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# A Method for Group Decision Making Based on Interval-Valued Intuitionistic Fuzzy Geometric Distance Measures

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**Abstract.** In this paper, at first, we develop some new geometric distance measures for interval-valued intuitionistic fuzzy information, including the interval-valued intuitionistic fuzzy weighted geometric distance (IVIFWGD) measure, the interval-valued intuitionistic fuzzy ordered weighted geometric distance (IVIFOWGD) measure and the interval-valued intuitionistic fuzzy hybrid weighted geometric distance (IVIFHWGD) measure. Also, several desirable properties of these new distance measures are studied and a numerical example is given to show application of the distance measure to pattern recognition problems. And then, based on the developed distance measures a consensus reaching process with interval-valued intuitionistic fuzzy preference information for group decision making is proposed. Finally, an illustrative example with interval-valued intuitionistic fuzzy information is given.

**Key words:** interval-valued intuitionistic fuzzy set, weighted geometric, distance measure, consensus reaching process, group decision making.

## 1. Introduction

In many topical fields, including pattern recognition, medical diagnosis, group decision making, supply chain management and so on, distance measure is a commonly used tool for measuring the deviations of different arguments. Over the last decades, many authors focused on distance measures and the applications refer to Bogart (1975), Kaufmann (1975), Kacprzyk (1997), Szmidt and Kacprzyk (2000), Zwick *et al.* (1987), Bolton *et al.* (2008), Xu (2010a, 2010b), Xu and Yager (2009), Merigó and Yager (2013), Merigó (2013), Zeng (2013), Peng *et al.* (2014). Most of the existing distance measures in the literature are the weighed distance measures, such as some well-known distance measures including the weighted Hamming distance (WHD) measure and the weighted Euclidean distance (WED) measure. However, these distance measures only take the importance of each deviation value into consideration. Motivated by the idea of the ordered weighted averaging (OWA) operator (Yager, 1988), Xu and Chen (2008) developed the ordered

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weighted distance (OWD) measure, which emphasizes the importance of the ordered position of the given individual distances instead of weighting arguments themselves. Also, the prominent characteristic of the OWD measure is that it can relieve (or intensify) the influence of unduly large or small deviations on the aggregation results by assigning low (or high) weights of them. For further research on other distance measures based on the OWA operator, please see, for example, Yager (2010), Merigó and Casanovas (2010, 2011), Merigó and Gil-Lafuente (2010), Xu (2007b, 2010a), Xu and Xia (2011), Zeng *et al.* (2013).

However, the distance measures above are used to deal with the situation where the input information is expressed in exact numerical numbers rather than other types of variables. In fact, as the increasing complexity of our real life, in many situations, the given information is expressed in the form of vague and imprecise variables because of time pressure, people's limited expertise related to the problem domain and so on. Atanassov (1986, 1989) introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set (Zadeh, 1965). Later, Xu (2007a, 2010b) and Xu and Yager (2006) proposed the notion of intuitionistic fuzzy numbers (IFNs), which is characterized by a membership degree and a non-membership degree. And they also developed the intuitionistic fuzzy weighted averaging (IFWA) operator, the intuitionistic fuzzy weighted geometric (IFWG) operator and the intuitionistic fuzzy hybrid averaging (IFHA) operator. Xu (2007b) developed some similarity measures of intuitionistic fuzzy sets. Based on the idea of the OWD measure and the intuitionistic fuzzy information, Zeng (2013) developed some intuitionistic fuzzy weighted distance measures including intuitionistic fuzzy ordered weighted distance (IFOWD) measure and intuitionistic fuzzy hybrid weighted distance (IFHWD) measure. Peng *et al.* (2014) proposed an approach to group decision making based on some intuitionistic fuzzy weighted geometric distance measures. However, the IFS has its limitation due to insufficiency in information availability, it may not be likely to identify exact values for the membership and non-membership degrees of an element to a given set. Thus, Atanassov and Gargov (1989) proposed an interval-valued intuitionistic fuzzy set (IVIFS), which is characterized by a membership function and a non-membership function whose values are intervals rather than real numbers. Moreover, the IVIFS provides a more reasonable mathematical framework to process the imperfect information. The research on the distance measures and operators under intuitionistic fuzzy and interval-valued intuitionistic fuzzy environment and its applications has attracted substantial attention (Bi *et al.*, 2015; Bustince *et al.*, 2000; Deschrijver and Kerre, 2003; Hwang *et al.*, 2012; Szmidt and Kacprzyk, 2000, 2001; Vlachos and Sergiadis, 2007; Wang, 2009; Zhao and Wei, 2013; Liang and Shi, 2003; Li and Cheng, 2002; Xu and Wang, 2012; Yu and Xu, 2013; Xu and Xia, 2011; Ye, 2010; Zeng and Su, 2011; Xu and Yager, 2011).

