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SIMULATION AND OPTIMIZATION OF RADAR SEARCH STRATEGIES

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Abstract. The results of investigation of the resource management in radar search are presented in the paper. The time for search of manoeuvring targets is minimized by optimal distribution of radar power among the space directions and by optimization of search parameters.

The problem of the optimal control of radar search is extremely complicated in the general case and in real situations, therefore we have compared only some strategies (e.g., one stage cyclic strategy, various multistage strategies).

In some simple cases (e.g., motionless targets) optimal parameters of multistage strategies may be found but in the general case the efficiency of strategies may be evaluated with the help of statistical simulation. The simulation time was essentially reduced by some simplifications of models, by the forecast of discrete coordinates of the targets and by the use of averaged values.

The usage of the proposed strategies enables us to reduce the time of search by 2-3 times. Those strategies may be executed in real time.

Key words: optimal distribution, radar search, multistage strategy.

1. Introduction. Radar search strategies of manoeuvring targets in a three-dimensional space are investigated. The purpose of investigations is to minimize the average time until the moment at which all targets are detected with a given false alarm probability and the probability of detection.

Some assumptions will be made on the search region, on radio signals and control parameters.

The space region in which the targets are searched is divided into a number of cells, because the technical possibilities of radar equipment to separate two neighboring targets are limited. We have $I = m_a * m_b$ directions (where m_a is the number of cells along the bearing angle, m_b is the number of cells along the elevation angle) and m is the number of cells in each direction.

We suppose that the number of targets is much smaller than the number of cells in the search region and that the targets are uniformly distributed and mutually independent in the region.

Let the radar set have a possibility to send a package of some radio impulses of the fixed equal amplitude and equal power to each direction. The power of impulses is constant all the time and the time required for changing the search direction may be neglected. Therefore, the resource of a radar system may be characterized by the summary number of impulses.

The impulses sent in some direction may be reflected from the targets in each cell of this direction. The stochastic independent noise (the stationary Gaussian noise) is added to the reflected useful signal. The noise is uniform for all cells and has the Rayleigh envelope V with the density (see Akimov, 1989):

$$P_n(V, \sigma_1) = V/\sigma_1^2 \exp(-V^2/2\sigma_1^2), \tag{1}$$

where σ_1^2 is the variance of noise.

We also assume that the reflected impulses are received in a coherent way, therefore the summary amplitude of the useful signal is equal to

$$A = tA_1,$$

where t is the number of sent impulses; A_1 is the amplitude of a single reflected impulse, which depends on the distance to the target. This dependence is determined by the signal-to-noise ratio.

The signal-to-noise ratio of power is

$$q = \frac{A_1^2}{2\sigma_1^2}$$

in the case of the fixed amplitude A_1. The ratio q depends on the distance to the target according to the range equation:

$$q = (C_D/d)^4, \tag{2}$$

where C_D is the constant, depending on the characteristics of the atmosphere and targets, on the power of a transmitter and so on.

The envelope X of the reflected signal mixed with noise has the density corresponding to the generalized Rayleigh distribution law (see Akimov, 1989 and Levin, 1966):

$$p_s(X,\sigma,A) = (X/\sigma^2)\exp[-(X^2 + A^2)/2\sigma^2]I_0(XA/\sigma^2), \quad (3)$$

where $\sigma^2 = t\sigma_1^2$, I_0 – a modified Bessel function (see Korn, 1973).

The reflected signal is detected by checking two statistical hypotheses: H_0 – the envelope of the reflected signal is distributed according to law (1) or H_1 – it is distributed according to law (3). The decision rule depends on the probability of the first order error Q_0 (in our case the false alarm probability) and the probability of the second order error β. The probability of detection $P_0 = 1 - \beta$.

We have the possibility to control some parameters of search strategies during the search time period. The parameters are:

- the search direction $i \in \{\overline{1,I}\}$;
- the number t of impulses in the package, transmitted to the direction i;
- the probabilities P_0 and Q_0 of the direction i.

The summary false alarm probability for each cell obtained after the whole search period must be smaller than the given level Q of the probability. The summary probability of detection of each target must be greater than the given level P of the probability.

