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New Coding Algorithm Based on Variable-Length Codewords for Piecewise Uniform Quantizers

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Abstract. This paper presents a model for signal compression, which consists of a piecewise uniform quantizer and a new lossless coder. The model is designed in a general manner, i.e. for any symmetrical signal distribution; this general theory is applied to design models for Gaussian and Laplacian distributions. Rigorous mathematical derivation of the expression for the bit-rate is presented. Forward adaptation of the model is done for non-stationary signals. Theory is proved by simulations in MATLAB and by an experiment with a real speech signal. The most important advantages of the model are low complexity and good performances – it satisfies G.712 standard for the speech transmission quality with 6.18 bps (bits per sample), which is significantly smaller than 8 bps required for quantizers used in PSTN (public switched telephone network) defined with G.711 standard.

Key words: piecewise uniform quantizer, lossless code, forward adaptation, Gaussian and Laplacian distributions.

1. Introduction

Modern telecommunication systems are based on digital processing and transmission. Quantization and source coding are main parts of the process of A/D (analog-to-digital) conversion of analog signals. Hence, quantizers and source coders are very significant part of modern telecommunication systems. Amount of data, transmitted over telecommunication systems or stored in data centres, has been rapidly increased, especially data obtained from multimedia signals (speech, audio, pictures, video). Due to limited resources (channel capacity or memory capacity), data compression (i.e. decreasing of the bit-rate) becomes an important demand of digital transmission. It can be achieved by an appropriate design of the quantizer and by using some lossless (entropy) source code.

Real signals can be modelled with some probability density function. According to Jayant and Noll (1984), the most used probability density functions for the modelling of real signals are symmetrical probability density functions (e.g. Gaussian, Laplacian and uniform). Therefore, symmetrical probability density functions are of the most practical importance.

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Two main types of scalar quantizers are uniform and nonuniform quantizers (Jayant and Noll, 1984; Gersho and Gray, 1992). The uniform quantizer is very simple for realization and optimal for signals with uniform distribution; hence, it is not optimal for the most real signals (speech, images, etc.) which have some nonuniform distribution (Laplacian, Gaussian, etc.). These signals should be quantized with the nonuniform quantizer (Jayant and Noll, 1984; Na, 2008, 2011), where the density of quantization levels is proportional to the probability density function (pdf) of the input signal. In this way, the nonuniform quantizer is adjusted to the statistical characteristics of the input signal and achieves higher SQNR (signal-to-quantization noise ratio) than the uniform quantizer for the same number of levels, but its main drawback is high complexity.

The piecewise uniform quantizer (Kazakos and Makki, 2008; Jeong and Gibson, 1995; Kuhlmann and Bucklew, 1988; Jovanović and Perić, 2011; Saito *et al.*, 1996) consists of several regions; in each region a different uniform quantizer is applied. The piecewise uniform quantizer is a combination of the uniform and the nonuniform quantizers and it takes good characteristics from both of them: its complexity is small (near to the complexity of the uniform quantizer), but it can achieve values of SQNR near to the SQNR of the nonuniform quantizer. It can be considered as a generalized quantizer, whose special cases are uniform and nonuniform quantizers. It allows high flexibility: by choosing the number of uniform regions and the number of levels in each uniform region, performances of the piecewise uniform quantizer can be very well adjusted to the required performances for some application.

Output quantization levels are coded, assigning one codeword to each level. For the aim of compression, variable-length codes (also called entropy or lossless codes) can be used for the coding of quantization levels. The main principle of lossless codes is to code highly probable quantization levels with shorter codewords and less probable quantization levels with longer codewords (Salomon, 2007). The most used lossless code is the Huffman code (Salomon, 2007; Stabno and Wrembel, 2009; Wu *et al.*, 2012; Qi *et al.*, 2012; Polpetta and Banelli, 2012), but its main drawback is high complexity, especially for high number of quantization levels, since the code tree should be formed. Another drawback of the Huffman code is the fact that probabilities of all quantization levels have to be known before the coding process, which is not always easy to provide. Some simple lossless codes were proposed in Perić *et al*. (2009, 2010, 2011). Lossless codes in Perić *et al*. (2009, 2010) use two overlapping coding ranges: one narrow range and one wide range equal to the range of the quantizer. The lossless code presented in Perić *et al.* (2011) uses three coding ranges: narrow, medium and wide (equal to the range of the quantizer). For all these lossless codes, fixed-length codewords are used in each coding range. However, lengths of codewords for different coding ranges are different, i.e. the length of codewords for some coding range depends on the width of this coding range (longer codewords correspond to wider coding ranges).

Most of real signals are non-stationary, which means that their variances are changed in time. For these signals, the best solution is to use adaptive quantizers, whose parameters are adapted to variance changes. There are two types of adaptive quantizers: forward (Jayant and Noll, 1984; Perić *et al.* 2009, 2013) and backward (Jayant and Noll, 1984).

In Perić *et al.* (2009) it was highlighted that the forward adaptive quantizers can achieve about 1 dB higher SQNR compared to backward adaptive quantizers. Model in Perić *et al.* (2013) consists of two ranges; two different companding functions are used in those two ranges; in each range fixed-rate codewords are used but lengths of codewords for those two ranges are different.

This paper introduces a model that consists of a piecewise uniform quantizer and a new lossless code. This lossless code is very simple for realization and it represents an improved modification of the lossless codes presented in Perić *et al.* (2009, 2011). This lossless code uses two coding ranges: narrow (equal to the first segment of the piecewise uniform quantizer) and wide (equal to the range of the piecewise uniform quantizer, which contains L coding intervals). There are several significant improvements compared to lossless codes in Perić *et al.* (2009, 2011): i) the lossless code proposed in this paper contains L coding intervals (L is an arbitrary integer), while the lossless codes in Perić *et al.* (2009, 2011) have two and three coding intervals, respectively; therefore, the lossless code proposed in this paper is much more general but also much more flexible (it gives us the opportunity to choose the number of coding intervals which is the best for some specific application) than lossless codes in Perić *et al.* (2009, 2011); ii) the lossless code in this paper is designed for any symmetric distribution (Gaussian, Laplacian, etc.) while codes in Perić *et al.* (2009, 2011) were designed only for Laplacian distribution; therefore, the lossless code in this paper can be applied for much more types of signals than codes in Perić *et al.* (2009, 2011); iii) for the wide coding range, variable-length codewords are used which allows further decreasing of the bit-rate; iv) hierarchical coding is used which significantly simplifies the decoding process. Furthermore, an important contribution of this paper is a rigorous mathematical derivation of expression for the bit-rate. The design of the model is done by minimization of the bit-rate under the condition that SQNR is greater than some minimum value $\mathrm{SQNR}_{\mathrm{min}}$ which is chosen based on the specific application. The design of the model is performed in a general manner, i.e. for any symmetrical probability density function of the input signal and for an arbitrary number of regions of the piecewise uniform quantizer. After that, this general theory is applied to design the model for signals with Gaussian and Laplacian distribution (since a large number of signals (speech, images, audio, etc.) can be modelled with these two distributions). A simulation of the model is performed in MATLAB for Gaussian and Laplacian distribution. Performances obtained by the simulation are matched very well with the theoretically calculated performances, which confirms the validity of the developed theory. The analysis of performances of the model in a wide range of variances of the input signal is presented. The forward adaptation of the model is done for non-stationary signals. An experiment is performed, applying the model with the forward adaptation to the speech signal. It is shown that the experimental results are matched very well with the theoretically obtained results, which proves the correctness of the developed theory.

The main advantage of this model is small complexity (since both the piecewise uniform quantizer and the new lossless code have small complexity). Besides, this model can achieve very good performances. For example, this model can satisfy the G.712 standard (ITU-T, 2001) for the quality of the speech transmission with 6.18 bps. Therefore, our model achieves the decreasing of the bit-rate of 1.82 bps compared to quantizers defined with the G.711 standard (ITU-T, 1972), which are used in PSTN network. Also, our model achieves the decreasing of the bit-rate for 0.49 bps compared to the model in Perić *et al.* (2011), for 0.25 bps compared to the model in Perić *et al.* (2010) and for 0.12 bps compared to the model proposed in Perić *et al.* (2013).

