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Minimum Mean Square Error Estimators for the Exponential SSALT Model

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Abstract. This paper presents minimum mean square error (MMSE) estimators for mean life and failure rate of Exponential distribution model based on failure censored step-stress accelerated life-testing (SSALT) data. The MMSE estimators are drived by revising the corresponding unbiased estimators in terms of mean square error (MSE). Two theorems prove mathematically the fact that MSE of the resulting MMSE estimators are smaller than that of the corresponding unbiased estimators. The results show that the MMSE estimators are more efficient than the unbiased estimators and maximum likelihood estimators (MLEs) in small and moderate sample size.

Key words: Step-stress accelerated life-testing (SSALT), exponential distribution, mean life, failure rate, mean square error (MSE).

1. Introduction

Accelerated life testing (ALT) is commonly practiced in product life testing and analysis to reduce operation time and costs as products may have high reliability under normal conditions (Brumen *et al.*, 2014; Miller and Nelson, 1983; Wu and Yu, 2005) because time and costs are expensive (Zhou *et al.*, 2012). The objective of ALT is to improve the performance and reliability of products by shorting the period between product design and release time (Nelson, 1980; Mao and Wang, 1997). Failure data collected from ALT may be used to estimate some product characteristics, such as mean lifetime, failure rate and reliability, etc (Lawless, 1982; Zhou *et al.*, 2013a). Step-stress accelerated life testing (SSALT) is a special type of ALT in which stress levels are increased during the test period in a specified discrete sequence (Nelson, 1990). SSALT allows the stress setting of a test unit to be changed at pre-specified times or upon the occurrence of a fixed number of

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failures. The former is called SSALT with Type-I censoring and the latter is called SSALT with Type-II censoring. However, SSALT should be further developed to reduce operation time and costs.

In SSALT model, many researchers assumed that failure time (life time) follows Exponential distribution (Sarhan *et al.*, 2012). For example, Miller and Nelson (1983) presented a simple SSALT plan assuming the exponential life distribution. Khamis (1997) proposed an optimal m-step SSALT design with k stress variables under the exponential life distribution. Tang *et al.* (1999) discussed optimum plans for two-parameter Exponential distribution. Wang (2006) obtained unbiased estimators for Exponential distribution based on SSALT censored data. Balakrishnan and Rasouli (2008) obtained conditional maximum likelihood estimators (MLEs) of two exponential mean parameters by exact inference for two exponential populations under joint Type-II censoring. Ling *et al.* (2009) obtained the MLE of the model parameters assuming an exponentially distributed life of test units and a cumulative exposure model. Due to the simplicity and practicability of Exponential distribution, we further investigate the estimators for parameters of SSALT model assuming the exponential life distribution based on mean square error (MSE) criterion. The resulting estimators are called minimum mean square error (MMSE) estimators.

The rest of this paper is organized as follows. Section 2 gives some basic assumptions and lemmas related with exponential SSALT model and MSE criterion for accessing the estimators. Section 3 derives the MMSE estimators for the exponential SSALT model. Section 4 concludes the paper.

2. SSALT Model and Optimization Criterion

2.1. *Basic Assumptions*

Under the condition of Exponential distribution, statistical inference for SSALT depends on the following assumptions:

(1) For any stress level, the life time of a test unit follows an Exponential distribution with the following cumulative distribution function (CDF):

$$f(t) = 1 - \exp(t/\theta), \quad t > 0,$$

where θ is the mean life of the test unit at stress level.

(2) The mean life of a test unit is a log-linear function of stress level x, that is,

$$\log(\theta) = \alpha + \beta x,$$

where α and β are the unknown parameters depending on the nature of the test unit and the test method.

(3) If two stress levels $x_1 < x_2$, then $\theta_1 < \theta_2$, where θ_1 and θ_2 are the mean lives of the test unit at stress levels x_1 and x_2, respectively.

(4) A cumulative exposure model holds: the remaining life depends on the current cumulative failure probability and current stress level regardless of how the probability is accumulated (Nelson, 1980; Wang, 2010; Zhou *et al.*, 2013b).