The most investigated search strategies are when the number m of cells in each direction is equal to 1 (see Kuzmin, 1986; Sosulin, 1987; Vlasov, 1989). A sequential criterion is more efficient in this case. Optimal search strategies under some assumptions were developed by Sosulin (1987).

A more complicated case $m > 1$ is less investigated in the analytical way. The difficulties of the case depend on the fact that all the cells of some direction get the same signals but with diffe nt characteristics. The possibilities to investigate in the analytical way are limited, therefore, the statistical simulation is used. The simulation leads to the conclusion that Neyman-Pearson search strategies are more preferable in the case $m > 1$. Two-stage Neyman-Pearson strategies are especially effective. If we use a greater false alarm probability at the first stage and only some directions are examined at the second (see Kuzmin, 1986), then the search time may be reduced from 25% to 40%. We have used the information from previous stages more completely therefore the search time was reduced by 70%.

Two main approaches may be distinguished in the optimization of search strategies. The first is the static case when one can assume the targets to be motionless during the period of investigation. In this simplified case we were able to construct the analytical expression of the dependence of the average resource t on search parameters and to optimize the parameters. A more real dynamic case when the motion of targets is taken into account has been investigated by statistical simulation and optimal search parameters have been obtained.

2. Static case.

2.1. One-stage cyclic Neyman-Pearson strategy. This simple widely used strategy carries out the search in all I directions in some consecutive order. The package of t impulses is used for each direction, the detection is carried out on the basis of the Neyman-Pearson criterion with the probabilities Q_0 and P_0, which are uniform for all directions (P_0 is calculated for the most distant cell). The summary false alarm probability $Q = Q_0$ and the summary probability of detection $P \geqslant P_0$, because each cell is examined only once with one package of impulses and the targets are motionless.

The statistical hypothesis H_0 is accepted according to the Neyman-Pearson criteria if the envelope of the reflected signal z (normalized to σ) is less than the normalized threshold z_p. Otherwise the hypothesis H_1 is accepted. The value of the threshold may be obtained from (1), if $z = v/\sigma$:

$$Q_0 = \int_{z_p}^{\infty} \sigma p_n(v, \sigma)\, dz = \int_{z_p}^{\infty} z \exp(-z^2/2)\, dz.$$

Hence

$$z_p = \sqrt{2\ln(1/Q_0)} \ . \tag{4}$$

Usually the ratio for the summary signal is $A/\sigma > 3$. Then the generalized Rayleigh density function (3) may be approximated by the Gaussian density function with mean A and variance σ^2 (see Levin, 1966). Therefore, for the most remote cells we have:

$$P_0 = \int_{z_p}^{\infty} \sigma p_s(v, \sigma, A)dz$$
$$= \frac{1}{\sqrt{2\Pi}} \int_{z_p}^{\infty} \exp\left[-(z - a\sqrt{t})^2/2\right]dz = \Phi(a\sqrt{t} - z_p), \tag{5}$$

where $a = A_1/\sigma_1$ is the signal-to-noise ratio for the voltage of the signal reflected from the most remote cell; Φ is the Gaussian distribution function.

After a substitution of (2) and (4) into (5) we get the necessary number of impulses for one direction

$$t = \frac{1}{a^2}\left[\Phi^{-1}(P_0) + z_p\right]^2 = \frac{d_{\max}^4}{2C_D^4}\left[\Phi^{-1}(P_0) + \sqrt{2\ln(1/Q_0)}\right]^2, \tag{6}$$

where $d_{\max}$ is the maximal range in the space region. Here we leave out of account the discretization of t.

Equations (4), (6) determine the search parameters for a single direction. If the number of targets is unknown the search period ends after all I directions were examined and after the total number of impulses $t_\Sigma = It$ was sent. t_Σ depends on the probabilities Q_0 and P_0 but it does not depend on the coordinates and number of targets.

2.2. Multistage strategies. The division of the Neyman-Pearson search procedure to several stages under certain conditions may reduce the summary search time.