This paper is organized in the following way. In Section 2, the model which consists of the piecewise uniform quantizer and the new lossless coder is described; rigorous mathematical derivation of the expression for the bit-rate is presented and the model is designed for an arbitrary symmetrical probability density function of the input signal; after that, this general analysis is applied to design the model for signals with Gaussian and Laplacian distributions; numerical and simulation results are presented; performances of the model in the wide range of variances are analysed. In Section 3, the forward adaptation of the model is done; also, an experiment is performed, applying the model with the forward adaptation on the real speech signal. Section 4 concludes the paper.

2. The Description of the Model

Let $p(x,\sigma)$ denotes the probability density function of the input signal, where σ^2 denotes the variance (power) of the signal. The analysis of the model will be done in the general manner, i.e. for an arbitrary symmetrical probability density function $p(x,\sigma)$. The design of quantizers is always done for some referent variance, usually for the unit variance $\sigma^2 = 1$, without loss of generality. Let $p(x) \equiv p(x, \sigma = 1)$ denotes the probability density function for the unit variance $\sigma^2 = 1$, which will be used during the design process. Quantizer for any other variance $\sigma^2 \neq 1$ can be easily obtained from the quantizer designed for $\sigma^2 = 1$, multiplying thresholds and representation levels with σ (Jayant and Noll, 1984).

Signals with zero-mean value will be considered, without loss of generality. This is the standard approach in the design of quantizers (Jayant and Noll, 1984). If the input signal has a non-zero mean, we can remove it by subtracting mean-value from the input, and add it back after quantization.

The model consists of the midrise piecewise uniform quantizer (with even number of levels) and the new lossless coder.

2.1. *The Piecewise Uniform Quantizer*

The piecewise uniform quantizer has N quantization levels. These N levels are grouped into L regions. In each region there is one uniform quantizer. Let $x_{\max}$ denotes the maximal amplitude of the piecewise uniform quantizer and t_i, $i = 1, \dots, L-1$, denote amplitude boundaries between regions. It means that the first region is placed in the interval $I_1 = [-t_1, t_1)$; the i-th region, $i = 2, \dots, L-1$, is placed in the interval $l_i = [-t_i, -t_{i-1}) \cup [t_{i-1}, t_i)$; the L-th region is placed in the interval $I_L^* = [-x_{\max}, -t_{L-1}) \cup [t_{L-1}, x_{\max}]$. Let N_i, $i = 1, \dots, L$ denotes the number of levels in the i-th region. Values of N_i have to be powers of two, i.e. $N_i = 2^{r_i}$, where r_i, $i = 1, \dots, L$, are integers. This condition is important for the application of the new lossless code. We have that $\sum_{i=1}^{L} N_i = N$.

Let $\Delta_1 = 2t_1/N_1$ denotes the stepsize of the uniform quantizer in the first region; $\Delta_i = 2(t_i - t_{i-1})/N_i$, $i = 2, \dots, L-1$ denotes the stepsize of the uniform quantizer in the i-th region; $\Delta_L = 2(x_{\max} - t_{L-1})/N_L$ denotes the stepsize of the uniform quantizer in the L-th region. Let P_i, $i = 1, \dots, L-1$, denotes the probability of the i-th region and P_L^* denotes the probability of the L-th region. These probabilities are defined as:

$$P_1 = 2\int_0^{t_1} p(x)\,dx, \tag{1}$$

$$P_i = 2\int_{t_{i-1}}^{t_i} p(x)\,dx, \quad i = 2, \dots, L-1, \tag{2}$$

$$P_L^* = 2\int_{t_{L-1}}^{x_{\max}} p(x)\,dx. \tag{3}$$

A quantizer is defined with its thresholds and representation levels. Let $x_{i,j}$ $(j = 0, \dots, N_i/2)$ denote positive thresholds and $y_{i,j}$ $(j = 1, \dots, N_i/2)$ denote positive representation levels in the i-th region $(i = 1, \dots, L)$ of the proposed quantizer. Positive thresholds and representation levels can be calculated as $x_{1,j} = j \cdot \Delta_1$, $(j = 0, \dots, N_1/2)$; $x_{i,j} = t_{i-1} + j \cdot \Delta_i$, $(i = 2, \dots, L;\ j = 0, \dots, N_i/2)$; $y_{1,j} = (j - \frac{1}{2}) \cdot \Delta_1$, $(j = 1, \dots, N_1/2)$; $y_{i,j} = t_{i-1} + (j - \frac{1}{2}) \cdot \Delta_i$, $(i = 2, \dots, L;\ j = 1, \dots, N_i/2)$. We have that $x_{i,N_i/2} = x_{i+1,0} = t_i$, $(i = 1, \dots, L-1)$ and $x_{L,N_L/2} = x_{\max}$. Since the pdf is symmetrical, negative thresholds and representation levels are symmetrical to their positive counterparts. The area $|x| \leqslant x_{\max}$ is the granular region while the area $|x| > x_{\max}$ is the overload region.

During the quantization process, an irreversible error is made, which is expressed with the distortion. The total distortion D is the sum of the granular distortion D_g (in the granular region) and the overload distortion D_{ov} (in the overload region). Based on Jayant and Noll (1984), D_g can be expressed as $D_g = \sum_{i=1}^{L-1} \frac{\Delta_i^2}{12} P_i + \frac{\Delta_L^2}{12} P_L^*$. Hence, the total distortion D can be expressed as:

$$D = \sum_{i=1}^{L-1} \frac{\Delta_i^2}{12} P_i + \frac{\Delta_L^2}{12} P_L^* + D_{ov}. \tag{4}$$

To calculate D_{ov} we will consider the positive overload region $x > x_{\max}$. All points x from the positive overload region $(x > x_{\max})$ are mapped to the last positive representation level $y_{L,N_L/2} = t_{L-1} + (\frac{N_L}{2} - \frac{1}{2}) \cdot \Delta_L = t_{L-1} + \frac{N_L}{2} \cdot \Delta_L - \frac{\Delta_L}{2} = x_{\max} - \frac{\Delta_L}{2}$. For some arbitrary $x > x_{\max}$, power of the quantization error is $(x - (x_{\max} - \Delta_L/2))^2$. The overload distortion (as the average power of the quantization error in the overload region) is obtained by the statistical averaging over all x from the overload region:

$$D_{ov} = 2\int_{x_{\max}}^{\infty} p(x)\big(x - (x_{\max} - \Delta_L/2)\big)^2 dx. \tag{5}$$

The term 2 in front of the integral denotes the contribution of the negative overload region. The quality of the output signal of the quantizer is defined with the signal-to-quantization noise ratio (SQNR). For the unit variance $\sigma = 1$, SQNR is defined as:

$$\mathrm{SQNR[dB]} = -10\log_{10} D\ . \tag{6}$$

Let us define the interval $I_L = (-\infty, -t_{L-1}) \cup [t_{L-1}, \infty)$, whose probability is:

$$P_L = 2\int_{t_{L-1}}^{\infty} p(x)dx. \tag{7}$$

We have that $\sum_{i=1}^{L} P_i = 1$.

2.2. *New Coding Algorithm*

Output levels of the piecewise uniform quantizer are coded with the new lossless code presented in this subsection. This lossless code is an improved version of the codes presented in Perić *et al.* (2009, 2011); improvements are explained in Introduction. The lossless code works on the frame-by-frame basis. Frames of M samples of the input signals are formed. Let $F = \{x_1, \ldots, x_M\}$ denotes the set of samples of one frame. At the beginning of each frame, one control bit C is transmitted. The new lossless coding algorithm is defined in the following way:

If all M samples in one frame belong to the interval I_1 (the probability of this event is P_1^M), **then**:

- The control bit C is set to zero, i.e. $C = 0$;
- All samples in the frame are coded with $r_1 = \log_2 N_1$ bits.

