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Interval Type-2 Fuzzy c-Control Charts: An Application in a Food Company

Sevil ŞENTÜRK[1], Jurgita ANTUCHEVICIENE[2]*
[1]*Department of Statistics, Anadolu University, Eskisehir, Turkey*
[2]*Department of Construction Technology and Management*
Vilnius Gediminas Technical University, Lithuania
e-mail: sdeligoz@anadolu.edu.tr, jurgita.antucheviciene@vgtu.lt

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Abstract. Many papers exist on ordinary fuzzy control charts in literature in order to consider the vagueness and uncertainty in observation data. These are on both variable and attribute control charts. Several extensions of fuzzy sets have appeared in literature since ordinary fuzzy sets emerged. Type-2 fuzzy sets are one of these extensions. Type-2 fuzzy sets take into account the imprecision of membership functions in three dimensions. The aim of this paper is to develop interval type-2 fuzzy control charts for number of nonconformities, briefly c-control charts. In this paper, the theoretical structure of interval type-2 fuzzy c-control charts is proposed for the first time and the application is implemented in a food company.

Key words: fuzzy control charts, interval type-2 fuzzy sets, c-control charts, nonconformity, process control.

1. Introduction

Statistical process control (SPC) is a method employing statistical techniques to monitor the process. In its turn, control charts are one of the major tools for SPC. W.A. Shewhart introduced the control charts in the 1920s. The purpose of the proposed Shewhart's control charts was to observe and monitor a process for shifts in mean and variance of a separate quality characteristic.

The two main types of primary control charts can be distinguished, namely variable control charts and attribute control charts. If the quality characteristics are measured on numerical scales, then control charts for variable, like $\bar{X}$ chart for the process average and R or S charts for process variability can be used. If the quality characteristics are presented in a qualitative form, then attribute control charts can be used to evaluate the process, like p (fraction of nonconforming), np (nonconforming units), c (number of nonconformities) or u (nonconformities per unit) control charts (Montgomery, 1991).

In the traditional Shewhart control charts, data that come from process or measurement systems consists of crisp values. But this data includes "uncertainty" or "vagueness" due

*Corresponding author.

to some difficulties while obtaining data handled from operators or process records. In that case, fuzzy control charts are useful tools to evaluate the process with fuzzy data. Concepts and techniques of fuzzy control charts contribute towards dealing with uncertainty or imprecision while monitoring the process.

In literature, fuzzy set theory was firstly introduced by Zadeh (1965). Fuzzy control charts have been well re-searched by using ordinary fuzzy sets. Several of the most significant papers on fuzzy variable control charts and their applications are worth considering. Rowlands and Wang (2000) proposed fuzzy SPC methods based on the application of fuzzy logic to the SPC zone rules, while El-Shal and Morris (2000) developed a methodology for control charts by integrating them with fuzzy logic and SPC zone rules. The presented SPC zone rules methodology is provided in order to reduce false alarms and to detect the real faults. Zarandi *et al.* (2008) suggested a combination of fuzzified sensitivity criteria and fuzzy adaptive sampling rules that can be applied in a case when there is a need to determine the sample size as well as the sample interval of the control charts. In addition, some fuzzy control charts based on transformation techniques have been presented. Erginel (2008) presented the fuzzy individual and moving range control charts with α-cuts by considering the fuzziness. Şentürk and Erginel (2009) developed α-cut fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts for the first time and improved the α-cuts fuzzy $\tilde{\bar{\bar{X}}} - \tilde{R}$ and $\tilde{\bar{\bar{X}}} - \tilde{S}$ control charts together with α-level fuzzy midrange transformation techniques. Additionally, Şentürk (2010) developed a fuzzy regression control charts based on an α-cut approximation. Poongodi and Muthulakshmi (2015) used α-cut approach for analysing the fuzzy queueing model via fuzzy control charts in their paper. Gildeh and Shafiee (2015) investigated the effects of autocorrelation on fuzzy average and moving range control charts. Wang and Hyrnıewıcz (2015) considered a nonparametric Shewhart chart for fuzzy data. They used the bootstrap approach to calculate the quantile of fuzzy mean. Chen and Huang (2016) used fuzzy zones and fuzzy rules while constructing the fuzzy zone control charts. Kaya *et al.* (2017) developed a fuzzy individual and fuzzy moving range control chart designed to monitor stock prices.

A number of publications on fuzzy attribute control charts and their applications have been also published. Raz and Wang (1990) generated the probabilistic approach and the membership approach for constructing control charts for quality assurance, if the observations are collected in the form of attribute data. Wang and Raz (1990) developed a generic approach for constructing an attribute control chart. They used attribute data and several methods for calculating the values that represent sample means and determine the centre line and control limits. Kanagawa *et al.* (1993) modified the given control chart by considering the process average and process variability. Gülbay *et al.* (2004) proposed α-cut control chart to regulate the tightness of the inspection to attribute with triangular fuzzy numbers with the aim to reflecting the vagueness of the data. Gülbay and Kahraman (2006a) developed the new method to treat attribute data using a fuzzy c-control chart without defuzzification. Also, they developed a fuzzy control chart for the number of nonconforming units under the conditions of vague data and using the probabilities of fuzzy events. Gülbay and Kahraman (2006b) proposed a direct fuzzy approach to the fuzzy control chart. Their contributions to the fuzzy control chart were based on fuzzy transformation methods. Şentürk *et al.* (2011) showed a theoretical structure of fuzzy u control charts with applications. Wang *et al.* (2014) generated the weighted possibilistic mean (WMP) and weighted internal possibilistic mean (WIVPM) for fuzzy attribute data. Also, they developed a new fuzzy c-control chart with WPM and WIVPM. Hou *et al.* (2016) proposed a necessity and possibility measurement rules for the fuzzy control chart in their paper.