Let the strategies of each consequent stage be independent, i.e., the values of signals of each stage are not memorized and are not used in the next stages. The selection of search directions is independent of the results of search in other directions. Therefore, we may restrict ourselves by investigating the search strategies separately for each direction. The directions which were not rejected in the previous stages are examined using the probabilities P_i and Q_i of the i-th stage with the parameters determined from (4) and (6). In these formulas we take the maximal distance in which the useful signal was detected in the previous stage instead of d_{max}.

We minimize the average number of summary impulses

$$t_\Sigma = t_1 + \sum_{i=2}^{k} \bar{t}_i, \tag{7}$$

where $\bar{t}_i$ is the average number of impulses in the i-th stage, k is the number of stages. The average number of summary impulses t_Σ depends on the distribution of search resources between the stages. The distribution may be controlled by changing the probabilities P_i and Q_i for the i-th stage.

Let us restrict ourselves by the strategies with uniform false alarm probabilities Q_i for all cells. The summary false alarm probability is equal to the product of all $Q_i (i = \overline{1,k})$ because the noise is not correlated. The next inequality must be fulfilled:

$$\prod_{i=1}^{k} Q_i \leqslant Q_0, \tag{8}$$

$$0 \leqslant Q_i \leqslant 1, \qquad i = \overline{1,k}. \tag{9}$$

It can be proved that the minimum of t_Σ is achieved when

$$\prod_{i=1}^{k} Q_i = Q_0, \quad Q_0 \leqslant Q_i \leqslant 1, \quad i = \overline{1,k}, \tag{10}$$

since by (6), $\bar{t}_k$ decreases when increasing Q_k. In a similar way the probability of detection P_{ij} in the i-th stage and j-th cell (on condition that the signal be detected in the $(i-1)$-th stage) must satisfy the next inequalities:

$$\prod_{i=1}^{k} P_{ij} \geqslant P_0, \quad P_0 \leqslant P_{ij} \leqslant 1, \quad i = \overline{1,k}, \quad j = \overline{1,m}. \tag{11}$$

Therefore we have the problem of minimization of t_Σ under the conditions (10) and (11). We investigated the relatively simple cases $m = 1$ and $k = 2$ (at $m \geqslant 1$) with a small amount of controlled variables.

2.3. Multistage strategies for $m = 1$. For simplicity let us denote $P_i = P_{i1}$ and n be the number of targets in the region of the space. Then the probability of detection under the threshold $z_i = \sqrt{2\ln(1/Q_i)}$ in the i-th stage is equal to

$$R_i = \frac{1-n}{I}\prod_{j=1}^{i} Q_j + \frac{n}{I}\prod_{j=1}^{i} P_j. \qquad (12)$$

The corresponding numbers of impulses are

$$t_i = CF(P_i, Q_i), \quad i = \overline{1,k}, \qquad (13)$$

where $C = \frac{d_{\max}^4}{2C_D^4}$,

$$F(P_i, Q_i) = \left[\Phi^{-1}(P_i) + \sqrt{2\ln(1/Q_i)}\right]^2.$$

Then the average number of impulses used in the k-stage strategy for a single direction is equal to

$$t_{\Sigma k} = CF(P_1, Q_1) + C\sum_{i=2}^{k} R_{i-1}F(P_i, Q_i).$$

The efficiency of the multistage strategy may be measured by the ratio:

$$E(P^k, Q^k) = \frac{t_{\Sigma 1}}{t_{\Sigma k}} = E(P^k, Q^k)$$
$$= \frac{F(P_0, Q_0)}{F(P_1, Q_1) + \sum_{i=2}^{k} R_{i-1}F(P_i, Q_i)}, \qquad (14)$$

where $P^4 = (P_1, ..., P_k)$, $Q^k = (Q_1, ..., Q_k)$ and t is the average number of impulses used in the one-stage strategy.

It is important that the ratio E does not depend on the size of the space region or on the parameters of a radar set incorporated in the constant C.