2.2. *Basic Lemmas*

We now consider the basic models under SSALT with the progressive Type-II censoring provided by Wang (2006). Suppose that all n test units are initially placed at the lowest stress level x_1 and run until failure r_1 units occur. At t_{1,r_1}, the stress level is increased to x_2 $(x_1 < x_2)$. The test is continued, triggering stress level changes at times t_{i,r_i}. At x_k, the test is terminated after r_k units have failed. At the stress level x_i, r_i failure times of test units are observed, which are $t_{i,j}$ $(i = 1, 2, \ldots, k,\ j = 1, 2, \ldots, r_1)$. We write

$$T_1 = \sum_{j=1}^{r_1} t_{1,r_1} + (n - r_1)$$

and

$$T_i = \sum_{j=1}^{r_1} (t_{i,j} - t_{i-1,j-1}) + \left(n - \sum_{j=1}^{i} r_i\right)(t_{i,r_i} - t_{i-1,r_i-1}), \quad (i = 1, 2, \ldots, k).$$

Obviously, statistics $T_1, T_2, \ldots, T_k$ are independent. Moreover, we write

$$\Psi(x) = d \log\big(\Gamma(x)\big)/dx, \qquad \Psi'(x) = d^2 \log\big(\Gamma(x)\big)/d^2x.$$

Let $U_i = \ln(T_i) - \Psi(r_i)$ $(i = 1, 2, \ldots, k)$, then the expectation and the variance of U_i are as follows:

$$E(U_i) = \ln(\theta_i) = \alpha + \beta x_i, \qquad \mathrm{Var}(U_i) = \Psi'(x) \quad (i = 1, 2, \ldots, k).$$

Mao and Wang (1997) introduced the linear unbiased estimators of the unknown parameters α and β by Gauss–Markov theorem, which are given as follows:

$$\hat{\alpha} = \frac{GH - IM}{EG - I^2}, \qquad \hat{\beta} = \frac{EM - IH}{EG - I^2}.$$

where

$$E = \sum_{i=1}^{k} \left[\Psi'(r_i)\right]^{-1}, \qquad I = \sum_{i=1}^{k} \left[\Psi'(r_i)\right]^{-1} x_i, \qquad G = \sum_{i=1}^{k} \left[\Psi'(r_i)\right]^{-1} x_i^2,$$

$$H = \sum_{i=1}^{k} \left[\Psi'(r_i)\right]^{-1} U_i, \qquad M = \sum_{i=1}^{k} \left[\Psi'(r_i)\right]^{-1} x_i U_i.$$

Hence, the estimators of the mean life θ_0 and the failure rate λ_0 at the design stress x_0 under normal operating conditions are given as follows:

$$\theta_0 = \exp(\hat{\alpha} + \hat{\beta} x_0), \qquad \lambda_0 = \exp(-\hat{\alpha} - \hat{\beta} x_0).$$

Two lemmas related with the exponential SSALT model are given as follows Wang (2006).

Lemma 1. *Let* $D_i = \frac{G-(x_0+x_i)I+x_0 x_i E}{\Psi'(r_i)(EG-I^2)}$ $(i = 1, 2, \ldots, k)$ *and* θ_0 *be the mean life at the design stress* x_0

(1) *If* $r_i + D_i > 0$ $(i = 1, 2, \ldots, k)$*, then the unbiased estimator of* θ_0 *is*

$$\tilde{\theta_0} = \hat{\theta_0} \exp\left(\sum_{i=1}^{k} D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_1)}{\Gamma(r_i + D_i)};$$

(2) *If* $r_i + 2D_i > 0$ $(i = 1, 2, \ldots, k)$*, then the variance of* $\tilde{\theta_0}$ *is*

$$\mathrm{Var}(\tilde{\theta_0}) = \left(\prod_{i=1}^{k} \frac{\Gamma(r_i)\Gamma(r_i + 2D_i)}{\Gamma^2(r_i + D_i)} - 1\right)\theta_0^2.$$

Lemma 2. *Let* $D_i = \frac{G-(x_0+x_i)I+x_0 x_i E}{\Psi'(r_i)(EG-I^2)}$ $(i = 1, 2, \ldots, k)$ *and* λ_0 *be the failure rate at the design stress* x_0.

(1) *If* $r_i - D_i > 0$ $(i = 1, 2, \ldots, k)$*, then the unbiased estimator of* λ_0 *is*

$$\tilde{\lambda_0} = \hat{\lambda_0} \exp\left(-\sum_{i=1}^{k} D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_1)}{\Gamma(r_i - D_i)};$$

(2) *If* $r_i - 2D_i > 0$ $(i = 1, 2, \ldots, k)$*, then the variance of* $\tilde{\lambda_0}$ *is*

$$\mathrm{Var}(\tilde{\lambda_0}) = \left(\prod_{i=1}^{k} \frac{\Gamma(r_i)\Gamma(r_i - 2D_i)}{\Gamma^2(r_i - D_i)} - 1\right)\lambda_0^2.$$

The proof of Lemma 1 and Lemma 2 sees the reference (Wang, 2006).