If there is at least one sample x_j in the frame F which does not belong to the interval I_1 (the probability of this event is $(1 - P_1^M)$), **then**:

- The control bit C is set to one, i.e. $C = 1$;
- An arbitrary sample x_k from the frame F is coded with the hierarchical coding, in the following way. If x_k belongs to the interval $I_i, i = 1, \ldots, L$, the codeword for x_k has the form $\underbrace{1\ldots1}_{i-1}0\underbrace{x\ldots x}_{r_i}$. The length of the codeword for $x_k \in I_i, i = 1, \ldots, L$, is $(l_i + r_i)$ bits, where $l_i = i$. The first l_i bits are used to code the interval I_i, while the last r_i bits represent the natural binary code of the position of x_k within the interval I_i ($r_i = \log_2 N_i$). Zero after $(i - 1)$ ones is inserted to ensure that no codeword is a prefix of any other codeword.

The average bit-rate can be written as:

$$R = P(\text{all } M \text{ samples from } F \in I_1) \cdot R_1 + P(\exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) \cdot R_2 + \frac{1}{M}. \tag{8a}$$

Now, all terms from (8a) will be analysed. $P(\text{all } M \text{ samples from } F \in I_1)$ is the probability that all M samples from the frame F belong to the interval I_1. We have that

$$P(\text{all } M \text{ samples from } F \in I_1) = P_1^M. \tag{8b}$$

R_1 is the average bit-rate (i.e. the average number of bits per sample) in the case that all M samples from F belong to I_1. In this case, all samples are coded with r_1 bits, therefore the average number of bits per one sample is:

$$R_1 = r_1. \tag{8c}$$

$P(\exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1)$ denotes the probability that there is at least one sample x_j from frame F which does not belong to the interval I_1. We have that

$$P(\exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) = 1 - P_1^M. \tag{8d}$$

R_2 is the average bit-rate (i.e. the average number of bits per sample) in the case that there is at least one sample x_j from F which does not belong to I_1. In this case, as it was explained above, if an arbitrary sample x_k from F belongs to the interval I_i (the probability of this event is equal to the conditional probability $P(x_k \in I_i | \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1)$), then x_k is coded with $(l_i + r_i)$ bits, $(i = 1, \ldots, L)$. The average number of bits per one sample in this case (i.e. the average bit-rate R_2) is obtained by averaging over all L intervals, as:

$$R_2 = \sum_{i=1}^{L} (l_i + r_i) P(x_k \in I_i | \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1). \tag{8e}$$

The last term $1/M$ in (8) represents the contribution of the control bit C to the bit-rate increase. There is one control bit per frame of M samples, which is equal to $1/M$ bits per one sample.

According to expressions (8a)–(8e), the following expression for the average bit-rate R is obtained:

$$R = P_1^M r_1 + \left(1 - P_1^M\right) \sum_{i=1}^{L} (l_i + r_i) P(x_k \in I_i | \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) + \frac{1}{M}. \tag{8f}$$

Theorem 1. *The conditional probabilities from* (8f) *are determined with the following expressions*:

$$P(x_k \in I_1 | \exists \textit{ at least one } x_j \textit{ from } F \therefore x_j \notin I_1) = \frac{P_1(1 - P_1^{M-1})}{1 - P_1^M}, \tag{9}$$

$$P(x_k \in I_i | \exists \textit{ at least one } x_j \textit{ from } F \therefore x_j \notin I_1) = \frac{P_i}{1 - P_1^M}, \quad i = 2, \ldots, L. \tag{10}$$

Proof. The conditional probabilities from (8f) can be expressed as:

$$\begin{aligned} &P(x_k \in I_i | \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) \\ &\quad = \frac{P(x_k \in I_i \wedge \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1)}{P(\exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1)}, \quad i = 1, \dots, L. \end{aligned} \tag{11}$$

According to (8d), it follows that:

$$\begin{aligned} &P(x_k \in I_i | \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) \\ &\quad = \frac{P(x_k \in I_i \wedge \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1)}{1 - P_1^M}, \quad i = 1, \dots, L. \end{aligned} \tag{12}$$

Now, we have to find joint probabilities $P(x_k \in I_i \wedge \exists$ at least one x_j from $F \therefore x_j \notin I_1)$, $i = 1, \dots, L$. Let us define the set $F_k = F \backslash x_k = \{x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_M\}$, which contains $M - 1$ samples. Let $\mathcal{F}_{k,i}^l$, $l = 1, \dots, M-1$; $i = 1, \dots, L$, denotes the class of all sets F_k where exactly l samples belong to the interval I_i and the rest of $(M - 1 - l)$ samples do not belong to the interval I_i. We have to choose l samples of $(M - 1)$ samples, which can be done in $\binom{M-1}{l}$ ways. Probability that all l samples belong to I_i is P_i^l and probability that all the rest $(M - 1 - l)$ samples do not belong to I_i is $(1 - P_i)^{M-1-l}$. Therefore, it is obtained that:

$$P\left(F_k \in \mathcal{F}_{k,i}^l\right) = \binom{M-1}{l} P_i^l (1 - P_i)^{M-1-l}. \tag{13}$$

The following identity is valid:

$$\sum_{l=0}^{M-1} P\left(F_k \in \mathcal{F}_{k,i}^l\right) = \sum_{l=0}^{M-1} \binom{M-1}{l} P_i^l (1 - P_i)^{M-1-l} = 1. \tag{14}$$

For $i = 1$ we have that:

$$\begin{aligned} &P(x_k \in I_1 \wedge \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) \\ &\quad = P(x_k \in I_1) \sum_{l=0}^{M-2} P\left(F_k \in \mathcal{F}_{k,1}^l\right) \\ &\quad = P_1 \sum_{l=0}^{M-2} \binom{M-1}{l} P_1^l (1 - P_1)^{M-1-l} \\ &\quad = P_1 \left(1 - \binom{M-1}{M-1} P_1^{M-1} (1 - P_1)^0\right) \\ &\quad = P_1\left(1 - P_1^{M-1}\right). \end{aligned} \tag{15}$$

For $i = 2, \dots, L$ it is obtained that:

$$\begin{aligned} &P(x_k \in I_i \wedge \exists \text{ at least one } x_j \text{ from } F \therefore x_j \notin I_1) \\ &\quad = P(x_k \in I_i) \sum_{l=0}^{M-1} P\big(F_k \in \mathcal{F}_{k,1}^{l}\big) \\ &\quad = P_i \sum_{l=0}^{M-1} \binom{M-1}{l} P_1^l (1 - P_1)^{M-1-l} = P_i. \end{aligned} \tag{16}$$

Based on (12), (15) and (16), expressions (9) and (10) are obtained; hence, the theorem is proved. □

Theorem 2. *The average bit-rate can be expressed with the following expression*:

$$R = P_1^M r_1 + (l_1 + r_1) P_1 \big(1 - P_1^{M-1}\big) + \sum_{i=2}^{L} (l_i + r_i) P_i + \frac{1}{M}. \tag{17}$$

Proof. Substituting expressions (9) and (10) for conditional probabilities into (8f), it is obtained that:

$$R = P_1^M r_1 + \big(1 - P_1^M\big)\left[(l_1 + r_1)\frac{P_1(1 - P_1^{M-1})}{1 - P_1^M} + \sum_{i=2}^{L} (l_i + r_i)\frac{P_i}{1 - P_1^M}\right] + \frac{1}{M}. \tag{18}$$

After some basic mathematical steps we obtain (17), which proves the theorem. □

At the beginning of the design process, the minimal acceptable value of SQNR, denoted as $\mathrm{SQNR}_{\mathrm{min}}$, is defined. The value of $\mathrm{SQNR}_{\mathrm{min}}$ depends on the application (e.g. for speech transmission, $\mathrm{SQNR}_{\mathrm{min}} = 34$ dB). Also, the value of the parameter L is defined in advance. The design of the model is performed by numerical minimization of the average bit-rate R, with the constraint that $\mathrm{SQNR} \geqslant \mathrm{SQNR}_{\mathrm{min}}$. As a result of the design process, optimal values of parameters N_i $(i = 1, \dots, L)$, t_i $(i = 1, \dots, L-1)$, $x_{\max}$ and M are obtained.