The objective of this paper is to extend the above mentioned distance measures and operators to accommodate interval-valued intuitionistic fuzzy environment. For this purpose, we shall develop some weighted geometric distance measures under interval-valued intuitionistic fuzzy environment, such as the interval-valued intuitionistic fuzzy weighted geometric distance (IVIFWGD) measure, the interval-valued intuitionistic fuzzy ordered weighted geometric distance (IVIFOWGD) measure and the interval-valued intuitionistic fuzzy hybrid weighted geometric distance (IVIFHWGD) measure. These developed distance measures are very suitable to deal with the situations where the input arguments are represented in interval-valued intuitionistic fuzzy numbers. Moreover, the distance measures can alleviate (or intensify) the influence of unduly large (or small) deviations on the aggregation results by assigning low (or high) weights of them. To do so, this paper is structured as follows. In Section 2, we review the weighted distance, the ordered weighted geometric (OWG) operator, the weighted geometric distance (WGD) measure and the ordered weighted geometric distance (OWGD) measure. In Section 3, we develop the IVIFWGD measure, the IVIFOWGD measure and the IVIFHWGD measure, and study their various properties. In Section 4, we propose an approach to establish a consensus reaching process for group decision making based on the developed distance measures. In Section 5, an illustrative example is given to verify the proposed approach and to demonstrate the practicality and effectiveness, and the main conclusions of the paper are summarized in Section 6.

## 2. Preliminaries

In this section, we briefly review the OWG operator and some distance measures. Xu and Da (2002) developed the ordered weighted geometric (OWG) operator based on the OWA operator (Yager, 1988). The two operators as well as the weighted harmonic averaging operator have been investigated by many authors (Cheng *et al.*, 2009; Herrera *et al.*, 2003; Li *et al.*, 2015; Wei, 2010; Peng *et al.*, 2012; Xu and Da, 2002, 2003; Xu, 2005; Wang and Chin, 2011; Wang and Wang, 2013). The OWG operator is defined as follows:

Definition 1. An OWG operator of dimension $n$ is a mapping OWG: $R_+^n \to R_+$ that has an associated $n$ vector $w = (w_1, w_2, \dots, w_n)^T$ such that $w_j \in [0, 1]$, $\sum_{j=1}^n w_j = 1$ and

$$OWG_w(a_1, \dots, a_n) = \prod_{j=1}^{n} b_j^{w_j}, \tag{1}$$

where $b_j$ is the $j$th largest of the $a_i$.

Based on the most widely used distances including the weighted Hamming distance (WHD), the weighted Euclidean distance (WED) and the geometric mean, a weighted geometric distance (WGD) is defined as follows. For two collections of arguments $A = \{a_1, a_2, \dots, a_n\}$ and $B = \{b_1, b_2, \dots, b_n\}$.

Definition 2. A weighted geometric distance (WGD) of dimension $n$ is a mapping WGD: $R_+^n \to R_+$ that has an associated weighting $n$ vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ such that

$\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$ and

$$WGD = \left( \prod_{i=1}^{n} \big(|a_i - b_i|\big)^{\lambda\omega_i} \right)^{1/\lambda}, \quad \lambda > 0. \tag{2}$$

(1) If $\lambda = 1$, the WGD measure is called a weighted Hamming geometric distance (WHGD) measure:

$$WHGD(A, B) = \prod_{i=1}^{n} \big(|a_i - b_i|\big)^{\omega_i}, \tag{3}$$

(2) If $\lambda = 2$, the WGD measure is called a weighted Euclidean geometric distance (WEGD) measure:

$$WEGD(A, B) = \sqrt{\prod_{i=1}^{n} (a_i - b_i)^{2\omega_i}}. \tag{4}$$

The above weighted distance measures take only the given individual distances into consideration. Motivated by the idea of the ordered weighted averaging, Xu and Chen (2008) developed an ordered weighted distance (OWD) measure and an ordered weighted geometric distance (OWGD) measure.

Definition 3. Let $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be two collections of real numbers, and $d(a_j, b_j) = |a_j - b_j|$ be the distance between $a_j$ and $b_j$, then

$$OWD(A, B) = \left( \sum_{j=1}^{n} w_j \big(d(a_{\sigma(j)}, b_{\sigma(j)})\big)^{\lambda} \right)^{1/\lambda}, \tag{5}$$

is called an ordered weighted distance (OWD) between $A$ and $B$, in which $\lambda > 0$, $w = (w_1, w_2, \ldots, w_n)^T$ is the weighted vector of the ordered position of the $d(a_{\sigma(j)}, b_{\sigma(j)})$, where $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$, and $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is any permutation of $(1, 2, \ldots, n)$, such that

$$d(a_{\sigma(j-1)}, b_{\sigma(j-1)}) \geqslant d(a_{\sigma(j)}, b_{\sigma(j)}). \tag{6}$$

Definition 4. Let $A = \{a_1, a_2, \ldots, a_n\}$ and $B = \{b_1, b_2, \ldots, b_n\}$ be two collections of real numbers, and $d(a_j, b_j) = |a_j - b_j|$ be the distance between $a_j$ and $b_j$, then

$$OWGD(A, B) = \left( \prod_{j=1}^{n} \big(d(a_{\sigma(j)}, b_{\sigma(j)})\big)^{\lambda w_j} \right)^{1/\lambda}, \tag{7}$$

is called an ordered weighted geometric distance (OWGD) between $A$ and $B$.

Table 1
The measures in different situations.

| | $\lambda = 1$ | $\lambda = 2$ |
|---|---|---|
| WGD | WHGD | WEGD |
| OWGD | OWHGD | OWEGD |

(1) If $\lambda = 1$, the OWGD measure is called an ordered weighted Hamming geometric distance (OWHGD) measure:

$$OWHGD(A, B) = \prod_{j=1}^{n} \left(d(a_{\sigma(j)}, b_{\sigma(j)})\right)^{w_j}, \tag{8}$$

(2) If $\lambda = 2$, the OWGD measure is called an ordered weighted Euclidean geometric distance (OWEGD) measure:

$$OWEGD(A, B) = \sqrt{\prod_{j=1}^{n} \left(d(a_{\sigma(j)}, b_{\sigma(j)})\right)^{2w_j}}. \tag{9}$$

The measures mentioned above can be presented in Table 1.

