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Some Generalized Interval-Valued 2-Tuple Linguistic Correlated Aggregation Operators and Their Application in Decision Making

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Abstract. With respect to multi-attribute decision making under uncertain linguistic environment, a new interval-valued 2-tuple linguistic representation model is introduced. To deal with the situation where the elements in a set are interdependent, several generalized interval-valued 2-tuple linguistic correlated aggregation operators are defined. It is worth pointing out that some interval-valued 2-tuple linguistic operators based on additive measures are special cases of our operators. Meanwhile, several special cases and desirable properties are discussed. Furthermore, models based on the correlation coefficient are constructed, by which the optimal weight vector can be obtained. Moreover, an approach to multi-attribute group decision making with uncertain linguistic information is developed. Finally, an example is selected to show the effectivity and feasibility of the developed procedure.

Key words: multi-attribute decision making, interval-valued 2-tuple linguistic variable, Choquet integral, generalized Shapley function.

1. Introduction

Decision making is one of the most common activities for human beings, which is defined in uncertain, vague and imprecise situations. Thus, the experts usually express their preferences using fuzzy variables, such as fuzzy sets (Zadeh, 1965), type-2 fuzzy sets (Zadeh, 1973), intuitionistic fuzzy sets (Atanassov, 1986), and hesitant fuzzy sets (Torra, 2010). All these fuzzy tools are defined for quantitative situations. However, in many situations, it is more suitable to express the experts' preferences using qualitative variables rather than quantitative variables (Zadeh, 1975a, 1975b, 1975c), which is usually expressed by linguistic variables, such as "light", "fair", and "heavy". To well deal with qualitative variables, different kinds of linguistic variables are presented, such as linguistic variables (Zadeh, 1975a, 1975b, 1975c), uncertain linguistic variables (Xu, 2004a, 2004c), and hesitant fuzzy linguistic term sets (Rodríguez *et al.*, 2012).

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Since linguistic variables were first introduced by Zadeh (1975a, 1975b, 1975c), the fuzzy linguistic approach has been applied in many fields and applications, such as decision making (Ben-Arieh and Chen, 2006; Dong *et al.*, 2009a; Martínez *et al.*, 2010; Merigó *et al.*, 2010; Wang, 2013; Xu, 2004b; Zha, 2013), medical care (Becker, 2001), military system (Wu and Mendel, 2010), risk assessment (Liu *et al.*, 2010), engineering evaluation (Martínez *et al.*, 2005, 2007), and social choice (Garcia-Lapresta *et al.*, 2010). At present, there are main two kinds of computing methods for linguistic variables. One uses the membership function of linguistic terms (Degani and Bortolan, 1988; Martin and Klir, 2006), the other is based on the predefined ordinal scales (Yager, 1981). It needs to point out that model based on the predefined ordinal scales has been widely applied to decision-making processes for its easy calculation and application (Delgado *et al.*, 1993; Herrera and Martínez, 2000a; Wei, 2010, 2011a; Xu, 2004b, 2004c, 2004d, 2006, 2007; Xu *et al.*, 2013; Yager, 1993, 1995).

The 2-tuple fuzzy linguistic model (Herrera and Martínez, 2000a), an effective computational approach for computing with words (CW), can well deal with the process of linguistic information. Since it was first proposed by Herrera and Martínez (2000a), it has provided very good results in many fields and applications (Dong *et al.*, 2009a; Herrera and Martínez, 2000b; Herrera-Viedma *et al.*, 2007; Li *et al.*, 2009; Liu *et al.*, 2011; Moreno *et al.*, 2005; Yu, 2009; Zeng *et al.*, 2012). As Martínez and Herrera (2012) noted: "this success would not have been possible without methodologies to carry out the processes of computing with words (CW) that implies the use of linguistic information". As a key aspect of CW, many 2-tuple linguistic aggregation operators have been proposed, such as the 2-tuple linguistic weighted averaging (TLWA) operator (Herrera and Martínez, 2000a), the 2-tuple linguistic ordered weighted averaging (TLOWA) operator (Herrera and Martínez, 2000a), the extended 2-tuple linguistic weighted averaging (ET-LWA) operator (Herrera and Martínez, 2000a), the 2-tuple ordered weighted geometric (TOWG) operator (Jiang and Fan, 2003), the extended 2-tuple linguistic ordered weighted averaging (ET-LOWA) operator (Zhang and Fan, 2006), the 2-tuple hybrid weighted averaging (T-HWA) operator (Xu, 2004c), the 2-tuple linguistic weighted geometric averaging (TLWGA) operator (Xu and Huang, 2008), the extended 2-tuple linguistic ordered weighted geometric (ET-LOWG) operator (Wei, 2010), the induced generalized 2-tuple linguistic ordered weighted averaging (IG-2TLOWA) operator (Wei, 2011a), the 2-tuple linguistic power averaging (2TLPA) operator (Xu and Wang, 2011), the 2-tuple linguistic power ordered weighted averaging (2TLPOWA) operator (Xu and Wang, 2011), the dependent 2-tuple ordered weighted averaging (D2TOWA) operator (Wei and Zhao, 2012), the dependent 2-tuple ordered weighted geometric (D2TOWG) operator (Wei and Zhao, 2012), and the induced 2-tuple linguistic generalized ordered weighted averaging (2-TILGOWA) operator (Xu, 2004c). To cope with the situation where the elements in a set are correlative, some 2-tuple Choquet integral operators are defined, such as the 2-tuple linguistic induced quasi-arithmetic Choquet integral aggregation (Quasi-2-TLICIA) operator (Merigó and Gil-Lafuente, 2013), and the generalized 2-tuple correlated averaging operator (Xu, 2004d). Because the 2-tuple linguistic variable only addresses the situation where the attribute value of alternative is one linguistic term from the predefined ordinal

scales. This makes it insufficient to completely express the expert's personal judgment on the attribute values of alternatives.

To cope with this issue, several other types of linguistic variables are presented, such as uncertain linguistic variables (Xu, 2004a) and hesitant fuzzy linguistic term sets (Rodríguez *et al.*, 2012). With respect to uncertain linguistic variables, similar to the 2-tuple fuzzy linguistic model (Herrera and Martínez, 2000a) for linguistic variables, Lin *et al.* (2009) introduced the interval-valued 2-tuple linguistic model to deal with uncertain linguistic information that avoids the information loss and distortion in the process of uncertain linguistic information. Later, Zhang (2012, 2013) further researched the interval-valued 2-tuple linguistic model and defined some interval-valued 2-tuple linguistic aggregation operators, such as the interval-valued 2-tuple weighted average (IVTWA) operator, the interval-valued 2-tuple ordered weighted average (IVTOWA) operator, the generalized interval-valued 2-tuple weighted average (GIVTWA) operator, and the generalized interval-valued 2-tuple ordered weighted average (GIVTOWA) operator. At present, the interval-valued 2-tuple linguistic aggregation operators are all based on the assumption that the elements in a set are independent. However, in many real decision-making problems, there is usually a degree of interdependence between elements. For example, "we are to evaluate three companies according to three attributes: economic benefits, environment benefits, social benefits, we want to give more importance to environment benefits than to economic benefits or social benefits, but on the other hand we want to give some advantage to companies that are good in environment benefits and in any of economic benefits and social benefits". In this situation, the aggregation operators based on additive measures seem to be insufficient. The purpose of this paper is to define some new generalized interval-valued 2-tuple linguistic aggregation operators using the Choquet integral and the generalized Shapley function that address the situation in which the elements in a set are interdependent. Meanwhile, some important cases and desirable properties are considered. Furthermore, models for the optimal weight vector are constructed. Then, an approach to uncertain linguistic multi-attribute group decision making with incomplete weight information and interactive characteristics is developed.

This paper is organized as follows: Section 2 introduces several relative concepts, such as interval-valued 2-tuple linguistic variables, interval-valued 2-tuple linguistic representation models, the Choquet integral and the generalized Shapley function. Section 3 defines several generalized interval-valued 2-tuple linguistic aggregation operators that are based on the Choquet integral and the generalized Shapley function. Meanwhile, several special cases and desirable properties are studied. Section 4 constructs several models based on the correlation coefficient, by which the optimal vector on an attribute set, on an expert set and on their ordered sets can be respectively obtained. Furthermore, an approach to multi-attribute group decision making with uncertain linguistic information is developed. Section 5 uses an example to illustrate the concrete application of the procedure. Conclusion is made in the last section.

2. Basic Concepts

2.1. *Interval-Valued 2-Tuple Linguistic Variables*

The linguistic approach is an approximate technique to represent qualitative aspects using linguistic variables. Let $S = \{s_i \mid i = 0, 1, \ldots, t\}$ be a linguistic term set with odd cardinality. Any label s_i represents a possible value for a linguistic variable, and it should satisfy the following characteristics (Herrera and Martínez, 2000a):

(1) The set is ordered: $s_i > s_j$, if $i > j$;
(2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \geqslant s_j$;
(3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \leqslant s_j$;
(4) A negation operator: $neg(s_i) = s_j$ such that $j = t - i$.

For example, the linguistic term set S can be expressed by $S = \{s_0$: very light, s_1: light, s_2: fair, s_3: heavy, s_4: very heavy$\}$.

To represent the linguistic assessment information, Herrera and Martínez (2000a) introduced the 2-tuple linguistic representation model that is based on the concept of symbolic translation. A 2-tuple linguistic variable is composed by a linguistic term and a real number, denoted by a 2-tuple (s_i, α_i) with s_i being a linguistic term from predefined linguistic term set S and $\alpha_i \in [0.5, 0.5)$.