Additionally, many researchers studied fuzzy rule based method to construct the fuzzy control charts. Kaya and Kahraman (2011) firstly derived the fuzzy rule method for evaluating the fuzzy control charts in their paper. Erginel (2014) presented a fuzzy *p* control chart and a fuzzy *np* control chart by using decision rules. Khademi and Amırzadeh (2014) proposed a direct fuzzy approach for fuzzy control charts without any defuzzification. They defined fuzzy unnatural pattern rules based on the fuzzification of the crisp rules.

The mentioned type-1 fuzzy set uses a single membership function of data, while type-2 fuzzy set uses upper and lower membership functions. So type-2 fuzzy sets membership function is three dimensional. Type-2 fuzzy sets and their applications are well documented in literature. Zadeh (1975) used the extension of type-1 fuzzy sets for developing the concept of a type-2 fuzzy sets. Mizumoto and Tanaka (1976) gave some properties and set theoretic operations of fuzzy type-2 sets. Karnik and Mendel (2001) presented the centroid of type-2 fuzzy sets and developed an algorithm for interval-type-fuzzy sets. Also, Karnik and Mendel (2001) discussed the set operations on type-2 fuzzy sets, algebraic operations, properties of membership grades of type-2 sets, type-2 relations and their compositions. Mendel and John (2002) established a small set of terms for type-2 fuzzy sets. Also, they formulated a theorem for type-2 fuzzy sets and showed how it can be applied to derive formulas for the union, intersection, and complement of type-2 fuzzy sets without using the extension principle. Mendel *et al.* (2006) demonstrated that all the results that needed to implement an interval type-2 fuzzy logic systems can be obtained by using type-1 fuzzy systems mathematics. Also, Mendel developed an interval type-2 fuzzy system. The interval type-2 fuzzy systems are able to model higher levels of uncertainty comparing to type-1 fuzzy logic systems. This enables efficient development of control systems and to model human decision making.

Fuzzy multiple criteria decision making methods based on type-2 fuzzy sets have increasingly been considered in the past years due to their flexibility in modelling uncertain decision environment (Keshavarz Ghorabaee *et al.*, 2016a). Regarding control charts, although there are many papers on ordinary fuzzy control charts, no paper on type-2 fuzzy control charts has appeared so far. Modelling the fuzzy control charts with type-2/interval type-2 fuzzy sets may contribute to more accuracy in monitoring and evaluating the process because of taking into account the imprecision in the membership function of the data. The current paper is the first study of interval type-2 fuzzy control charts. In this paper, the theoretical structure of the interval type-2 fuzzy c-control chart is developed and presented in Section 3. In addition, the application of interval type-2 fuzzy c-control is implemented with real world data and presented in Section 4.

2. Interval Type-2 Fuzzy Sets

The theoretical base of type-1 fuzzy set theory was first introduced by Zadeh (1965). The concept of a type-2 fuzzy set was also introduced by Zadeh (1975). In the type-1 fuzzy sets, each element has a degree of membership function valued in the interval [0, 1] (Zadeh, 1965) and it is two-dimensional. Consequently, type-1 fuzzy sets cannot effectively model the uncertainties because in general their membership functions are completely crisp. Conversely, type-2 fuzzy sets are able to model the vagueness, because their membership functions really include fuzziness. The latter membership functions are three-dimensional, which means that the trace of uncertainty is included.

The three-dimensions of type-2 fuzzy sets procure additional degrees of freedom for directly modelling the uncertainty (Mendel and John, 2002). Interval type-2 fuzzy sets are a special case of type-2 fuzzy sets. It is observed that the mentioned interval type-2 fuzzy sets are the most preferred type-2 fuzzy sets in scientific publications, because computations with interval type-2 fuzzy sets are rather simple and manageable. For these reasons, most researchers use interval type-2 fuzzy sets in the real world (Mendel *et al.*, 2006; Keshavarz Ghorabaee *et al.*, 2016b).

Further in this section, definitions of type-2 fuzzy sets, interval type-2 fuzzy sets and arithmetic operators for given two trapezoidal interval type-2 fuzzy sets are represented hereinafter.