However, the distance measures can only be used in situations where the input arguments are the exact numerical values. In the next section, we shall extend the WGD measure and the OWGD measure to accommodate the situation in which the input arguments are expressed as interval-valued intuitionistic fuzzy information.

## 3. The Interval-Valued Intuitionistic Fuzzy Geometric Distance Measures

Atanassov and Gargov (1989) developed the interval-valued intuitionistic fuzzy set (IVIFS), which is an extension of the intuitionistic fuzzy set (IFS) proposed by Atanassov (1986). The IVIFS is characterized by a membership function and a non-membership function whose values are intervals rather than real numbers. And the IVIFS provides a more reasonable mathematical framework to process the imprecise facts or imperfect information. The research on the IVIFS and its applications has received more and more attention over the last two decades, please see, for example, Liu (2013a, 2013a), Wei (2008, 2010), Xu (2007a, 2007b, 2007d, 2010a, 2010b), Xu and Chen (2007), Xu and Yager (2006, 2009). The IVIFS can be defined as follows:

Definition 5. Let a set $X = \{x_1, x_2, \dots, x_n\}$ be fixed, an interval-valued intuitionistic fuzzy set (IVIFS) $A$ in $X$ is an object having the form:

$$A = \left\{\left\langle x, \mu_A(x), \nu_A(x)\right\rangle \,\middle|\, x \in X\right\}, \tag{10}$$

where $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ with the condition

$$\sup \mu_A(x) + \sup \nu_A(x) \leqslant 1, \quad \forall x \in X.$$

The interval-valued numbers $\mu_A(x)$ and $\nu_A(x)$ represent the interval-valued membership degree and interval-valued non-membership degree of the element $x$ to the set $A$, respectively.

Especially, if

$$\inf \mu_A(x) = \sup \mu_A(x), \qquad \inf \nu_A(x) = \sup \nu_A(x),$$

then the interval-valued intuitionistic fuzzy set (IVIFS) is reduced to an intuitionistic fuzzy set (IFS).

For computational convenience, the pair $(\mu_A(x), \nu_A(x))$ denoted by $\alpha = ([a,b],[c,d])$ is called an interval-valued intuitionistic fuzzy number (IVIFN) (Xu and Chen, 2007; Xu, 2010b), where

$$[a,b] \in [0,1], \qquad [c,d] \in [0,1], \qquad b + d \leqslant 1.$$

Definition 6. Let $\alpha = ([a,b],[c,d])$ be an IVIFN, a score function and an accuracy function (Xu, 2007d, 2010b) of an interval-valued intuitionistic fuzzy value can be represented as follows, respectively:

$$S(\alpha) = \frac{1}{2}(a - c + b - d), \quad S(\alpha) \in [-1,1], \tag{11}$$

$$H(\alpha) = \frac{1}{2}(a + c + b + d), \quad H(\alpha) \in [0,1]. \tag{12}$$

Moreover, an order relation between IVIFNs is proposed (Xu, 2007d, 2010b).

Let $\alpha_1$ and $\alpha_2$ be two IVIFNs, if the score function $S(\alpha_1) < S(\alpha_2)$, then $\alpha_1$ is smaller than $\alpha_2$, denoted by $\alpha_1 < \alpha_2$; if $S(\alpha_1) = S(\alpha_2)$, then

(1) If $H(\alpha_1) < H(\alpha_2)$, then $\alpha_1$ is smaller than $\alpha_2$, denoted by $\alpha_1 < \alpha_2$;
(2) If $H(\alpha_1) = H(\alpha_2)$, then $\alpha_1$ and $\alpha_2$ represent the same information, denoted by $\alpha_1 = \alpha_2$.

Let $\alpha = ([a,b],[c,d])$, $\alpha_1 = ([a_1,b_1],[c_1,d_1])$ and $\alpha_2 = ([a_2,b_2],[c_2,d_2])$ be any three IVIFNs, based on the notion of the interval-valued intuitionistic fuzzy numbers (Xu, 2010b), we can define the operational laws as follows:

(1) $\alpha_1 \oplus \alpha_2 = ([a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2], [c_1c_2, d_1d_2])$;
(2) $\lambda\alpha = ([1 - (1-a)^\lambda, 1 - (1-b)^\lambda], [c^\lambda, d^\lambda])$, $\lambda > 0$;
(3) $\alpha_1 \otimes \alpha_2 = ([a_1a_2, b_1b_2], [c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2])$;
(4) $\alpha^\lambda = ([a^\lambda, b^\lambda], [1 - (1-c)^\lambda, 1 - (1-d)^\lambda])$, $\lambda > 0$.

Obviously, the operational result above are still interval-valued intuitionistic fuzzy numbers.

Xu (2007c, 2010b) defined the distance measure between the two IVIFNs $\alpha_1$ and $\alpha_2$ as following:

Definition 7. Let $\alpha_1$ and $\alpha_2$ be two IVIFNs, then

$$d_{IVIFD}(\alpha_1, \alpha_2) = \frac{1}{4}\big(|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|\big), \tag{13}$$

is called the interval-valued intuitionistic fuzzy distance (IVIFD) between $\alpha_1$ and $\alpha_2$.