The model is designed for the signals with the unit variance, $\sigma^2 = 1$. If the variance of the signal σ^2 is not equal to 1 ($\sigma^2 \neq 1$), the model has to be designed for this value of the variance. The good thing is the fact that if we need to design the quantizer for some $\sigma^2 \neq 1$, we can obtain this quantizer from the quantizer designed for $\sigma^2 = 1$, by multiplying all thresholds and representation levels with σ.

2.3. *Model Design for Gaussian and Laplacian Distributions and Numerical Results*

The previously described theory was general, i.e. it was developed for an arbitrary symmetrical probability density function $p(x)$. In telecommunications, the most used probability density functions for signal modelling are Gaussian and Laplacian probability density functions. Therefore, in this subsection, the piecewise uniform quantizer and the lossless coder will be designed for those two probability density functions.

2.3.1. *Model Design for the Gaussian Probability Density Function*
The zero-mean Gaussian probability density function is defined with the expression:

$$p(x,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{x^2}{2\sigma^2}\right). \tag{19}$$

The model is developed for the unit variance $\sigma^2=1$, using the following probability density function:

$$p(x)=\frac{1}{\sqrt{2\pi}}\exp\left(-x^2/2\right). \tag{20}$$

Putting (20) into (1)–(3) and (7), the following expressions for probabilities are obtained:

$$P_1=\operatorname{erf}(t_1/\sqrt{2}), \tag{21}$$

$$P_i=\operatorname{erf}(t_i/\sqrt{2})-\operatorname{erf}(t_{i-1}/\sqrt{2}),\quad i=2,\ldots,L-1, \tag{22}$$

$$P_L^*=\operatorname{erf}(x_{\max}/\sqrt{2})-\operatorname{erf}(t_{L-1}/\sqrt{2}), \tag{23}$$

$$P_L=\operatorname{erfc}(t_{L-1}/\sqrt{2}). \tag{24}$$

Putting (20) into (5), expression for the overload distortion D_{ov} becomes:

$$D_{ov}=2\left(\frac{(-x_{\max}+\Delta_L)}{\sqrt{2\pi}}\exp\left(-\frac{x_{\max}^2}{2}\right)+\frac{1}{8}\left((-2x_{\max}+\Delta_L)^2+4\right)\operatorname{erfc}\left(\frac{x_{\max}}{\sqrt{2}}\right)\right). \tag{25}$$

Using (21)–(25), (4), (6) and (17), performances of the model (SQNR and R) can be calculated.

The design of the model for Gaussian distribution is done for different values of L (L = 2, 3, 4) and $\text{SQNR}_{\min}$ ($\text{SQNR}_{\min}=40$ dB, 34 dB, 30 dB and 25 dB). Obtained values of parameters (N_i, t_i, $x_{\max}$ and M), as well as values of performances (SQNR and R) calculated using the previously developed theory, are presented in Tables 1, 2 and 3. Simulation of the model is done in MATLAB, using the generator of random numbers for the Gaussian distribution. Values of SQNR and R obtained by the simulation are also presented in Tables 1, 2 and 3.

2.3.2. *Model Design for the Laplacian Probability Density Function*
The zero-mean Laplacian probability density function is defined with the expression:

$$p(x,\sigma)=\frac{1}{\sqrt{2}\sigma}e^{-\frac{\sqrt{2}|x|}{\sigma}}. \tag{26}$$

Table 1
Numerical results for the Gaussian distribution for $L = 2$.

$SQNR_{min}$ [dB]	N_1	N_2	t_1	x_{max}	M	Theory R [bps]	Theory SQNR [dB]	Simulation R [bps]	Simulation SQNR [dB]
40	128	128	2.21	4.53	7	7.32	40	7.33	40.06
34	64	64	2.21	4.28	7	6.32	34	6.31	34.05
30	32	32	1.70	3.72	4	5.56	30	5.57	30.04
25	16	16	1.44	3.30	3	4.72	25	4.72	25.04

Table 2
Numerical results for the Gaussian distribution for $L = 3$.

$SQNR_{min}$ [dB]	N_1	N_2	N_3	t_1	t_2	x_{max}	M	Theory R [bps]	Theory SQNR [dB]	Simulation R [bps]	Simulation SQNR [dB]
40	64	32	128	1.06	1.60	4.22	3	7.19	40	7.20	40.11
34	32	16	64	1.09	1.62	3.96	3	6.16	34	6.16	34.05
30	32	8	32	1.73	2.21	4.03	4	5.51	30	5.51	30.03
25	16	4	16	1.52	1.95	3.60	3	4.64	25	4.64	25.06

Table 3
Numerical results for the Gaussian distribution for $L = 4$.

$SQNR_{min}$ [dB]	N_1	N_2	N_3	N_4	t_1	t_2	t_3	x_{max}	M	Theory R [bps]	Theory SQNR [dB]	Simulation R [bps]	Simulation SQNR [dB]
40	64	16	32	128	1.08	1.38	1.91	4.37	3	7.12	40	7.12	40.05
34	32	8	16	64	1.09	1.40	1.91	4.11	3	6.11	34	6.12	34.08
30	16	8	16	32	0.91	1.34	2.00	3.90	3	5.48	30	5.50	30.05
25	16	4	8	16	1.56	1.99	2.58	4.01	4	4.63	25	4.63	25.02

Since the design of the model is done for the unit variance $\sigma^2 = 1$, the following probability density function should be used:

$$p(x) = \frac{1}{\sqrt{2}} e^{-\sqrt{2}|x|}. \tag{27}$$

Putting (27) into (1)–(3) and (7) the following expressions for probabilities are obtained:

$$P_1 = 1 - \exp(-\sqrt{2}t_1), \tag{28}$$

$$P_i = \exp(-\sqrt{2}t_{i-1}) - \exp(-\sqrt{2}t_i), \quad i = 2, \ldots, L-1, \tag{29}$$

$$P_L^* = \exp(-\sqrt{2}t_{L-1}) - \exp(-\sqrt{2}x_{\max}), \tag{30}$$

$$P_L = \exp(-\sqrt{2}t_{L-1}). \tag{31}$$

Putting (27) into (5), expression for the overload distortion D_{ov} becomes:

$$D_{ov} = \frac{1}{4}\big(4 + 2\sqrt{2}\Delta_L + \Delta_L^2\big)\exp(-\sqrt{2}x_{\max}). \tag{32}$$

Table 4
Numerical results for the Laplacian distribution for $L = 2$.

$SQNR_{min}$ [dB]	N_1	N_2	t_1	x_{max}	M	Theory R [bps]	Theory SQNR [dB]	Simulation R [bps]	Simulation SQNR [dB]
40	128	256	2.09	9.11	5	7.49	40	7.50	40.10
34	32	256	1.14	8.16	3	6.42	34	6.41	34.03
30	32	128	1.76	7.91	4	5.71	30	5.70	30.03
25	16	64	1.56	6.87	4	4.84	25	4.81	25.02

Table 5
Numerical results for the Laplacian distribution for $L = 3$.

$SQNR_{min}$ [dB]	N_1	N_2	N_3	t_1	t_2	x_{max}	M	Theory R [bps]	Theory SQNR [dB]	Simulation R [bps]	Simulation SQNR [dB]
40	64	64	256	1.04	2.10	9.12	3	7.20	40	7.23	40.16
34	32	32	128	1.07	2.10	8.26	3	6.18	34	6.22	34.05
30	32	16	64	1.69	2.69	7.99	4	5.61	30	5.60	29.97
25	16	8	32	1.48	2.35	6.83	3	4.73	25	4.74	25.07

Table 6
Numerical results for the Laplacian distribution for $L = 4$.

