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OPTIMIZATION OF THE COLLISION RESOLUTION ALGORITHM

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Abstract. The problem we are dealing with is following. There exist certain number of nodes η, transmitting messages at random time moments. If time interval between messages transmitted by different nodes are less than some given value, a collision occurs. We can fix the collision, but we cannot determine the nodes engaged in the collision. The hierarchical decomposition of the nodes is used to resolve the collision. At every hierarchical level, a subset of nodes "suspected" as participating in the collision is divided in a certain number of groups. There is a time period given to every group, at which messages can be transmitted. This proceeds while no more collisions occurs. This paper covers the problems of selecting a number of groups, to minimize the longest collision resolution time, as well as average collision resolution time.

Key words: multiple access, local area network, collision resolution, optimization.

1. Introduction. The problem of message collision occurs in the local area networks (LAN) operating according to Carrier Sense Multiple Access/Collision Detection (CSMA/CD) protocols (see for example, Liu and Rouse, 1984; Flint, 1986). Among installed LAN more than 50% of networks support this protocol (Personal computers development tendencies, 1989). With the load increase, LAN can become totally blocked due to collisions, thought the network's throughput are sufficient to cope with the load. An another shortage of LAN operating according to CSMA/CD protocol, also caused by collisions, is indeterminate response time. These are the main obstacles for LAN application in the real time information processing.

Several modifications of CSMA/CD protocol are proposed to overcome difficultis caused by collisions (Bersky, 1989; Flint 1989). If there are no collision, LAN is operating according to conventional CSMA/CD protocol. When a collision occurs, system switches to programmed time sharing mode. In this mode a certain number of nodes is attributed to every time slot. If no more collisions occur system switches back to conventional CSMA/CD mode, otherwise the time sharing mode continues. In this particular case a time sharing is carried out only for the nodes, attributed to the time slot at which the collision occured. The such hierarchical decomposition is continued until collision persist.

In general the number of time slots and the number of nodes attributed to one particular slot can be set to some selected values. The main task of this paper is to find optimal decomposition of nodes set and to assign them to the time slots to minimize the collision resolution time. We shall estimate a collision resolution time through the number of time slots the LAN needs to transmit messages.

2. Main assumptions. We shall take the following assumptions to solve this problem:

1) the number of nodes is large enough;
2) an equal number of nodes is attributed to every time slot;
3) the time sharing at a current hierarchical level is carried out to the end, independently from collision detection (if such occurs);
4) only two nodes participates in each collision.

Now we will briefly comment these assumptions. The first assumption is accepted to minimize some relations. For example if it holds, and η - is number of nodes, then ratio $(\eta - 1)/\eta$ approximately equals to 1. The second assumptions means, that at every hierarchical level i, number of nodes η_i can be divided by the number of time slots μ_i without reminder. If this assumption is no true for real system, we can always add dummy nodes to satisfy it. The third assumption mean, that independently on the collision detection or resolution at current hierarchical level time sharing persist up to the end. If the collision is resolved or detected not in the last time slot, this means some redundant use of time slots. Though if number of slots is small (this will be proved further), the result will not be distorted essentially. Moreover, such operating mode can be useful when multiple collision occurs. In the fourth assumption we say, that at every collision participates only 2 nodes. Sometimes it is not true for the very heavy loaded networks. But if a network protocol operates according to the third assumption in most cases the critical time of collision resolution is determined by collision resolution between two nodes.

In accordance to the introduced assumptions we shall solve two problems:

1) minimizing of the maximal collision resolution time;
2) minimizing of the average collision resolution time.

Let us introduce several essential relations. Lets $\eta_0 = \eta$ be a number of nodes in the network, η_i $(i = \overline{1,m})$ a number of nodes attributed to one time slot on the i-th hierarchical level, m – a maximal number of hierarchical levels needed for conflict resolution and μ_i $(i = \overline{1,m})$ a number of time slots on i-th hierarchical level. Between $\eta_0, \eta_i, \eta_{i-1}$ and μ_i the following relationships exist:

$$\eta_i = \eta_{i-1}/\mu_i, \qquad (i = \overline{1,m}), \tag{1}$$

$$\eta_0 = \mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_m. \tag{2}$$

In the last hierarchical level m only 1 point is attributed to each time slot:

$$\eta_m = 1. \tag{3}$$

3. Optimization of the maximal collision resolution time. The minimization of the maximal collision resolution time is equivalent to the minimization of the sum:

$$\sum_{i}^{m} \mu_i \rightarrow \min, \tag{4}$$

where restrictions for μ_i are given by relationships (1) – (3).