Based on the interval-valued intuitionistic fuzzy distance (IVIFD) and the ordered weighted geometric distance (OWGD), in the following, we shall develop an interval-valued intuitionistic fuzzy weighted geometric distance (IVIFWGD) measure and an interval-valued intuitionistic fuzzy ordered weighted geometric distance (IVIFOWGD) measure.

Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle \mid x \in X\}$ be two interval-valued intuitionistic fuzzy sets on $X = \{x_1, x_2, \ldots, x_n\}$, we let $A(x) = \alpha$ and $B(x) = \beta$, then the interval-valued intuitionistic fuzzy sets $A$ and $B$ can be denoted by $A = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ and $B = \{\beta_1, \beta_2, \ldots, \beta_n\}$. Then we can calculate the geometric distance between the interval-valued intuitionistic fuzzy sets $A$ and $B$ utilizing the IVIFD between $\alpha_i$ and $\beta_i$, $i = 1, 2, \ldots, n$.

The following form:

$$d_{IVIFWGD}(A, B) = \left(\prod_{j=1}^{n}\big(d_{IVIFD}(\alpha_j, \beta_j)\big)^{\lambda\omega_j}\right)^{1/\lambda}, \tag{14}$$

which is called an interval-valued intuitionistic fuzzy weighted geometric distance (IVIFWGD) between $A$ and $B$. In the case of $\lambda = 1$ and $\lambda = 2$, the IVIFWGD measure is reduced to the IVIFWGHD measure (15) and the IVIFWGED measure (16), respectively.

Definition 8. Let $A = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ and $B = \{\beta_1, \beta_2, \ldots, \beta_n\}$ be two sets of interval-valued intuitionistic fuzzy values, then

$$d_{IVIFWGHD}(A, B) = \prod_{j=1}^{n}\big(d_{IVIFD}(\alpha_j, \beta_j)\big)^{\omega_j}, \tag{15}$$

is called an interval-valued intuitionistic fuzzy weighted geometric Hamming distance (IVIFWGHD) between $A$ and $B$.

Definition 9. Let $A=\{\alpha_1,\alpha_2,\dots,\alpha_n\}$ and $B=\{\beta_1,\beta_2,\dots,\beta_n\}$ be two sets of interval-valued intuitionistic fuzzy values, then

$$d_{IVIFWGED}(A,B)=\sqrt{\prod_{j=1}^{n}\big(d_{IVIFD}(\alpha_j,\beta_j)\big)^{2\omega_j}}, \tag{16}$$

is called an interval-valued intuitionistic fuzzy weighted geometric Euclidean distance (IVIFWGED) between $A$ and $B$, where $\omega=(\omega_1,\omega_2,\dots,\omega_n)^T$ is the weighting vector of the $d_{IVIFD}(\alpha_j,\beta_j)$ such that $\omega_j\in[0,1]$, $\sum_{j=1}^{n}\omega_j=1$.

Based on the OWGD measure (7) and the IVIFWGD measure (14), an interval-valued intuitionistic fuzzy ordered weighted geometric distance (IVIFOWGD) measure is proposed as follows:

Definition 10. Let $A=\{\alpha_1,\alpha_2,\dots,\alpha_n\}$ and $B=\{\beta_1,\beta_2,\dots,\beta_n\}$ be two sets of interval-valued intuitionistic fuzzy values, then

$$d_{IVIFOWGD}(A,B)=\left(\prod_{j=1}^{n}\big(d_{IVIFD}(\alpha_{\sigma(j)},\beta_{\sigma(j)})\big)^{\lambda w_j}\right)^{1/\lambda}, \tag{17}$$

is called an interval-valued intuitionistic fuzzy ordered weighted geometric distance (IVIFOWGD) between $A$ and $B$, where $w=(w_1,w_2,\dots,w_n)^T$ is the weighted vector of the ordered position of the $d(\alpha_{\sigma(j)},\beta_{\sigma(j)})$, with the condition $w_j\in[0,1]$ and $\sum_{j=1}^{n}w_j=1$. $(\sigma(1),\sigma(2),\dots,\sigma(n))$ is any permutation of $(1,2,\dots,n)$, such that $d(\alpha_{\sigma(j-1)},\beta_{\sigma(j-1)})\geqslant d(\alpha_{\sigma(j)},\beta_{\sigma(j)})$.

(1) If $\lambda=1$, the IVIFOWGD measure is called an interval-valued intuitionistic fuzzy ordered weighted geometric Hamming distance (IVIFOWGHD) measure:

$$d_{IVIFOWGHD}(A,B)=\prod_{j=1}^{n}\big(d_{IVIFD}(\alpha_{\sigma(j)},\beta_{\sigma(j)})\big)^{w_j}, \tag{18}$$

(2) If $\lambda=2$, the IVIFOWGD measure is called an interval-valued intuitionistic fuzzy ordered weighted geometric Euclidean distance (IVIFOWGED) measure:

$$d_{IVIFOWGED}(A,B)=\sqrt{\prod_{j=1}^{n}\big(d_{IVIFD}(\alpha_{\sigma(j)},\beta_{\sigma(j)})\big)^{2w_j}}. \tag{19}$$