$SQNR_{min}$ [dB]	N_1	N_2	N_3	N_4	t_1	t_2	t_3	x_{max}	M	Theory R [bps]	Theory SQNR [dB]	Simulation R [bps]	Simulation SQNR [dB]
40	64	32	64	256	1.07	1.66	2.77	9.79	3	7.09	40	7.10	40.06
34	32	16	32	128	1.07	1.67	2.74	8.89	3	6.09	34	6.09	34.09
30	16	16	16	64	0.87	1.64	2.54	7.84	3	5.42	30	5.42	30.00
25	8	8	8	32	0.76	1.45	2.25	6.73	3	4.60	25	4.62	25.08

Using (28)–(32), (4), (6) and (17), performances of the model (SQNR and R) can be calculated.

The model for Laplacian distribution is designed for different values of L ($L = 2, 3, 4$) and $SQNR_{min}$ ($SQNR_{min} = 40$ dB, 34 dB, 30 dB and 25 dB). Theoretically obtained values of parameters (N_i, t_i, x_{max} and M) and performances (SQNR and R) are presented in Tables 4, 5 and 6. Simulation of the model is done in MATLAB, using the generator of random numbers for the Laplacian distribution. Values of SQNR and R obtained by the simulation are also presented in Tables 4, 5 and 6. We can see from Tables 1 to 6 that values of SQNR and R obtained by the theory and by the simulation are matched very well, which proves the correctness of the developed theory.

From Tables 1–6 we can see that the bit-rate R decreases with the increase of L, for the same SQNR. On the other hand, the complexity of the model increases with the increase of L. Therefore, a compromise between the complexity and compression has to be found. We propose the model with $L = 4$ regions as a very good solution.

2.4. *The Performances of the Model in the Wide Range of Variances of the Input Signal*

As it was said before, parameters of the model (N_i, t_i, x_{max}, Δ_i and M) are determined for the unit variance $\sigma^2 = 1$. Since the variance of the input signal can vary in time, it is important to calculate the performances of the model when the input signal variance σ^2 is not matched to the variance for which the model is designed. Probabilities $P_i(\sigma)$, $i = 1, \ldots, L$ and $P_L^*(\sigma)$ are obtained from (1)–(3) and (7), changing $p(x)$ with $p(x, \sigma)$. Based on (4), the expression for the total distortion $D(\sigma)$ becomes

$$D(\sigma) = \sum_{i=1}^{L-1} \frac{\Delta_i^2}{12} P_i(\sigma) + \frac{\Delta_L^2}{12} P_L^*(\sigma) + D_{ov}(\sigma), \tag{33}$$

where the overload distortion $D_{ov}(\sigma)$ is calculated by changing $p(x)$ with $p(x, \sigma)$ in (5). The signal-to-quantization noise ratio is calculated with the expression:

$$\mathrm{SQNR}(\sigma) = 10 \log_{10} \frac{\sigma^2}{D(\sigma)} \text{ [dB]}. \tag{34}$$

Based on (17), the bit-rate is calculated with the expression:

$$\begin{aligned} R(\sigma) = {} & \big(P_1(\sigma)\big)^M r_1 + (l_1 + r_1) \cdot P_1(\sigma) \cdot \big(1 - \big(P_1(\sigma)\big)^{M-1}\big) \\ & + \sum_{i=2}^{L} (l_i + r_i) \cdot P_i(\sigma) + \frac{1}{M}. \end{aligned} \tag{35}$$

For the Gaussian distribution $p(x, \sigma)$ defined with (19), the following expressions are obtained:

$$P_1(\sigma) = \mathrm{erf}\big(t_1/(\sqrt{2}\sigma)\big), \tag{36}$$

$$P_i(\sigma) = \mathrm{erf}\big(t_i/(\sqrt{2}\sigma)\big) - \mathrm{erf}\big(t_{i-1}/(\sqrt{2}\sigma)\big), \quad i = 2, \ldots, L-1, \tag{37}$$

$$P_L^*(\sigma) = \mathrm{erf}\big(x_{\max}/(\sqrt{2}\sigma)\big) - \mathrm{erf}\big(t_{L-1}/(\sqrt{2}\sigma)\big), \tag{38}$$

$$P_L(\sigma) = \mathrm{erfc}\big(t_{L-1}/(\sqrt{2}\sigma)\big), \tag{39}$$

$$D_{ov}(\sigma) = \frac{2\sigma(\Delta_L - x_{\max})}{\sqrt{2\pi}} \exp^{-\frac{x_{\max}^2}{2\sigma^2}} + \left(\left(\frac{\Delta_L}{2} - x_{\max}\right)^2 + \sigma^2\right) \mathrm{erfc}\left(\frac{x_{\max}}{\sqrt{2}\sigma}\right). \tag{40}$$

Putting (36)–(40) into (33)–(35) we can calculate $\mathrm{SQNR}(\sigma)$ and $R(\sigma)$ for the Gaussian distribution, for any value of σ.

For the Laplacian distribution $p(x, \sigma)$ defined with (26), the following expressions are obtained:

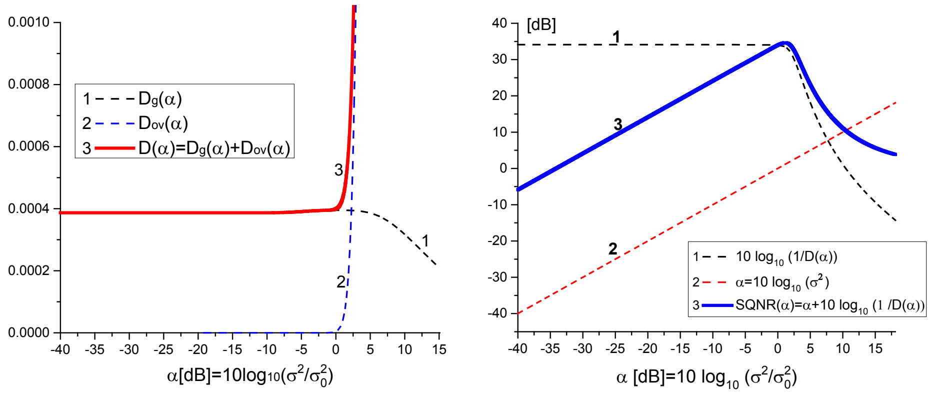
$$P_1(\sigma) = 1 - \exp(-\sqrt{2} t_1/\sigma), \tag{41}$$

$$P_i(\sigma) = \exp(-\sqrt{2} t_{i-1}/\sigma) - \exp(-\sqrt{2} t_i/\sigma), \quad i = 2, \ldots, L-1, \tag{42}$$

$$P_L^*(\sigma) = \exp(-\sqrt{2} t_{L-1}/\sigma) - \exp(-\sqrt{2} x_{\max}/\sigma), \tag{43}$$

$$P_L(\sigma) = \exp(-\sqrt{2} t_{L-1}/\sigma), \tag{44}$$

$$D_{ov}(\sigma) = \frac{1}{4}\big(4\sigma^2 + 2\sqrt{2}\Delta_L \sigma + \Delta_L^2\big) \exp(-\sqrt{2} x_{\max}/\sigma). \tag{45}$$



(a) (b)

Fig. 1. $D(\alpha)$ and $\mathrm{SQNR}(\alpha)$ in the wide range of input variances for the Gaussian distribution, for the model with the following parameters: $L = 4$, $N_1 = 32$, $N_2 = 8$, $N_3 = 16$ and $N_4 = 64$.

Putting (41)–(45) into (33)–(35), we can calculate $\mathrm{SQNR}(\sigma)$ and $R(\sigma)$ for the Laplacian distribution, for any value of σ.

