A VIKOR-Based Approach to Group Decision Making With Uncertain Preference Ordinals and Incomplete Weight Information

Zhen ZHANG[1]*, Chonghui GUO

Institute of Systems Engineering, Dalian University of Technology, Dalian 116024, PR China
e-mail: zhangzhen@mail.dlut.edu.cn, zhen.zhang@dlut.edu.cn

Abstract. In this paper, we focus on group decision making problems with uncertain preference ordinals, in which the weight information of decision makers is completely unknown or partly unknown. First of all, the consistency and deviation measures between two uncertain preference ordinals are defined. Based on the two measures, a multi-objective optimization model which aims to maximize the deviation of each decision maker's judgements and the consistency among different decision makers' judgements is established to obtain the weights of decision makers. The compromise solution method, i.e. the VIKOR method is then extended to derive the compromise solution of alternatives for group decision making problems with uncertain preference ordinals. Finally, three examples are utilized to illustrate the feasibility and effectiveness of the proposed approach.

Key words: group decision making, uncertain preference ordinal, incomplete weight information, VIKOR.

1. Introduction

Group decision making is a common activity occurring in our daily life, which usually needs a group of decision makers or experts to get involved (Hwang and Yoon, 1987; Zeng *et al.*, 2013; Liu and Liu, 2014; Wan and Dong, 2014). In general, there are usually two types of group decision making problems. One is called multi-attribute group decision making which requires decision makers to provide their assessments with regard to some attributes (Zavadskas and Turskis, 2011; Antucheviciene *et al.*, 2011; Balezentis *et al.*, 2012; Liu, 2012; Liu and Wang, 2014), and the other is group decision making based on preference structures. For the latter, decision makers are required to express their preference using utility values (Tanino, 1990), preference ordinals (Chiclana *et al.*, 1996), multiplicative preference relations (Saaty, 1977), fuzzy preference relations (Orlovsky, 1978) or linguistic preference relations (Herrera *et al.*, 1995; Zhang and Guo, 2014b). In the last decades, dozens of approaches have been proposed to deal with group decision making problems with different preference structures

---

*Corresponding author.

(Herrera-Viedma *et al.*, 2002; Ma *et al.*, 2006; Bregar *et al.*, 2008; Sadeghi *et al.*, 2012; Zhang and Guo, 2014a).

Compared with other preference structures, the preference ordinal is easier to be accepted and utilized by decision makers due to its simplicity, which only needs experts to give the ranking of alternatives. However, due to the uncertainty and complexity of the decision environment or lack of knowledge, decision makers sometimes cannot provide exact preference ordinals over alternatives and the preference information is usually provided in the form of uncertain preference ordinals (González-Pachón and Romero, 2001; Fan *et al.*, 2010; Xu, 2013). The uncertain preference ordinal (also called interval preference ordering), was initially introduced by González-Pachón and Romero (2001) as a collection of positive integer ranges given for providing the possible order positions of a set of alternatives. A typical example was presented in Fan and Liu (2010) as follows: a consumer wants to buy a car among four colour cars (black, white, blue and yellow). The preference information provided by the consumer, for instance, "the black one is ranked top 2 and the blue one is second or third" is a typical form of uncertain preference ordinals.

The use of uncertain preference ordinals is becoming more and more common in the real world, and the research on group decision making with uncertain preference ordinals has received more and more attention. González-Pachón and Romero (2001) proposed an interval goal programming-based approach to aggregating individual uncertain preference ordinals under group decision making environment. González-Pachón *et al.* (2003) also established an interval goal programming model to deal with multi-criteria decision making problems, in which uncertain preference ordinals and pairwise comparison matrix are involved. Wang *et al.* (2005) developed a linear programming-based approach to obtain the ranking from a set of preference ordinals by estimating a utility interval for each alternative and each preference ordinal. To aggregate uncertain preference ordinals and cardinal preferences, González-Pachón and Romero (2009) defined a collective choice function to measure the compromise consensus degree and established a goal programming model to derive the ranking of alternatives. Recently, Fan *et al.* (2010) transformed uncertain preference ordinals into probability vectors and established an optimization model based on the collective probability matrix of alternatives to obtain the ranking of alternatives. By considering different cases for comparing uncertain preference ordinals, Fan and Liu (2010) defined the possibility degree formulae to compare two uncertain preference ordinals and established some optimization models to derive the ranking of alternatives. You *et al.* (2012) developed an assignment method to deal with group decision making problems with uncertain preference ordinals by transforming uncertain preference ordinals into ordinal frequencies. For group decision making with interval utility values and uncertain preference ordinals, Xu and Cai (2013) developed a consensus model which can help decision makers reach consensus effectively.

Most of the above mentioned work addresses the case in which the weight information of decision makers is completely known. If the weight information of decision makers is partly unknown or completely unknown, the approaches will not work well. Indeed, in the process of group decision making, many scholars have assumed that all the experts have the same importance. However, due to the culture and education background,

the familiarity with the alternatives and the evaluation level of decision makers, there are significant differences among the decision quality for different decision makers. Consequently, sometimes it is unreasonable to assign equal weights to different decision makers. How to determine the weights of decision makers is quite important for group decision making problems (Xu and Cai, 2012). In recent years, some approaches have been proposed to determine the weights of decision makers. For instance, for multi-attribute group decision making problems with real numbers or interval numbers, Xu (2011) established a quadratic programming model, which can maximize the consensus among decision makers, to determine the weights of decision makers. Chen *et al.* (2011) investigated the compatibility of uncertain additive linguistic preference relations and established a mathematical model to obtain the weights of experts based on the criterion of minimizing the compatibility in group decision making. Yue (2011, 2012) employed the TOPSIS (technique for order preference by similarity to ideal solution) method and the projection method to determine the weights of decision makers under different environments. Although some work has been conducted on the determination of decision makers' weights, little work is devoted to determine the weights of decision makers for group decision making with uncertain preference ordinals. In a recent work, for group decision making with uncertain preference ordinals, Xu (2013) established a nonlinear programming model by minimizing the divergences between the individual uncertain preferences and the group's opinions to determine decision makers' relative importance weights. However, the proposed method usually assigns equal weights to different decision makers and cannot distinguish the importance of decision makers effectively (Xu *et al.*, 2014). Thus, there is a need to develop some other methods to derive the weights of decision makers.

In Xu (2013)'s work, the TOPSIS method is utilized to derive the final ranking of alternatives. However, the TOPSIS method determines a solution with the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution, but it does not take the relative importance degrees of these distances into account (Opricovic and Tzeng, 2004). As suggested by Xu (2013), the compromise solution method may be utilized to deal with such group decision making problems. The VIKOR (vlsekriterijumska optimizacija i kompromisno resenje in serbian) method proposed by Opricovic (1998) can determine compromise solutions for a problem with conflicting criteria and help the decision makers to reach a final decision. The main idea of the VIKOR method is to obtain compromise solutions based on the L_p metric which was used as an aggregation function in the compromise programming by Yu (1973). Due to its distinct characteristics and capabilities, the VIKOR method has been extended to other environments and different variants of the VIKOR method have been developed, such as fuzzy VIKOR (Fouladgar *et al.*, 2012; Kaya and Kahraman, 2011), linguistic VIKOR (Liu and Wu, 2012), intuitionistic fuzzy VIKOR (Park *et al.*, 2011) and hesitant fuzzy VIKOR (Liao and Xu, 2013). In addition, the VIKOR method has been applied to solve different practical decision making problems. For instance, Yücenur and Demirel (2012) determined the priority ranking of alternative Turkish insurance companies for the evaluating of suitability of their purchasable based on the fuzzy VIKOR. Liu *et al.* (2013a) developed an IOWA-based VIKOR method to reflect the complex attitudinal character of the decision

maker and applied it for the selection of materials. Liu *et al.* (2013b) proposed a VIKOR-based fuzzy multi-criteria decision making method to assess health-care waste disposal methods. Alimardani *et al.* (2013) utilized the step-wise weight assessment ratio analysis (SWARA) and the VIKOR method for supplier selection in an agile environment. Shen *et al.* (2014) combined the VIKOR method and DANP for solving glamour stock selection problem based on fundamental analysis. Rezaie *et al.* (2014) utilized an integrated fuzzy AHP-VIKOR method to evaluate the performance of Iranian cement firms. Although the VIKOR method is quite useful for dealing with multi-criteria (group) decision making problems, there is no work that extends it to deal with group decision making problems based on uncertain preference ordinals. Therefore, in addition to proposing a method to determine the weights of decision makers, this paper also extends the VIKOR method to group decision making with uncertain preference ordinals.

