

## PSYCHOANALITIC MODELS AS DYNAMICAL SYSTEMS

Zinovy V. REYTBLETT

Dept. of Mathematics, Illinois Institute of Technology  
Chicago, IL 60616, USA

Cesar SCIAMMARELLA

Dept. of Aerospace Eng., Illinois Institute of Technology  
Chicago, IL USA

Esther E. SCIAMMARELLA

SILA Mental Health Program, Chicago, IL USA

Leonor TORRES

Lacano Argentina, Buenas Aires, Argentina

**Abstract.** The Freudian psychoanalysis in its modern form assumes that activities of a patient depend on his physical and mental state ("energy"), and result in maintaining his life, in useless waste of his energy ("symptoms"), and in ("useful") contribution to the society and to patient's energy level.

Another contribution to patient's energy level comes from the society. It comprises life amenities, medication, and "education". Patient's mental state is characterized by two parameters, "symbolic" and "imaginary". Both parameters affect the outcome of patient activities, and are affected by contributions to his "energy".

A mathematical description of this model as a dynamical system is presented. Significance of obtained solutions for psychoanalysis is discussed.

**Key words:** Freudian psychoanalysis, mathematical modeling.

**1. Introduction.** The Freudian psychoanalysis (Freud, 1893; Freud, 1966) is a phenomenological theory that considers the "mind" as a black box with the input in the form of life amenities, sensory perceptions, and direct social means (such as "education", "medication", etc.) impacting on an individual.

The output of the black box, coupled with the "pulsion of the body", results in maintaining the body physical conditions ("survival"), in production of statements and actions that are considered useless, "wasteful" by the social establishment (so-called "symptoms"), and in the production of statements and actions considered "normal", "useful" by the society (Maci, 1979; Maci, 1983) (Fig. 1).

We postulate that the rate at which the output is generated is proportional to the "energy" level of a patient. A specific distribution of the output of the "mind" into the three above categories ("autopreservation", "symptoms", and the "useful output") depends on the physical and mental state of an individual. The mental state in a common version of modern psychoanalysis is characterized by its general level and by the ratio of so-called "imaginary" and "symbolic" components of data stored in the brain (including the "rules", that is, the software that processes the data) (Lacan, 1977; Lacan, 1988).

The modern psychoanalysis has not yet developed an unconditional test for measurement of these components. The "imaginary" component is associated, primarily, with unconscious thinking, memorization of "pictures", emotions etc. The "symbolic" component is associated with conscience and memorization of abstract information.

The rate at which the input is acquired by an individual and is eventually fed into the black box depends on the physical and mental state of the individual ("energy"), on the availability of the amenities, education, medication, and on the feedback from the black box output.

With certain simplifying assumptions, measures of introduced notions and their change rates satisfy certain balance equations which can be reduced to ordinary differential equations. Thus, the Freudian psychoanalytic model of human behavior can be formulated as a dynamical system (Lorenz, 1963).

**2. Mathematical formulation of Freudian models.** The mental state of a patient is associated with a certain parameter ("Mental State Measure")  $x$ ,  $x = x(t)$ , where  $t$  is time. The "nor-