Definition 1. (See Herrera and Martínez, 2000a.) Let β be the result of an aggregation of the indices of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation, $\beta \in [0, t]$, with t being the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, t]$ and $\alpha \in [0.5, 0.5)$, then α is called a symbolic translation.

Definition 2. (See Herrera and Martínez, 2000a.) Let $S = \{s_0, s_1, \ldots, s_t\}$ be a linguistic term set, and $\beta \in [0, t]$ be a number value representing the aggregation result of linguistic symbolic, then the 2-tuple linguistic variable that expresses the equivalent information to β is obtained with the following function Δ:

$$\Delta : [0, t] \to S \times [0.5, 0.5),$$

$$\Delta(\beta) = (s_i, \alpha_i), \quad \text{with } \begin{cases} s_i, & i = round(\beta), \\ \alpha_i = \beta - i, & \alpha_i \in [-0.5, 0.5), \end{cases}$$

where $round(\cdot)$ is the usual round operation, s_i has the closest index label to β and α_i is the value of the symbolic translation.

Definition 3. (See Herrera and Martínez, 2000a.) Let $S = \{s_0, s_1, \ldots, s_t\}$ be a linguistic term set, and (s_i, α_i) be a 2-tuple linguistic variable. There is always a function Δ^{-1}:

$$\Delta^{-1} : S \times [0.5, 0.5) \to [0, t],$$

$$\Delta^{-1}(s_i, \alpha_i) = i + \alpha_i = \beta.$$

Later, Chen and Tai (2005) introduced another form of the 2-tuple linguistic representation model.

DEFINITION 4. (See Chen and Tai, 2005.) Let $S = \{s_i \mid i = 0, 1, \ldots, t\}$ be a linguistic term set with odd cardinality, then any $\beta \in [0, 1]$ can be transformed into a 2-tuple linguistic variable, denoted by

$$\Delta(\beta) = (s_i, \alpha) \quad \text{with} \begin{cases} s_i, & i = round(\beta * t), \\ \alpha = \beta - i/t, & \alpha \in [-0.5/t, 0.5/t). \end{cases}$$

From Definition 4, one can conclude that any 2-tuple linguistic variable (s_i, α) can be converted into a crisp value $\beta \in [0, 1]$, denoted by $\Delta^{-1}(s_i, \alpha) = i/t + \alpha$. This representation model can deal with linguistic decision-making problems with multi-granularity linguistic term sets.

Similar to Herrera and Martínez (2000a), Lin *et al.* (2009) introduced the concept of interval-valued 2-tuple linguistic variables. Later, Zhang (2012) gave the definition of interval-valued 2-tuple linguistic variables as follows:

DEFINITION 5. (See Zhang, 2012.) Let $S = \{s_i \mid i = 0, 1, \ldots, t\}$ be a linguistic term set with odd cardinality. An interval-valued 2-tuple linguistic variable is composed of two linguistic terms and two numbers, denoted by $[(s_i, \alpha_1), (s_j, \alpha_2)]$, where $i \leqslant j$ and $\alpha_1 \leqslant \alpha_2$ if $i = j$, $s_i(s_j)$ represents the linguistic label in S and $\alpha_1(\alpha_2)$ is the value of the symbolic translation. Equivalently, any interval $[\beta_1, \beta_2] \subseteq [0, 1]$, $\beta_1 \leqslant \beta_2$, can be expressed by the interval-valued 2-tuple linguistic variable as follows:

$$\Delta\big([\beta_1, \beta_2]\big) = \big[(s_i, \alpha_1), (s_j, \alpha_2)\big], \quad \text{with} \begin{cases} s_i, & i = round(\beta_1 * t), \\ s_j, & j = round(\beta_2 * t), \\ \alpha_1 = \beta_1 - i/t, & \alpha_1 \in [-0.5/t, 0.5/t), \\ \alpha_2 = \beta_2 - j/t, & \alpha_2 \in [-0.5/t, 0.5/t). \end{cases}$$

From Definition 5, one can conclude that any interval-valued 2-tuple linguistic variable $[(s_i, \alpha_1), (s_j, \alpha_2)]$ can be converted into an interval $[\beta_1, \beta_2] \subseteq [0, 1]$, $\beta_1 \leqslant \beta_2$, denoted by $\Delta^{-1}([(s_i, \alpha_1), (s_j, \alpha_2)]) = [i/t + \alpha_1, j/t + \alpha_2]$. From Definition 5, we know that the value of α is small. For example, if $t = 8$, then $\alpha \in [0.0625, 0.0625)$. To extend the range of the value α, similar to Herrera and Martínez (2000a), we introduce another interval-valued 2-tuple linguistic representation model as follows:

DEFINITION 6. Let $S = \{s_i \mid i = 0, 1, \ldots, t\}$ be a linguistic term set with odd cardinality. An interval-valued 2-tuple linguistic variable is composed of two linguistic terms and two numbers, denoted by $[(s_i, \alpha_1), (s_j, \alpha_2)]$, where $(s_i, \alpha_1) \leqslant (s_j, \alpha_2)$ if $i + \alpha_1 \leqslant j + \alpha_2$, $s_i(s_j)$ is the linguistic term in S and $\alpha_1(\alpha_2)$ is the value of the symbolic translation. Furthermore, any interval $[\beta_1, \beta_2] \subseteq [0, 1]$, $\beta_1 \leqslant \beta_2$, can be expressed by interval-valued 2-tuple linguistic variable as follows:

$$\Delta\big([\beta_1,\beta_2]\big)=\big[(s_i,\alpha_1),(s_j,\alpha_2)\big],\quad \text{with}\ \begin{cases} s_i, & i=round(\beta_1*t),\\ s_j, & j=round(\beta_2*t),\\ \alpha_1=\beta_1 t-i, & \alpha_1\in[-0.5,0.5),\\ \alpha_2=\beta_2 t-j, & \alpha_2\in[-0.5,0.5). \end{cases}$$

From Definition 6, one can conclude that any interval-valued 2-tuple linguistic variable $[(s_i,\alpha_1),(s_j,\alpha_2)]$ can be converted into an interval $[\beta_1,\beta_2]\subseteq[0,1]$, denoted by $\Delta^{-1}([(s_i,\alpha_1),(s_j,\alpha_2)])=[(i+\alpha_1)/t,(j+\alpha_2)/t]$.

Remark 1. Without special explanation, in this paper we adopt the interval-valued 2-tuple linguistic representation model given in Definition 6. This representation model is convenient to compare interval-valued 2-tuple linguistic variables from different multi-granularity linguistic term sets.

Example 1. Let $S=\{s_i\mid i=0,1,\ldots,6\}$ be the predefined linguistic term set. For the interval $[\beta_1,\beta_2]=[0.4,0.7]$, by Definition 5 we have $\Delta([\beta_1,\beta_2])=[(s_2,0.07),(s_4,0.03)]$. On the other hand, from Definition 6 we obtain $\Delta([\beta_1,\beta_2])=[(s_2,0.4),(s_4,0.2)]$.

For any interval-valued 2-tuple linguistic variable $A=[(s_i,\alpha_1),(s_j,\alpha_2)]$, similar to Zhang (2012), the score function is defined to get the score of A, denoted by $S(A)=(i+j+\alpha_1+\alpha_2)/2t$, and the accuracy function is given to evaluate the accuracy degree of A, expressed by $H(A)=(j+\alpha_2-i-\alpha_1)/2t$. Furthermore, an order relationship, for any two interval-valued 2-tuple linguistic variables $A=[(s_i,\alpha_1),(s_j,\alpha_2)]$ and $B=[(s_k,\alpha_3),(s_l,\alpha_4)]$, is defined as follows:

If $S(A)<S(B)$, then $A<B$.

If $S(A)=S(B)$, then $\begin{cases} H(A)=H(B)\Rightarrow A=B,\\ H(A)<H(B)\Rightarrow A>B. \end{cases}$

From the expressions of the score and accuracy functions, one can derive that $S(A)=(\beta_1+\beta_2)/2$ and $H(A)=(\beta_2-\beta_1)/2$ with $\Delta^{-1}(A)=[\beta_1,\beta_2]$.

2.2. *The Choquet Integral and the Generalized Shapley Function*

As many researchers (Grabisch, 1996; Meng *et al.*, 2014a, 2014b, 2014c, 2015; Meng and Chen, 2015a, 2015b, 2016; Meng and Zhang, 2014; Tan and Chen, 2010, 2011; Xu, 2010; Xu and Xia, 2011; Yager, 2003; Zhang *et al.*, 2011) have noted, in many situations where the elements are interdependent. In this case, the aggregation operators based on additive measures seem to be insufficient, whereas the aggregation operators based on fuzzy measures (or non-additive measures) seem to well cope with this issue. This subsection reviews the concepts of the Choquet integral and the generalized Shapley function.

Definition 7. (See Sugeno, 1974.) A fuzzy measure on a finite set $N=\{1,2,\ldots,n\}$ is a set function $\mu: P(N)\to[0,1]$ satisfying

(1) $\mu(\emptyset)=0,\mu(N)=1$,

(2) If $A,B\in P(N)$ with $A\subseteq B$, then $\mu(A)\leqslant\mu(B)$, where $P(N)$ is the power set of N.