The rest of this paper is organized as follows. In Section 2, some preliminaries related to uncertain preference ordinals are provided. In Section 3, some optimization models are established to determine the weights of decision makers. Moreover, the VIKOR method is extended to deal with group decision making problems with uncertain preference ordinals. Afterwards, three examples are presented to illustrate the proposed approach in Section 4. The proposed approach is compared with some existing approaches in Section 5. Finally, some conclusions are provided in Section 6.

2. Preliminaries

In this section, some preliminaries about uncertain preference ordinals are reviewed.

DEFINITION 1. (See Fan *et al.*, 2010.) Let Z^+ be the set of positive integers. An uncertain preference ordinal $\tilde{r}$ is expressed as $\tilde{r} = \{r^L, r^L + 1, \ldots, r^U\}$, where $r^L, r^L + 1, \ldots, r^U \in Z^+$, $r^L \leqslant r^U$ and r^L, r^U are the lower bound and upper bound of $\tilde{r}$, respectively. Particularly, if $r^L = r^U$, then $\tilde{r}$ will be reduced to a preference ordinal.

By Definition 1, we know that $\tilde{r}$ is a discrete set. For simplicity, we denote $\tilde{r}$ as $\tilde{r} = [r^L, r^U]$.

For example, assume that there are five alternatives to be considered. If a decision maker considers the possible ranking of an alternative be the first, second and third, then the evaluation information of the decision maker can be denoted by an uncertain preference ordinal $\tilde{r} = \{1, 2, 3\}$. For simplicity, we denote it as $[1, 3]$. Obviously, the smaller the ranking, the better the alternative.

In what follows, we define the consistency and deviation measures between two uncertain preference ordinals.

DEFINITION 2. Let $\tilde{r}_1$ and $\tilde{r}_2$ be two uncertain preference ordinals, then the consistency between $\tilde{r}_1$ and $\tilde{r}_2$ is defined as

$$con(\tilde{r}_1, \tilde{r}_2) = \frac{\#(\tilde{r}_1 \cap \tilde{r}_2)}{\max\{\#(\tilde{r}_1), \#(\tilde{r}_2)\}}, \tag{1}$$

where $\#(\tilde{r})$ denotes the number of the elements in $\tilde{r}$.

Obviously, we have the following properties:

(1) $0 \leqslant con(\tilde{r}_1, \tilde{r}_2) \leqslant 1$;
(2) $con(\tilde{r}_1, \tilde{r}_2) = 1$ if and only if $r_1 = r_2$;
(3) $con(\tilde{r}_1, \tilde{r}_2) = con(\tilde{r}_2, \tilde{r}_1)$.

Accordingly, the deviation between $\tilde{r}_1$ and $\tilde{r}_2$ is defined as

$$dev(\tilde{r}_1, \tilde{r}_2) = 1 - con(\tilde{r}_1, \tilde{r}_2) = 1 - \frac{\#(\tilde{r}_1 \cap \tilde{r}_2)}{\max\{\#(\tilde{r}_1), \#(\tilde{r}_2)\}}. \tag{2}$$

EXAMPLE 1. Let $\tilde{r}_1 = [2, 4]$, $\tilde{r}_2 = [2, 3]$, $\tilde{r}_3 = [3, 5]$ be three uncertain preference ordinals, then we can use Definition 2 to calculate the consistency between any two uncertain preference ordinals as follows.

As $\tilde{r}_1 \cap \tilde{r}_2 = \{2, 3, 4\} \cap \{2, 3\} = \{2, 3\}$, $\tilde{r}_1 \cap \tilde{r}_3 = \{2, 3, 4\} \cap \{3, 4, 5\} = \{3, 4\}$, $\tilde{r}_2 \cap \tilde{r}_3 = \{2, 3\} \cap \{3, 4, 5\} = \{3\}$, we have

$$con(\tilde{r}_1, \tilde{r}_2) = \frac{\#(\tilde{r}_1 \cap \tilde{r}_2)}{\max\{\#(\tilde{r}_1), \#(\tilde{r}_2)\}} = \frac{2}{3} \approx 0.667,$$

$$con(\tilde{r}_1, \tilde{r}_3) = \frac{\#(\tilde{r}_1 \cap \tilde{r}_3)}{\max\{\#(\tilde{r}_1), \#(\tilde{r}_3)\}} = \frac{2}{3} \approx 0.667,$$

$$con(\tilde{r}_2, \tilde{r}_3) = \frac{\#(\tilde{r}_2 \cap \tilde{r}_3)}{\max\{\#(\tilde{r}_2), \#(\tilde{r}_3)\}} = \frac{1}{3} \approx 0.333.$$

Similarly, we have $dev(\tilde{r}_1, \tilde{r}_2) = dev(\tilde{r}_1, \tilde{r}_3) = 1 - 0.667 = 0.333$, $dev(\tilde{r}_2, \tilde{r}_3) = 1 - 0.333 = 0.667$.

DEFINITION 3. Let $\tilde{r}_1 = [r_1^L, r_1^U]$ and $\tilde{r}_2 = [r_2^L, r_2^U]$ be two uncertain preference ordinals, then the distance between $\tilde{r}_1$ and $\tilde{r}_2$ is defined as

$$dis(\tilde{r}_1, \tilde{r}_2) = \frac{1}{2}\left(\left|r_1^L - r_2^L\right| + \left|r_1^U - r_2^U\right|\right). \tag{3}$$

3. Group Decision Making with Uncertain Preference Ordinals

In this section, an approach to group decision making with uncertain preference ordinals is proposed. For convenience of analysis, we first give a description of the group decision making problem. Let $A = \{A_1, A_2, \ldots, A_n\}$, $n \geqslant 2$ be the set of alternatives, where A_i is the ith alternative, $i = 1, 2, \ldots, n$, and $E = \{E_1, E_2, \ldots, E_m\}$, $m \geqslant 2$ be the set of decision makers, where E_j is the jth decision maker, $j = 1, 2, \ldots, m$. The weight vector of the decision makers is $w = (w_1, w_2, \ldots, w_m)^T$, where w_j denotes the importance of the jth decision maker such that $\sum_{j=1}^{m} w_j = 1$, $0 \leqslant w_j \leqslant 1$, $j = 1, 2, \ldots, m$. For the ith alternative, the jth decision maker gives his/her uncertain preference ordinal evaluation as $\tilde{r}_{ij} = [r_{ij}^L, r_{ij}^U]$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$. In addition, the weight information of the decision makers is completely unknown or partly unknown. For the latter case, the partly

known weight information may involve the following five forms (Kim and Han, 1999; Zhang and Guo, 2012):

(1) $\{w_i \geqslant w_j\}$;
(2) $\{w_i - w_j \geqslant \alpha_i\}$, $\alpha_i > 0$;
(3) $\{w_i \geqslant \beta_i w_j\}$, $\beta_i \in [0, 1]$;
(4) $\{w_i - w_j \geqslant w_k - w_l\}$, $j \neq k \neq l$;
(5) $\{\gamma_i \leqslant w_i \leqslant \gamma_i + \varepsilon_i\}$, $\varepsilon_i > 0$,

where $i \neq j$, $i, j, k, l \in \{1, 2, \ldots, m\}$. These formats of incomplete weight information can construct a weight vector space W, which means that the weight vector of the decision makers $w = (w_1, w_2, \ldots, w_m)^{\mathrm{T}} \in W$.