Thus we have the problem of mathematical programming:

$$\max_{Q_i, P_i, i=\overline{1,k}} E(P^k, Q^k), \tag{15}$$

$$\prod_{i=1}^{k} Q_i = Q_0, \qquad Q_0 \leqslant Q_i \leqslant 1, \tag{16}$$

$$\prod_{i=1}^{k} P_i \geqslant P_0, \qquad P_0 \leqslant P_i \leqslant 1. \tag{17}$$

The number $t_{\Sigma k}$ monotonously increases with an increase of each $P_i, i = \overline{1,k}$, therefore the maximal value of E must be achieved when inequalities (17) are replaced by equalities:

$$\prod_{i=1}^{k} P_i = P_0, \qquad P_0 \leqslant P_i \leqslant 1, \qquad i = \overline{1,k}. \tag{18}$$

It must be admitted that in real situations the dependence (12,14) of the efficiency E on the number of targets n may be neglected, because $n/I << 1$. The error of approximation of the values of R_i, $i = \overline{1,k-1}$ by the value

$$\prod_{j=1}^{i} Q_j$$

is equal to

$$\frac{n\left[\prod_{j=1}^{i} P_j - \prod_{j=1}^{i} Q_j\right]}{I \prod_{j=1}^{i} Q_j}.$$

Therefore, the approximation is acceptable if

$$Q_0/Q_k >> n/I, \tag{19}$$

especially at the point of maximal E, and we use the approximated efficiency $\overline{E}$ instead of E.

After replacing equalities (16) and (18) the problem of optimization of search may be formulated as mathematical programming problem of $2(k-1)$ variables:

$$\max \overline{E}(P^{k-1}, Q^{k-1}),$$

$$\prod_{i=1}^{k} Q_i \geqslant Q_0, \qquad Q_0 \leqslant Q_i \leqslant 1, \qquad (20)$$

$$\prod_{i=1}^{k-1} P_i \geqslant P_0, \qquad P_0 \leqslant P_i \leqslant 1, \qquad (21)$$

where $P_k = P_0 / \prod_{i=1}^{k-1} P_i, Q_k = Q_0 / \prod_{i=1}^{k-1} Q_i$.

The algorithm of variable metrics (by Tiešis, 1975; 1983), which enables us to take into account the constraints of type (20), was used to solve the problem. The constraints (20) were involved into the optimization with the help of the penalty function (see Tiešis, 1984). The objective function after a transformation is

$$\hat{E}(P^{k-1}, Q^{k-1}) = \overline{E}(\alpha P^{k-1}, \beta Q^{k-1}) + \rho,$$

where

$$\alpha = \begin{cases} 1, & \text{if } \prod_{i=1}^{k-1} P_i \geqslant P_0 \\ \left(P_0 / \prod_{i=1}^{k-1} P_i\right)^{1/(k-1)} & \text{otherwise,} \end{cases}$$

$$\beta = \begin{cases} 1, & \text{if } \prod_{i=1}^{k-1} Q_i \geqslant Q_0 \\ \left(Q_0 / \prod_{i=1}^{k-1} Q_i\right)^{1/(k-1)} & \text{otherwise,} \end{cases} \qquad (22)$$

$$\rho = \begin{cases} 0, & \text{if } \alpha = 1 \text{ and } \beta = 1 \\ [(P^{k-1}, Q^{k-1}) - (\alpha P^{k-1}, \beta Q^{k-1})]^2 & \text{otherwise.} \end{cases}$$

The maximum values of E are presented in Table 1 for $Q_0 = 10^{-6}$, for various values P_0 and for two-stage and three-stage strategies.

Table 1. The efficiency of optimal strategies (in times in comparison with the one-stage strategy)

Strategy	$p_0 = 0.5$	$p_0 = 0.6$	$p_0 = 0.7$	$p_0 = 0.8$	$p_0 = 0.9$
Two-stage	3.31	3.13	2.95	2.77	2.56
Three-stage	4.08	3.78	3.50	3.23	2.92

We may see that the efficiency of two-stage strategies as compared to the one-stage strategy is relatively high. The additional efficiency of using the three-stage strategy is small. Therefore the use of two stages is most rational.