DEFINITION 8. (See Murofushi and Sugeno, 1989.) Let f be a positive real-valued function on X, and μ be a fuzzy measure on $A = \{x_1, x_2, \ldots, x_n\}$. The discrete Choquet integral of f with respect to μ is defined by

$$C_\mu\big(f(x_{(1)}), f(x_{(2)}), \ldots, f(x_{(n)})\big) = \sum_{i=1}^{n} f(x_{(i)})\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big),$$

where $(\cdot)$ indicates a permutation on N such that $f(x_{(1)}) \leqslant f(x_{(2)}) \leqslant \cdots \leqslant f(x_{(n)})$, and $A_{(i)} = \{x_{(i)}, \ldots, x_{(n)}\}$ with $A_{(n+1)} = \emptyset$.

From Definition 8, we know that the Choquet integral degenerates to the OWA operator if there are no interactions. Based on the fuzzy measure, the Choquet integral addresses the interactions between elements. However, it only considers the importance of the ordered positions and gives the interactions between adjacent coalitions $A_{(i)}$ and $A_{(i+1)}$, $i = 1, 2, \ldots, n$.

The generalized Shapley function proposed by Marichal (2000) is an effective tool to reflect the interactions between coalitions in game theory, denoted by

$$\Phi_S(\mu, N) = \sum_{T \subseteq N \setminus S} \frac{(n-s-t)!t!}{(n-s+1)!}\big(\mu(S \cup T) - \mu(T)\big) \quad \forall S \subseteq N,$$

where n, t and s are the cardinalities of N, T and S, respectively.

From Definition 7, one can easily derive that $\Phi_S(\mu, N) \geqslant 0$ for any $S \subseteq N$. Furthermore, for any ordered set $A_{(i)}$, we have $\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N)) = 1$ and $\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N) \geqslant 0$, $i = 1, 2, \ldots, n$. It means that $\{\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N)\}_{i=1,2,\ldots,n}$ is a weight vector (or probability measure). When $s = 1$, it derives the well-known Shapley function (Shapley, 1953)

$$\varphi_i(\mu, N) = \sum_{S \subseteq N \setminus i} \frac{(n-s-1)!s!}{n!}\big(\mu(S \cup i) - \mu(S)\big) \quad \forall i \in N.$$

From the expression of the generalized Shapley function, we know that it is an expectation value of the interactions between the coalition S and any coalition $T \subseteq N \setminus S$.

3. Some Generalized Interval-Valued 2-Tuple Linguistic Choquet Aggregation Operators

At present, there are few researches about the interval-valued 2-tuple linguistic aggregation operators based on fuzzy measures that restrict the application of interval-valued 2-tuple linguistic variables. To deal with the situation where the elements in a set are interdependent, this section defines several generalized interval-valued 2-tuple linguistic Choquet aggregation operators.

DEFINITION 9. Let $X = \{[(l_1, \alpha_1^l), (r_1, \alpha_1^r)], [(l_2, \alpha_2^l), (r_2, \alpha_2^r)], \ldots, [(l_n, \alpha_n^l), (r_n, \alpha_n^r)]\}$ be a set of interval-valued 2-tuple linguistic variables, and μ be a fuzzy measure on the ordered set $N = \{1, 2, \ldots, n\}$. The GIVTLCWA operator is defined by

$$\begin{aligned}
&\mathrm{GIVTLCWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&= \Delta\Bigg(\Bigg[\sum_{i=1}^{n} \big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)\Delta^{-1}\big(l_{(i)}, \alpha_{(i)}^l\big)^{\lambda}, \\
&\quad \sum_{i=1}^{n} \big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)\Delta^{-1}\big(r_{(i)}, \alpha_{(i)}^r\big)^{\lambda}\Bigg]^{\frac{1}{\lambda}}\Bigg),
\end{aligned}$$

where $\lambda \in R^+$, $(\cdot)$ is a permutation on N with $[(l_{(i)}, \alpha_{(i)}^l), (r_{(i)}, \alpha_{(i)}^r)]$ being the jth least value of $[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]$, $i = 1, 2, \ldots, n$, and $A_{(i)} = \{i, i+1, \ldots, n\}$ with $A_{(n+1)} = \emptyset$.

REMARK 2. If there are no interactions between the ordered positions, then the GIVTLCWA operator degenerates to the generalized interval-valued 2-tuple ordered weighted average (GIVTOWA) operator (Zha, 2013)

$$\begin{aligned}
&\mathrm{GIVTOWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&= \Delta\Bigg(\Bigg[\sum_{i=1}^{n} w_{(i)}\Delta^{-1}\big(l_{(i)}, \alpha_{(i)}^l\big)^{\lambda}, \sum_{i=1}^{n} w_{(i)}\Delta^{-1}\big(r_{(i)}, \alpha_{(i)}^r\big)^{\lambda}\Bigg]^{\frac{1}{\lambda}}\Bigg).
\end{aligned}$$

REMARK 3. If $\lambda = -1$, then the GIVTLCWA operator degenerates to the interval-valued 2-tuple linguistic Choquet weighted harmonic (IVTLCWH) operator

$$\begin{aligned}
&\mathrm{IVTLCWH}\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big] \\
&= \Delta\Bigg(\Bigg[\Bigg(\sum_{i=1}^{n} \frac{\mu(A_{(i)}) - \mu(A_{(i+1)})}{\Delta^{-1}(l_{(i)}, \alpha_{(i)}^l)}\Bigg)^{-1}, \Bigg(\sum_{i=1}^{n} \frac{\mu(A_{(i)}) - \mu(A_{(i+1)})}{\Delta^{-1}(r_{(i)}, \alpha_{(i)}^r)}\Bigg)^{-1}\Bigg]\Bigg).
\end{aligned}$$

REMARK 4. If $\lambda \to 0$, then the GIVTLCWA operator degenerates to the interval-valued 2-tuple linguistic Choquet weighted geometric mean (IVTLCWGM) operator

$$\begin{aligned}
&\mathrm{IVTLCWGM}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&= \Delta\Bigg(\Bigg[\prod_{i=1}^{n} \Delta^{-1}\big(l_{(i)}, \alpha_{(i)}^l\big)^{\mu(A_{(i)}) - \mu(A_{(i+1)})}, \prod_{i=1}^{n} \Delta^{-1}\big(r_{(i)}, \alpha_{(i)}^r\big)^{\mu(A_{(i)}) - \mu(A_{(i+1)})}\Bigg]\Bigg).
\end{aligned}$$

REMARK 5. If $\lambda = 1$, then the GIVTLCWA operator degenerates to the interval-valued 2-tuple linguistic Choquet weighted averaging (IVTLCWA) operator

$$\begin{aligned}&\mathrm{IVTLCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\big(\mu(A_{(i)})-\mu(A_{(i+1)})\big)\Delta^{-1}\big(l_{(i)},\alpha_{(i)}^l\big),\sum_{i=1}^{n}\big(\mu(A_{(i)})-\mu(A_{(i+1)})\big)\\&\qquad\times\Delta^{-1}\big(r_{(i)},\alpha_{(i)}^r\big)\Bigg]\Bigg).\end{aligned}$$

Remark 6. If $\lambda=2$, then the GIVTLCWA operator degenerates to the interval-valued 2-tuple linguistic quadratic Choquet weighted averaging (IVTLQCWA) operator

$$\begin{aligned}&l\mathrm{IVTLQCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\big(\mu(A_{(i)})-\mu(A_{(i+1)})\big)\Delta^{-1}\big(l_{(i)},\alpha_{(i)}^l\big)^2,\sum_{i=1}^{n}\big(\mu(A_{(i)})-\mu(A_{(i+1)})\big)\\&\qquad\times\Delta^{-1}\big(r_{(i)},\alpha_{(i)}^r\big)^2\Bigg]^{\frac{1}{2}}\Bigg).\end{aligned}$$

Remark 7. If $\lambda\to+\infty$, then the GIVTLCWA operator degenerates to the Max operator

$$\begin{aligned}&\mathrm{GIVTLCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\max_{i=1}^{n}\big[\big(l_i,\alpha_i^l\big),\big(r_i,\alpha_i^r\big)\big],\end{aligned}$$

and if $\lambda\to-\infty$, then the GIVTLCWA operator degenerates to the Min operator

$$\begin{aligned}&\mathrm{GIVTLCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\min_{i=1}^{n}\big[\big(l_i,\alpha_i^l\big),\big(r_i,\alpha_i^r\big)\big].\end{aligned}$$

Remark 8. If $(l_i,\alpha_i^l)=(r_i,\alpha_i^r)$ for each $i=1,2,\ldots,n$, then the GIVTLCWA operator degenerates to the generalized 2-tuple correlated averaging (GTCA) operator (Yang and Chen, 2012)

$$\begin{aligned}&\mathrm{GTCA}\big((l_1,\alpha_1),(l_2,\alpha_2),\ldots,(l_n,\alpha_n)\big)\\&\quad=\Delta\Bigg(\sum_{i=1}^{n}\big(\mu(A_{(i)})-\mu(A_{(i+1)})\big)\Delta^{-1}\big(l_{(i)},\alpha_{(i)}^l\big)^{\lambda}\Bigg)^{\frac{1}{\lambda}}.\end{aligned}$$

Similar to the 2-tuple linguistic induced quasi-arithmetic Choquet integral aggregation (Quasi-2- TLICIA) operator (Merigó and Gil-Lafuente, 2013), the Quasi-GIVTLCWA (Q-GIVTLCWA) operator is defined as follows:

DEFINITION 10. Let $X = \{[(l_1, \alpha_1^l), (r_1, \alpha_1^r)], [(l_2, \alpha_2^l), (r_2, \alpha_2^r)], \ldots, [(l_n, \alpha_n^l), (r_n, \alpha_n^r)]\}$ be a set of interval-valued 2-tuple linguistic variables, and μ be a fuzzy measure defined on the ordered set $N = \{1, 2, \ldots, n\}$. The Q-GIVTLCWA operator is defined by

$$\begin{aligned}
&\text{Q-GIVTLCWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&= \Delta\Bigg(\Bigg[g^{-1}\Bigg(\sum_{i=1}^{n}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)g\big(\Delta^{-1}(l_{(i)}, \alpha_{(i)}^l)\big)\Bigg), \\
&\quad g^{-1}\Bigg(\sum_{i=1}^{n}\big(\mu(A_{(i)}) - \mu(A_{(i+1)})\big)g\big(\Delta^{-1}\big(r_{(i)}, \alpha_{(i)}^r\big)\big)\Bigg)\Bigg]\Bigg),
\end{aligned}$$

where g is a strictly continuous monotonic function such that $g : [0, 1] \to R$, $(\cdot)$ is a permutation on N with $[(l_{(i)}, \alpha_{(i)}^l), (r_{(i)}, \alpha_{(i)}^r)]$ being the jth least value of $[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]$, $i = 1, 2, \ldots, n$, and $A_{(i)} = \{i, i + 1, \ldots, n\}$ with $A_{(n+1)} = \emptyset$.

REMARK 9. If $g(x) = x^\lambda$, $x \in [0, 1]$, then the Q-GIVTLCWA operator reduces to the GIVTLCWA operator.

Now, let us consider several desirable properties of the GIVTLCWA operator.

Theorem 1. *Let $X = \{[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]\}_{i\in N}$ and $Y = \{[(\tau_i, \varepsilon_i^l), (\pi_i, \varepsilon_i^r)]\}_{i\in N}$ be two sets of interval-valued 2-tuple linguistic variables and μ be a fuzzy measure defined on the ordered set $N = \{1, 2, \ldots, n\}$.*

(i) *Commutativity: let $X' = \{[(l_i', \alpha_i^{l\prime}), (r_i', \alpha_i^{r\prime})]\}_{i\in N}$ be a permutation of $X = \{[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]\}_{i\in N}$. Then*

$$\begin{aligned}
&\text{GIVTLCWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&\quad = \text{GIVTLCWA}\big(\big[\big(l_1', \alpha_1^{l\prime}\big), \big(r_1', \alpha_1^{r\prime}\big)\big], \big[\big(l_2', \alpha_2^{l\prime}\big), \big(r_2', \alpha_2^{r\prime}\big)\big], \ldots, \\
&\qquad\qquad \big[\big(l_n', \alpha_n^{l\prime}\big), \big(r_n', \alpha_n^{r\prime}\big)\big]\big);
\end{aligned}$$

(ii) *Comonotonicity: let $(\cdot)$ is a permutation on N with $[(l_{(i)}, \alpha_{(i)}^l), (r_{(i)}, \alpha_{(i)}^r)]$ and $[(\tau_{(i)}, \varepsilon_{(i)}^l), (\pi_{(i)}, \varepsilon_{(i)}^r)]$ being the jth least values of $[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]$ and $[(\tau_i, \varepsilon_i^l), (\pi_i, \varepsilon_i^r)]$, $i \in N$, respectively. If $(l_{(i)}, \alpha_{(i)}^l) \leqslant (\tau_{(i)}, \varepsilon_{(i)}^l)$ and $(r_{(i)}, \alpha_{(i)}^r) \leqslant (\pi_{(i)}, \varepsilon_{(i)}^r)$, then*

$$\begin{aligned}
&\text{GIVTLCWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&\quad \leqslant \text{GIVTLCWA}\big(\big[\big(\tau_1, \varepsilon_1^l\big), \big(\pi_1, \varepsilon_1^r\big)\big], \big[\big(\tau_2, \varepsilon_2^l\big), \big(\pi_2, \varepsilon_2^r\big)\big], \ldots, \\
&\qquad\qquad \big[\big(\tau_n, \varepsilon_n^l\big), \big(\pi_n, \varepsilon_n^r\big)\big]\big);
\end{aligned}$$

(iii) *Idempotency: if $[(l_i, \alpha_i^l), (r_i, \alpha_i^r)] = [(l, \alpha^l), (r, \alpha^r)]$ for each $i \in N$, then*

$$\begin{aligned}
&\text{GIVTLCWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big) \\
&\quad = \big[\big(l, \alpha^l\big), \big(r, \alpha^r\big)\big];
\end{aligned}$$

(iv) *Boundary:*

$$\begin{aligned}&\left[\min_{i=1}^{n}\left(l_i,\alpha_i^l\right),\min_{i=1}^{n}\left(r_i,\alpha_i^r\right)\right]\\&\quad\leqslant \mathrm{GIVTLCWA}\left(\left[\left(l_1,\alpha_1^l\right),\left(r_1,\alpha_1^r\right)\right],\left[\left(l_2,\alpha_2^l\right),\left(r_2,\alpha_2^r\right)\right],\ldots,\left[\left(l_n,\alpha_n^l\right),\left(r_n,\alpha_n^r\right)\right]\right)\\&\quad\leqslant\left[\max_{i=1}^{n}\left(l_i,\alpha_i^l\right),\max_{i=1}^{n}\left(r_i,\alpha_i^r\right)\right].\end{aligned}$$

Proof. Because $((\mu(A_{(i)})-\mu(A_{(i+1)})))_{i\in N}$ is a weight vector, namely, $\mu(A_{(i)})-\mu(A_{(i+1)})\geqslant 0$ and $\sum_{i=1}^{n}(\mu(A_{(i)})-\mu(A_{(i+1)}))=1$, one can easily obtain the conclusions. □

Although the GIVTLCWA operator reflects the interactions between elements, it only considers the importance of the ordered positions and gives the interactions between the adjacent coalitions $A_{(i)}$ and $A_{(i+1)}$, $i=1,2,\ldots,n$. To eliminate these disadvantages, the generalized interval-valued 2-tuple linguistic Shapley Choquet weighted averaging (GIVTLSCWA) operator is defined as follows:

Definition 11. Let $X=\{[(l_1,\alpha_1^l),(r_1,\alpha_1^r)],[(l_2,\alpha_2^l),(r_2,\alpha_2^r)],\ldots,[(l_n,\alpha_n^l),(r_n,\alpha_n^r)]\}$ be a set of interval-valued 2-tuple linguistic variables, and Φ be the associated generalized Shapley function for the fuzzy measure μ on the ordered set $N=\{1,2,\ldots,n\}$. The GIVTLSCWA operator is defined by

$$\begin{aligned}&\mathrm{GIVTLSCWA}\left(\left[\left(l_1,\alpha_1^l\right),\left(r_1,\alpha_1^r\right)\right],\left[\left(l_2,\alpha_2^l\right),\left(r_2,\alpha_2^r\right)\right],\ldots,\left[\left(l_n,\alpha_n^l\right),\left(r_n,\alpha_n^r\right)\right]\right)\\&=\Delta\left(\left[\sum_{i=1}^{n}\left(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)\right)\left(\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}\left(l_{(i)},\alpha_{(i)}^l\right)\right)^{\lambda},\right.\right.\\&\qquad\left.\left.\sum_{i=1}^{n}\left(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)\right)\left(\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}\left(r_{(i)},\alpha_{(i)}^r\right)\right)^{\lambda}\right]^{\frac{1}{\lambda}}\right),\end{aligned}$$

where $\lambda\in R^+$, $\varphi_{x_i}(\rho,X)$ is the Shapley value for the fuzzy measure ρ on X with $x_i=[(l_i,\alpha_i^l),(r_i,\alpha_i^r)]$, $i=1,2,\ldots,n$, $(\cdot)$ is a permutation on N with $\varphi_{x_{(i)}}(\rho,X)[(l_{(i)},\alpha_{(i)}^l),(r_{(i)},\alpha_{(i)}^r)]$ being the jth least value of $\varphi_{x_i}(\rho,X)[(l_i,\alpha_i^l),(r_i,\alpha_i^r)]$, $i=1,2,\ldots,n$, and $A_{(i)}=\{i,i+1,\ldots,n\}$ with $A_{(n+1)}=\emptyset$.

From Definition 11, we know that the GIVTLSCWA operator does not only consider the importance of elements and their ordered positions but also reflect their interactions. However, the GIVTLSCWA operator does not satisfy idempotency that makes the rationality of this operator be questioned. Hence, we further present the following generalized interval-valued 2-tuple linguistic normalized Shapley Choquet weighted averaging (GIVTLNSCWA) operator.