Definition 1. A type-2 fuzzy set (T2 FS) $\tilde{A}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{A}}$ presented as follows (Mendel *et al.*, 2016):

$$\tilde{A} = \big\{\big((x,u), \mu_{\tilde{A}}(x,u)\big) \,\big|\, x \in X,\ u \in [0,1]\big\}, \tag{1}$$

$$I_x = \big\{u \in [0,1] \,\big|\, \big(\mu_{\tilde{A}}(x,u) > 0\big)\big\}. \tag{2}$$

An interval type-2 fuzzy set (IT2 FS) is a type-2 fuzzy sets $\tilde{A}$ such that

$$I_x = \big\{u \in [0,1] \,\big|\, \mu_{\tilde{A}}(x,u) = 1\big\}. \tag{3}$$

Then interval type-2 fuzzy set is called a closed interval type-2 fuzzy set (CIT2 FS) if I_x is closed interval for every $x \in X$.

Definition 2. The upper membership function and the lower membership function of interval type-2 fuzzy set (see Fig. 1), and type-1 membership functions are given respectively (Chen and Lee, 2010).

A trapezoidal interval type-2 fuzzy set

$$\tilde{A}_i = \big(A_i^U, A_i^L\big) = \begin{pmatrix} \big(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(A_i^U), H_2(A_i^U)\big), \\ \big(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(A_i^L), H_2(A_i^L)\big) \end{pmatrix} \tag{4}$$

where A_i^U and A_i^L are type-1 fuzzy sets, $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U, a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L$ are the reference points of the interval type-2 fuzzy $\tilde{A}_i$, $H_j(A_i^U)$; denotes the membership value of the

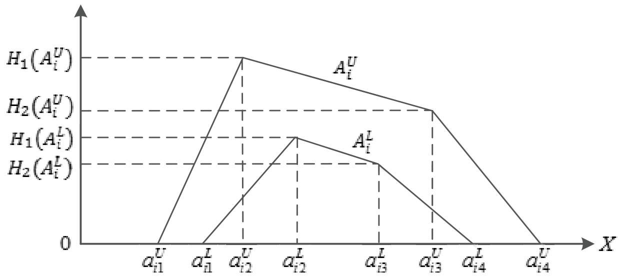

Fig. 1. The membership functions of the interval type-2 fuzzy set $\tilde{A}$.

element $a_{i(j+1)}^U$ in the upper trapezoidal membership function A_i^U, $1 \leqslant j \leqslant 2$, $H_j(A_i^L)$ indicates the membership value of the element $a_{i(j+1)}^L$ in the lower trapezoidal membership function A_i^L, $1 \leqslant j \leqslant 2$, $H_1(A_i^U)$, $H_2(A_i^U)$, $H_1(A_i^L)$, $H_2(A_i^L) \in [0, 1]$, $1 \leqslant i \leqslant n$.

Let $\tilde{A}_1$ and $\tilde{A}_2$ are two trapezoidal interval type-2 fuzzy sets:

$$\tilde{A}_1 = \left(A_1^U, A_1^L\right) = \begin{pmatrix} \left(a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)\right), \\ \left(a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(A_1^L), H_2(A_1^L)\right) \end{pmatrix}, \tag{5}$$

$$\tilde{A}_2 = \left(A_2^U, A_2^L\right) = \begin{pmatrix} \left(a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(A_2^U), H_2(A_2^U)\right), \\ \left(a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(A_2^L), H_2(A_2^L)\right) \end{pmatrix}. \tag{6}$$

The arithmetic operators for given two trapezoidal interval type-2 fuzzy sets are as follows (Chen and Lee, 2010):

Addition operation:

$$\begin{aligned} \tilde{A} \oplus \tilde{A}_2 &= \left(A_1^U, A_1^L\right) \oplus \left(A_2^U, A_2^L\right) \\ &= \begin{pmatrix} \left(a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U\right); \\ \min\left(H_1(A_1^U); H_1(A_2^U)\right), \min\left(H_2(A_1^U), H_2(A_2^U)\right), \\ \left(a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L\right); \\ \min\left(H_1(A_1^L); H_1(A_2^L)\right), \min\left(H_2(A_1^L); H_2(A_2^L)\right) \end{pmatrix}. \end{aligned} \tag{7}$$

Subtraction operation:

$$\begin{aligned} \tilde{A}_1 \ominus \tilde{A}_2 &= \left(A_1^U, A_1^L\right) \ominus \left(A_2^U, A_2^L\right) \\ &= \begin{pmatrix} \left(a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U\right); \\ \min\left(H_1(A_1^U); H_1(A_2^U)\right), \min\left(H_2(A_1^U); H_2(A_2^U)\right), \\ \left(a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L\right); \\ \min\left(H_1(A_1^L); H_1(A_2^L)\right), \min\left(H_2(A_1^L); H_2(A_2^L)\right) \end{pmatrix}. \end{aligned} \tag{8}$$

Multiplication operation:

$$\tilde{A}_1 \otimes \tilde{A}_2 = \left(A_1^U, A_1^L\right) \otimes \left(A_2^U, A_2^L\right)$$
$$\approx \begin{pmatrix} \left(a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U\right); \\ \min\left(H_1(A_1^U); H_1(A_2^U)\right), \min\left(H_2(A_1^U), H_2(A_2^U)\right), \\ \left(a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L\right); \\ \min\left(H_1(A_1^L); H_1(A_2^L)\right), \min\left(H_2(A_1^L), H_2(A_2^L)\right) \end{pmatrix}. \tag{9}$$

Arithmetic operations with crisp value k:

$$kx\tilde{A}_1 = \begin{pmatrix} \left(k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U; H_1(A_1^U), H_2(A_1^U)\right), \\ \left(k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L; H_1(A_1^L), H_2(A_1^L)\right) \end{pmatrix}, \tag{10}$$

$$\frac{\tilde{A}_1}{k} = \begin{pmatrix} \left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U; H_1(A_1^U), H_2(A_1^U)\right), \\ \left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times a_{14}^L; H_1(A_1^L), H_2(A_1^L)\right) \end{pmatrix}, \tag{11}$$

where $k > 0$.