2.4. Two-stage strategy for $m > 1$with discrete t. We deal here with the case $t_1, t_2 \in T$, where T is a set of integers. So the problem (7), (10), (11) is of such a form:

$$\begin{aligned} &\min t_\Sigma = \min(t_1 + \bar{t}_2), \\ &Q_1 Q_2 = Q_0 \ , \\ &P_{1j} P_{2j} \geqslant P_0, \ P_0 \leqslant P_{ij} \leqslant 1, \ i = 1,2; \ \ j = \overline{1,m}. \end{aligned} \tag{23}$$

We assume that the number of impulses for the one-stage cyclic strategy $t = L(t)$, where L is the function of discretization $L : R \to T$, such that $L(ta) = tL(a)$. We have by (6), that the number of impulses for the j-th cell and for the second stage is

$$t_{2j} = t \cdot L\left((d_j/d_{\max})^4 \frac{F(P_0/P_{1j}, \ Q_0/Q_1)}{F(P_0, Q_0)} \right) , \tag{24}$$

where d_j is the range up to the j-th cell and

$$P_{1j} = \Phi\left(\sqrt{(t_1/t)(d_{\max}/d_j)^4 F(P_0, Q_0)} - \sqrt{2\ln(1/Q_1)}\right).$$

The probability of detection in j-th cell and in first stage is

$$R_{1j} = Q_1\left(\frac{Im - n}{Im}\right) + P_{1j}\frac{n}{Im} \cong Q_1.$$

So the average number of impulses in the second stage is

$$\bar{t}_2 = t \cdot \bar{t}_2^*(t/t_1, Q_1),$$

where

$$\bar{t}_2^*(t/t_1, Q_1) = \sum_{j=1}^{m} \left[(t_{2j}/t) \cdot R_{1j} \prod_{i=j+1}^{m} (1 - R_{1i}) \right]$$
$$\cong Q_1 \sum_{j=1}^{m} t_{2j} (1 - Q_1)^{m-1}. \quad (25)$$

Problem (23) is transformed by (24) and (25) into such a problem

$$\max_{t/t_1, Q_1} \{E\} = \max_{t/t_1, Q_1} \left\{ \frac{t/t_1}{1 - (t/t_1) \cdot \bar{t}_2^*(t/t_1, Q_1)} \right\}.$$

The parameters t/t_1 and Q_1 are used as control parameters. The parameters of the second stage t/t_2 and Q_2 may be found from the constraints of the problem (23) and from equation (24).

Note that the number of impulses in the second stage depends on the maximal distance in which the useful signal was detected in the first stage.

The maximum values of efficiency $E = t/t_\Sigma$ for various m, P_0 and $Q_0 = 10^{-6}$ are presented in Table 2.

Table 2. The efficiency of optimal discrete two-stage strategies (in times in comparison with the one-stage strategy)

m	$p_0 = 0.5$	$p_0 = 0.7$	$p_0 = 0.9$
10	2.68	2.08	1.96
100	1.93	1.87	1.72
1000	1.51	1.30	1.01

We may see that the discretization of t decreases the efficiency. But the discretization of t is caused by technological constraints.

The discretization of m essentially increases the precision of search.

3. Dynamic case. Contrary to the static case we assume that the targets change their space coordinates during the search period. We consider these changes to be greater than the size of space cells, however, on the other hand, the targets move through some limited number of cells during the search period. Therefore, we may introduce two main assumptions on the movement of targets, which considerably simplifies the algorithms and programs of statistical simulation:

1) the targets don't change their altitudes;

2) they don't change the movement direction.

The investigation of dynamic situations is carried out mainly by means of statistical simulation. The simulation of the target movement is the main part of the simulation system.

3.1. The search region and its surroundings. The region of a space in which the radar search is carried out is illustrated in Fig. 1 and Fig. 2. A horizontal projection of the search region ABCD is presented in Fig. 1, and a vertical one CDEF is presented in Fig. 2. The region is restricted by four planes, stretching through the point O, and by two spherical surfaces with the center at the same point O. The characteristics of the region are:

1) maximal distance $d_{\max}$;

2) minimal distance $d_{\min}$;

3) width along the bearing angle $\Delta\alpha$;

4) width along the elevation angle $\Delta\varepsilon$.

The horizontal plane OZ divides the angle $\Delta\varepsilon$ in half (Fig. 2).