Definition 12. Let $X=\{[(l_1,\alpha_1^l),(r_1,\alpha_1^r)],[(l_2,\alpha_2^l),(r_2,\alpha_2^r)],\ldots,[(l_n,\alpha_n^l),(r_n,\alpha_n^r)]\}$ be a set of interval-valued 2-tuple linguistic variables, and Φ be the associated generalized Shapley function for the fuzzy measure μ on the ordered set $N=\{1,2,\ldots,n\}$. The

GIVTLNSCWA operator is defined by

$$\begin{aligned}&\mathrm{GIVTLNSCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))(\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}(l_{(i)},\alpha_{(i)}^l))^{\lambda}}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)^{\lambda}},\\&\qquad\sum_{i=1}^{n}\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))(\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}(r_{(i)},\alpha_{(i)}^r))^{\lambda}}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)^{\lambda}}\Bigg]^{\frac{1}{\lambda}}\Bigg),\end{aligned}$$

where $\lambda\in R^+$, $\varphi_{x_i}(\rho,X)$ is the Shapley value for the fuzzy measure ρ on X with $x_i=[(l_i,\alpha_i^l),(r_i,\alpha_i^r)]$, $i=1,2,\ldots,n$, $(\cdot)$ is a permutation on N with $\varphi_{x_{(i)}}(\rho,X)[(l_{(i)},\alpha_{(i)}^l),(r_{(i)},\alpha_{(i)}^r)]$ being the jth least value of $\varphi_{x_i}(\rho,X)[(l_i,\alpha_i^l),(r_i,\alpha_i^r)]$, $i=1,2,\ldots,n$, and $A_{(i)}=\{i,i+1,\ldots,n\}$ with $A_{(n+1)}=\emptyset$.

Remark 10. If there are no interactions between the elements as well as their ordered positions, then the GIVTLNSCWA operator degenerates to the generalized interval-valued 2-tuple linguistic hybrid weighted averaging (GIVTLHWA) operator:

$$\begin{aligned}&\mathrm{GIVTLHWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\frac{w_{(i)}(\omega_{x_{(i)}}\Delta^{-1}(l_{(i)},\alpha_{(i)}^l))^{\lambda}}{\sum_{i=1}^{n}w_{(i)}\omega_{x_{(i)}}{}^{\lambda}},\sum_{i=1}^{n}\frac{w_{(i)}(\omega_{x_{(i)}}\Delta^{-1}(r_{(i)},\alpha_{(i)}^r))^{\lambda}}{\sum_{i=1}^{n}w_{(i)}\omega_{x_{(i)}}{}^{\lambda}}\Bigg]^{\frac{1}{\lambda}}\Bigg).\end{aligned}$$

Remark 11. If $\lambda=-1$, then the GIVTLNSCWA operator degenerates to the interval-valued 2-tuple linguistic normalized Shapley Choquet weighted harmonic (IVTLNSCWH) operator

$$\begin{aligned}&\mathrm{IVTLNSCWH}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\quad=\Delta\Bigg(\Bigg[\Bigg(\sum_{i=1}^{n}\frac{\vartheta_{x_{(i)}}}{\Delta^{-1}(l_{(i)},\alpha_{(i)}^l)}\Bigg)^{-1},\Bigg(\sum_{i=1}^{n}\frac{\vartheta_{x_{(i)}}}{\Delta^{-1}(r_{(i)},\alpha_{(i)}^r)}\Bigg)^{-1}\Bigg]\Bigg),\end{aligned}$$

where $\vartheta_{x_{(i)}}=\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))/\varphi_{x_{(i)}}(\rho,X)}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))/\varphi_{x_{(i)}}(\rho,X)^{\lambda}}$, $i=1,2,\ldots,n$.

Remark 12. If $\lambda\to 0$, then the GIVTLNSCWA operator degenerates to the interval-valued 2-tuple linguistic Shapley Choquet weighted geometric mean (IVTLSCWGM) operator

$$\begin{aligned}&\mathrm{IVTLSCWGM}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\&\qquad=\Delta\Bigg(\Bigg[\prod_{i=1}^{n}\Delta^{-1}\big(l_{(i)},\alpha_{(i)}^l\big)^{\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)},\\&\qquad\quad\prod_{i=1}^{n}\Delta^{-1}\big(r_{(i)},\alpha_{(i)}^r\big)^{\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)}\Bigg]\Bigg).\end{aligned}$$

REMARK 13. If $\lambda = 1$, then the GIVTLNSCWA operator degenerates to the interval-valued 2-tuple linguistic normalized Shapley Choquet weighted averaging (IVTLNSCWA) operator

$$\begin{aligned}
&\mathrm{IVTLNSCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\
&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}(l_{(i)},\alpha_{(i)}^l)}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)},\\
&\qquad\sum_{i=1}^{n}\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}(r_{(i)},\alpha_{(i)}^r)}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)}\Bigg]\Bigg).
\end{aligned}$$

REMARK 14. If $\lambda = 2$, then the GIVTLNSCWA operator degenerates to the interval-valued 2-tuple linguistic normalized quadratic Shapley Choquet weighted averaging (IVTLNQSCWA) operator:

$$\begin{aligned}
&\mathrm{IVTLNQSCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\
&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))(\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}(l_{(i)},\alpha_{(i)}^l))^2}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)^2},\\
&\qquad\sum_{i=1}^{n}\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))(\varphi_{x_{(i)}}(\rho,X)\Delta^{-1}(r_{(i)},\alpha_{(i)}^r))^2}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)^2}\Bigg]^{\frac{1}{2}}\Bigg).
\end{aligned}$$

REMARK 15. If $\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)=\frac{1}{n}$, $i=1,2,\ldots,n$, and $\lambda=1$, then the GIVTLNSCWA operator degenerates to the interval-valued 2-tuple linguistic Shapley weighted averaging (IVTLSWA) operator:

$$\begin{aligned}
&\mathrm{IVTLSWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\
&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\varphi_{x_i}(\rho,X)\Delta^{-1}\big(l_i,\alpha_i^l\big),\varphi_{x_i}(\rho,X)\Delta^{-1}\big(r_i,\alpha_i^r\big)\Bigg]\Bigg).
\end{aligned}$$

REMARK 16. If $\varphi_{x_i}(\rho,X)=\frac{1}{n}$, $i=1,2,\ldots,n$, and $\lambda=1$, then the GIVTLNSCWA operator degenerates to the generalized interval-valued 2-tuple linguistic Shapley Choquet weighted averaging (GIVTLSCWA) operator

$$\begin{aligned}
&\mathrm{GIVTLSCWA}\big(\big[\big(l_1,\alpha_1^l\big),\big(r_1,\alpha_1^r\big)\big],\big[\big(l_2,\alpha_2^l\big),\big(r_2,\alpha_2^r\big)\big],\ldots,\big[\big(l_n,\alpha_n^l\big),\big(r_n,\alpha_n^r\big)\big]\big)\\
&\quad=\Delta\Bigg(\Bigg[\sum_{i=1}^{n}\big(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)\big)\Delta^{-1}\big(l_{(i)},\alpha_{(i)}^l\big)^{\lambda},\\
&\qquad\sum_{i=1}^{n}\big(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N)\big)\Delta^{-1}\big(r_{(i)},\alpha_{(i)}^r\big)^{\lambda}\Bigg]^{\frac{1}{\lambda}}\Bigg).
\end{aligned}$$

REMARK 17. If $(l_i, \alpha_i^l) = (r_i, \alpha_i^r)$, $i = 1, 2, \ldots, n$, then the GIVTLNSCWA operator degenerates to the generalized 2-tuple linguistic normalized Shapley Choquet weighted averaging (GTLNSCWA) operator

$$\begin{aligned}&\mathrm{GTLNSCWA}\big(\big(l_1, \alpha_1^l\big), \big(l_2, \alpha_2^l\big), \ldots, \big(l_n, \alpha_n^l\big)\big)\\&= \Delta\left(\sum_{i=1}^{n} \frac{(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N))(\varphi_{x_{(i)}}(\rho, X)\Delta^{-1}(l_{(i)}, \alpha_{(i)}^l))^{\lambda}}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N))\varphi_{x_{(i)}}(\rho, X)^{\lambda}}\right)^{\frac{1}{\lambda}}.\end{aligned}$$

Since $\left(\frac{(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)^{\lambda}}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu,N)-\Phi_{A_{(i+1)}}(\mu,N))\varphi_{x_{(i)}}(\rho,X)^{\lambda}}\right)_{i\in N}$ is a weight vector, by Theorem 1 one can easily derive that the GIVTLNSCWA operator satisfies Commutativity, Comonotonicity, Idempotency, and Boundary.