3. Interval Type-2 Fuzzy c-Control Chart

c-control chart is related to the number of nonconformities that can be described as cracked, hole, stained etc. If the number of nonconformities is more important than the number of conforming/nonconforming of product, c-control chart is an available tool for monitoring the process when the sample size is constant.

The number of defects or nonconformities that occur in the sample are distributed with Poisson distribution, that is:

$$p(x) = \frac{e^{-c}c^x}{x!}, \tag{12}$$

where c is the number of nonconformities, and $c > 0$ is the parameter of the Poisson distribution. Both the mean and the variance of the Poisson distribution are equal to the parameter c.

Classical c-control chart limits were proposed by Shewhart. They can be presented by equations as follows (Montgomery, 1991):

$$\begin{aligned} UCL_c &= c + 3\sqrt{c}, \\ CL_c &= c, \\ LCL_c &= c - 3\sqrt{c}, \end{aligned} \tag{13}$$

where UCL is the upper control limit, CL is the centre line and LCL is the lower control limit of c control chart.

Table 1
Interval type-2 fuzzy number of nonconformities.

c					
$\tilde{c}_{1.} = [(a_{11}^U$	a_{12}^U	a_{13}^U	$a_{14}^U;$	$H_1(A_1^U)$	$H_2(A_1^U))$
$(a_{11}^L$	a_{12}^L	a_{13}^L	$a_{14}^L;$	$H_1(A_1^L)$	$H_2(A_1^L))]$
$\tilde{c}_{2.} = [(a_{21}^U$	a_{22}^U	a_{23}^U	$a_{24}^U;$	$H_1(A_2^U)$	$H_2(A_2^U))$
$(a_{21}^L$	a_{22}^L	a_{23}^L	$a_{24}^L;$	$H_1(A_2^L)$	$H_2(A_2^L))]$
$\vdots$				$\vdots$	
$\tilde{c}_{n.} = [(a_{n1}^U$	a_{n2}^U	a_{n3}^U	$a_{n4}^U;$	$H_1(A_n^U)$	$H_2(A_n^U))$
$(a_{n1}^L$	a_{n2}^L	a_{n3}^L	$a_{n4}^L;$	$H_1(A_n^L)$	$H_2(A_n^L))]$

If c is not known from population, it can be estimated from a sample, as shown below:

$$E[c] = \overline{c} \tag{14}$$

and

$$\overline{c} = \frac{\sum_{i=1}^{m} c_i}{m}, \tag{15}$$

where the expected value of c equals to the mean of nonconformities in a sample.

In literature, the fuzzy approach to the c control chart was first introduced by Gülbay *et al.* (2004), Gülbay and Kahraman (2006a). They described the fuzzy c-control chart by using direct fuzzy approach and transformation techniques. However, interval type-2 fuzzy c-control chart is proposed for the first time in this paper.

3.1. *Interval Type-2 Fuzzy c-Control Chart for Trapezoidal Fuzzy Number*

Although fuzzy control charts are introduced by many authors in literature, interval type-2 fuzzy control charts have not been handled up to now. In contrast to the fuzzy control chart, interval type-2 fuzzy control chart considers the fuzzy membership functions that have the grades themselves while constructing the limits of the control chart. The structure of the interval type-2 fuzzy c-control charts is constructed as described below in the paper.

The number of nonconformities is presented in Table 1.

Where $\tilde{c}_{i.}$ expresses the interval type-2 trapezoidal fuzzy number of nonconformities for the i. sample and is presented as

$$\tilde{c}_{i.} = \begin{bmatrix} (a_{i1}^U & a_{i2}^U & a_{i3}^U & a_{i4}^U; & H_1(A_i^U) & H_2(A_i^U)) \\ (a_{i1}^L & a_{i2}^L & a_{i3}^L & a_{i4}^L; & H_1(A_i^L) & H_2(A_i^L)) \end{bmatrix}.$$