From the IVIFWGD measure (14) and the IVIFOWGD measure (17), we know that the IVIFWGD measure weights the given variable distances while the IVIFOWGD measure weights the ordered positions of the given variable distances instead of weighting the variable distances themselves. Thus, weights represent different aspects in both distance measures. To overcome the drawback, an interval-valued intuitionistic fuzzy hybrid weighted geometric distance (IVIFHWGD) measure is proposed as follows:

Definition 11. Let $A = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ and $B = \{\beta_1, \beta_2, \ldots, \beta_n\}$ be two sets of interval-valued intuitionistic fuzzy values, then

$$d_{IVIFHWGD}(A, B) = \left( \prod_{j=1}^{n} \left(D_{IVIFD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)})\right)^{w_j} \right)^{1/\lambda}, \tag{20}$$

is called an interval-valued intuitionistic fuzzy hybrid weighted geometric distance (IVIFHWGD) measure between $A$ and $B$, where $w = (w_1, w_2, \ldots, w_n)^T$ is the weighting vector associated with the IVIFHWGD measure, $D_{IVIFD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)})$ is the $j$th largest of $D_{IVIFD}(\alpha_j, \beta_j)$ ($D_{IVIFD}(\alpha_j, \beta_j) = ((d_{IVIFD}(\alpha_j, \beta_j))^{\lambda\omega_j})^n$), $j = 1, 2, \ldots, n$, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weighting vector of the $d_{IVIFD}(\alpha_j, \beta_j)$ such that $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$, $n$ is the balancing coefficient.

Let $w = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$ and $\omega = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, respectively, we can have:

Remark 1. The IVIFWGD measure and the IVIFOWGD measure are special cases of the IVIFHWGD measure, respectively.

Remark 2. The IVIFHWGD measure generalizes both the IVIFWGD measure and the IVIFOWGD measure and reflects the importance degrees of both the given variable distances and their ordered positions.

Remark 3. The IVIFHWGD measure weights the given variable distances at first, and then reorders the weighted variable distances in descending order and weights these ordered variable distances by the IVIFHWGD weights. And then, we process these variable distances into a collective one under the parameter $\lambda$.

Remark 4. The IVIFHWGD measure can relieve (or intensify) the influence of unduly large or small difference individual on the aggregation results by assigning them low (or high) weights.

Remark 5. It is worth pointing out that the geometric distance measures mentioned above can be viewed as the generalization of some widely used distance measures when dealing with interval-valued intuitionistic fuzzy situations. However, if the input arguments are the extreme form such as $([1, 1], [0, 0])$ and $([0, 0], [1, 1])$ from two sets of IVIFVs, for the time being the distance between them is 1. On the other hand, if the input arguments exactly equal each other, for the time being the distance is 0. In the two situations above the aggregating procedure by using the geometric distance measures are not well considered due to the characteristics of geometric average. And then, the distance measures based on arithmetic average may be the better choice.

Next, a numerical example is given to show application of the developed distance measures to pattern recognition problems.

EXAMPLE 1. Assume that there exist three patterns, which are represented by IVIFVs in the feature space $X = \{x_1, x_2, x_3, x_4, x_5\}$:

$$A_1 = \big\{\big([0.400, 0.600], [0.100, 0.200]\big), \big([0.500, 0.500], [0.400, 0.500]\big), \big([0.400, 0.500], [0.200, 0.400]\big), \big([0.200, 0.300], [0.600, 0.700]\big), \big([0.500, 0.700], [0.100, 0.200]\big)\big\}$$

$$A_2 = \big\{\big([0.300, 0.500], [0.200, 0.400]\big), \big([0.400, 0.400], [0.200, 0.300]\big), \big([0.200, 0.600], [0.100, 0.300]\big), \big([0.100, 0.300], [0.400, 0.500]\big), \big([0.200, 0.300], [0.500, 0.500]\big)\big\}$$

$$A_3 = \big\{\big([0.200, 0.800], [0.100, 0.200]\big), \big([0.300, 0.500], [0.100, 0.300]\big), \big([0.300, 0.500], [0.300, 0.100]\big), \big([0.300, 0.400], [0.500, 0.500]\big), \big([0.600, 0.700], [0.100, 0.300]\big)\big\}$$

and the weight vector of the feature space $X = \{x_1, x_2, x_3, x_4, x_5\}$ is

$$\omega = (0.150, 0.200, 0.250, 0.100, 0.300).$$

We consider a sample $B$, which is represented by IVIFVs will be recognized, where

$$B = \big\{\big([0.200, 0.400], [0.400, 0.500]\big), \big([0.400, 0.500], [0.200, 0.300]\big), \big([0.400, 0.500], [0.500, 0.500]\big), \big([0.200, 0.300], [0.400, 0.700]\big), \big([0.300, 0.500], [0.300, 0.400]\big)\big\}$$

By utilizing Eq. (13), we can get

$$d_{IVIFD}(\alpha_{11}, \beta_1) = 0.250, \quad d_{IVIFD}(\alpha_{12}, \beta_2) = 0.125, \quad d_{IVIFD}(\alpha_{13}, \beta_3) = 0.100,$$
$$d_{IVIFD}(\alpha_{14}, \beta_4) = 0.050, \quad d_{IVIFD}(\alpha_{15}, \beta_5) = 0.200;$$
$$d_{IVIFD}(\alpha_{21}, \beta_1) = 0.125, \quad d_{IVIFD}(\alpha_{22}, \beta_2) = 0.025, \quad d_{IVIFD}(\alpha_{23}, \beta_3) = 0.225,$$
$$d_{IVIFD}(\alpha_{24}, \beta_4) = 0.075, \quad d_{IVIFD}(\alpha_{25}, \beta_5) = 0.150;$$
$$d_{IVIFD}(\alpha_{31}, \beta_1) = 0.250, \quad d_{IVIFD}(\alpha_{32}, \beta_2) = 0.050, \quad d_{IVIFD}(\alpha_{33}, \beta_3) = 0.175,$$
$$d_{IVIFD}(\alpha_{34}, \beta_4) = 0.125, \quad d_{IVIFD}(\alpha_{35}, \beta_5) = 0.200.$$

Without loss of generality, here we take into consideration the case of $\lambda = 1, 2$.

