal of Computational Intelligence Systems*, 8, 701–712.

Xu, Z. S. (2004). *Uncertain Multiple Attribute Decision Making: Methods and Applications*. Tsinghua University Press, Beijing.

Yager, R.R. (1995). An approach to ordinal decision making. *International Journal of Approximate Reasoning*, 12, 237–261.

Zadeh, L.A. (1975). The concept of a linguistic variable and its application to approximate reasoning. *Information Sciences, Part I, II, III* (8, 9), 199–249, 301–357, 143–180.

Zeng, S., Chen, J., Li, X. (2016a). A hybrid method for pythagorean fuzzy multiple-criteria decision making. *International Journal of Information Technology & Decision Making*, 15, 403–422.

Zeng, S., Su, W., Zhang, C. (2016b). Intuitionistic fuzzy generalized probabilistic ordered weighted averaging operator and its application to group decision making. *Technological and Economic Development of Economy*, 22, 177–193.

Zeng, S., Palacios-Marques, D., Zhu, F. (2016c). A new model for interactive group decision making with intuitionistic fuzzy preference relations. *Informatica*, 27(4), 911–928.

Zhang, X. (2015). An integrated maximizing consistency and multi-choice goal programming approach for hybrid multiple criteria group decision making based on interval-valued intuitionistic fuzzy number. *Informatica*, 26(4), 705–726.

Zhang, X., Xu, Z. (2014a). The TODIM analysis approach based on novel measured functions under hesitant fuzzy environment. *Knowledge-Based Systems*, 61, 48–58.

Zhang, X., Xu, Z. (2014b). Interval programming method for hesitant fuzzy multi-attribute group decision making with incomplete preference over alternatives. *Computers & Industrial Engineering*, 75, 217–229.

Zhang, X., Xu, Z. (2015). Soft computing based on maximizing consensus and fuzzy TOPSIS approach to interval-valued intuitionistic fuzzy group decision making. *Applied Soft Computing*, 26, 42–56.

Zhang, X., Xu, Z., Wang, H. (2015). Heterogeneous multiple criteria group decision making with incomplete weight information: A deviation modeling approach. *Information Fusion*, 25, 49–62.

Zhang, X., Xu, Z., Liu, M. (2016). Hesitant trapezoidal fuzzy QUALIFLEX method and its application in the evaluation of green supply chain initiatives. *Sustainability*, 8(9), 952.

Zhao, X.F., Lin, R., Wei, G.W. (2014). Hesitant triangular fuzzy information aggregation based on Einstein operations and their application to multiple attribute decision making. *Expert Systems with Applications*, 41, 1086–1094.

X. Zhang received the BS and MS degrees from Jiangxi University of Finance & Economics and the PhD degree from Southeast University, China, in 2009, 2011 and 2015. His current research interests include fuzzy multiple criteria decision making, clustering analysis and group decision making, etc. He has contributed several pertinent papers in *Information Sciences*, *International Journal of Systems Science*, *Information Fusion*, *Knowledge-Based Systems*, *Applied Soft Computing*, *Computers & Industrial Engineering*, *INFORMATICA*, *OR Spectrum*, and so on.