$\overline{c}$ shows the mean of interval type-2 fuzzy sample (n is the sample size) as follows:

$$\overline{c}_{a_{.1}^U} = \frac{\sum_{i=1}^{n} a_{i1}^U}{n}, \quad \overline{c}_{a_{.2}^U} = \frac{\sum_{i=1}^{n} a_{i2}^U}{n}, \quad \overline{c}_{a_{.3}^U} = \frac{\sum_{i=1}^{n} a_{i3}^U}{n}, \quad \overline{c}_{a_{.4}^U} = \frac{\sum_{i=1}^{n} a_{i4}^U}{n}, \tag{16}$$

$$\overline{c}_{a^L_{.1}} = \frac{\sum_{i=1}^{n} a^L_{i1}}{n}, \quad \overline{c}_{a^L_{.2}} = \frac{\sum_{i=1}^{n} a^L_{i2}}{n}, \quad \overline{c}_{a^L_{.3}} = \frac{\sum_{i=1}^{n} a^L_{i3}}{n}, \quad \overline{c}_{a^L_{.4}} = \frac{\sum_{i=1}^{n} a^L_{i4}}{n}. \quad (17)$$

The fuzzy upper control limit can be calculated by incorporating Eq. (13) and the addition operation based on Eq. (7), and for the fuzzy lower control limit – based on Eq. (8), and by using the subtraction operation for interval type-2 trapezoidal fuzzy number:

$$\begin{aligned} UCL_c = \Big[&\overline{c}_{a^U_{.1}} + 3\sqrt{\overline{c}_{a^U_{.1}}}, \overline{c}_{a^U_{.2}} + 3\sqrt{\overline{c}_{a^U_{.2}}}, \overline{c}_{a^U_{.3}} + 3\sqrt{\overline{c}_{a^U_{.3}}}, \overline{c}_{a^U_{.4}} + 3\sqrt{\overline{c}_{a^U_{.4}}}; \\ &\min\big(H_1\big(A^U_i\big), H_2\big(A^U_i\big)\big), \overline{c}_{a^L_{.1}} + 3\sqrt{\overline{c}_{a^L_{.1}}}, \overline{c}_{a^L_{.2}} + 3\sqrt{\overline{c}_{a^L_{.2}}}, \overline{c}_{a^L_{.3}} + 3\sqrt{\overline{c}_{a^L_{.3}}}, \overline{c}_{a^U_{.4}} \\ &+ 3\sqrt{\overline{c}_{a^L_{.4}}}; \min\big(H_1\big(A^L_i\big), H_2\big(A^L_i\big)\big)\Big], \end{aligned} \quad (18)$$

$$CL_c = \begin{bmatrix} \overline{c}_{a^U_{.1}}, \overline{c}_{a^U_{.2}}, \overline{c}_{a^U_{.3}}, \overline{c}_{a^U_{.4}}; \min\big(H_1(A^U_1), H_2(A^U_1)\big), \\ \overline{c}_{a^L_{.1}}, \overline{c}_{a^L_{.2}}, \overline{c}_{a^L_{.3}}, \overline{c}_{a^L_{.4}}; \min\big(H_1(A^L_1), H_2(A^L_1)\big) \end{bmatrix}, \quad (19)$$

$$\begin{aligned} LCL_c = \Big[&\overline{c}_{a^U_{.1}} - 3\sqrt{\overline{c}_{a^U_{.4}}}, \overline{c}_{a^U_{.2}} - 3\sqrt{\overline{c}_{a^U_{.3}}}, \overline{c}_{a^U_{.3}} - 3\sqrt{\overline{c}_{a^U_{.2}}}, \overline{c}_{a^U_{.4}} - 3\sqrt{\overline{c}_{a^U_{.1}}}; \\ &\min\big(H_1\big(A^U_1\big), H_2\big(A^U_1\big)\big), \overline{c}_{a^L_{.1}} - 3\sqrt{\overline{c}_{a^L_{.4}}}, \overline{c}_{a^L_{.2}} - 3\sqrt{\overline{c}_{a^L_{.3}}}, \overline{c}_{a^L_{.3}} - 3\sqrt{\overline{c}_{a^L_{.2}}}, \overline{c}_{a^L_{.4}} \\ &- 3\sqrt{\overline{c}_{a^L_{.1}}}; \min\big(H_1\big(A^L_1\big), H_2\big(A^L_1\big)\big)\Big], \end{aligned} \quad (20)$$

where the upper control limit, the centre line and the lower control limit of interval type-2 fuzzy c-control chart are given by the above equations and by using interval type-2 fuzzy set theory, respectively.

3.2. *Defuzzification Method for Interval Type-2 Fuzzy c-Control Chart*

The previous type-1 fuzzy control chart studies used transformation techniques to convert the fuzzy numbers into crisp real numbers. These crisp values are also called representative values. Four well-known methods for defuzzification of a fuzzy subset are usually applied, namely fuzzy mode, fuzzy midrange, fuzzy median and fuzzy average. These four methods are well-known in descriptive statistics (Raz and Wang, 1990) and are not described in detail in the current paper.