(1) If $\lambda = 1$, we have

$$D_{IVIFD}(\alpha_{11}, \beta_1) = 0.354, \quad D_{IVIFD}(\alpha_{12}, \beta_2) = 0.125, \quad D_{IVIFD}(\alpha_{13}, \beta_3) = 0.056,$$
$$D_{IVIFD}(\alpha_{14}, \beta_4) = 0.224, \quad D_{IVIFD}(\alpha_{15}, \beta_5) = 0.089.$$

Table 2
The distance between $A_i$ $(i = 1, 2, 3)$ and $B$.

| | $d(A_1, B)$ | $d(A_2, B)$ | $d(A_3, B)$ |
|---|---|---|---|
| IVIFWGD | 0.138 | 0.105 | 0.145 |
| IVIFOWGD | 0.129 | 0.103 | 0.151 |
| IVIFHWGD | 0.136 | 0.115 | 0.146 |

The weight vector $w = (0.110, 0.240, 0.300, 0.240, 0.110)^T$ associated with the IVIFHWGD measure, which is derived by using the Gaussian distribution based method, for more details, refer to Xu (2005). Then we can get the interval-valued intuitionistic fuzzy hybrid weighted geometric distance between $A_1$ and $B$:

$$\begin{aligned} d_{IVIFHWGD}(A_1, B) &= \prod_{j=1}^{5} (D_{IVIFD}(\alpha_{\sigma(j)}, \beta_{\sigma(j)}))^{w_j} \\ &= 0.354^{0.110} \times 0.224^{0.240} \times 0.125^{0.300} \times 0.089^{0.240} \times 0.056^{0.110} \\ &= 0.136. \end{aligned}$$

Similarly, we have

$$d_{IVIFHWGD}(A_2, B) = 0.115, \qquad d_{IVIFHWGD}(A_3, B) = 0.146,$$

thus

$$d_{IVIFHWGD}(A_2, B) = \min_{1 \leqslant i \leqslant 3} \{d_{IVIFHWGD}(A_i, B)\} \quad (\lambda = 1).$$

(2) If $\lambda = 2$, similar to the calculation process above, we have

$$d_{IVIFHWGD}(A_2, B) = \min_{1 \leqslant i \leqslant 3} \{d_{IVIFHWGD}(A_i, B)\} \quad (\lambda = 2).$$

As we can see, the results of both (1) and (2) show that the sample $B$ belongs to the pattern $A_2$. Moreover, the numerical results of the distance measure between $A_i$ $(i = 1, 2, 3)$ and $B$ are consistent. In fact, we can confirm the truth of the consensus results no matter what case of the value of $\lambda$ is taken into consideration. The conclusion can be easily proven, and thus omitted.

By utilizing IVIFWGD measure (14) and the IVIFOWGD measure (17) to calculate the distances between the given patterns $A_i$ $(i = 1, 2, 3)$ and the sample $B$, we derive the corresponding distances (Table 2).

As we can see from Table 2, the results derived by both the IVIFWGD measure and the IVIFOWGD measure show that the sample $B$ belongs to the pattern $A_2$. Furthermore, among the IVIFWGD, IVIFOWGD and IVIFHWGD measures, the IVIFHWGD measure can not only reflect the importance of each given argument, but consider the importance of the ordered position of the argument. Thus, the IVIFHWGD measure remains preferences as IVIFVs $X$ with respect to a criterion in the final decision results.

## 4. An Approach to Reach Consensus of Group Decision Making Based on the IVIFHWGD Measure

Let us consider a group decision making with interval-valued intuitionistic fuzzy information. Let $X=\{x_1,x_2,\dots,x_n\}$ be a discrete set of alternatives, $d_k\in D$ $(k=1,2,\dots,m)$ be the set of decision makers, and $u=(u_1,u_2,\dots,u_m)^T$ be the weight vector of DMs, with the condition $u_k\geqslant 0$, $\sum_{k=1}^m u_k=1$. The DMs provide their preferences with interval-valued intuitionistic fuzzy values $\alpha_{kj}$ $(j=1,2,\dots,n)$ over all the alternatives $x_j\in X$ respect to a criterion. For convenience, the preference vectors of all the DMs $d_k$ are denoted by:

$$\alpha_k=(\alpha_{k1},\alpha_{k2},\dots,\alpha_{kn}),\quad k=1,2,\dots,m. \tag{21}$$

Next, based on the decision information above, we shall propose an approach to reaching consensus of group opinions by using the IVIFHWGD measure.