2.3. *Optimization Criterion*

In recent years, the estimators that fall outside the tradition of linear unbiased estimator have been concentrated on in the literature. One biased estimator that has received considerable attention is the MMSE estimator. Often, it is found that inducing biases may significantly reduce MSE of the estimators. For example, Theil (1971) showed that the MMSE estimator is the best in terms of MSE among the class of linear homogeneous estimators. Liski *et al.* (1993) provided a unified discussion of MMSE estimator and illustrated the different interpretations of MMSE estimator that can arise. MSE is a key criterion in accessing the performance of the estimators and selecting an appropriate estimator in statistical models (DeGroot, 1980), which is defined as

$$\mathrm{MSE}(\hat{\theta}) = E(\hat{\theta} - \theta)^2 = \mathrm{Var}(\hat{\theta}) + \big(E(\theta) - \theta\big)^2$$

where $\hat{\theta}$ is the estimator of the unknown parameter θ. MSE equals the sum of the variance and the squared bias of the estimator. Therefore, a biased estimator should have lower MSE in practical terms. Among unbiased estimators, minimizing MSE is equivalent to minimizing the variance.

In ALT, MSE have been considered as the optimization criterion for accessing the performance of the resulting estimators of the unknown parameters (Sarhan *et al.*, 2012; Wang, 2006; Balakrishnan and Rasouli, 2008; Ling *et al.*, 2009; Ismail, 2012; Dey and Dey, 2014; Dey and Pradhan, 2014). However, MMSE estimators are not taken into account for ALT in the existing literature. In this paper, we will derive the optimal estimators for mean life and failure rate of the exponential SSALT model, while MSE of the optimal estimators at the design stress are minimized. The resulting estimators are called MMSE estimators.

3. MMSE Estimators

In this section, we will derive the optimal estimators for mean life and failure rate of the exponential SSALT model, while MSE of the optimal estimators at the design stress is minimized. The resulting estimators are called MMSE estimators.

Theorem 1. *Let* $D_i = \frac{G-(x_0+x_i)I+x_0x_iE}{\Psi'(r_i)(EG-I^2)}$ $(i = 1, 2, \ldots, k)$ *and* θ_0 *be the mean life at the design stress* x_0.

(1) *If* $r_i + 2D_i > 0$ $(i = 1, 2, \ldots, k)$, *then the MMSE estimator of* θ_0 *is*

$$\bar{\theta_0} = \hat{\theta_0} \exp\left(\sum_{i=1}^{k} D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_i + D_i)}{\Gamma(r_i + 2D_i)};$$

(2) *If* $r_i + 2D_i > 0$ $(i = 1, 2, \ldots, k)$, *then the variance of* $\bar{\theta_0}$ *is*

$$\mathrm{Var}(\bar{\theta_0}) = \left(\prod_{i=1}^{k} \frac{\Gamma^2(r_i + D_i)}{\Gamma(r_i + 2D_i)\Gamma(r_i)} - \frac{\Gamma^4(r_i + D_i)}{\Gamma^2(r_i + 2D_i)\Gamma^2(r_i)}\right)\theta_0^2;$$

(3) *If* $r_i + 2D_i > 0$ $(i = 1, 2, \ldots, k)$ *and* $\tilde{\theta_0}$ *is the unbiased estimator of* θ_0 *in Lemma 1, then*

$$\mathrm{MSE}(\bar{\theta_0}) < \mathrm{MSE}(\tilde{\theta_0}).$$

Proof. (1) From Lemma 1, $\tilde{\theta_0}$ is an unbiased estimator of θ_0, then

$$E(\tilde{\theta_0}) = E\left(\hat{\theta_0} \exp\left(\sum_{i=1}^{k} D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_i)}{\Gamma(r_i + D_i)}\right)\theta_0$$

we have

$$E(\hat{\theta_0}) = \exp\left(-\sum_{i=1}^{k} D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_i + D_i)}{\Gamma(r_i)} \theta_0.$$

Since

$$\mathrm{Var}(\tilde{\theta_0}) = \left(\prod_{i=1}^{k} \frac{\Gamma(r_i)\Gamma(r_i + 2D_i)}{\Gamma^2(r_i + D_i)} - 1\right) \theta_0^2.$$