In the type-2 fuzzy sets studies, several defuzzification techniques are proposed for reduction process. One of the most used type reduction methods is centroid of a type-2 fuzzy set techniques that was proposed by Mendel *et al.* (2006), also the indices method by Niewiadomski *et al.* (2006) and the Best Nonfuzzy Performance (BNP) by Tsaur *et al.* (2002). Kahraman *et al.* (2014) modified the BNP method for application with trapezoidal type-2 fuzzy sets. The method is given as below (Kahraman *et al.*, 2014):

$$DIT2^U_{Trap(i)} = \frac{(a^U_{i4} - a^U_{i1}) + (H_2(A^U_1)a^U_{i2} - a^U_{i1}) + (H_1(A^U_1)a^U_{i3} - a^U_{i1})}{4} + a^U_{i1}, \quad (21)$$

$$DIT2^L_{Trap(i)} = \frac{(a^L_{i4} - a^L_{i1}) + (H_2(A^L_1)a^L_{i2} - a^L_{i1}) + (H_1(A^L_1)a^L_{i3} - a^L_{i1})}{4} + a^L_{i1}, \quad (22)$$

$$DIT2_{Trap(i)} = \frac{DIT2^U_{Trap(i)} + DIT2^L_{Trap(i)}}{2}; \quad i = 1, 2, \ldots, n, \quad (23)$$

where $H_1(A^U_1)$ and $H_2(A^U_1)$ are the maximum membership degrees of the upper membership function; a^U_{i4} and a^U_{i1} are the largest and the least possible values of the upper membership function, respectively; a^U_{i2} and a^U_{i3} are the second and third parameters of the upper membership function; a^L_{i4} and a^L_{i1} are the largest and the least possible values of the lower membership function; a^L_{i2} and a^L_{i3} are the second and third parameters of the lower membership function.

A modified BNP method for trapezoidal type-2 fuzzy sets is adopted to the interval type-2 fuzzy control charts for defuzzification in the following way:

$$CDIT2^U_{Trap(i)} = \frac{(\overline{c}_{a^U_{.4}} - \overline{c}_{a^U_{.1}}) + (H_2(A^U_1)\overline{c}_{a^U_{.2}} - \overline{c}_{a^U_{.1}}) + (H_1(A^U_1)\overline{c}_{a^U_{.3}} - \overline{c}_{a^U_{.1}})}{4} + \overline{c}_{a^U_{.1}}, \quad (24)$$

$$CDIT2^L_{Trap(i)} = \frac{(\overline{c}_{a^L_{.4}} - \overline{c}_{a^L_{.1}}) + (H_2(A^L_1)\overline{c}_{a^L_{.2}} - \overline{c}_{a^L_{.1}}) + (H_1(A^L_1)\overline{c}_{a^L_{.3}} - \overline{c}_{a^L_{.1}})}{4} + \overline{c}_{a^L_{.1}}, \quad (25)$$

$$CDIT2_{Trap(i)} = \frac{CDIT2^U_{Trap(i)} + CDIT2^L_{Trap(i)}}{2}, \quad (26)$$

where $CDIT2_{Trap(i)}$ represents the defuzzification value of interval type-2 fuzzy number of nonconformities.

4. An Application for Interval Type-2 Fuzzy c-Control Chart

The application is conducted in one of the biggest food companies in Turkey. It produces biscuits, cake, wafers etc. This company has ISO 9001: quality management systems and an ISO 22000: HACCP certification. Foods are sometimes packaged erroneously during the packaging process. These recurring mistakes are "the letters on package are not in the right place", "the printing quality of letters is not appropriate" and "gluing failure of the package". In this situation, the packaging does not conform to acceptable standards. Our study considers the number of nonconformities in this packaging process and monitors them with c-control charts. The data are collected from packaging process by operators visually checking the packets. The number of nonconformities are determined and recorded on a sheet. But in this collecting process, some uncertainty and vagueness can occur due to the operators' judgments, because the control is made by visual checking. Human cognitive decisions play an important role, therefore fuzzy control charts modelled by membership functions are inevitable tools for these uncertainties. Also, the fuzzy

Table 2
The interval type-2 trapezoidal fuzzy number of nonconformities for packaging process.

$\tilde{c}_i$: The interval type-2 trapezoidal fuzzy number of nonconformities for i. sample
$\tilde{c}_1 = [(2, 3, 4, 5; 1, 1) \quad (1, 2, 3, 4; 0.9, 0.5)]$
$\tilde{c}_2 = [(3, 5, 6, 7; 1, 1) \quad (2, 4, 5, 6; 0.6, 0.5)]$
$\tilde{c}_3 = [(3, 4, 5, 6; 1, 1) \quad (2, 3, 4, 5; 0.7, 0.5)]$
$\tilde{c}_4 = [(2, 3, 4, 6; 1, 1) \quad (1, 2, 3, 5; 0.8, 0.6)]$
$\tilde{c}_5 = [(2, 3, 5, 6; 1, 1) \quad (1, 2, 4, 5; 0.6, 0.5)]$
$\tilde{c}_6 = [(4, 6, 6, 7; 1, 1) \quad (3, 4, 4, 6; 0.7, 0.6)]$
$\tilde{c}_7 = [(4, 5, 6, 7; 1, 1) \quad (3, 4, 5, 6; 0.9, 0.6)]$
$\tilde{c}_8 = [(2, 3, 5, 6; 1, 1) \quad (1, 2, 4, 5; 0.7, 0.6)]$
$\tilde{c}_9 = [(3, 5, 5, 6; 1, 1) \quad (2, 4, 4, 5; 0.9, 0.5)]$
$\tilde{c}_{10} = [(3, 4, 4, 6; 1, 1) \quad (2, 3, 4, 5; 0.8, 0.5)]$
$\tilde{c}_{11} = [(2, 3, 4, 5; 1, 1) \quad (1, 2, 3, 4; 0.9, 0.7)]$
$\tilde{c}_{12} = [(3, 5, 6, 7; 1, 1) \quad (2, 4, 5, 6; 0.8, 0.6)]$
$\tilde{c}_{13} = [(2, 3, 6, 6; 1, 1) \quad (1, 2, 4, 5; 0.7, 0.5)]$
$\tilde{c}_{14} = [(3, 4, 5, 6; 1, 1) \quad (2, 3, 4, 5; 0.9, 0.7)]$
$\tilde{c}_{15} = [(4, 5, 5, 7; 1, 1) \quad (3, 4, 5, 6; 0.9, 0.8)]$
$\tilde{c}_{16} = [(2, 4, 5, 7; 1, 1) \quad (1, 3, 4, 6; 0.8, 0.6)]$
$\tilde{c}_{17} = [(2, 3, 3, 5; 1, 1) \quad (1, 2, 3, 4; 0.9, 0.5)]$
$\tilde{c}_{18} = [(2, 3, 4, 6; 1, 1) \quad (1, 2, 3, 5; 0.9, 0.7)]$