**Step 1:** Calculate the collective preference vector $\alpha=(\alpha_1,\alpha_2,\dots,\alpha_n)$ by using the interval-valued intuitionistic fuzzy weighted geometric operator (Xu, 2007d), and we have

$$\alpha_j=\alpha_{1j}^{u_1}\otimes\alpha_{2j}^{u_2}\otimes\cdots\otimes\alpha_{mj}^{u_m},\quad i=1,2,\dots,m;\ j=1,2,\dots,n. \tag{22}$$

**Step 2:** Calculate the distance $d_{IVIFD}(\alpha_{kj},\alpha_j)$ between each preference value $\alpha_{kj}$ given by the decision maker $d_k$ and the corresponding collective preference with interval-valued intuitionistic fuzzy value $\alpha_j$ by using Eq. (13).

**Step 3:** Calculate the IVIFHWGD measure between the preference vectors $\alpha_k$ and $\alpha$ by using Eq. (20):

$$d_{IVIFHWGD}(\alpha_k,\alpha)=\left(\prod_{j=1}^n\left(D_{IVIFD}(\alpha_{\sigma(kj)},\alpha_{\sigma(j)})\right)^{w_j}\right)^{1/\lambda}, \tag{23}$$

where $w_j=(w_1,w_2,\dots,w_n)^T$ is the weighting vector associated with the IVIFHWGD measure, can be derived by using the Gaussian distribution based method (Xu, 2005), $D_{IVIFD}(\alpha_{\sigma(kj)},\alpha_{\sigma(j)})$ is the $j$th largest of the weighted distance $D_{IVIFD}(\alpha_{kj},\alpha_j)$ $(D_{IVIFD}(\alpha_{kj},\alpha_j)=((d_{IVIFD}(\alpha_{kj},\alpha_j))^{\lambda\omega_j})^n$, $j=1,2,\dots,n)$, and $\omega=(\omega_1,\omega_2,\dots,\omega_n)^T$ is the weighting vector of the $d_{IVIFD}(\alpha_{kj},\widetilde{\alpha}_j)$ such that $\omega_j\in[0,1]$, $\sum_{j=1}^n\omega_j=1$.

**Step 4:** Discussion on the consensus reaching process for group decision making:

- **Case 1:** If all $d_{IVIFHWGD}(\alpha_k,\alpha)\leqslant\rho$ $(k=1,2,\dots,m)$, where $\rho$ is the threshold value of acceptable consensus, then the group is of acceptable consensus.

Table 3
Decision matrix with IVIFVs.

| | $d_1$ | $d_2$ | $d_3$ |
|---|---|---|---|
| $x_1$ | ([0.300, 0.400], [0.500, 0.500]) | ([0.400, 0.500], [0.100, 0.200]) | ([0.400, 0.600], [0.200, 0.300]) |
| $x_2$ | ([0.500, 0.600], [0.200, 0.300]) | ([0.600, 0.700], [0.100, 0.200]) | ([0.400, 0.500], [0.400, 0.500]) |
| $x_3$ | ([0.300, 0.400], [0.400, 0.500]) | ([0.500, 0.600], [0.200, 0.300]) | ([0.300, 0.400], [0.500, 0.500]) |
| $x_4$ | ([0.500, 0.600], [0.200, 0.200]) | ([0.600, 0.700], [0.100, 0.100]) | ([0.500, 0.600], [0.200, 0.300]) |

- **Case 2:** If there exists some $k_0$, such that $d_{IVIFHWGD}(\alpha_{k_0}, \alpha) > \rho$, then we shall return $\alpha_{k_0}$ (together with $\alpha$ as a reference) to the decision maker $d_k$ for revaluation, and repeat this consensus reaching process until $d_{IVIFHWGD}(\alpha_{k_0}, \alpha) \leqslant \rho$ or the number of rounds reach the maximum which is predefined by the group so as to avoid stagnation.

**Step 5:** By utilizing Eqs. (11) and (12), we can have the score function and accuracy function. According to them, we can rank all of the alternatives.

## 5. Illustrative Example

In order to demonstrate the application of the developed approach to group decision making with interval-valued intuitionistic fuzzy information, we consider the decision making problem of evaluating university faculty for tenure and promotion (Xu, 2007a, 2007b, 2007c, 2007d; Xu and Chen, 2008). One main criterion used is "teaching". There are four faculty candidates $x_j \in X$ ($j = 1, 2, 3, 4$) and three DMs $d_k \in D$ ($k = 1, 2, 3$) (whose weighting vector is $u = (0.200, 0.500, 0.300)^T$). Suppose the threshold value of acceptable consensus is $\rho = 0.100$. And each decision maker $d_k$ provides his/her preferences with interval-valued intuitionistic fuzzy values $\alpha_{kj}$ over all the faculty candidates $x_j$, as listed in Table 3.

Calculate the collective preference vector by using

$$\alpha_j = \alpha_{1j}^{u_1} \otimes \alpha_{2j}^{u_2} \otimes \alpha_{3j}^{u_3}, \quad j = 1, 2, 3, 4$$

in which, the preferences of all the DMs $d_k \in D$ ($k = 1, 2, 3$) are denoted by the following vector forms for computational convenience:

$$\begin{aligned}
&(\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}) \\
&\quad = \big(([0.300, 0.400], [0.500, 0.500]\big), \big([0.500, 0.600], [0.200, 0.300]\big) \\
&\qquad \big([0.300, 0.400], [0.400, 0.500]\big), \big([0.500, 0.600], [0.200, 0.200])\big), \\
&(\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}) \\
&\quad = \big(([0.400, 0.500], [0.100, 0.200]\big), \big([0.600, 0.700], [0.100, 0.200]\big) \\
&\qquad \big([0.500, 0.600], [0.200, 0.300]\big), \big([0.600, 0.700], [0.100, 0.100])\big),
\end{aligned}$$

$$\begin{aligned}&(\alpha_{31}, \alpha_{32}, \alpha_{33}, \alpha_{34})\\&\quad = \big(\big([0.400, 0.600], [0.200, 0.300]\big), \big([0.400, 0.500], [0.400, 0.500]\big)\\&\qquad \big([0.300, 0.400], [0.500, 0.500]\big), \big([0.500, 0.600], [0.200, 0.300]\big)\big).\end{aligned}$$