We simulate the movement of targets only in the search region. But during the search period some new targets may appear from the surroundings of the region. Therefore, there arises a necessity of simulation of additional targets in the surroundings. The width Δd of the surroundings must be chosen according to the maximal speed of targets and the search period. If we choose the width Δd too large , we shall have to simulate a great number of targets. Too small Δd may cause errors in the simulation of new targets.

3.2. Generation of the target coordinates. We must generate the coordinates of the targets randomly and uniformly distributed in the region and surroundings. The next algorithm of generation is suggested.

1. The uniformly distributed coordinates of the targets are generated both in the region and in the surroundings.

2. The coordinates are checked whether they are in the region

or in the surroundings.

3. The process of generation must stop after the required number of targets is generated in the region.

The first stage of generation in the region and surroundings uses new characteristics, different from the four ones presented above. The differences are (see Fig. 1, Fig. 2):

1) a new center O' is used instead of the center O;

2) a new maximal distance

$$d'_{\max} = d_{\max} + \Delta d\left(1 + \frac{1}{\sin\frac{\Delta\alpha}{2}}\right)$$

is used instead of $d_{\max}$.

The characteristics $d_{\min}$, $\Delta\alpha$ and $\Delta\varepsilon$ remain the same.

The generation of random uniformly distributed values α and ε is rather simple. If ξ is a uniformly distributed random number in the interval (0,1), then

$$\alpha' = \xi\Delta\alpha - \frac{\Delta\alpha}{2},$$

$$\varepsilon' = \xi\Delta\alpha - \frac{\Delta\alpha}{2}.$$

The third coordinate – distance d' – must be generated with the density increasing as square of the distance. It is easy to verify that if ξ is a uniformly distributed random number in the interval (0,1), then

$$d' = \sqrt[3]{d^3_{\min} + \xi(d^3_{max} - d^3_{\min})}.$$

These polar coordinates are calculated with respect to the center O'. In order to transform the coordinates to the center O we transform them to the orthogonal system:

$$x = d'\cos\varepsilon'\cos\alpha' - \frac{\Delta d}{\sin\frac{\Delta\alpha}{2}},$$