The problem to be solved is to determine the weight vector of the decision makers and select the best alternative based on the information available.

When the weight information of the decision makers is completely unknown, Xu (2013) proposed a formula to determine the weight vector of the decision makers as follows.

Let

$$
\tilde{Q} = \begin{pmatrix}
\sum_{i=1}^{n} m((r_{i1}^L)^2 + (r_{i1}^U)^2) & \sum_{i=1}^{n} m(r_{i1}^L r_{i2}^L + r_{i1}^U r_{i2}^U) & \cdots & \sum_{i=1}^{n} m(r_{i1}^L r_{im}^L + r_{i1}^U r_{im}^U) \\
\sum_{i=1}^{n} m(r_{i2}^L r_{i1}^L + r_{i2}^U r_{i1}^U) & \sum_{i=1}^{n} m((r_{i2}^L)^2 + (r_{i2}^U)^2) & \cdots & \sum_{i=1}^{n} m(r_{i2}^L r_{im}^L + r_{i2}^U r_{im}^U) \\
\vdots & \vdots & \vdots & \vdots \\
\sum_{i=1}^{n} m(r_{im}^L r_{i1}^L + r_{im}^U r_{i1}^U) & \sum_{i=1}^{n} m(r_{im}^L r_{i2}^L + r_{im}^U r_{i2}^U) & \cdots & \sum_{i=1}^{n} m((r_{im}^L)^2 + (r_{im}^U)^2)
\end{pmatrix},
$$

(4)

$$
\tilde{\rho} = \left(\sum_{k=1}^{m} \sum_{i=1}^{n} (r_{ik}^L r_{i1}^L + r_{ik}^U r_{i1}^U), \sum_{k=1}^{m} \sum_{i=1}^{n} (r_{ik}^L r_{i2}^L + r_{ik}^U r_{i2}^U), \ldots, \right.
$$
$$
\left. \sum_{k=1}^{m} \sum_{i=1}^{n} (r_{ik}^L r_{im}^L + r_{ik}^U r_{im}^U) \right)^{\mathrm{T}},
$$

(5)

and $e = (1, 1, \ldots, 1)^{\mathrm{T}}$, then the weight vector w can be derived as Xu (2013)

$$
w = \frac{\tilde{Q}^{-1} e (1 - e^{\mathrm{T}} \tilde{Q}^{-1} \tilde{\rho})}{e^{\mathrm{T}} \tilde{Q}^{-1} \tilde{e}} + \tilde{Q}^{-1} \tilde{\rho}.
$$

(6)

For Eq. (6), we have the following theorem (Xu *et al.*, 2014).

Theorem 1. *The weight vector obtained by Eq.* (6) *is* $w = (1/m, 1/m, \ldots, 1/m)^{\mathrm{T}}$.

By Theorem 1, it can be concluded that equal weights $(1/m)$ are assigned to the decision makers, which seems meaningless to some extent. Although Xu *et al.* (2014) proposed an alternative method to determine the weight vector of decision makers, this method cannot deal with the situation when the weight information is partly unknown.

To determine the weight vector more objectively, an approach is proposed in the following section. The basic ideas are as follows: (1) For a single decision maker, if there exists less deviation among the alternatives from his/her opinion, then he/she will contribute less to the ranking result. Thus the weight vector should be determined to maximize the deviation among the evaluation of the alternatives for each decision maker. (2) On the other hand, for a group decision making problem, the opinion on the alternatives among different decision makers should be as close to each other as possible. If a decision maker's evaluation is more consistent with others, more weight should be assigned to the decision maker. Based on the above ideas, we can determine the weight vector of the decision makers as follows.

For the jth decision maker, the average deviation between his/her evaluation on the ith alternative and other alternatives can be calculated as

$$d_{ij} = \frac{1}{n-1} \sum_{\substack{l=1 \\ l \neq i}}^{n} dev(r_{ij}, r_{lj}) = \frac{1}{n-1} \sum_{\substack{l=1 \\ l \neq i}}^{n} \left(1 - \frac{\#(\tilde{r}_{ij} \cap \tilde{r}_{lj})}{\max\{\#(\tilde{r}_{ij}), \#(\tilde{r}_{lj})\}} \right). \tag{7}$$

The overall deviation degree among the jth decision maker's evaluation on all the alternatives is calculated as

$$d_{j} = \frac{1}{n} \sum_{i=1}^{n} d_{ij} = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{\substack{l=1 \\ l \neq i}}^{n} \left(1 - \frac{\#(\tilde{r}_{ij} \cap \tilde{r}_{lj})}{\max\{\#(\tilde{r}_{ij}), \#(\tilde{r}_{lj})\}} \right). \tag{8}$$

Similarly, for the ith alternative, the average consistency degree between the jth decision maker's evaluation and other decision makers' evaluation can be calculated as

$$s_{ij} = \frac{1}{m-1} \sum_{\substack{l=1 \\ l \neq j}}^{m} con(\tilde{r}_{ij}, \tilde{r}_{il}) = \frac{1}{m-1} \sum_{\substack{l=1 \\ l \neq j}}^{m} \frac{\#(\tilde{r}_{ij} \cap \tilde{r}_{il})}{\max\{\#(\tilde{r}_{ij}), \#(\tilde{r}_{il})\}}. \tag{9}$$

The overall consistency degree between the jth decision maker's evaluation and other decision makers' evaluation on all the alternatives is calculated as

$$s_{j} = \frac{1}{n} \sum_{i=1}^{n} s_{ij} = \frac{1}{n(m-1)} \sum_{i=1}^{n} \sum_{\substack{l=1 \\ l \neq j}}^{m} \frac{\#(\tilde{r}_{ij} \cap \tilde{r}_{il})}{\max\{\#(\tilde{r}_{ij}), \#(\tilde{r}_{il})\}}. \tag{10}$$

To determine the weight vector of the decision makers, we establish the following optimization model:

$$\max \ d(w) = \sum_{j=1}^{m} w_j d_j$$
$$s(w) = \sum_{j=1}^{m} w_j s_j$$
$$\text{s.t. } w = (w_1, w_2, \ldots, w_m)^{\mathrm{T}} \in W; \quad \text{(M-1)}$$
$$\sum_{j=1}^{m} w_j = 1;$$
$$w_j \geqslant 0, j = 1, 2, \ldots, m.$$

By using the simple weighted sum method, the above model (M-1) is transformed into a single-objective optimization model as

$$\max \ z(w) = \sum_{j=1}^{m} w_j \big(\lambda d_j + (1 - \lambda)s_j\big)$$
$$\text{s.t. } w = (w_1, w_2, \ldots, w_m)^{\mathrm{T}} \in W; \quad \text{(M-2)}$$
$$\sum_{j=1}^{m} w_j = 1;$$
$$w_j \geqslant 0, j = 1, 2, \ldots, m.$$

In the model (M-2), the parameter λ reflects the compromise between the deviation of a single decision maker's evaluation and the consistency among different decision makers' evaluation. If $\lambda > 0.5$, the deviation of a single decision maker's evaluation is more preferred; if $\lambda < 0.5$, the consistency among different decision makers' evaluation is more preferred. Particularly, if $\lambda = 1$, only the deviation of a single decision maker's evaluation is considered; if $\lambda = 0$, only the consistency among different decision makers' evaluation is considered. The value of λ can be set according to actual situations. By solving the model (M-2), the weight vector of the decision makers can be derived.