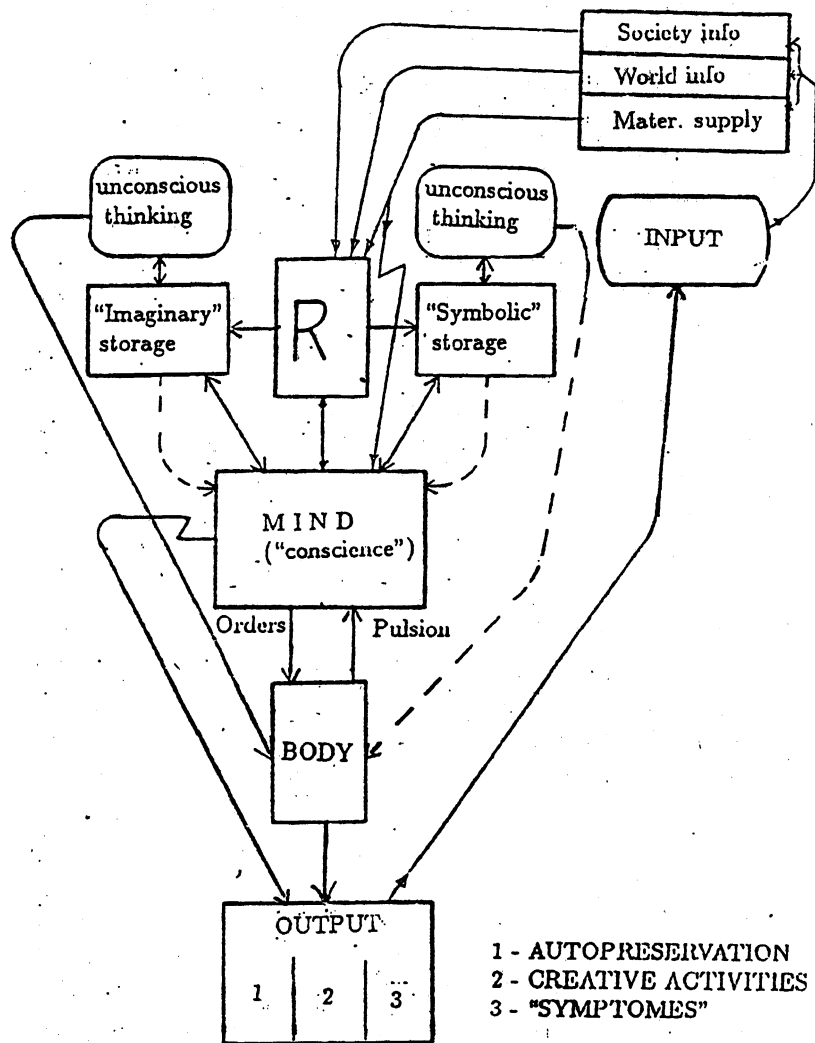


Fig. 1. Generalized model of a patient.

mal" state corresponds to  $x = x_0 = x(t_0)$ , and larger values of  $|x - x_0|$  correspond to higher degrees of mental "abnormality".

We introduce measures of imaginary and symbolic components,  $i(t)$  and  $s(t)$  where  $i + s = 1$ .

If the total amount of information stored in the brain is  $u(t)$ , then, in the first approximation, the dimensionless equations for  $x(t)$  are assumed as

$$x = x_0 + (s(t) - s_0(t))[1 + u(t)/u_0(t)],$$

where  $s_0(t)$  and  $u_0(t)$  are "normal", average values at the age  $t$ .

Activities of a patient are categorized into three generalized categories:

- 1) physiological activities aimed at self-preservation as of a biological organism;
- 2) "wasteful" activities ("symptoms") which may only decrease  $x - x_0$ ;
- 3) "useful" activities that increase  $x - x_0$ .

The "production" rates from these activities are denoted as  $a_1(t)$ ,  $a_2(t)$ ,  $a_3(t)$ , respectively.

The potential of a patient to perform these activities is called  $y$ ,  $y = y(t)$ <sup>1</sup>.

In the present paper we assume that  $a_k(t)$  are linear differential operators of the first order with the coefficients depending on  $x(t)$ .

$$a_k = \beta_k(x)y + \gamma_k(x)y' + \dots$$

Patient's energy,  $y(t)$ , is replenished via interaction with the environment, and its supply rate is assumed, also, to depend on the patient activities and his mental state,  $x(t)$ .

Thus, it is assumed that the rate of change of energy,  $y'(t)$  satisfies the balance equation,

$$y'(t) = \sum_k^1 a_k + f_1, \quad (1)$$

<sup>1</sup> the term "energy" is used for brevity.

where  $c_k^1$  depend on  $x$ , and, therefore, on  $u$  and  $s$ . The function  $f_1(t)$ , and functions  $f_2(t)$  and  $f_3(t)$  introduced below describe the impact of environment.

The rate of change of information stored in brain,  $u'(t)$ , is assumed to depend on the environment (health care, "education", etc.), on patient's energy, and on his mental state. Thus, it is assumed that  $u'(t)$  satisfies the following (linearized) equation:

$$u'(t) = \sum c_k^2 a_k + f_2, \quad (2)$$

where  $c_k^2$  are certain constants pertinent to a particular patient.

The question of a reasonable model for the symbolic component change rate was decided in favor of a generalized S-shaped curve for given constant parameters:

$$s' = s(\sum c_k^3 a_k + f_3 - qs). \quad (3)$$

Thus, the system of three differential equations describing the Freudian model as a dynamical system has been formed.

**3. Normal and quasi-normal child's development.** This process is characterized by balanced "imaginary" to "symbolic" ratio, so that  $s(t)$  changes relatively slowly and the model can be simplified by assuming that  $s'(t) = 0$ .

If the society impact is assumed as almost constant in time, then the system of Eq. 2, 3 becomes autonomous. Moreover, this system with further simplifications has solutions similar to "logistic" curves. Small perturbations on the society impact do not change the solution qualitatively.

Such behavior is in line with generally accepted views on phases of children's development. Both, the "energy" and the amount of information stored in the brain, increase similarly to S-shaped curves.

**4. Patients with periodic remission.** Another important limiting case takes place when the "energy"  $y(t)$  is almost constant so that the Eq. 1 can be dropped out. Then, with some simplifying assumptions, Eq. 2, 3 form a system of two linear differential

equations of the first order, solution of which may exhibit either periodic (with bounded or unbounded amplitudes) or exponential character. With the given initial conditions, the choice of timing and intensity of the society impact (medication, surgery, psychoanalytic sessions, etc.) becomes crucial for an advanced prognosis. It appears that the case presents the only opportunity for experimental determination of the parameters of the model peculiar to a given patient via measurement of patient's responses to the known quantified society impact.

**5. General case.** Selected numerical solutions of the non-linear system Eq. 1 - 3 even with simplified assumptions exhibit very complicated behavior. With certain simplifications the system resembles Lorenz's system (Lorenz, 1963), and becomes exactly a Lorenz's system under certain conditions.

The numerical solutions in the general case confirm complicated basins, and strange attractors appear to exist.

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