Similar to the Q-GIVTLCWA operator, the Quasi-GIVTLNSCWA (Q-GIVTLNSCWA) operator is defined as follows:

DEFINITION 13. Let $X = \{[(l_1, \alpha_1^l), (r_1, \alpha_1^r)], [(l_2, \alpha_2^l), (r_2, \alpha_2^r)], \ldots, [(l_n, \alpha_n^l), (r_n, \alpha_n^r)]\}$ be a set of interval-valued 2-tuple linguistic variables, and Φ be the associated generalized Shapley function for the fuzzy measure μ on the ordered set $N = \{1, 2, \ldots, n\}$. The Q-GIVTLNSCWA operator is defined by

$$\begin{aligned}&\text{Q-GIVTLNSCWA}\big(\big[\big(l_1, \alpha_1^l\big), \big(r_1, \alpha_1^r\big)\big], \big[\big(l_2, \alpha_2^l\big), \big(r_2, \alpha_2^r\big)\big], \ldots, \big[\big(l_n, \alpha_n^l\big), \big(r_n, \alpha_n^r\big)\big]\big)\\&= \Delta\left(\left[g^{-1}\left(\sum_{i=1}^{n} \frac{(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N))g(\varphi_{x_{(i)}}(\rho, X)\Delta^{-1}(l_{(i)}, \alpha_{(i)}^l))}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N))g(\varphi_{x_{(i)}}(\rho, X))}\right),\right.\right.\\&\left.\left. g^{-1}\left(\sum_{i=1}^{n} \frac{(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N))g(\varphi_{x_{(i)}}(\rho, X)\Delta^{-1}(r_{(i)}, \alpha_{(i)}^r))}{\sum_{i=1}^{n}(\Phi_{A_{(i)}}(\mu, N) - \Phi_{A_{(i+1)}}(\mu, N))g(\varphi_{x_{(i)}}(\rho, X))}\right)\right]\right),\end{aligned}$$

where g is a strictly continuous monotonic function such that $g : [0, 1] \to R$, $\varphi_{x_i}(\rho, X)$ is the Shapley value for the fuzzy measure ρ on X with $x_i = [(l_i, \alpha_i^l), (r_i, \alpha_i^r)]$, $i = 1, 2, \ldots, n$, $(\cdot)$ is a permutation on N with $\varphi_{x_{(i)}}(\rho, X)[(l_{(i)}, \alpha_{(i)}^l), (r_{(i)}, \alpha_{(i)}^r)]$ being the jth least value of $\varphi_{x_i}(\rho, X)[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]$, $i = 1, 2, \ldots, n$, and $A_{(i)} = \{i, i + 1, \ldots, n\}$ with $A_{(n+1)} = \emptyset$.

REMARK 18. If $g(x) = x^{\lambda}$, $x \in [0, 1]$, then the Q-GIVTLNSCWA operator reduces to the GIVTLNSCWA operator.

4. An Approach to Multi-Attribute Decision Making

As we know, in decision making another hot topic is how to obtain the weight vector. Zha (2013) introduced a method to determinate the weight vector using the precision function

that considers the higher degree of precision of the interval-valued 2-tuple linguistic variables the bigger weights will be. Although this method is simple, the author does not give more explanations. As we know, the elements' importance is mainly determined by object self, such as the decision makers and the attributes. Take a special case to illustrate; if all interval-valued 2-tuple linguistic variables have the same degree of precision, then they have the same weight that equals to $1/n$. This seems to be unreasonable.

4.1. *Models of the Optimal Weight Vectors*

Similar to the correlation coefficient of interval-valued intuitionistic fuzzy sets (Bustince and Burillo, 1995), we define the following correlation coefficient of interval-valued 2-tuple linguistic variables.

DEFINITION 14. Let $X = \{[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]\}_{i\in N}$ and $Y = \{[(\tau_i, \varepsilon_i^l), (\pi_i, \varepsilon_i^r)]\}_{i\in N}$ be two sets of interval-valued 2-tuple linguistic variables with l_i, r_i, τ_i and π_i being the linguistic terms in $S = \{s_0, s_1, \ldots, s_t\}$. The correlation coefficient of X and Y is defined by

$$K(X, Y) = \frac{C(X, Y)}{\sqrt{E(X) \cdot E(Y)}},$$

where $N = \{1, 2, \ldots, n\}$, $C(X, Y) = \sum_{i=1}^{n} \frac{\Delta^{-1}(l_i,\alpha_i^l)\Delta^{-1}(\tau_i,\alpha_i^l)+\Delta^{-1}(r_i,\alpha_i^r)\Delta^{-1}(\pi_i,\alpha_i^r)}{2}$ is the correlation between X and Y, $E(X) = \sum_{i=1}^{n} \frac{(\Delta^{-1}(l_i,\alpha_i^l)^2+\Delta^{-1}(r_i,\alpha_i^r)^2)}{2}$ and $E(Y) = \sum_{i=1}^{n} \frac{(\Delta^{-1}(\tau_i,\alpha_i^l)^2+\Delta^{-1}(\pi_i,\alpha_i^r)^2)}{2}$ are the informational energies of X and Y, respectively.

Proposition 1. *The correlation coefficient $K(X, Y)$ between $X = \{[(l_i, \alpha_i^l), (r_i, \alpha_i^r)]\}_{i\in N}$ and $Y = \{[(\tau_i, \varepsilon_i^l), (\pi_i, \varepsilon_i^r)]\}_{i\in N}$ satisfies the following conditions*:

(i) $0 \leqslant K(X, Y) \leqslant 1$;
(ii) $K(X, Y) = K(Y, X)$;
(iii) $K(X, Y) = 1$, *if* $X = Y$, *namely,* $[(l_i, \alpha_i^l), (r_i, \alpha_i^r)] = [(\tau_i, \varepsilon_i^l), (\pi_i, \varepsilon_i^r)]$ *for each* $i = 1, 2, \ldots, n$.

Proof. By the Cauchy–Schwarz inequality, it is not difficult to get the conclusion (i). Furthermore, from the definition of the correlation coefficient one can easily get the conclusion (ii) and (iii). □

Consider a multi-attribute group decision-making problem, in which the attribute preference values take the form of uncertain linguistic variables. Suppose there are m alternatives $A = \{a_1, a_2, \ldots, a_m\}$ and n attributes $C = \{c_1, c_2, \ldots, c_n\}$, which are evaluated by q experts, $E = \{e_1, e_2, \ldots, e_q\}$. Assume that $R^k = (\bar{s}_{ij}^k)_{m\times n}$ is the uncertain linguistic decision matrix given by the expert e_k, where $s_{ij}^k = [s_{lij}^k, s_{rij}^k]$ is the uncertain linguistic preference value of the alternative $a_i \in A$ for the attribute $c_j \in C$, and s_{lij}^k and s_{rij}^k belong to the predefined linguistic term set $S = \{s_i \mid i = 0, 1, \ldots, t\}$.

- **Models for the optimal weight vector on the expert set**

Let $\bar{R} = (\bar{s}_{ij})_{m\times n}$ be the average interval-valued 2-tuple linguistic decision matrix, with $\bar{s}_{ij} = [\Delta(\frac{1}{q}\sum_{k=1}^{q}\Delta^{-1}(s_{lij}^k, 0)), \Delta(\frac{1}{q}\sum_{k=1}^{q}\Delta^{-1}(s_{rij}^k, 0))]$ for all $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. Calculate the correlation coefficient $K(\bar{s}_j, \bar{s}_j^k)$ between $\bar{s}_j = (\bar{s}_{ij})_{i=1,\ldots,m}$ and $\bar{s}_j^k = ([(s_{lij}^k, 0), (s_{rij}^k, 0)])_{i=1,\ldots,m}$ for all $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, q$. If the weight information on the expert set is not exactly known, then the following model for the optimal weight vector w on the expert set E is established.

$$\max \sum_{k=1}^{q}\sum_{j=1}^{n} w_{e_k} K\left(\bar{s}_j, \bar{s}_j^k\right),$$

$$\text{s.t.} \begin{cases} \sum_{k=1}^{q} w_{e_k} = 1, \\ w_{e_k} \geqslant 0, \quad k = 1, 2, \ldots, q, \\ w_{e_k} \in W_{e_k}, \quad k = 1, 2, \ldots, q \end{cases} \tag{1}$$

where W_{e_k} is the range of the expert e_k.

In model (1), if we delete the condition $w_{e_k} \in W_{e_k}$, $k = 1, 2, \ldots, q$, then model for the optimal weight vector on the expert set where the weight information is completely unknown is obtained. From model (1), one can conclude that the closer an expert's evaluation values are to the other experts', the larger the weight measure is. This can avoid the unduly high or low evaluation values induced by the experts' limited knowledge or expertise.

Model (1) is based on the assumption that the importance of the experts is independent. However, the experts' importance is a relative value that is influenced by the other experts. Considering the interdependence between the experts, the following model for the optimal fuzzy measure ρ on E is established.

$$\max \sum_{k=1}^{q}\sum_{j=1}^{n} \varphi_{e_k}(\rho, E) K\left(\bar{s}_j, \bar{s}_j^k\right),$$

$$\text{s.t.} \begin{cases} \rho(E) = 1, \\ \rho(S) \leqslant \rho(T), \quad \forall S, T \subseteq E \text{ s.t. } S \subseteq T, \\ \rho(e_k) \in W_{e_k}, \quad k = 1, 2, \ldots, q, \\ \rho(e_k) \geqslant 0, \quad k = 1, 2, \ldots, q, \end{cases} \tag{2}$$

where $\varphi_{e_k}(\rho, E)$ is the Shapley value of the expert e_k.

Here we use the experts' Shapley values as their importance that overall consider their interactions. If there are no interactions, then model (2) degenerates to model (1).

- **Models for the optimal weight vector on the ordered set Q**

For each $j = 1, 2, \ldots, n$, reorder the correlation coefficient $K(\bar{s}_j, \bar{s}_j^k)$, $k \in Q = \{1, 2, \ldots, q\}$, in increasing order such that $K(\bar{s}_j, \bar{s}_j^{(k)}) \leqslant K(\bar{s}_j, \bar{s}_j^{(k+1)})$, where $(\cdot)$ is a permutation on $Q = \{1, 2, \ldots, q-1\}$. The following model for the optimal weight vector ω on the ordered set Q is constructed.

$$\max \sum_{k=1}^{q} \sum_{j=1}^{n} \omega_k K\left(\bar{s}_j, \bar{s}_j^{(k)}\right),$$
$$\text{s.t.} \begin{cases} \sum_{k=1}^{q} \omega_k = 1, \\ \omega_k \geqslant 0, \quad k = 1, 2, \ldots, q, \\ \omega_k \in W_k, \quad k = 1, 2, \ldots, q, \end{cases} \tag{3}$$

where W_k is the range of the kth's position.