$$y = d'\cos\varepsilon'\sin\varepsilon',$$

$$z = d'\sin\varepsilon'.$$

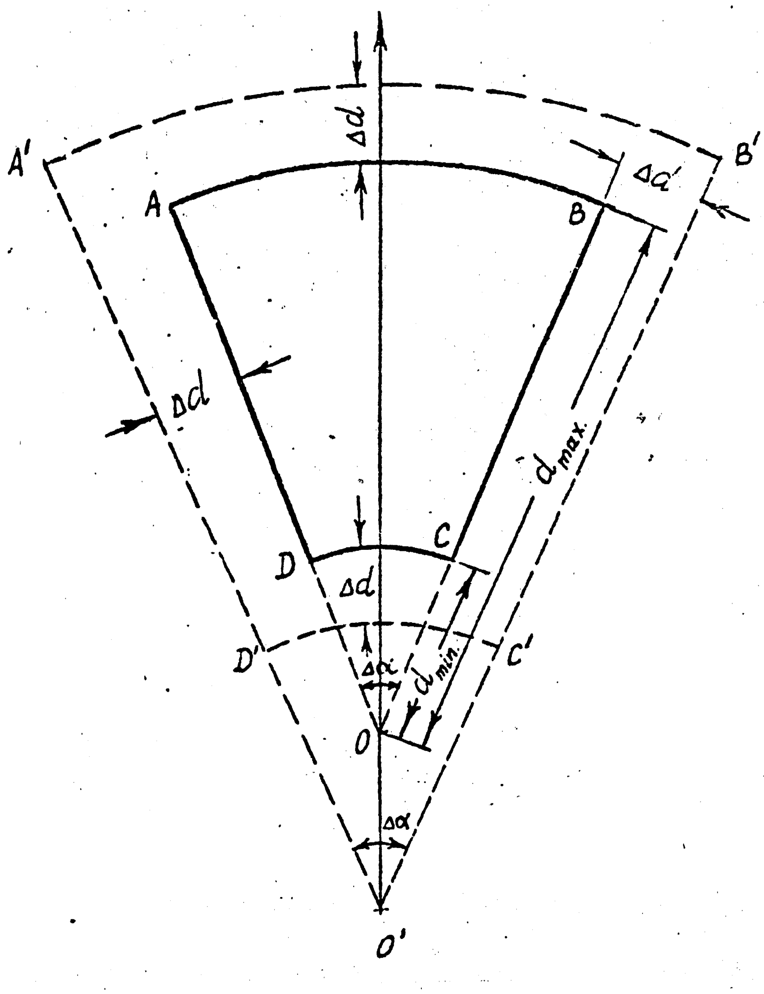

Fig. 1. A horizontal projection of the search region.

The polar coordinates with the center O are transformed according to the next formulas:

$$\alpha = \operatorname{arctg}\frac{y}{x},$$
$$d = \sqrt{x^2 + y^2 + z^2},$$
$$\varepsilon = \arcsin\frac{z}{d}.$$

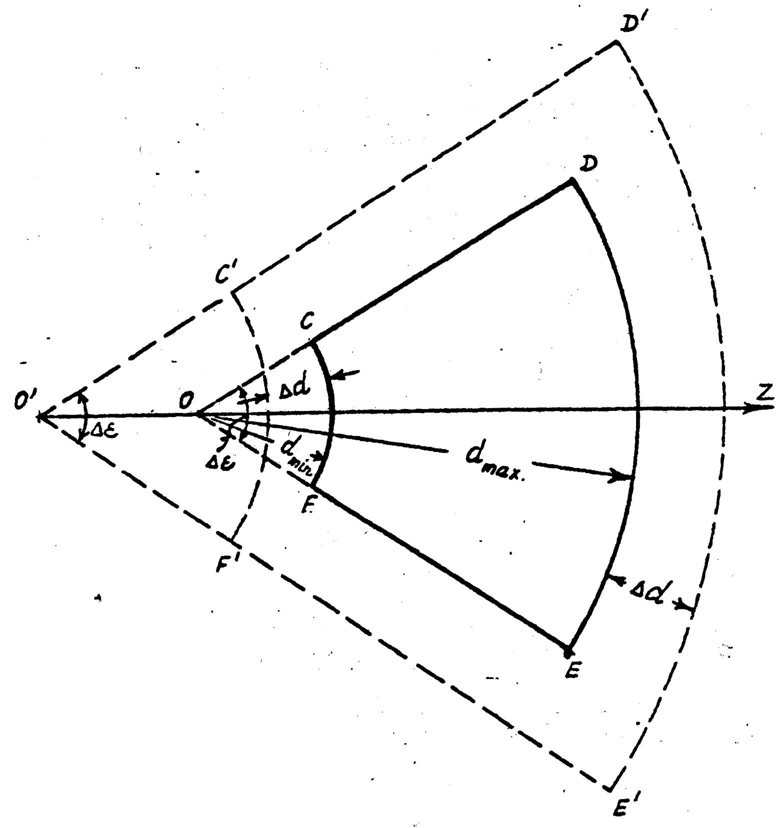

Fig. 2. A vertcal projection of the search region.

The verification of the polar coordinates whether they are in the region is equivalent to a simultaneous satisfaction of such conditions:

$$-\frac{\Delta\alpha}{2} \leqslant \alpha \leqslant \frac{\Delta\alpha}{2},$$
$$-\frac{\Delta\varepsilon}{2} \leqslant \varepsilon \leqslant \frac{\Delta\varepsilon}{2},$$
$$d_{\min} \leqslant d \leqslant d_{\max}.$$