And we can have

$$\begin{aligned}\alpha &= (\alpha_1, \alpha_2, \alpha_3, \alpha_4)\\&= \big(\big([0.380, 0.510], [0.230, 0.300]\big), \big([0.510, 0.610], [0.220, 0.320]\big)\\&\quad \big([0.390, 0.490], [0.340, 0.410]\big), \big([0.550, 0.650], [0.150, 0.180]\big)\big).\end{aligned}$$

Calculate the distance $d_{IVIFD}(\alpha_{kj}, \alpha_j)$ of each preference value $\alpha_{kj}$ and the collective preference value $\alpha_j$ by using Eq. (13), and we can have

$$\begin{aligned}&d_{IVIFD}(\alpha_{11}, \alpha_1) = 0.165, \quad d_{IVIFD}(\alpha_{12}, \alpha_2) = 0.015, \quad d_{IVIFD}(\alpha_{13}, \alpha_3) = 0.083,\\&d_{IVIFD}(\alpha_{14}, \alpha_4) = 0.043, \quad d_{IVIFD}(\alpha_{21}, \alpha_1) = 0.065, \quad d_{IVIFD}(\alpha_{22}, \alpha_2) = 0.105,\\&d_{IVIFD}(\alpha_{23}, \alpha_3) = 0.118, \quad d_{IVIFD}(\alpha_{24}, \alpha_4) = 0.058, \quad d_{IVIFD}(\alpha_{31}, \alpha_1) = 0.035,\\&d_{IVIFD}(\alpha_{32}, \alpha_2) = 0.145, \quad d_{IVIFD}(\alpha_{33}, \alpha_3) = 0.108, \quad d_{IVIFD}(\alpha_{34}, \alpha_4) = 0.068.\end{aligned}$$

Without loss of generality, let $\lambda = 1$ and $\omega = (0.200, 0.350, 0.300, 0.150)^T$, the weight vector associated with the interval-valued intuitionistic fuzzy hybrid weighted geometric distance measure $w = (0.155, 0.345, 0.345, 0.155)^T$, which is derived by the Gaussian distribution based method (Xu, 2005). Then we calculate the IVIFHWGD measure between $\alpha_k$ and $\alpha$ by using Eq. (23):

$$\begin{aligned}&d_{IVIFHWGD}(\alpha_1, \alpha) = 0.060, \quad d_{IVIFHWGD}(\alpha_2, \alpha) = 0.091,\\&d_{IVIFHWGD}(\alpha_3, \alpha) = 0.081.\end{aligned}$$

As we can see, all $d_{IVIFHWGD}(\alpha_k, \alpha) \leqslant \rho = 0.100$ $(k = 1, 2, 3)$, that is all the distances are less than the predefined threshold value of acceptable consensus, which indicates that the group reaches consensus or the group is of acceptable consensus.

Note that if there existed some $k_0$, such that $d_{IVIFHWGD}(\alpha_{k_0}, \alpha) > 0.100$, we would need to return $\alpha_{k_0}$ (together with $\alpha$ as a reference) to the decision maker $d_{k_0}$ for revaluation.

Furthermore, the process of group decision making reaches consensus in the case of $\lambda = 2$ similar to the case of $\lambda = 1$.

Based on the collective preference vector by utilizing the interval-valued intuitionistic fuzzy weighted geometric operator above and Eqs. (11) and (12), we also can have

$$S(\alpha_1) = 0.180, \qquad S(\alpha_2) = 0.290, \qquad S(\alpha_3) = 0.065, \qquad S(\alpha_4) = 0.435.$$

According to $S(\alpha_j)$ $(j = 1, 2, 3, 4)$, we can rank all of the alternatives:

$$x_4 \succ x_2 \succ x_1 \succ x_3$$

thus, the best choice is $x_4$.

## 6. Conclusions

In this paper, we have developed some new geometric distance measures with interval-valued intuitionistic fuzzy information, including the IVIFWGD measure, the IVIFOWGD measure, the IVIFHWGD measure and so on.

The IVIFHWGD measure can be used in situations where the input arguments are IVIFVs and it reflects the importance degrees of both the given interval-valued intuitionistic fuzzy variables and their ordered positions. Also, it can alleviate the influence of unduly large (or small) deviations on the results by assigning them low (or high) weights.

Moreover, we have studied several desirable properties of the new distance measures and investigated the application to pattern recognition problem. And finally, we have developed an approach to establish a consensus reaching process for group decision making based on the new distance measures.

In future research, we expect to extend the developed distance measures to deal with the situations where the input arguments are expressed in other fuzzy information including triangular intuitionistic fuzzy numbers and uncertain pure linguistic labels. We will also develop different types of applications such as medical diagnosis, data mining, image segmentation and so on.

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