Obviously, the case when the weight information is completely unknown can be considered as a special case of the model (M-2) by eliminating the constraint $w \in W$. However, unreasonable result will be yielded. Since the model is a linear programming model, the weight will be derived as 1 for w_j with the largest coefficient in the objective function and 0 for other w_j. To avoid such situation, we use a nonlinear constraint $\sum_{j=1}^{m} w_j^2 = 1$ to replace the linear constraint $\sum_{j=1}^{m} w_j = 1$ (Wang, 1998; Wu and Chen, 2007). After obtaining the solution, we can normalize the weight vector to satisfy $\sum_{j=1}^{m} w_j = 1$. Therefore, the optimization model (M-2) can be revised as

$$\max \ z(w) = \sum_{j=1}^{m} w_j \big(\lambda d_j + (1 - \lambda)s_j\big)$$
$$\text{s.t. } \sum_{j=1}^{m} w_j^2 = 1; \quad \text{(M-3)}$$
$$w_j \geqslant 0, \ j = 1, 2, \ldots, m.$$

To solve the model (M-3), the following Lagrangian function is constructed:

$$L(w, \theta) = \sum_{j=1}^{m} w_j \left(\lambda d_j + (1 - \lambda) s_j \right) + \frac{1}{2} \theta \left(\sum_{j=1}^{m} w_j^2 - 1 \right). \tag{11}$$

Let $\dfrac{\partial L}{\partial w_j} = 0$, then

$$\lambda d_j + (1 - \lambda) s_j + \theta w_j = 0, \quad j = 1, 2, \ldots, m,$$

i.e.

$$w_j = -\frac{\lambda d_j + (1 - \lambda) s_j}{\theta}, \quad j = 1, 2, \ldots, m. \tag{12}$$

As $\sum_{j=1}^{m} w_j^2 = 1$, it follows that

$$\frac{1}{\theta^2} \sum_{j=1}^{m} \left(\lambda d_j + (1 - \lambda) s_j \right)^2 = 1. \tag{13}$$

By Eq. (13),

$$\theta = - \sqrt{\sum_{j=1}^{m} \left(\lambda d_j + (1 - \lambda) s_j \right)^2}. \tag{14}$$

Substituting Eq. (14) into Eq. (12), we have

$$w_j = \frac{\lambda d_j + (1 - \lambda) s_j}{\sqrt{\sum_{h=1}^{m} (\lambda d_h + (1 - \lambda) s_h)^2}}, \quad j = 1, 2, \ldots, m.$$

After normalization, the weight vector can be derived as

$$w_j^* = \frac{w_j}{\sum_{i=1}^{m} w_i} = \frac{\lambda d_j + (1 - \lambda) s_j}{\sum_{i=1}^{m} (\lambda d_i + (1 - \lambda) s_i)}, \quad j = 1, 2, \ldots, m. \tag{15}$$

Once the weight vector of the decision makers is determined, the VIKOR method (Opricovic, 1998) can be extended to obtain compromise solutions for group decision making problems with uncertain preference ordinals. Considering the group decision problems mentioned above, the L_p metric over the alternatives for uncertain preference ordinals is defined as follows:

$$L_{p,i} = \left(\sum_{j=1}^{m} \left(w_j \frac{dis(r_j^*, r_{ij})}{dis(r_j^*, r_j^-)} \right)^p \right)^{1/p}, \quad 1 \leqslant p \leqslant \infty, \ i = 1, 2, \ldots, n, \tag{16}$$

where r_j^* and r_j^- are the positive ideal value and negative ideal value for each decision maker's judgement. Obviously, $r_j^* = [1, 1]$, $r_j^- = [n, n]$, $j = 1, 2, \ldots, m$.

Based on Eq. (16), the compromise solutions can be derived as follows.

Step 1: Calculate the group utility measure (S_i) and individual regret measure (R_i) for each alternative by

$$
\begin{aligned}
S_i &= \sum_{j=1}^{m} w_j \frac{dis(r_j^*, r_{ij})}{dis(r_j^*, r_j^-)} = \frac{1}{n-1} \sum_{j=1}^{m} w_j dis(r_j^*, r_{ij}) \\
&= \frac{1}{2(n-1)} \sum_{j=1}^{m} w_j (|r_{ij}^L - 1| + |r_{ij}^U - 1|) \\
&= \frac{1}{2(n-1)} \sum_{j=1}^{m} w_j (r_{ij}^L + r_{ij}^U - 2), \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
R_i &= \max_j \left\{ w_j \frac{dis(r_j^*, r_{ij})}{dis(r_j^*, r_j^-)} \right\} = \frac{1}{n-1} \max_j \left\{ w_j dis(r_j^*, r_{ij}) \right\} \\
&= \frac{1}{2(n-1)} \max_j \left\{ w_j (|r_{ij}^L - 1| + |r_{ij}^U - 1|) \right\} \\
&= \frac{1}{2(n-1)} \max_j \left\{ w_j (r_{ij}^L + r_{ij}^U - 2) \right\}, \quad i = 1, 2, \ldots, n.
\end{aligned} \tag{18}
$$

Step 2: Calculate the compromise measure for each alternative by

$$
Q_i = v \frac{S_i - S^*}{S^- - S^*} + (1 - v) \frac{R_i - R^*}{R^- - R^*}, \quad i = 1, 2, \ldots, n, \tag{19}
$$

where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$, and v is introduced as the weight of the strategy of "the majority of criteria" (or "the maximum group utility"). Without loss of generality, the value of v is usually set to 0.5.

Step 3: Rank the alternatives according to S_i, R_i and Q_i, respectively in ascending order, i.e. the smaller the value, the better the alternative. Thereby, three ranking lists can be obtained.

Step 4: Determine the compromise solution(s). The alternative with the smallest value of Q_i (we denote it as A_p) is the compromise solution if the following two conditions are satisfied: (1) acceptable advantage: $Q_q - Q_p \geqslant \frac{1}{n-1}$, where Q_q is the second smallest value of Q_i, $i = 1, 2, \ldots, n$; (2) acceptable stability: A_p should be ranked the best by S_i or R_i.

If only condition (2) is not satisfied, the compromise solution set is $\{A_p, A_q\}$. If condition (1) is not satisfied, solve the inequality $Q_M - Q_p < \frac{1}{n-1}$, and the compromise solution set is the alternatives whose Q_i values are between Q_p and Q_M.

Table 1
Evaluation information of the six candidates.