data are collected as interval type-2 fuzzy numbers from the process. In this application, the fuzzy numbers of nonconformities are modelled with interval type-2 fuzzy c-control charts using trapezoidal membership functions. Collected data are shown in Table 2 as interval type-2 trapezoidal fuzzy numbers.

Also, the averages of the interval type-2 fuzzy number of nonconformities are calculated and given as follows:

$$\overline{c}_{a^U_{.1}} = 2.66; \quad \overline{c}_{a^U_{.2}} = 3.94; \quad \overline{c}_{a^U_{.3}} = 4.88; \quad \overline{c}_{a^U_{.4}} = 6.16, \tag{27}$$

$$\overline{c}_{a^L_{.1}} = 1.66; \quad \overline{c}_{a^L_{.2}} = 2.88; \quad \overline{c}_{a^L_{.3}} = 4; \quad \overline{c}_{a^L_{.4}} = 5.07. \tag{28}$$

The interval type-2 fuzzy upper and lower control limits and centre line of the fuzzy c-control chart are calculated by using Eqs. (18)–(20):

$$UCL_c = \begin{bmatrix} (7.55, 9.89, 11.50, 13.60; 1, 1), \\ (5.52, 7.97, 10, 11.82; ; 0.6, 0.5) \end{bmatrix}, \tag{29}$$

$$CL_c = [(2.66, 3.94, 4.88, 6.16; 1, 1), (1.66, 2.88, 4, 5.07; 0.6, 0.5)], \tag{30}$$

$$LCL_c = \begin{bmatrix} (-4.78, -2.68, -1.07, 1.26; 1, 1), \\ (-5.09, -3.12, -1.09, 1.20; 0.6, 0.5) \end{bmatrix}. \tag{31}$$

Calculating the limits of interval type-2 fuzzy c-control chart, then c-control limits are defuzzified by using proposed modified BNP method for fuzzy control charts. The operations of defuzzification for interval type-2 fuzzy control limits and centre line are given as Eqs. (32)–(40) as follows:

$$CDIT2^U_{Trap} = \frac{(13.60 - 7.55) + (1 * 9.89 - 7.55) + (1 * 11.50 - 7.55)}{4} + 7.55$$
$$= 10.635, \quad (32)$$

$$CDIT2^L_{Trap} = \frac{(11.82 - 5.52) + (0.5 * 7.97 - 5.52) + (0.6 * 10 - 5.52)}{4} + 5.52$$
$$= 6.831, \quad (33)$$

$$UCL\_CDIT2_{Trap} = \frac{6.831 + 10.635}{2} = 8.733, \quad (34)$$

$$CDIT2^U_{Trap} = \frac{(6.16 - 2.66) + (1 * 3.94 - 2.66) + (1 * 4.88 - 2.66)}{4} + 2.66$$
$$= 4.410, \quad (35)$$

$$CDIT2^L_{Trap} = \frac{(5.07 - 1.66) + (0.5 * 2.88 - 1.66) + (0.6 * 4 - 1.66)}{4} + 1.66$$
$$= 2.642, \quad (36)$$

$$CL\_CDIT2_{Trap} = \frac{2.642 + 4.410}{2} = 3.526, \quad (37)$$

$$CDIT2^U_{Trap}$$
$$= \frac{(1.26 + 4.78) + (1 * (-2.68) - (-4.78)) + (1 * (-1.07) - (-4.78))}{4} + (-4.78)$$
$$= -1.817, \quad (38)$$

$$CDIT2^L_{Trap}$$
$$= \frac{(1.20 + 5.09) + (0.5 * (-3.12) - (-5.09)) + (0.6 * (-1.09) - (-5.09))}{4} + (-5.09)$$
$$= -1.526, \quad (39)$$

$$LCL\_CDIT2_{Trap} = \frac{-1.817 + 1.526}{2} = -0.291 < 0. \quad (40)$$

After defuzzification of the control limits of c-control charts, the number of nonconformities in each sample is also defuzzified by using modified BNP method as given below. The example of defuzzification for the first sample is given in Eqs. (41)–(46). The obtained results for all samples are given in Table 3.