For a given problem we does not know neither m, nor μ_i ($i = \overline{1,m}$), thought it is known, that if m are fixed the following relationship is true:

$$\mu_1 = \mu_2 = \ldots = \mu_m = \overset{*}{\mu}. \tag{5}$$

So the (4) can be expressed as:

$$m \cdot \eta_0^{1/m} \rightarrow \min. \tag{6}$$

From the (6) we get an optimal number of hierarchical levels to minimize the longest collision resolution time:

$$m = \ln \eta_0. \tag{7}$$

By taking into account (2) and (7) the optimal number of time slots at the every hierarchical level can be evaluated in the following way:

$$\overset{*}{\mu} = \eta_0^{1/m} = \eta_0^{1/\ln \eta_0} = e \approx 3. \tag{8}$$

The received result can be formulated as a theorem:

Theorem 1. *To minimize the longest collision resolution time at every hierarchical level the set of nodes have to be divided into 3 equal parts. In this case the maximal number of hierarchical levels is* $\ln \eta_0$, *and the number of time slots is approximately equal to* $3 \ln \eta_0$.

4. Average collision resolution time optimization. The average collision resolution time minimization is much more complicated. In this case we have to deal with probabilities, which evaluate possibilities of collision resolution at every hierarchical level. To get these probabilities some work have to be done.

Let's p'_i means probability, that some node is attributed to the some time slot at a hierarchical level i ($i = \overline{1,m}$) if we know, that collision is not resolved at previous levels. p'_i can be evaluated through η_i and μ_i in the following way:

$$p'_i = (\eta_{i-1}/\mu_i) \cdot 1/\eta_{i-1} = 1/\mu_i. \tag{9}$$

Lets p_i'' means probability that some other node is attributed to the same time slot. This probability can be expressed:

$$p_i'' = p_i' \cdot (\eta_{i-1}/\mu_i - 1)/(\eta_{i-1} - 1). \tag{10}$$

In accordance with the first assumption probability p_i'' approximately can be evaluated by the following expression:

$$p_i'' \approx p_i' \cdot 1/\mu_i = 1/\mu_i^2. \tag{11}$$

If certain two nodes are ready to send messages p_i'' means probability, that they both will be attributed to the same time slot and a collision occurs in that particular time slot. A number of time slots at i-th hierarchical level is μ_i, therefore a probability p_i, that at this hierarchical level occurs collision can be estimated as probability, that collision occurs in any time slot:

$$p_i = \sum_{j=1}^{\mu_i} p_i'' = \sum_{j=1}^{\mu_i} 1/\mu_i^2 = 1/\mu_i. \tag{12}$$

Probabilities p_i $(i = 1, \overline{m-1})$ would be used to evaluate average number of time slots required for collision resolution. Fot this purpose we would write recurrent expressions for an average number $\overset{*}{\mu}_i$ of time slots needed for the collision resolution, on the i-th and lower levels, if we are aware, that collision was not resolved on previous levels. The first part of that expresion forms production of number of time slots on the i-th level and of probability, that collision will be resolved at that level $(1-p_i)$. The second part consist of number of required time slots if collision is not resolved on current level multiplied by probability of this event.

For the first level we can write the following recurrent expression:

$$\mu = \overset{*}{\mu}_1 = (1 - p_1) \cdot \mu_1 + p_1 \cdot (\mu_1 + \overset{*}{\mu}_2) = \mu_1 + \overset{*}{\mu}_2 \, p_1. \tag{13}$$

Taking into account the (12) and (13) we can write:

$$\mu = \overset{*}{\mu}_1 = \mu_1 + \overset{*}{\mu}_2 / \mu_1. \tag{14}$$

For every i-th level:

$$\overset{*}{\mu}_i = (1-p_i)\mu_i + p_i(\mu_i + \overset{*}{\mu}_{i+1}) = \mu_i + \overset{*}{\mu}_{i+1}/\mu_i, \quad (i = 1, \overline{m-1}). \quad (15)$$

The (14) and (15) allows us to express μ in the following way:

$$\mu = \mu_1 + [\mu_2 + (\mu_3 + \ldots)/\mu_2]/\mu_1. \quad (16)$$

Taking into account the relation (2) between η_0 and μ_i the optimization of average number of time slots can be expressed as the following optimization problem:

$$\begin{cases} \mu = \mu_1 + \mu_2/\mu_1 + \mu_3/(\mu_2 \cdot \mu_3) + \ldots \\ \qquad + \mu_m/(\mu_1 \cdot \ldots \cdot \mu_{m-1}) \to \min, \\ \eta_0 = \mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_m. \end{cases} \quad (17)$$

In the (17) the variables μ_i are unknown as well as the number m itself.

The key for the solution can be found from the certain relation between μ_i and μ_{i-1}, which we can get from the (17) fixing m and applying the method of Lagrange (see, for example, Taha, 1982). After some exercising we can get, the relation between μ_i and μ_{i-1} in this following form:

$$\mu_i = \mu_{i-1}^2/2. \quad (18)$$

From this relation follows, that μ_1 for every i, can be expressed as certain function of μ_1:

$$\mu_i = \mu_1^{\alpha_i}/2^{\alpha_i - 1}, \quad (19)$$

where $\alpha_i = 2^{i-1}$.