E_i	Alternatives					
	A_1	A_2	A_3	A_4	A_5	A_6
E_1	[3, 4]	[1, 2]	[2, 2]	[2, 4]	[5, 6]	[1, 3]
E_2	[4, 5]	[3, 4]	[1, 4]	[1, 3]	[4, 6]	[4, 5]
E_3	[5, 6]	[3, 4]	[1, 2]	[2, 3]	[4, 6]	[5, 6]
E_4	[3, 4]	[2, 3]	[1, 3]	[2, 3]	[5, 6]	[4, 5]

REMARK 1. The above method is proposed to deal with group decision making problems with uncertain preference ordinals. Since the preference ordinals (a permutation of $\{1, 2, \ldots, n\}$) can be considered as a special case of uncertain preference ordinals, our method will also work well if the evaluation of decision makers are provided by preference ordinals. In this case, it is obvious that $d_1 = d_2 = \cdots = d_m$ and d_j contributes nothing to the determination of the weight. Thereby, we can let $\lambda = 0$ for the models (M-2), (M-3) and Eq. (15). By solving the models (M-2) and (M-3), we can also obtain the weight vector of the decision makers. After that, the extended VIKOR method can also be utilized to derive the compromise solution.

4. Illustrative Examples

In this section, we utilize three examples to illustrate the proposed group decision making approach.

EXAMPLE 2. The quality of staff is an important factor for building high-level universities. In order to enhance the quality of staff, a school in a university of China intends to recruit new teachers. After primary selection, six candidates are selected to enter the interview stage. After having a face to face interview with the six candidates, four professors of the school provide their preference over the six candidates based on the eduction background, research experiences, publications and the interview performance of the candidates. The preference information is provided in the form of uncertain preference ordinals as shown in Table 1. To select the most appropriate candidate, the proposed decision making approach is adopted.

CASE 1. The weight information of the four professors is completely unknown. In this case, we utilize Eq. (15) to derive the weight vector of the four professors.

By Eqs. (8) and (10), we can calculate $d_1 = 0.7444$, $d_2 = 0.6$, $d_3 = 0.7556$, $d_4 = 0.6889$; $s_1 = 0.4120$, $s_2 = 0.5741$, $s_3 = 0.5185$, $s_4 = 0.6065$. Without loss of generality, let $\lambda = 0.5$. By Eq. (15), the weight information of the four professors is obtained as $w = (0.2360, 0.2396, 0.2600, 0.2644)^{\text{T}}$.

Let $\nu = 0.5$. By Eqs. (17), (18) and (19), we can calculate the group utility measure (S_i), individual regret measure (R_i) and compromise measure (Q_i) for each candidate as $S_1 = 0.6519$, $S_2 = 0.3527$, $S_3 = 0.1980$, $S_4 = 0.2996$, $S_5 = 0.85$, $S_6 = 0.6340$;

Table 2
The values of S_i, R_i and Q_i for the six candidates when $w = (0.2360, 0.2396, 0.2600, 0.2644)^{\mathrm{T}}$.

Alternatives	S_i		R_i		Q_i	
	Value	Rank	Value	Rank	Value	Rank
A_1	0.6519	5	0.2340	4	0.8363	5
A_2	0.3527	3	0.13	3	0.2937	3
A_3	0.1980	1	0.0719	1	0	1
A_4	0.2996	2	0.0944	2	0.1458	2
A_5	0.85	6	0.2379	6	1	6
A_6	0.6340	4	0.2340	4	0.8226	4

$R_1 = 0.2340$, $R_2 = 0.13$, $R_3 = 0.0719$, $R_4 = 0.0944$, $R_5 = 0.2379$, $R_6 = 0.2340$; $Q_1 = 0.8363$, $Q_2 = 0.2937$, $Q_3 = 0$, $Q_4 = 0.1458$, $Q_5 = 1$, $Q_6 = 0.8226$. From Table 2, the ranking of the alternatives can be observed based on the values of S_i, R_i and Q_i. The ranking of the candidates is $A_3 \succ A_4 \succ A_2 \succ A_6 \succ A_1 \succ A_5$ according to the values of S_i and Q_i, and the ranking is $A_3 \succ A_4 \succ A_2 \succ A_6 \sim A_1 \succ A_5$ according to the values of R_i.

As $Q_4 - Q_3 = 0.1458 - 0 = 0.1458 < \frac{1}{6-1} = 0.2$, the inequality $Q_M - Q_3 < \frac{1}{6-1} = 0.2$ needs to be solved and we have $Q_M < 0.2$. Therefore, $\{A_3, A_4\}$ is the compromise solution set according to the two judgement conditions.

CASE 2. The weight information of the four professors is partly unknown. Assume that the weight information is provided as follows: $w_4 \geqslant 0.4w_2$, $w_2 - w_3 \geqslant 0.1$, $0.2 \leqslant w_1 \leqslant 0.3$, $0.1 \leqslant w_4 \leqslant 0.25$.

In this case, we establish the following optimization model to derive the weight vector of the four professors:

$$
\begin{aligned}
\max \ z(w) = & \big(0.7444\lambda + 0.4120(1 - \lambda)\big)w_1 + \big(0.6\lambda + 0.5741(1 - \lambda)\big)w_2 \\
& + \big(0.7556\lambda + 0.5185(1 - \lambda)\big)w_3 + \big(0.6889\lambda + 0.6065(1 - \lambda)\big)w_4 \\
\text{s.t. } & 0.4w_2 - w_4 \leqslant 0, w_2 - w_3 \geqslant 0.1 \\
& 0.2 \leqslant w_1 \leqslant 0.3, w_2, w_3 \geqslant 0, 0.1 \leqslant w_4 \leqslant 0.25 \\
& w_1 + w_2 + w_3 + w_4 = 1.
\end{aligned}
\tag{20}
$$

Let $\lambda = 0.5$. By solving the model (20), the weight vector is obtained as $w = (0.2, 0.325, 0.225, 0.25)^{\mathrm{T}}$.

Let $\nu = 0.5$. By Eqs. (17), (18) and (19), we can calculate the group utility measure (S_i), individual regret measure (R_i) and compromise measure (Q_i) for each candidate as $S_1 = 0.6550$, $S_2 = 0.3700$, $S_3 = 0.2100$, $S_4 = 0.2875$, $S_5 = 0.8450$, $S_6 = 0.6450$; $R_1 = 0.2275$, $R_2 = 0.1625$, $R_3 = 0.0975$, $R_4 = 0.0800$, $R_5 = 0.2600$, $R_6 = 0.2275$; $Q_1 = 0.7601$, $Q_2 = 0.3552$, $Q_3 = 0.0486$, $Q_4 = 0.0610$, $Q_5 = 1$, $Q_6 = 0.7522$. From Table 3, we find that the ranking of the candidates is $A_3 \succ A_4 \succ A_2 \succ A_6 \succ A_1 \succ A_5$ according to the values of S_i and Q_i and that the ranking is $A_4 \succ A_3 \succ A_2 \succ A_6 \sim A_1 \succ A_5$ according to the values of R_i. Since $Q_4 - Q_3 = 0.0610 - 0.0486 = 0.0124 < \frac{1}{6-1} = 0.2$, the inequality $Q_M - Q_3 < \frac{1}{6-1} = 0.2$ needs to be solved and we have $Q_M < 0.2486$. Therefore, $\{A_3, A_4\}$ is the compromise solution set.

Table 3

The values of S_i, R_i and Q_i for the six candidates when $w = (0.2, 0.325, 0.225, 0.25)^{\mathrm{T}}$.

Alternatives	S_i		R_i		Q_i	
	Value	Rank	Value	Rank	Value	Rank
A_1	0.6550	5	0.2275	4	0.7601	5
A_2	0.3700	3	0.1625	3	0.3552	3
A_3	0.2100	1	0.0975	2	0.0486	1
A_4	0.2875	2	0.0800	1	0.0610	2
A_5	0.8450	6	0.2600	6	1	6
A_6	0.6450	4	0.2275	4	0.7522	4

Table 4

Evaluation information of the four transnational corporations.