$$DIT2^U_{Trap(i)} = \frac{(a^U_{i4} - a^U_{i1}) + (H_2(A^U_1)a^U_{i2} - a^U_{i1}) + (H_1(A^U_1)a^U_{i3} - a^U_{i1})}{4} + a^U_{i1}, \quad (41)$$

$$DIT2^U_{Trap(1)} = \frac{(5 - 2) + (1 * 3 - 1) + (1 * 4 - 2)}{4} + 2 = 3.500, \quad (42)$$

Table 3
Defuzzified values of the number of nonconformities for each sample.

$DIT2^U_{Trap(i)}$	$DIT2^L_{Trap(i)}$	$DIT2_{Trap(i)}$
3.500	2.325	2.912
5.250	3.700	4.475
4.500	3.225	3.862
3.750	2.550	3.150
4.000	2.575	3.287
5.750	4.075	4.912
5.500	4.350	4.925
4.250	2.675	3.462
4.750	3.450	4.100
4.250	3.275	3.762
3.500	2.375	2.937
5.000	3.900	4.450
4.250	2.650	3.450
4.500	3.375	3.937
5.250	4.400	4.825
4.500	2.900	3.700
3.250	2.325	2.787
3.750	2.375	3.062

$$DIT2^L_{Trap} = \frac{(a^L_{i4} - a^L_{i1}) + (H_2(A^L_1)a^L_{i2} - a^L_{i1}) + (H_1(A^L_1)a^L_{i3} - a^L_{i1})}{4} + a^L_{i1}, \tag{43}$$

$$DIT2^L_{Trap(1)} = \frac{(4-1) + (0.5 * 2 - 1) + (0.9 * 3 - 1)}{4} + 1 = 2.325, \tag{44}$$

$$DIT2_{Trap(i)} = \frac{DIT2^U_{Trap(i)} + DIT2^L_{Trap(i)}}{2}, \tag{45}$$

$$DIT2_{Trap(1)} = \frac{2.325 + 3.500}{2} = 2.912. \tag{46}$$

The defuzzified control limits of interval type-2 fuzzy c-control charts and defuzzified value for each sample are compared for the evaluation of process control. If the defuzzified values for all samples are in the control limits, the packaging process is “in-control”. If one defuzzified value from the sample is out of control limits, the packaging process is “out of-control”. These calculations and decisions of process are given in Table 4.

According to Table 4, all samples are within control limits, so the packaging process is “in control”.

5. Conclusion

When data is collected as the interval type-2 fuzzy numbers from process, the classical c-control chart is not suitable for monitoring the process. In this situation, the interval type-2 fuzzy control charts should be used. The main contribution of the current study to

Table 4
Decisions of process for type-2 fuzzy c-control charts.

Sample no.	$LCL\_CDIT2_{Trap} < DIT2_{Trap(i)} < UCL\_CDIT2_{Trap}$
1	0 < 2.912 < 8.733
2	0 < 4.475 < 8.733
3	0 < 3.862 < 8.733
4	0 < 3.150 < 8.733
5	0 < 3.287 < 8.733
6	0 < 4.912 < 8.733
7	0 < 4.925 < 8.733
8	0 < 3.462 < 8.733
9	0 < 4.100 < 8.733
10	0 < 3.762 < 8.733
11	0 < 2.937 < 8.733
12	0 < 4.450 < 8.733
13	0 < 3.450 < 8.733
14	0 < 3.937 < 8.733
15	0 < 4.825 < 8.733
16	0 < 3.700 < 8.733
17	0 < 2.787 < 8.733
18	0 < 3.062 < 8.733

the existent literature is that it proposes using an interval type-2 fuzzy c-control chart for the first time.

The application is implemented into the packaging process of the food sector. Data are collected from 18 samples as interval type-2 fuzzy numbers from the process because of the imprecise collecting methodology, and the limits of interval type-2 fuzzy c-control charts are calculated. Then the above proposed defuzzification method is applied to the fuzzy control limits, and a decision is made about the packaging process.

In further research, other control charts can be modelled by using interval type-2 fuzzy sets theory when data are incomplete and vague.

Another alternative for further research is to use other extensions of fuzzy sets for control charts such as intuitionistic fuzzy sets, hesitant fuzzy sets, neutrosophic sets, etc.

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S. Şentürk is an assoc. professor at the Department of Statistics, Anadolu University, Turkey. She received her PhD from Anadolu University, in 2006. She is interested in statistics, quality control, fuzzy sets, decision-making theories and multi-criteria analysis.

J. Antucheviciene is a professor at the Department of Construction Technology and Management at Vilnius Gediminas Technical University, Lithuania. She received her PhD in 2005. Her research interests include multi-criteria analysis, decision-making theories and decision support systems, sustainable development, construction management and investment.