3.3. Generation of the direction of the target movement. The distribution of probabilities for movement directions must be nonuniform. The movements in a relatively short period of search may take place only in the horizontal plane. The movements to the center (the attack on the radar set) or out of the center (the escaping manoeuvre) are more probable. The coefficient of nonuniformity C_n is the ratio between the largest and the smallest probabilities for all the angles of movement directions. We use the next density of probabilities for the directions:

$$p(\phi) = \begin{cases} p_0 - \psi p_1, & \text{if } 0 \leqslant \psi < \frac{\pi}{2} \\ p_0 - (\pi - \psi), & \text{if } \frac{\pi}{2} \leqslant \Psi < \pi \\ p_0 - (\psi - \pi)p_1 & \text{if } \pi \leqslant \Psi < \frac{3\pi}{2} \\ p_0 - (2\pi - \psi), & \text{if } \frac{3\pi}{2} \leqslant \psi < 2\pi, \end{cases}$$

where $\psi = \phi - \alpha, \alpha$ is the bearing angle of the target,

$$p_0 = \frac{C_n}{\pi(C_n + 1)},$$
$$p_1 = \frac{2(C_n - 1)}{\pi^2(C_n + 1)}.$$

The function $p(\phi)$ is actually the density of probabilities because $\int_0^{2\pi} p(\phi)d\phi = 1$ for various values of C_n and α.

Then we may generate random values of the angle

$$\phi = p^{-1}(\xi),$$

where P^{-1} is the inverse function of $P(\phi) = \int_0^{\phi} p(x)dx$, ξ is a uniformly distributed random number in the interval (0,1), using the next formula:

$$\phi = \begin{cases} \dfrac{p_0 - \sqrt{p_0^2 - 2p_1\xi}}{p_1}, & \text{if } 0 < \xi \leqslant 0,25 \\ \pi - \dfrac{p_0 - \sqrt{p_0^2 - 2p_1(0,5 - \xi)}}{p_1}, & \text{if } 0,25 < \xi \leqslant 0,5 \\ \pi + \dfrac{p_0 - \sqrt{p_0^2 - 2p_1(\xi - 0,5)}}{p_1}, & \text{if } 0,5 < \xi \leqslant 0,75 \\ 2\pi - \dfrac{p_0 - \sqrt{p_0^2 - 2p_1(1 - \xi)}}{p_1}, & \text{if } 0,75 < \xi \leqslant 1. \end{cases}$$

3.4. Calculations of the coordinates of the moving targets. During the simulation process we must calculate the discrete coordinates of the moving targets at some moments of time. The process consists of:

1) the orthogonal coordinates of targets are calculated on the base of:

a) the coordinates at the time $t = 0$;

b) the velocity of the targets;

c) the direction of the movement;

d) time t from the beginning of simulation;

2) the coordinates are transformed to the polar coordinates;

3) the discrete coordinates, i.e., the numbers of cells are calculated.

The coordinates must be calculated at each step during the simulation, because the targets may change their discrete coordinates, go out of the region or go into the region. Therefore a straightforward process of simulation requires much computer time for real radar search situations. For example, only one statistical experiment for 10 targets and the use of a relatively simple strategy require about 30 minutes of PC AT computer time.

We used the forecast of time moments when the targets change their discrete coordinates or go out of the region or go into the region.

3.5. Results of simulation. The search of moving targets was simulated using the Neyman-Pearson and two-stage strategies. The process of simulation terminated when all the targets were found with the probability P_0. The result of one process of simulation was the time until termination. The search in one direction was not simulated, but the average time of the search was calculated analytically both in the case of absence and in the case of location of a target. We assume that the target is motionless during the search in one direction. Also, we assume that there are 900 search directions and 5 targets in the search area. The efficiency of the two-stage strategy in comparison with the Neyman-Pearson strategy is presented in Table 3 (the results are averaged

Table 3. The efficiency of the two-stage strategy (dynamic case) (in times in comparison with the one-stage strategy)

m	$p_0 = 0.5$	$p_0 = 0.9$
10	2.77	1.94
100	2.05	1.73
1000	1.57	0.91

from 10 simulation processes).

We see that the values of efficiency are almost like that in the state case.

4. Conclusions. Multi-stage strategies for radar search were designed and investigated. The two-stage strategy in the case of distance measuring ($m > 1$) is about 3 times faster than a simple Neyman-Pearson strategy if the search is not precise ($P_0 = 0.5$, $m = 10$). Both strategies are approximately equivalent in the case of precise search ($P_0 = 0.5$, $m = 1000$).

The simulation system for the movement of targets and search strategies was designed. The simulation of complicated dynamic situations shows that the results of analytical investigations may be effectively used in dynamic cases.

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