E_i	Alternatives			
	A_1	A_2	A_3	A_4
E_1	[2, 3]	[1, 1]	[2, 4]	[3, 4]
E_2	[3, 4]	[1, 2]	[1, 2]	[3, 4]
E_3	[4, 4]	[2, 3]	[2, 3]	[1, 2]
E_4	[1, 1]	[2, 3]	[4, 4]	[2, 3]
E_5	[2, 3]	[1, 3]	[1, 2]	[4, 4]

EXAMPLE 3. (See Fan and Liu, 2010; Xu, 2013.) Eastsoft is one of the top five software companies in China. It offers a rich portfolio of businesses, mainly including industry solutions, product engineering solutions, and related software products and platform and services. It is dedicated to becoming a globally leading IT solutions and services provider through continuous improvement of organization and process, competence development of leadership and employees, and alliance and open innovation. To improve the operation and competitiveness capability in the global market, Eastsoft plans to establish a strategic alliance with a transnational corporation. After numerous consultations, four transnational corporations would like to establish a strategic alliance with Eastsoft; they are HP (A_1), PHILIPS (A_2), EMC (A_3), and SAP (A_4). To select the desirable strategic alliance partner, five experts (E_1, E_2, E_3, E_4, and E_5) are invited to participate in the decision analysis, who come from the operation management department, the engineering management department, the finance department, the human resources department, and the business process outsourcing department of Eastsoft, respectively. The preference information of the five experts is provided in the form of uncertain preference ordinals (see Table 4). In what follows, we will utilize the proposed method to select the best strategic alliance partner.

By Eqs. (8) and (10), we have $d_1 = 0.6944$, $d_2 = 0.6667$, $d_3 = 0.6667$, $d_4 = 0.8333$, $d_5 = 0.6944$, $s_1 = 0.3750$, $s_2 = 0.4688$, $s_3 = 0.3021$, $s_4 = 0.25$, $s_5 = 0.4167$.

Without loss of generality, let $\lambda = 0.5$. By Eq. (15), we can obtain $w = (0.1992, 0.2115, 0.1805, 0.2018, 0.2070)^{\mathrm{T}}$.

Let $\nu = 0.5$. By Eqs. (17), (18) and (19), the group utility measure (S_i), individual regret measure (R_i) and compromise measure (Q_i) for each alternative are calculated as

Table 5
The values of S_i, R_i and Q_i for the four transnational corporations.

Alternatives	S_i		R_i		Q_i	
	Value	Rank	Value	Rank	Value	Rank
A_1	0.5598	3	0.1805	2	0.7186	2
A_2	0.2954	1	0.1009	1	0	1
A_3	0.4946	2	0.2018	3	0.7344	3
A_4	0.6803	4	0.2070	4	1	4

Table 6
Preference ordinal information of the four factors.

E_i	Factors			
	A_1	A_2	A_3	A_4
E_1	2	1	3	4
E_2	3	2	1	4
E_3	4	2	3	1
E_4	1	2	4	3

$S_1 = 0.5598$, $S_2 = 0.2954$, $S_3 = 0.4946$, $S_4 = 0.6803$; $R_1 = 0.1805$, $R_2 = 0.1009$, $R_3 = 0.2018$, $R_4 = 0.2070$; $Q_1 = 0.7186$, $Q_2 = 0$, $Q_3 = 0.7344$, $Q_4 = 1$. From Table 5, the ranking of the alternatives can be derived as follows. The ranking is $A_2 \succ A_3 \succ A_1 \succ A_4$ according to the values of S_i, and the ranking is $A_2 \succ A_1 \succ A_3 \succ A_4$ according to the values of R_i and Q_i. Since $Q_1 - Q_2 = 0.7186 - 0 = 0.7186 > \frac{1}{4-1} = 0.333$, the compromise solution is A_2 according to the two judgement conditions. Therefore, PHILIPS (A_2) is the best strategic alliance partner.

If we utilize Xu (2013)'s approach, the weight vector is obtained as $w = (0.2, 0.2, 0.2, 0.2, 0.2)^{\mathrm{T}}$ and the ranking of the alternatives is $A_2 \succ A_3 \succ A_1 \succ A_4$. However, we can find that Xu (2013)'s approach obtains equal weight for each decision maker and cannot distinguish the importance of decision makers. If we only consider the deviation of a single decision maker's evaluation ($\lambda = 1$) or the consistency among different decision makers' evaluation ($\lambda = 0$), we can obtain the weight information as $w = (0.1953, 0.1875, 0.1875, 0.2344, 0.1953)^{\mathrm{T}}$ and $w = (0.2069, 0.2586, 0.1667, 0.1379, 0.2299)^{\mathrm{T}}$, respectively . Obviously the proposed approach can distinguish the importance of decision makers.

EXAMPLE 4. (See Xu, 2013.) In a supply chain management, the enterprise usually needs to establish a partnership to (1) lower the total cost of supply chain; (2) lower inventory of enterprises; (3) enhance information sharing of enterprises; (4) improve the interaction of enterprises; (5) obtain more competitive advantages for enterprises. In order to identify the most important factor, four experts E_k ($k = 1, 2, 3, 4$) (whose weight vector $w = (w_1, w_2, w_3, w_4)^{\mathrm{T}}$ is to determined) are asked to provide their preferences over four factors: respond time (delivery time) and supply capacity (A_1), quality and technology level (A_2), price and cost (A_3) and service level (A_4). The preference information provided by the four experts is expressed in exact preference ordinals, as shown in Table 6.

Table 7
The values of S_i, R_i and Q_i for the four factors.

Alternatives	S_i		R_i		Q_i	
	Value	Rank	Value	Rank	Value	Rank
A_1	0.5667	3	0.3	3	0.9091	3
A_2	0.2667	1	0.1	1	0	1
A_3	0.5333	2	0.2	2	0.6136	2
A_4	0.6333	4	0.3	3	1	4

Since preference ordinals can be considered as special cases of uncertain preference ordinals (i.e. the lower bound and upper bound are the same), we shall utilize the proposed approach to identify the most important factor. By Eq. (10), we have $s_1 = 0.1667$, $s_2 = 0.25$, $s_3 = 0.25$ and $s_4 = 0.1667$. Let $\lambda = 0$ and the weight information of the decision makers can be obtained as $w = (0.2, 0.3, 0.3, 0.2)^{\mathrm{T}}$. After that, we can calculate the group utility measure (S_i), individual regret measure (R_i) and compromise measure (Q_i) for each factor as $S_1 = 0.5667$, $S_2 = 0.2667$, $S_3 = 0.5333$, $S_4 = 0.6333$; $R_1 = 0.3$, $R_2 = 0.1$, $R_3 = 0.2$, $R_4 = 0.3$; $Q_1 = 0.9091$, $Q_2 = 0$, $Q_3 = 0.6136$, $Q_4 = 1$. From Table 7, the ranking of the alternatives can be observed. The ranking of the factors is $A_2 \succ A_3 \succ A_1 \succ A_4$ according to the values of S_i and Q_i, and the ranking is $A_2 \succ A_3 \succ A_1 \sim A_4$ according to the values of R_i. Since $Q_3 - Q_2 = 0.6136 - 0 = 0.6136 > \frac{1}{4-1} = 0.333$, the compromise solution is A_2. Therefore, quality and technology level (A_2) is the most important factor.

If Xu (2013)'s approach is utilized, the weight information is obtained as $w = (0.25, 0.25, 0.25, 0.25)^{\mathrm{T}}$, which also assigns equal weight to decision makers. Due to different weight, the ranking of the factors is $A_2 \succ A_1 \succ A_3 \succ A_4$, which is slightly different from the proposed approach. However, the most important factor is also A_2, which is the same as that derived the proposed approach.