The following step of a solution consist of putting the expression of μ_i in the (2):

$$\eta_0 = \mu_1 \cdot \mu_2 \cdot \ldots \cdot \mu_m = \mu_1^{\beta}/2^{\beta - m}, \quad (20)$$

where $\beta = \sum_1^m \alpha_i$.

From the (20) we get an evaluation of μ_1 through η_0, α_i and m:

$$\mu = \sqrt[\beta]{\eta_0 \cdot 2^{\beta - m}}. \qquad (21)$$

In the (21) β can be substituted by its value. It is easy to show that a sequence $\alpha_1, \alpha_2, \ldots, \alpha_m$ are geometrical progression with the first member equal to 1 and power equal to 2. Therefore the sum $\sum_1^m \alpha_i$ can be counted as a sum of geometrical progression:

$$\sum_1^m \alpha_i = 1 \cdot \frac{2^m - 1}{2 - 1} = 2^m - 1. \qquad (22)$$

From the (22) and (21) we can get:

$$\mu_1 = 2 \cdot (\eta_0 / 2^m)^{1/(2^m - 1)}. \qquad (23)$$

Relations (18) and (23) can be used to eliminate variables in a problem (17). Before doing this let's modify function in the (17). For this purpose lets multiply minimizing function by η_0. This doesn't change arguments of a solution, because for every problem η_0 is constant. By doing this we get F instead of μ as minimizing function:

$$F = \eta_0 \cdot \mu = \mu_1^2 \cdot \mu_2 \cdot \ldots \cdot \mu_m + \ldots + \mu_m^2. \qquad (24)$$

According to the (19) and (22) an every product in F can be expressed through μ_1 and m in the following way:

$$\mu_i^2 \cdot \mu_{i-1} \cdot \ldots \cdot \mu_m = \mu_1^{2^m} / 2^{2^m - m - 2 + i}. \qquad (25)$$

The relation (25) is true for every i and even for $i = m$ therefore F can be expressed like:

$$F = \sum_1^m \mu_1^{2^m} / 2^{2^m - m - 2 + i} = \mu_1^{2^m} \cdot \frac{2^m - 1}{2^{2^m - 2}}. \qquad (26)$$

If in the (26) μ_1 is substituted by its expression form the (23) we will get:

$$F = 4 \eta_0^{2^m/(2^m - 1)} \cdot 2^{-m \cdot 2^m/(2^m - 1)} \cdot (2^m - 1). \qquad (27)$$

From (27) we directly get, that total number of hierarchical levels m is:

$$m = \log_2 \eta_0. \tag{28}$$

By substituting m in the (23) by its expression from the (28) we get:

$$\mu_1 = 2 \cdot (\eta_0 / 2^{\log_2 \eta_0})^{1/(2^{\log_2 \eta_0} - 1)} = 2. \tag{29}$$

Eventually from (19) and (29) we get that:

$$\mu_1 = \mu_2 = \ldots = \mu_m = 2. \tag{30}$$

All the results can be stated as the following theorem:

Theorem 2. *To minimize the average time of collision resolution, the set of nodes in every hierarchical level have to be divided in 2 equal parts. In this case the average number μ of time slots required for the collision resolution is $4(\eta_0 - 1)/\eta_0$ and the maximal number of hierarchical levels is $\log_2 \eta_0$ and the average number of hierarchical levels is $2(\eta_0 - 1)/\eta_0$.*

5. Conclussions. Here we analyzed the hierarchical decomposition method of collisions resolution for the situation when the maximal and the average collision resolution time is minimized. Proved, that to minimize the maximal collision resolution time a set of nodes of every hierarchical level have to be divided in 3 equal parts. In this case number of time slots approximately equals to $3 \ln \eta_0$.

To minimize the average collisions resolutions time a set of nodes at every hierarchical level have to be divided in 2 equal parts. In this case the average number of time slots needed for collisions resolutions is $4(\eta_0 - 1)/\eta_0$ and the average number of hierarchical levels is $2(\eta_0 - 1)/\eta_0$.

REFERENCES

Bersky, D. (1989). Deterministic algorithm makes possible to use LAN in plants. *Electronika*, **21**, 3–4 (in Russian).

Flint, D. (1986). *The Data Ring Main. An Introduction to Local Area Networks.* Moscow. 359pp. (in Russian).

Liu, M.T., and D.M.Rouse (1984). A study of ring networks. In I.N.Dallas and E.B.Spratt (Eds.), *Ring Technology Local Area Networks,* Elseviere Science Publishers, North-Holland. pp. 1–39.

Personal Computers Development Tendencies. (1989). Computer Press, 2. pp. 2–25 (in Russian).

Taha, H.F. (1982). *Operations Research. An Introduction.* II. Moscow. 496pp. (in Russian).

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