Very often, the variance σ^2 is expressed in the logarithmic domain as $\alpha[\mathrm{dB}] = 10\log_{10}(\sigma^2/\sigma_0^2)$, where σ_0^2 is the referent variance. Without losing the generality, we assume that $\sigma_0^2 = 1$. In this case we have that $\alpha[\mathrm{dB}] = 10\log_{10}(\sigma^2)$, i.e. α is the power of the signal in the logarithmic domain. Functions $\mathrm{SQNR}(\alpha)$ and $R(\alpha)$ are obtained from functions $\mathrm{SQNR}(\sigma)$ and $R(\sigma)$, putting that $\sigma = 10^{\alpha[\mathrm{dB}]/20}$.

Figure 1 shows performances in the wide range of input variances for the Gaussian distribution, for the model with the following parameters: $L = 4$, $N_1 = 32$, $N_2 = 8$, $N_3 = 16$ and $N_4 = 64$. Figure 1(a) shows granular and overload distortions ($D_g(\alpha)$ and $D_{ov}(\alpha)$), as well as the total distortion ($D(\alpha) = D_g(\alpha) + D_{ov}(\alpha)$) in the wide range of input variances. $D_g(\alpha)$ is constant in the wide range of negative α, then slightly increases when α approaches 0 dB, continues to increase for very small values of positive α and then starts to decrease and continues to decrease as α becomes more and more positive. On the other hand, $D_{ov}(\alpha)$ is almost zero for negative α, but when α becomes higher than 0 dB $D_{ov}(\alpha)$ starts rapidly to increase; this rapid increasing of $D_{ov}(\alpha)$ is continued as α becomes more and more positive.

Now, we will give the explanation of this behaviour. For very small α ($\alpha < -10$ dB), which means for very small variance σ^2, the power of the signal is very small, hence the magnitude of the signal is very small, which means that almost all samples of the signal belong to the first interval I_1; therefore, only the first term $\frac{\Delta_1^2}{12}P_1$ in the expression for the granular distortion ($D_g = \sum_{i=1}^{L-1}\frac{\Delta_i^2}{12}P_i + \frac{\Delta_L^2}{12}P_L^*$) has some non-zero value while all other terms in this expression are negligible. Therefore, for very small α granular distortion is constant and equal to $\frac{\Delta_1^2}{12}P_1$.

As α increases (roughly speaking, for -10 dB $< \alpha < 1.5$ dB), the magnitude of samples increases; the most of samples are still in the interval I_1 (since for Gaussian distribution (as well as Laplacian distribution) small values of samples have higher probability), but the number of samples in intervals $I_2, \dots, I_L$ increases. The term $\frac{\Delta_1^2}{12}P_1$ is still dominant, but with the increase of α other terms ($\frac{\Delta_2^2}{12}P_2, \dots, \frac{\Delta_L^2}{12}P_L^*$) in the expression for D_g start to increase, which results in slight increase of D_g. For α much smaller than 0 dB, the number of samples in the overload region is negligible, therefore the overload distortion D_{ov} is equal to zero. However, as α approaches to 0 dB and becomes higher than 0 dB, the magnitude of samples increases and hence the number of samples in the overload region is getting bigger and bigger. Therefore, when α becomes positive, the overload distortion D_{ov} starts rapidly to increase. As α continues to increase, more and more samples are in the overload region, which means that less and less samples are in the granular region, hence the granular distortion D_g starts to decrease. For further increasing of α, D_{ov} continues to rapidly increase while D_g continues to decrease.

The total distortion D is almost constant for $\alpha < 0$ dB while for $\alpha > 0$ dB starts to increase rapidly. We can see that for $\alpha < 0$ dB, total distortion D is almost equal to the granular distortion D_g, since D_{ov} is almost equal to zero for negative α. On the other hand, for positive α, total distortion D is almost equal to the overload distortion D_{ov} since D_{ov} rapidly increases and very quickly reaches high values. We can say that the granular distortion D_g is dominant for negative α while the overload distortion D_{ov} is dominant for positive α.

According to the definition, $\mathrm{SQNR}(\sigma) = 10\log_{10}\frac{\sigma^2}{D(\sigma)} = 10\log_{10}\sigma^2 + 10\log_{10}\frac{1}{D(\sigma)}$. Therefore, $\mathrm{SQNR}(\alpha) = \alpha + 10\log_{10}\frac{1}{D(\alpha)}$, where $D(\alpha)$ is obtained by putting that $\sigma = 10^{\alpha[\mathrm{dB}]/20}$ into $D(\sigma)$. Figure 1(b) shows $\mathrm{SQNR}(\alpha)$ in the wide range of α, as well as its components α and $10\log_{10}\frac{1}{D(\alpha)}$. The behaviour of the second component ($10\log_{10}\frac{1}{D(\alpha)}$) can be understood considering the behaviour of $D(\alpha)$, which is shown in Fig. 1(a). Term $10\log_{10}\frac{1}{D(\alpha)}$ is almost constant for negative α since $D(\alpha)$ is almost constant for negative α. When α becomes positive, $D(\alpha)$ starts to increase rapidly; therefore $10\log_{10}\frac{1}{D(\alpha)}$ starts to decrease rapidly.

For negative α, $\mathrm{SQNR}(\alpha)$ increases since α increases and $10\log_{10}\frac{1}{D(\alpha)}$ is constant. For positive α, the first term α continues to increase while $10\log_{10}\frac{1}{D(\alpha)}$ starts to rapidly decrease. For some value of α (α^*), the decrease of $10\log_{10}\frac{1}{D(\alpha)}$ becomes more dominant than the increase of α, therefore, $\mathrm{SQNR}(\alpha)$ starts to decrease. α^* is the point where the behaviour of $\mathrm{SQNR}(\alpha)$ is changed (from increasing to decreasing), hence α^* is the point where $\mathrm{SQNR}(\alpha)$ has the maximum. For Fig. 1(d) we have that $\alpha^* = 1.12$.

Let $(\sigma^*)^2$ denotes the value of the variance where $\mathrm{SQNR}(\sigma)$ has the maximum, i.e. $\mathrm{SQNR}(\sigma^*) = \max(\mathrm{SQNR}(\sigma))$. The value of $(\sigma^*)^2$ depends on the parameters of the quantizer: L, N_i, $(i = 1, \dots, L)$, $x_{\max}$ and $\mathrm{SQNR}_{\min}$. The following connection exists: $\alpha^*[\mathrm{dB}] = 10\log_{10}((\sigma^*)^2/\sigma_0^2) = 10\log_{10}((\sigma^*)^2) - 10\log_{10}(\sigma_0^2)$. We can see that the expression for α^* consists of two terms. The first term ($10\log_{10}((\sigma^*)^2)$) depends on the parameters of the quantizer L, N_i $(i = 1, \dots, L)$, $x_{\max}$ and $\mathrm{SQNR}_{\min}$. The second term ($-10\log_{10}(\sigma_0^2)$) is constant since σ_0^2 is constant. For example, all quantizers from Tables 1–6 are designed for the same value of σ_0^2 ($\sigma_0^2 = 1$), but all those quantizers have different values of α^* since they have different values of parameters L, N_i $(i = 1, \dots, L)$, $x_{\max}$ and $\mathrm{SQNR}_{\min}$.

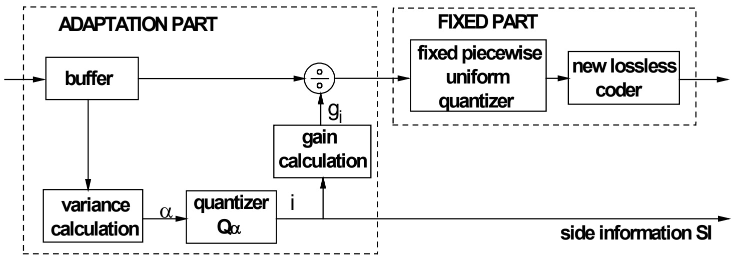


Fig. 2. The block schema of the forward adaptive quantizer.