Thus

$$\mathrm{Var}(\hat{\theta_0}) = \exp\left(-\sum_{i=1}^{k} 2D_i \Psi(r_i)\right) \left(\prod_{i=1}^{k} \frac{\Gamma^2(r_i + D_i)}{\Gamma^2(r_i)} \prod_{i=1}^{k} \frac{\Gamma(r_i)\Gamma(r_i + 2D_i)}{\Gamma^2(r_i + D_i)} - 1\right) \theta_0^2.$$

We now set $\bar{\theta_0} = c\hat{\theta_0}$ (c is undetermined constant), it follows that

$$\mathrm{Var}(\bar{\theta_0}) = c^2 \exp(-\sum_{i=1}^{k} 2D_i \Psi(r_i)) \left(\prod_{i=1}^{k} \frac{\Gamma^2(r_i + D_i)}{\Gamma^2(r_i)} \prod_{i=1}^{k} \frac{\Gamma(r_i)\Gamma(r_i + 2D_i)}{\Gamma^2(r_i + D_i)} - 1\right) \theta_0^2$$

and

$$\left(E(\bar{\theta_0}) - \theta_0\right)^2 = \left(c \exp(-\sum_{i=1}^{k} D_i \Psi(r_i)) \prod_{i=1}^{k} \frac{\Gamma(r_i + D_i)}{\Gamma(r_i)} - 1\right)^2 \theta_0^2.$$

According to the definition of MSE, we have

$$\begin{aligned} \mathrm{MSE}(\bar{\theta_0}) \ = \ & \left(c^2 \exp\left(-\sum_{i=1}^{k} 2D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_i + 2D_i)}{\Gamma(r_i)} \right. \\ & \left. - 2c \exp\left(\sum_{i=1}^{k} -D_i \Psi(r_i)\right) \prod_{i=1}^{k} \frac{\Gamma(r_i + D_i)}{\Gamma(r_i)} + 1\right) \theta_0^2. \end{aligned}$$

In order to obtain the optimal estimator in terms of MSE, we construct an optimization model as follows:

$$\mathrm{MinimizeMSE}(\bar{\theta_0}) = \mathrm{MinimizeMSE}(c\hat{\theta_0}).$$

Taking the derivative of $\mathrm{MSE}(\bar{\theta_0})$ with respect to c, and letting it be zero,

$$\frac{d(\mathrm{MSE}(\bar{\theta_0}))}{dc} = \left(2c\exp\left(-\sum_{i=1}^{k}2D_i\Psi(r_i)\right)\prod_{i=1}^{k}\frac{\Gamma(r_i+2D_i)}{\Gamma(r_i)}\right.$$
$$\left.-\,2\exp\big(-D_i\Psi(r_i)\big)\prod_{i=1}^{k}\frac{\Gamma(r_i+D_i)}{\Gamma(r_i)}\right)\theta_0^2 = 0.$$

Then, we obtain

$$c = \exp\left(\sum_{i=1}^{k}2D_i\Psi(r_i)\right)\prod_{i=1}^{k}\frac{\Gamma(r_i+D_i)}{\Gamma(r_i+2D_i)}.$$

Hence, the MMSE estimator of θ_0 is

$$\bar{\theta_0} = c\hat{\theta_0} = \theta_0\exp\left(\sum_{i=1}^{k}D_i\Psi(r_i)\right)\prod_{i=1}^{k}\frac{\Gamma(r_i+D_i)}{\Gamma(r_i+2D_i)}.$$

(2) The variance of $\bar{\theta_0}$ is

$$\mathrm{Var}(\bar{\theta_0}) = c^2\mathrm{Var}(\hat{\theta_0}) = \left(\prod_{i=1}^{k}\frac{\Gamma^2(r_i+D_i)}{\Gamma(r_i+2D_i)\Gamma(r_i)} - \frac{\Gamma^4(r_i+D_i)}{\Gamma^2(r_i+2D_i)\Gamma^2(r_i)}\right)\theta_0^2.$$

(3) According to the definition of MSE, we have

$$\mathrm{MSE}(\tilde{\theta_0}) = \left(\prod_{i=1}^{k}\frac{\Gamma(r_i)\Gamma(r_i+2D_i)}{\Gamma^2(r_i+D_i))} - 1\right)\theta_0^2$$

and

$$\mathrm{MSE}(\bar{\theta_0}) = \left(-\prod_{i=1}^{k}\frac{\Gamma^2(r_i+D_i)}{\Gamma(r_i+2D_i)\Gamma(r_i)} + 1\right)\theta_0^2.$$