Considering the interdependence between the ordered positions, the following model for the optimal weight vector μ on Q is constructed.

$$\max \sum_{k=1}^{q} \sum_{j=1}^{n} \varphi_k(\mu, Q) K\left(\bar{s}_j, \bar{s}_j^{(k)}\right),$$
$$\text{s.t.} \begin{cases} \mu(Q) = 1, \\ \mu(S) \leqslant \mu(T), \quad \forall S, T \subseteq Q \text{ s.t. } S \subseteq T, \\ \mu(k) \in W_k, \quad k = 1, 2, \ldots, q, \\ \mu(k) \geqslant 0, \quad k = 1, 2, \ldots, q, \end{cases} \tag{4}$$

where $\varphi_k(\mu, Q)$ is the Shapley value of the kth position.

In models (3) and (4), if we delete the condition $\omega_k \in W_k$ and $\mu(k) \in W_k$, $k = 1, 2, \ldots, q$, then models for the optimal weight vectors on Q where the weight information is completely unknown are obtained.

- **Models for the optimal weight vector on the attribute set**

Let $R = \left(x_{ij}\right)_{m \times n}$ be the comprehensive interval-valued 2-tuple linguistic decision matrix with $x_{ij} = [(l_{ij}, \alpha_{ij}^l), (r_{ij}, \alpha_{ij}^r)]$ for all $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. Let $X^+ = (x_1^+, x_2^+, \ldots, x_n^+)$ and $X^- = (x_1^-, x_2^-, \ldots, x_n^-)$ be respective the positive and negative ideal vectors, where $x_j^+ = [\max_{1 \leqslant i \leqslant m}(l_{ij}, \alpha_{ij}^l), \max_{1 \leqslant i \leqslant m}(r_{ij}, \alpha_{ij}^r)]$ and $x_j^- = [\min_{1 \leqslant i \leqslant m}(l_{ij}, \alpha_{ij}^l), \min_{1 \leqslant i \leqslant m}(r_{ij}, \alpha_{ij}^r)]$ for all $j = 1, 2, \ldots, n$. Calculate the correlation coefficient $K(x_j^+, x_{ij})$ between x_j^+ and x_{ij} as well as the correlation coefficient $K(x_j^-, x_{ij})$ between x_j^- and x_{ij} for all $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. For the benefit attribute c_j, let $d_{ij} = \frac{K(x_j^+, x_{ij})}{K(x_j^-, x_{ij}) + K(x_j^+, x_{ij})}$, and $d_{ij} = \frac{K(x_j^-, x_{ij})}{K(x_j^-, x_{ij}) + K(x_j^+, x_{ij})}$, otherwise.

If the weight information on the attribute set C is not exactly known, the following model for the optimal weight vector w on C is built.

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} w_{c_j},$$
$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} w_{c_j} = 1, \\ w_{c_j} \geqslant 0, \quad j = 1, 2, \ldots, n, \\ w_{c_j} \in W_{c_j}, \quad j = 1, 2, \ldots, n, \end{cases}$$

where W_{c_j} is the range of the attribute c_j.

Considering the interdependence between attributes, the following model for the optimal weight vector v on C is built.

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} \varphi_{c_j}(v, C), \tag{5}$$

$$\text{s.t.} \begin{cases} v(C) = 1, \\ v(S) \leqslant v(T), & \forall S, T \subseteq C \text{ s.t. } S \subseteq T, \\ v(c_j) \in W_{c_j}, & j = 1, 2, \ldots, n, \\ v(c_j) \geqslant 0, & j = 1, 2, \ldots, n, \end{cases} \tag{6}$$

where $\varphi_{c_j}(v, C)$ is the Shapley value of the kth position.

From models (5) and (6), one can easily get the corresponding model for the optimal weight vector on the attribute set C where the weight information is completely unknown.

- **Models for the optimal weight vector on the ordered set N**

For each $i = 1, 2, \ldots, m$, let $(\cdot)$ be a permutation on d_{ij}, $j = 1, 2, \ldots, n$, such that $d_{i(j)}$ being the jth least value of d_{ij}. Similar to model for the optimal weight vector on the ordered set Q, if the weight information on the ordered set N is incompletely known, the following model for the optimal weight vector ω on N is constructed.

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} d_{i(j)} \omega_j,$$

$$\text{s.t.} \begin{cases} \sum_{j=1}^{n} \omega_j = 1, \\ \omega_j \geqslant 0, & j = 1, 2, \ldots, n, \\ \omega_j \in W_j, & j = 1, 2, \ldots, n, \end{cases} \tag{7}$$

where W_j is the range of the jth position.

Considering the interdependence between the ordered positions, the following model for the optimal weight vector η on N is constructed.

$$\max \sum_{i=1}^{m} \sum_{j=1}^{n} d_{i(j)} \varphi_j(\eta, N),$$

$$\text{s.t.} \begin{cases} \eta(N) = 1 \\ \eta(S) \leqslant \eta(T), & \forall S, T \subseteq N \text{ s.t. } S \subseteq T, \\ \eta(j) \in W_j, & j = 1, 2, \ldots, n, \\ \eta(j) \geqslant 0, & j = 1, 2, \ldots, n, \end{cases} \tag{8}$$

where $\varphi_j(\eta, N)$ is the Shapley value of the jth position.

Deleting the conditions $\omega_j \in W_j$ and $\eta(j) \in W_j$, $j = 1, 2, \ldots, n$, in models (7) and (8), models for the optimal weight vector on the attribute set where the weight information is completely unknown are obtained.

4.2. *An Approach Based on the GIVTLNSCWA Operator*

This subsection develops an approach to multi-attribute group decision making under uncertain linguistic environment using GIVTLNSCWA operator. The main decision procedure can be described as follows:

Step 1: Transform the uncertain linguistic decision matrix $A^k = (\bar{s}_{ij}^k)_{m\times n}, k = 1, 2, \ldots, q$, into interval-valued 2-tuple linguistic decision matrix $R^k = (r_{ij}^k)_{m\times n}$, where $r_{ij}^k = [(s_{lij}^k, 0), (s_{rij}^k, 0)]$ is an interval-valued 2-tuple linguistic variable such that s_{lij}^k and s_{rij}^k belong to the predefined linguistic term set $S = \{s_i | i = 0, 1, \ldots, t\}$.

Step 2: Calculate the average interval-valued 2-tuple linguistic decision matrix $\bar{R} = (\bar{s}_{ij})_{m\times n}$.

Step 3: Utilize model (1) or (2) to calculate the optimal weight vector on the expert set E.

Step 4: Utilize model (3) or (4) to calculate the optimal weight vector on the ordered set Q.

Step 5: Use the GIVTLNSCWA operator to calculate the comprehensive interval-valued 2-tuple linguistic variable, we obtain the comprehensive interval-valued 2-tuple linguistic decision matrix $R = (x_{ij})_{m\times n}$ with $x_{ij} = [(s_{lij}, \alpha_{ij}^l), (s_{rij}, \alpha_{ij}^r)]$ for all $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

Step 6: Utilize model (5) or (6) to calculate the optimal weight vector on the attribute set C.

Step 7: Utilize model (7) or (8) to calculate the optimal weight vector on the ordered set N.

Step 8: Again use the GIVTLNSCWA operator to calculate the comprehensive interval-valued 2-tuple linguistic variables $x_{ij} = [(s_{lij}, \alpha_{ij}^l), (s_{rij}, \alpha_{ij}^r)]$ of the alternatives $a_i, i = 1, 2, \ldots, m$.

Step 9: According to the comprehensive interval-valued 2-tuple linguistic value $x_i = [(s_{i_1}, \alpha_{i_1}), (s_{i_2}, \alpha_{i_2})]$, $i = 1, 2, \ldots, m$, calculate the score $S(x_i) = (i_1 + i_2 + \alpha_{i_1} + \alpha_{i_2})/2t$ and the accuracy degree $H(x_i) = (i_2 - i_1 + \alpha_{i_2} - \alpha_{i_1})/2t$. Then, select the best one.

Step 10: End.

5. An Illustrative Example

In this section, we use a multi-attribute group decision-making problem of determining which kind of air-conditioning systems should be installed in a library (Yoon, 1989) to illustrate the proposed approach.