The model has to be used for variances $\alpha \leqslant \alpha^*$, since for $\alpha > \alpha^*$ we have that SQNR rapidly decreases due to the overload distortion. For variances $\alpha \leqslant \alpha^*$ which are of interest, $\mathrm{SQNR}(\alpha)$ is an increasing function.

3. Forward Adaptation of the Model and Its Application to the Speech Signal

In this section, the forward adaptation of the model is performed for non-stationary signals. After that, this forward adaptive model is applied to the speech signal.

3.1. *Forward Adaptation of the Model*

Forward adaptation is used to achieve almost constant SQNR in the wide range of input variances $\alpha \in (\alpha_{\min}, \alpha_{\max})$ [dB]. Let Δ_α [dB] $= \alpha_{\max} - \alpha_{\min}$ denotes the width of the variance range in the logarithmic domain where SQNR has to be constant.

Forward adaptive quantizer (Jayant and Noll, 1984; Nikolić and Perić, 2008; Perić *et al.*, 2013) consists of the adaptation part (which contains the input buffer, the block for the variance calculation, the quantizer Q_α for the variance quantization, the block for the gain calculation and the divider) and the fixed part (which contains the fixed piecewise uniform quantizer and the new lossless coder), which is presented in Fig. 2. The forward adaptive quantizer works on the frame-by-frame basis. Frames of M_1 samples of the input signal $(x_1, \ldots, x_{M_1})$ are loaded into the input buffer. The block for the variance calculation calculates the variance of samples in the input buffer as $\sigma^2 = \frac{1}{M_1}\sum_{q=1}^{M_1} x_q^2$. After that, the variance in the logarithmic domain is calculated as $\alpha[\mathrm{dB}] = 10\log_{10}\sigma^2$. Using quantizer Q_α, uniform quantization of the variance α in the logarithmic domain is done, hence quantizer Q_α is called the log-uniform quantizer. Let N_g denotes the number of levels of the quantizer Q_α. Using Q_α, the range of variances in the logarithmic domain $(\alpha_{\min}, \alpha_{\max})$ [dB] is uniformly divided into N_g intervals, with the quantization stepsize δ_α [dB] $= \Delta_\alpha/\mathrm{N_g}$. Let $\widehat{\alpha_i} = \alpha_{\min} + i\delta_\alpha$, $i = 0, \ldots, N_g$, denote thresholds of the quantizer Q_α. If variance α belongs to the interval $(\widehat{\alpha}_{i-1}, \widehat{\alpha_i})$, the index i is obtained on the output of the quantizer Q_α. This index is transmitted to the receiver as side information (SI) with $\log_2 N_g$ bits. Also, based on the index i, the gain $g_i = 10^{\widehat{\alpha_i}/20}$ is calculated in the block for the gain calculation. There are N_g different values of the gain g_i. All samples from the input buffer $(x_1, \ldots, x_{M_1})$ are divided with the gain g_i. After that, these samples pass through the fixed piecewise uniform quantizer. Output levels of this quantizer are coded using new lossless code.

We have to clarify something. The proposed model can be used in two ways: without adaptation and with adaptation. Model without adaptation is used for stationary signals (whose power is constant or almost constant over the time), which was described in Section 2. In the model without adaptation we have only one type of frame: coding frame of the length M, where M usually takes small values. Model with adaptation is used for non-stationary signals (whose power is significantly changed in time) and this model is analysed in this section. Forward adaptation is applied. In the model with forward adaptation we have two types of frames: frames for the forward adaptation with M_1 samples and coding frames with M samples. The reason for this is the fact that both forward adaptation and lossless code works on frame-by-frame basis. Usually, adaptation frames are much larger than coding frames, i.e. $M_1 \gg M$ (M_1 is of the order of several tens or hundreds, while M is usually smaller than 10). Firstly, input samples are grouped into large frames of M_1 samples for the forward adaptation; after that, each large frame is subdivided into small coding frames of M samples. The M_1 has to be divisible with M. Let us consider one example with $M_1 = 200$ and $M = 5$; firstly, the large adaptation frame of 200 samples is formed: $(x_1, \ldots, x_{200})$; after that, this large frame is subdivided into 40 small coding frames of 5 samples: $(x_1, \ldots, x_5), (x_6, \ldots, x_{10}), \ldots, (x_{196}, \ldots, x_{200})$.

For the forward adaptive quantizer functions $\mathrm{SQNR}(\alpha)$ and $R(\alpha)$ are periodic on the interval of variances $\alpha \in (\alpha_{\min}, \alpha_{\max})$, with the period δ_α. The average value of SQNR is obtained by averaging $\mathrm{SQNR}(\alpha)$ within the one period as:

$$\overline{\mathrm{SQNR}} = \frac{1}{\delta_\alpha} \int_{\alpha_{\max}-\delta_\alpha}^{\alpha_{\max}} \mathrm{SQNR}(\alpha)\mathrm{d}\alpha \text{ [dB]}. \tag{46}$$

The average value of the bit-rate $\overline{R}$ is obtained by averaging $R(\alpha)$ within the one period as:

$$\overline{R} = \frac{1}{\delta_\alpha} \int_{\alpha_{\max}-\delta_\alpha}^{\alpha_{\max}} R(\alpha)\mathrm{d}\alpha + \frac{\log_2 N_g}{M_1} \text{ [bps]}, \tag{47}$$

where the last term in (47) represents the rate increasing due to transmission of side information (SI).

Since $\mathrm{SQNR}(\alpha)$ and $R(\alpha)$ are periodic with the period δ_α, it is enough to consider only one period $\alpha \in (\alpha_{\max} - \delta_\alpha, \alpha_{\max})$ [dB] for the design of the forward adaptive model. Input parameters of the design process are $\mathrm{SQNR}_{\min}$, L, M_1 and N_g, i.e. values of these parameters are defined in advance. We use values of N_i $(i = 1, \ldots, L)$, obtained for $\sigma_0^2 = 1$ for the defined values of $\mathrm{SQNR}_{\min}$ and L (for Gaussian and Laplacian distributions, these values of N_i are given in Tables 1–6). The design is performed by the minimization of the average bit-rate $\overline{R}$ with the condition that $\mathrm{SQNR}(\alpha) \geqslant \mathrm{SQNR}_{\min}$ for $\alpha \in (\alpha_{\max} - \delta_\alpha, \alpha_{\max})$ [dB]. Since $\mathrm{SQNR}(\alpha)$ is the increasing function in the interval $\alpha \in (\alpha_{\max} - \delta_\alpha, \alpha_{\max})$ [dB], the previous condition becomes $\mathrm{SQNR}(\alpha = (\alpha_{\max} - \delta_\alpha)) \geqslant \mathrm{SQNR}_{\min}$. Therefore, the design of the model with the forward adaptation can be described as minimization of

$\overline{R}$ ∴ $\text{SQNR}(\alpha = (\alpha_{\max} - \delta_\alpha)) \geqslant \text{SQNR}_{\min}$. As a result of the design process, we obtain values of parameters $x_{\max}$, t_i, $(i = 1, \ldots, L - 1)$ and M.

3.2. *Application of the Model with the Forward Adaptation to the Speech Signal*

The model with the forward adaptation will be applied to the speech signal. Based on (Jayant and Noll, 1984, Fig. 5.10, p. 241), where 33.9 dB is given as CCITT requirement for the minimum value of SQNR (in the wide power range of almost 40 dB) in systems for speech transmission where high quality of speech is required, we choose: $\text{SQNR}_{\min} = 34$ dB (leaving the margin of 0.1 dB), $\alpha_{\min} = -40$ dB, $\alpha_{\max} = 0$ dB, $\Delta_\alpha = 40$ dB. During the design of quantizer and coder, transmission errors are usually not considered (i.e. it is supposed that transmission errors are small or that they are corrected with some techniques for error correction (e.g. with retransmission of error-corrupted data or by using error correction codes)). Therefore, the value of 34 dB for $\text{SQNR}_{\min}$ is used in systems with high speech quality with negligible transmission errors (i.e. with negligible channel distortion). Value $\text{SQNR}_{\min} = 34$ dB was often used in literature about speech quantization (Perić *et al.*, 2010, 2013; Nikolić and Perić, 2008).