According to $x + \frac{1}{x} = (\sqrt{x} - \frac{1}{\sqrt{x}})^2 > 0$, $(x > 0)$, we have

$$\mathrm{MSE}(\tilde{\theta_0}) - \mathrm{MSE}(\bar{\theta_0}) = \left(\prod_{i=1}^{k}\frac{\Gamma(r_i)\Gamma(r_i+2D_i)}{\Gamma^2(r_i+D_i))} + \prod_{i=1}^{k}\frac{\Gamma^2(r_i+D_i)}{\Gamma(r_i+2D_i)\Gamma(r_i)} - 2\right)\theta_0^2$$
$$> 0.$$

That is

$$\mathrm{MSE}(\bar{\theta_0}) < \mathrm{MSE}(\tilde{\theta_0}).$$

Thus, the proof is completed. □

Similarly, we can obtain the results of the failure rate λ_0 at the design stress x_0.

Theorem 2. *Let* $D_i = \frac{G-(x_0+x_i)I+x_0x_iE}{\Psi'(r_i)(EG-I^2)}$ $(i = 1, 2, \ldots, k)$ *and* λ_0 *be the failure rate at the design stress* x_0:

(1) *If* $r_i - 2D_i > 0$ $(i = 1, 2, \ldots, k)$*, then the MMSE estimator of* λ_0 *is*

$$\bar{\lambda}_0 = \hat{\lambda}_0 \exp(1)(-\sum_{i=1}^{k} D_i \Psi(r_i)(1)) \prod_{i=1}^{k} \frac{\Gamma(r_i - D_i)}{\Gamma(r_i - 2D_i)};$$

(2) *If* $r_i - 2D_i > 0$ $(i = 1, 2, \ldots, k)$*, then the variance of* $\bar{\lambda}_0$ *is*

$$\mathrm{Var}(\bar{\lambda}_0) = \left(\prod_{i=1}^{k} \frac{\Gamma^2(r_i - D_i)\Gamma(r_i - 2D_i)}{\Gamma^3(r_i)} - \prod_{i=1}^{k} \frac{\Gamma^4(r_i - D_i)}{\Gamma^4(r_i)} \right) \lambda_0^2.$$

(3) *If* $r_i - 2D_i > 0$ $(i = 1, 2, \ldots, k)$ *and* $\tilde{\lambda}_0$ *is the unbiased estimator of* λ_0 *in Lemma 2, then*

$$\mathrm{MSE}(\bar{\lambda}_0) < \mathrm{MSE}(\tilde{\lambda}_0).$$

Proof of Theorem 2 is similar with Theorem 1, thus we omit it here.

It is noted that Wang (2006) implemented Monte Carlo simulations investigation in small and moderate sample size and showed that the unbiased estimators given in Lemma 1 and Lemma 2 are more efficient than the corresponding MLEs. Theorem 1 and Theorem 2 illustrate that MSE of the resulting MMSE estimator is less than that of the corresponding unbiased estimators introduced by Wang (2006). Then, it holds that the MMSE estimators are more efficient than the corresponding unbiased estimators in small and moderate sample size. That is to say, for mean life and failure rate at a design stress, the MMSE estimators are more efficient than the MLEs and the unbiased estimators in small and moderate sample size. Therefore, in this paper, it is not necessary to implement Monte Carlo simulation to illustrate the performance and efficiency of the resulting MMSE estimators.

4. Conclusions

This paper obtains MMSE estimators of mean life and failure rate at a design stress under normal operating condition based on failure censored SSALT data. It is assumed that life time of the test units follows Exponential distribution and that the mean life of test units is a log-linear function of stress level. The optimization criterion is defined to minimize MSE of the optimal estimator at the design stress. Two theorems have proved mathematically the fact that MSE of the resulting MMSE estimators is smaller than that of the corresponding unbiased estimators. The results show that the resulting MMSE estimators are more efficient than the unbiased estimators and MLEs in small and moderate sample size in terms of MSE. In the future work, we further investigate the optimal censoring schemes and other important aspects from SSALT model.

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Eksponentinio SSALT modelio mažiausios vidutinių kvadratų klaidos įverčiai

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Šiame straipsnyje nagrinėjami įrengimų veikimo laiko įverčiai, siekiant sumažinti laiką tarp įrengimų projektavimo ir paleidimo į prekybą. Vertinami ir gedimų dažniai. Daroma prielaida, kad įrengimų veikimo trukmė yra pasiskirsčiusi eksponentiškai ir kad vidutinė funkcionavimo trukmė yra log-tiesinė funkcija, priklausanti nuo tam tikro reikšmingumo lygio. Pateikiami eksponentinio SSALT modelio mažiausios vidutinių kvadratų klaidos įverčiai.