5. Comparisons with Existing Approaches

In this section, we make some comparisons between the proposed approach with some existing ones.

First, we compare the weight determining method with Xu (2013) and Xu *et al.* (2014)'s methods. To derive the weight vector of decision makers, an optimization model is established to maximize both the deviation of each decision maker's judgements and the consistency among different decision makers' judgements. The proposed method can deal with the cases when the weight information is completely unknown or partly unknown. Through numerical examples, it can be observed that Xu (2013)'s method assigns equal weights to different decision makers and cannot distinguish the importance of decision makers. Although Xu *et al.* (2014)'s method can distinguish the importance of decision makers, it can only be used to deal with the cases when the weight information is completely unknown, and cannot deal with the cases when the weight information is partly unknown. In general, the weight determining method presented in our proposal has more extensive application prospects.

Second, we compare the ranking techniques for group decision making with uncertain preference ordinals. In the literature, the ranking of alternatives are usually derived by solving optimization models (Fan *et al.*, 2010; You *et al.*, 2012) or using the ranking approaches based on fuzzy preference relations (Fan and Liu, 2010). These two types of ranking techniques usually need to transform the uncertain preference ordinals into other types of information and sometimes also need to solve binary integer programming models, which may result in the loss of information and complex computations. While the proposed approach (i.e. the VIKOR approach) is based on the L_p metric of uncertain preference ordinals and does not need to transform the uncertain preference ordinals into other types of information, which can keep the original information as much as possible. Moreover, the VIKOR approach determines the compromise solutions by mutual concessions and can overcome the flaw of the TOPSIS method which is used for ranking in Xu (2013)'s method.

Third, like Xu (2013) and Xu *et al.* (2014)'s approaches, the proposed approach can also be used to deal with group decision making problems with exact preference ordinals, while Fan and Liu (2010)'s approach is only suitable for group decision making problems with uncertain preference ordinals.

To summarize, the proposed approach not only can determine the weight vector of decision makers, but also can derive compromise solutions of a group decision making problem. Therefore, it can be concluded that the proposed approach can deal with group decision making problems with (uncertain) preference ordinals effectively.

6. Conclusions

In this paper, group decision making problems with uncertain preference ordinals are investigated. First, an approach is proposed to determine the weights of decision makers based on the deviation of individual judgement and the consistency among the group's judgement, which can deal with the situations in which the weight information of decision makers is completely unknown and partly unknown. Second, the compromise solution approach, i.e. VIKOR method is extended to group decision making with uncertain preference ordinals. Through three examples, we find that the proposed weight determination approach can distinguish the importance of decision makers clearly and that the extended VIKOR method can derive compromise solution of alternatives effectively. For future research, we intend to extend the proposed approach to multi-attribute group decision making with uncertain preference ordinals.

References

Alimardani, M., Hashemkhani Zolfani, S., Aghdaie, M.H., Tamosaitiene, J. (2013). A novel hybrid SWARA and VIKOR methodology for supplier selection in an agile environment. *Technological and Economic Development of Economy*, 19(3), 533–548.

Antucheviciene, J., Zakarevicius, A., Zavadskas, E.K. (2011). Measuring congruence of ranking results applying particular MCDM methods. *Informatica*, 22(3), 319–338.

Balezentis, A., Baleentis, T., Brauers, W. (2012). MULTIMOORA-FG: a multi-objective decision making method for linguistic reasoning with an application to personnel selection. *Informatica*, 23(2), 173–190.

Bregar, A., Gyorkos, J., Juric, M.B. (2008). Interactive aggregation/disaggregation dichotomic sorting procedure for group decision analysis based on the threshold model. *Informatica*, 19(2), 161–190.

Chen, H., Zhou, L., Han, B. (2011). On compatibility of uncertain additive linguistic preference relations and its application in the group decision making. *Knowledge-Based Systems*, 24(6), 816–823.

Chiclana, F., Herrera, F., Herrera-Viedma, E., Poyatos, M.C. (1996). A classification method of alternatives for multiple preference ordering criteria based on fuzzy majority. *The Journal of Fuzzy Mathematics*, 4, 801–813.

Fan, Z.P., Liu, Y. (2010). An approach to solve group-decision-making problems with ordinal interval numbers. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 40(5), 1413–1423.

Fan, Z.P., Yue, Q., Feng, B., Liu, Y. (2010). An approach to group decision-making with uncertain preference ordinals. *Computers & Industrial Engineering*, 58(1), 51–57.

Fouladgar, M.M., Yazdani-Chamzini, A., Zavadskas, E.K., Yakhchali, S.H., Ghasempourabadi, M.H. (2012). Project portfolio selection using fuzzy AHP and VIKOR techniques. *Transformations in Business & Economics*, 11(1), 213–231.

González-Pachón, J., Romero, C. (2001). Aggregation of partial ordinal rankings: an interval goal programming approach. *Computers & Operations Research*, 28(8), 827–834.

González-Pachón, J., Romero, C. (2009). Aggregation of ordinal and cardinal preferences: a framework based on distance functions. *Journal of Multi-Criteria Decision Analysis*, 15(3–4), 79–85.

González-Pachón, J., Rodríguez-Galiano, M.I., Romero, C. (2003). Transitive approximation to pairwise comparison matrices by using interval goal programming. *Journal of the Operational Research Society*, 54(5), 532–538.

Herrera, F., Herrera-Viedma, E., Verdegay, J.L. (1995). A sequential selection process in group decision making with a linguistic assessment approach. *Information Sciences*, 85(4), 223–239.

Herrera-Viedma, E., Herrera, F., Chiclana, F. (2002). A consensus model for multiperson decision making with different preference structures. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 32(3), 394–402.

Hwang, C.L., Yoon, K. (1987). *Group Decision Making under Multiple Criteria: Methods and Applications*. Springer, New York.

Kaya, T., Kahraman, C. (2011). Fuzzy multiple criteria forestry decision making based on an integrated VIKOR and AHP approach. *Expert Systems with Applications*, 38(6), 7326–7333.

Kim, S.H., Han, C.H. (1999). An interactive procedure for multi-attribute group decision making with incomplete information. *Computers & Operations Research*, 26(8), 755–772.

Liao, H., Xu, Z. (2013). A VIKOR-based method for hesitant fuzzy multi-criteria decision making. *Fuzzy Optimization and Decision Making*, 12(4), 373–392.

Liu, P. (2012). An approach to group decision making based on 2-dimension uncertain linguistic information. *Technological and Economic Development of Economy*, 18(3), 424–437.

Liu, P., Liu, Y. (2014). An approach to multiple attribute group decision making based on intuitionistic trapezoidal fuzzy power generalized aggregation operator. *International Journal of Computational Intelligence Systems*, 7(2), 291–304.

Liu, P., Wang, Y. (2014). Multiple attribute group decision making methods based on intuitionistic linguistic power generalized aggregation operators. *Applied Soft Computing*, 17, 90–104.

Liu, P., Wu, X. (2012). A competency evaluation method of human resources managers based on multi-granularity linguistic variables and VIKOR method. *Technological and Economic Development of Economy*, 18(4), 696–710.

Liu, H., Mao, L., Zhang, Z., Li, P. (2013a). Induced aggregation operators in the VIKOR method and its application in material selection. *Applied Mathematical Modelling*, 37(9), 6325–6338.

Liu, H., Wu, J., Li, P. (2013b). Assessment of health-care waste disposal methods using a VIKOR-based fuzzy multi-criteria decision making method. *Waste Management*, 33(12), 2744–2751.