However, if transmission errors are not negligible, then some value of $\text{SQNR}_{\min}$ higher than 34 dB has to be used (what that value would be, depends on the level of transmission errors). On the other hand, maybe for some applications there is no need to achieve such high quality of speech, i.e. some lower level of quality is acceptable. For such applications, some value of $\text{SQNR}_{\min}$ smaller than 34 dB should be used. Also, for other types of signals, other values of $\text{SQNR}_{\min}$ should be used. Therefore, the value of $\text{SQNR}_{\min}$ depends on the type of signal and on the specific application. This paper describes the design of the model for any value of $\text{SQNR}_{\min}$; the value of 34 dB is chosen just as an example.

It is known that short time statistics of the speech signal can be modelled with the Gaussian distribution (Jayant and Noll, 1984). From this reason, the previously developed theory for the Gaussian distribution will be used.

The model with the following parameters is considered: $L = 4$, $M_1 = 198$, $N_1 = 32$, $N_2 = 8$, $N_3 = 16$ and $N_4 = 64$ (these values of N_1, N_2, N_3 and N_4 are taken from Table 3, for $\text{SQNR}_{\min} = 34$ dB and $L = 4$). Values of parameters (t_1, t_2, t_3, $x_{\max}$ and M) of the model, obtained by the optimization process, are presented in Table 7 for different values of N_g. Theoretical values of performances ($\overline{\text{SQNR}}$ and $\overline{R}$) calculated using the previously derived expressions for the Gaussian distribution are also shown in Table 7.

An experiment is performed, applying the model with the forward adaptation to the real speech signal. Performances obtained by the experiment are also presented in Table 7. We can see that values of $\overline{\text{SQNR}}$ and $\overline{R}$ obtained by the theory and by the experiment are matched very well, which proves the previously developed theory.

Figure 3 shows very small variations of SQNR (which means that SQNR is almost constant) in the wide range of variances for the model with the forward adaptation for $N_g = 64$. This is also evident from Table 7 where average SQNR, ($\overline{\text{SQNR}}$) differs from the minimal value of SQNR (34 dB) for only 0.25 dB.

Also, we can see from Table 7 that the proposed model can satisfy the G.712 standard with the bit-rate of 6.18 bps. This model is much better than the model defined with the

Table 7
Numerical and experimental results for the model with the forward adaptation.

N_g	t_1	t_2	t_3	$x_{\max}$	M	Theory		Experiment	
						$\overline{\text{SQNR}}$	$\overline{R}$	$\overline{\text{SQNR}}$	$\overline{R}$
32	0.93	1.20	1.67	3.57	3	34.42	6.26	34.65	6.01
64	1.01	1.30	1.79	3.83	3	34.25	6.21	34.88	5.95
128	1.05	1.35	1.85	3.97	3	34.13	6.18	34.42	5.92

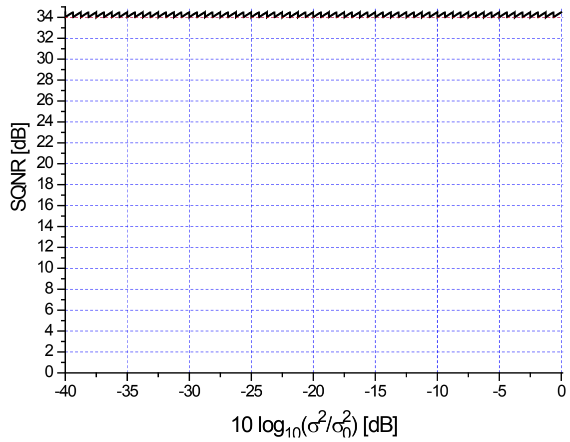


Fig. 3. SQNR in the wide range of variances for the model with the forward adaptation for $N_g = 64$.

G.711 standard (ITU-T, 1972) which requires 8 bps to satisfy the G.712 standard. Also, this model is better than the models presented in Perić *et al*. (2010, 2011, 2013) which satisfy the G.712 standard with the bit-rates of 6.67 bps, 6.43 bps and 6.30 bps, respectively.

4. Conclusion

The model for the quantization and coding of signals, which consists of the piecewise quantizer and new lossless coder, was presented in this paper. One of main characteristics of this model is low complexity, since both the piecewise uniform quantizer and new lossless coder are very simple for realization (new lossless code is much simpler than the Huffman code because it does not require the formation of the code tree or knowledge of probabilities of quantization levels). Another important advantage of the model is its flexibility, since it provides a great possibility of choice of the number of regions of the piecewise uniform quantizer and numbers of levels of these regions; in this way, the model can be adapted to the desired performances for any application.The design was done in a general manner, i.e. for any symmetrical signal distribution and for an arbitrary number of regions of the piecewise uniform quantizer. Rigorous mathematical derivation of the expression for the bit-rate was performed. Developed theory was applied for Gaussian and Laplacian distributions, since a lot of signals in telecommunications (e.g. speech, audio, images, video) can be modelled with those two distributions. Simulation of the model was done in MATLAB for Gaussian and Laplacian distributions. It was shown that numerical results obtained by theory and by simulation were matched very well. Since a lot of real signals are non-stationary, the forward adaptation of the model was performed. The model with the forward adaptation, designed for Gaussian distribution, was applied to the speech signal. Also, an experiment was performed using a real speech signal. It was shown that theoretical and experimental results were matched very well. The proposed model can achieve very good performances. For example, this model can satisfy the G.712 standard for the speech quality transmission with the bit-rate of 6.18 bps, achieving compression of 1.82 bps with respect to the model defined with the G.711 standard (ITU-T, 1972) which requires 8 bps to satisfy the G.712 standard.

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Glaudinimo algoritmas, pagrįstas atkarpomis tolygaus kvantatoriaus ir kintamo ilgio kodo naudojimu

Milan R. DINČIĆ, Zoran H. PERIĆ, Aleksandra Ž. JOVANOVIĆ

Straipsnio autoriai pristato pagerintą signalų glaudinimo algoritmą, pagrįstą nuosekliu kvantatoriaus ir koderio naudojimu. Glaudinamasis signalas pirmiausia kvantuojamas atkarpomis tolygiuoju kvantatoriumu, po to koduojamas autorių pasiūlytu nenuostolinguoju metodu su laisvai parenkamu kodavimo intervalų skaičiumi. Toks glaudinimas leidžia minimizuoti signalo duomenų srautą esant užduotai minimaliai galimai kvantavimo triukšmo ir naudingojo signalo santykio (KTNSS) vertei. Esminiai pasiūlyto metodo privalumai yra paprastumas ir žemas KTNSS lygis. Lyginant su ankstesniame autorių darbe suformuluotu glaudinimo algoritmu pasiektas 0.12 bps mažesnis duomenų srautas.

Straipsnyje analitiškai gaunama ir apibendrinama suglaudinto signalo duomenų srauto išraiška, tinkanti bet kuriam simetriniu reikšmių tikimybių pasiskirstymo dėsniu pasižymінčiam signalui. Nestacionariems signalams glaudinti suformuluota algoritmo versija su išankstinio adaptavimo tipo kvantatoriumi. Eksperimentinė pasiūlyto metodo patikra atlikta MATLAB aplinkoje su simuliuotomis Laplaso ir Gauso tipo sekomis bei su realiu šnekos signalu. Eksperimentų rezultatai atitinka teorines prielaidas ir patvirtina autorių suformuluoto signalų glaudinimo metodo efektyvumą.