Ma, J., Fan, Z.P., Jiang, Y.P., Mao, J.Y. (2006). An optimization approach to multiperson decision making based on different formats of preference information. *IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans*, 36(5), 876–889.

Opricovic, S. (1998). *Multicriteria Optimization of Civil Engineering Systems*. Faculty of Civil Engineering, Belgrade.

Opricovic, S., Tzeng, G.H. (2004). Compromise solution by MCDM methods: a comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455.

Orlovsky, S.A. (1978). Decision-making with a fuzzy preference relation. *Fuzzy Sets and Systems*, 1(3), 155–167.

Park, J.H., Cho, H.J., Kwun, Y.C. (2011). Extension of the VIKOR method for group decision making with interval-valued intuitionistic fuzzy information. *Fuzzy Optimization and Decision Making*, 10(3), 233–253.

Rezaie, K., Ramiyani, S.S., Nazari-Shirkouhi, S., Badizadeh, A. (2014). Evaluating performance of Iranian cement firms using an integrated fuzzy AHP-VIKOR method. *Applied Mathematical Modelling*, 38(21–22), 5033–5046.

Saaty, T.L. (1977). A scaling method for priorities in hierarchical structures. *Journal of Mathematical Psychology*, 15(3), 234–281.

Sadeghi, M., Rashidzadeh, M.A., Soukhakian, M.A. (2012). Using analytic network process in a group decision-making for supplier selection. *Informatica*, 23(4), 621–643.

Shen, K., Yan, M., Tzeng, G. (2014). Combining VIKOR-DANP model for glamor stock selection and stock performance improvement. *Knowledge-Based Systems*, 58, 86–97.

Tanino, T. (1990). On group decision making under fuzzy preferences. In: Kacprzyk, J., Fedrizzi, M. (Eds.), *Multiperson Decision Making Using Fuzzy Sets and Possibility Theory*, Kluwer, Dordrecht, pp. 172–185.

Wan, S., Dong, J. (2014). Multi-attribute group decision making with trapezoidal intuitionistic fuzzy numbers and application to stock selection. *Informatica*, 25(4), 663–697.

Wang, Y. (1998). Using the method of maximizing deviations to make decision for multiindices. *Systems Engineering and Electronics*, 20(7), 24–26, 31.

Wang, Y.M., Yang, J.B., Xu, D.L. (2005). A preference aggregation method through the estimation of utility intervals. *Computers & Operations Research*, 32(8), 2027–2049.

Wu, Z., Chen, Y. (2007). The maximizing deviation method for group multiple attribute decision making under linguistic environment. *Fuzzy Sets and Systems*, 158(14), 1608–1617.

Xu, Z. (2011). Attribute weights determination models for consensus maximization in multiple attribute group decision making. *International Journal of General Systems*, 40(7), 755–774.

Xu, Z. (2013). Group decision making model and approach based on interval preference orderings. *Computers & Industrial Engineering*, 64(3), 797–803.

Xu, Z., Cai, X. (2012). Minimizing group discordance optimization model for deriving expert weights. *Group Decision and Negotiation*, 21(6), 863–875.

Xu, Z., Cai, X. (2013). On consensus of group decision making with interval utility values and interval preference orderings. *Group Decision and Negotiation*, 22(6), 997–1019.

Xu, Y., Wang, H., Sun, H., Yu, D. (2014). A distance-based aggregation approach for group decision making with interval preference orderings. *Computers & Industrial Engineering*, 72, 178–186.

You, T., Fan, Z., Yu, Z. (2012). An assignment method for group decision making with uncertain preference ordinals. *Journal of Systems Science and Systems Engineering*, 21(2), 174–183.

Yu, P.L. (1973). A class of solutions for group decision problems. *Management Science*, 19(8), 936–946.

Yücenur, G.N., Demirel N.Ç. (2012). Group decision making process for insurance company selection problem with extended VIKOR method under fuzzy environment. *Expert Systems with Applications*, 39(3), 3702–3707.

Yue, Z. (2011). An extended TOPSIS for determining weights of decision makers with interval numbers. *Knowledge-Based Systems*, 24(1), 146–153.

Yue, Z. (2012). Approach to group decision making based on determining the weights of experts by using projection method. *Applied Mathematical Modelling*, 36(7), 2900–2910.

Zavadskas, E.K., Turskis, Z. (2011). Multiple criteria decision making (MCDM) methods in economics: an overview. *Technological and Economic Development of Economy*, 17(2), 397–427.

Zeng, S., Balezentis, T., Chen, J., Luo, G. (2013). A projection method for multiple attribute group decision making with intuitionistic fuzzy information. *Informatica*, 24(3), 485-503.

Zhang, Z., Guo, C. (2012). A method for multi-granularity uncertain linguistic group decision making with incomplete weight information. *Knowledge-Based Systems*, 26, 111–119.

Zhang, Z., Guo, C. (2014a). An approach to group decision making with heterogeneous incomplete uncertain preference relations. *Computers & Industrial Engineering*, 71, 27–36.

Zhang, Z., Guo, C. (2014b). Consistency-based algorithms to estimate missing elements for uncertain 2-tuple linguistic preference relations. *International Journal of Computational Intelligence Systems*, 7, 924–936.

Z. Zhang received his PhD in Management Science and Engineering from Dalian University of Technology, China, in 2014. He also holds a BS degree in Engineering Management from China University of Petroleum. He is now an assistant professor with the Institute of Systems Engineering, Faculty of Management and Economics, Dalian University of Technology. From October 2012 to November 2013, he was a visiting PhD student with the Centre for Quantum Computation & Intelligent Systems, University of Technology, Sydney. His current research interests include group decision making, multi-criteria decision making and bi-level decision making. His work has been published on refereed journals and conference proceedings, including *Knowledge-Based Systems, Computers & Industrial Engineering, International Journal of Computational Intelligence Systems and International Journal of Systems Science*, among others.

C. Guo received his BS degree in Mathematics from Liaoning University, China, in 1995, MS degree in Operational Research and Control Theory, and PhD degree in Management Science and Engineering from Dalian University of Technology, China, in 2002. He was a postdoctoral research fellow in the Department of Computer Science, Tsinghua University from 2002 to 2004. Now he is a professor of the Institute of Systems Engineering, Dalian University of Technology. His current research interests include machine learning, data mining, and group decision making. He has published more than 80 papers on refereed journals and conference proceedings, including *IEEE Transactions on Knowledge and Data Engineering, Knowledge-Based Systems, Computers & Industrial Engineering, Knowledge and Information Systems*, among others.

Grupinis sprendimų priėmimas su netiksliais pirmenybių vertinimais ir nevisa svorių informacija taikant VIKOR

Zhen ZHANG, Chonghui GUO

Šis straipsnis skirtas grupinio sprendimų priėmimo problemai, taikant netikslius pirmenybių vertinimus ir nevisą ekspertų svorių informaciją. Pirmiausia apibrėžiami suderinamumo ir nuokrypių tarp netikslių pirmenybių vertinimų matai. Pagal juos sudaromas daugiatikslio optimizavimo modelis, maksimizuojantis kiekvieno eksperto alternatyvų vertinimo nuokrypius ir suderinamumą tarp ekspertų vertinimų. Kompromisinis sprendimų metodas, t. y. VIKOR, papildomas tam, kad galėtų būti taikomas grupiniam sprendimų priėmimui su netiksliais pirmenybių vertinimais. Pateikiami trys pavyzdžiai, rodantys pasiūlyto metodo taikymo galimybes.