A city is planning to build a municipal library. One of the problems facing the city development commissioner is to determine which kind of air-conditioning systems should be installed in the library. The contractor offers five feasible alternatives, which might be adapted to the physical structure of the library. The offered air-conditioning system must take a decision according to four attributes: (1) performance c_1; (2) maintainability c_2; (3) flexibility c_3; and (4) safety c_4. The five possible alternatives a_j ($j = 1, 2, 3, 4, 5$) are to

be evaluated using the uncertain linguistic variables by three experts under the above four attributes on the predefined linguistic term set $S = \{s_i \mid i = 0, 1, \ldots, 6\}$, and construct, respectively, the decision matrices as listed in the following:

$$A^1 = \begin{pmatrix} [s_5, s_6] & [s_4, s_5] & [s_2, s_4] & [s_3, s_4] \\ [s_3, s_4] & [s_1, s_3] & [s_5, s_6] & [s_2, s_3] \\ [s_2, s_4] & [s_3, s_4] & [s_1, s_3] & [s_3, s_5] \\ [s_4, s_5] & [s_3, s_5] & [s_6, s_6] & [s_2, s_3] \\ [s_2, s_3] & [s_4, s_6] & [s_4, s_5] & [s_3, s_4] \end{pmatrix},$$

$$A^2 = \begin{pmatrix} [s_3, s_5] & [s_2, s_4] & [s_1, s_2] & [s_3, s_5] \\ [s_4, s_5] & [s_2, s_3] & [s_2, s_3] & [s_4, s_6] \\ [s_1, s_2] & [s_3, s_5] & [s_1, s_2] & [s_2, s_3] \\ [s_3, s_5] & [s_2, s_4] & [s_2, s_4] & [s_1, s_3] \\ [s_1, s_3] & [s_4, s_5] & [s_5, s_6] & [s_4, s_6] \end{pmatrix},$$

$$A^3 = \begin{pmatrix} [s_2, s_3] & [s_3, s_4] & [s_1, s_3] & [s_2, s_3] \\ [s_3, s_5] & [s_1, s_3] & [s_3, s_5] & [s_2, s_4] \\ [s_1, s_3] & [s_4, s_5] & [s_2, s_3] & [s_4, s_5] \\ [s_2, s_3] & [s_3, s_4] & [s_4, s_5] & [s_1, s_2] \\ [s_4, s_5] & [s_3, s_4] & [s_3, s_4] & [s_2, s_4] \end{pmatrix}.$$

Assume that the weight vector on the expert set E is provided by $W_E = ([0.2, 0.3], [0.4, 0.5], [0.3, 0.4])$ and the weight vector on the ordered set $Q = \{1, 2, 3\}$ is given by $W_Q = ([0.1, 0.3], [0.2, 0.4], [0.3, 0.5])$. Furthermore, the weight vector on the attribute set C is defined by $W_C = ([0.1, 0.3], [0.1, 0.2], [0.2, 0.4], [0.3, 0.5])$ and the weight vector on the ordered set $N = \{1, 2, 3, 4\}$ is provided by $W_N = ([0.1, 0.2], [0.15, 0.25], [0.2, 0.3], [0.25, 0.35])$. To obtain the best alternative(s), the following procedure is involved.

Step 1: From A^k, $k = 1, 2, 3$, we can easily get the associated interval-valued 2-tuple linguistic decision matrices, take A^1 for example,

$$R^1 = \begin{pmatrix} [(s_5, 0), (s_6, 0)] & [(s_4, 0), (s_5, 0)] & [(s_2, 0), (s_4, 0)] & [(s_3, 0), (s_4, 0)] \\ [(s_3, 0), (s_4, 0)] & [(s_1, 0), (s_3, 0)] & [(s_5, 0), (s_6, 0)] & [(s_2, 0), (s_3, 0)] \\ [(s_2, 0), (s_4, 0)] & [(s_3, 0), (s_4, 0)] & [(s_1, 0), (s_3, 0)] & [(s_3, 0), (s_5, 0)] \\ [(s_4, 0), (s_5, 0)] & [(s_3, 0), (s_5, 0)] & [(s_6, 0), (s_6, 0)] & [(s_2, 0), (s_3, 0)] \\ [(s_2, 0), (s_3, 0)] & [(s_4, 0), (s_6, 0)] & [(s_4, 0), (s_5, 0)] & [(s_3, 0), (s_4, 0)] \end{pmatrix}.$$

Step 2: Using R^k, $k = 1, 2, 3$, we get the following average interval-valued 2-tuple linguistic decision matrix

$$\bar{R} = \begin{pmatrix} [(s_3, 0.33), (s_5, -0.33)] & [(s_3, 0), (s_4, 0.33)] & [(s_1, 0.33), (s_3, 0)] & [(s_3, -0.33), (s_4, 0)] \\ [(s_3, 0.33), (s_5, -0.33)] & [(s_1, 0.33), (s_3, 0)] & [(s_3, 0.33), (s_5, -0.33)] & [(s_3, -0.33), (s_4, 0.33)] \\ [(s_1, 0.33), (s_3, 0)] & [(s_3, 0.33), (s_5, -0.33)] & [(s_1, 0.33), (s_3, -0.33)] & [(s_3, 0), (s_4, 0.33)] \\ [(s_3, 0), (s_4, 0.33)] & [(s_3, -0.33), (s_4, 0.33)] & [(s_4, 0), (s_5, 0)] & [(s_1, 0.33), (s_3, -0.33)] \\ [(s_2, 0.33), (s_4, -0.33)] & [(s_4, -0.33), (s_5, 0)] & [(s_4, 0), (s_5, 0)] & [(s_3, 0), (s_5, -0.33)] \end{pmatrix}.$$

Step 3: From steps 1 and 2, we obtain the correlation coefficients as follows:

$$\begin{aligned}
&\left(K\left(\bar{s}_1, \bar{s}_1^1\right), K\left(\bar{s}_2, \bar{s}_2^1\right), K\left(\bar{s}_3, \bar{s}_3^1\right), K\left(\bar{s}_4, \bar{s}_4^1\right)\right) = (0.978, 0.991, 0.988, 0.984),\\
&\left(K\left(\bar{s}_1, \bar{s}_1^2\right), K\left(\bar{s}_2, \bar{s}_2^2\right), K\left(\bar{s}_3, \bar{s}_3^2\right), K\left(\bar{s}_4, \bar{s}_4^2\right)\right) = (0.982, 0.991, 0.952, 0.971),\\
&\left(K\left(\bar{s}_1, \bar{s}_1^3\right), K\left(\bar{s}_2, \bar{s}_2^3\right), K\left(\bar{s}_3, \bar{s}_3^3\right), K\left(\bar{s}_4, \bar{s}_4^3\right)\right) = (0.949, 0.991, 0.99, 0.978).
\end{aligned}$$

According to model (2), the following linear programming model for the optimal fuzzy measure ρ on E is built.

$$\begin{aligned}
&\max 0.0128\big(\rho(e_1) - \rho(e_2, e_3)\big) - 0.009\big(\rho(e_2) - \rho(e_1, e_3)\big)\\
&\quad - 0.003\big(\rho(e_3) - \rho(e_1, e_2)\big) + 3.915,\\
&\text{s.t.} \begin{cases} \rho(e_1, e_2, e_3) = 1,\\ \rho(S) \leqslant \rho(T), \quad \forall S, T \subseteq \{e_1, e_2, e_3\} \text{ s.t. } S \subseteq T,\\ \rho(e_1) \in [0.2, 0.3], \quad \rho(e_2) \in [0.4, 0.5], \quad \rho(e_3) \in [0.3, 0.4]. \end{cases}
\end{aligned}$$

Solving the above model, we get

$$\begin{aligned}
&\rho(e_1) = \rho(e_3) = 0.3, \qquad \rho(e_2) = 0.4, \qquad \rho(e_2, e_3) = 0.4,\\
&\rho(e_1, e_2) = \rho(e_1, e_3) = \rho(e_1, e_2, e_3) = 1,
\end{aligned}$$

and the experts' Shapley values are $\varphi_{e_1}(\rho, E) = 0.52$, $\varphi_{e_2}(\rho, E) = 0.26$, $\varphi_{e_3}(\rho, E) = 0.22$.

Step 4: From the correlation coefficients given in step 3 and model (4), the following linear programming model for the optimal fuzzy measure μ on Q is built.

$$\begin{aligned}
&\max -0.0256\big(\mu(1) - \mu(2, 3)\big) + 0.01\big(\mu(2) - \mu(1, 3)\big)\\
&\quad + 0.017\big(\mu(3) - \mu(1, 2)\big) + 3.915,\\
&\text{s.t.} \begin{cases} \mu(1, 2, 3) = 1,\\ \mu(S) \leqslant \mu(T), \quad \forall S, T \subseteq \{1, 2, 3\} \text{ s.t. } S \subseteq T,\\ \mu(1) \in [0.1, 0.3], \quad \mu(2) \in [0.2, 0.4], \quad \mu(3) \in [0.3, 0.5]. \end{cases}
\end{aligned}$$

Solving the above model, we obtain

$$\begin{aligned}
&\mu(1) = 0.1, \qquad \mu(2) = \mu(1, 2) = 0.2, \qquad \mu(3) = \mu(1, 3) = 0.5,\\
&\mu(2, 3) = \mu(1, 2, 3) = 1,
\end{aligned}$$

and the ordered positions' generalized Shapley values are $\Phi_{\{1,2,3\}}(\mu, Q) = 1$, $\Phi_{\{2,3\}}(\mu, Q) = 0.95$, $\Phi_3(\mu, Q) = 0.63$.

Step 5: If $\lambda = 1$, by the GIVTLNSCWA operator the comprehensive interval-valued 2-tuple linguistic matrix is obtained as follows:

$$R = \begin{pmatrix} [(s_5,-0.473),(s_6,-0.276)] & [(s_4,-0.235),(s_5,-0.203)] & [(s_2,-0.203),(s_4,-0.235)] & [(s_3,-0.033),(s_4,0.448)] \\ [(s_4,-0.46),(s_5,-0.12)] & [(s_1,0.48),(s_3,0)] & [(s_5,-0.438),(s_6,-0.267)] & [(s_3,0.109),(s_5,-0.001)] \\ [(s_2,0.203),(s_4,-0.235)] & [(s_4,-0.441),(s_5,-0.105)] & [(s_1,0.032),(s_3,-0.48)] & [(s_3,0.395),(s_5,-0.082)] \\ [(s_4,-0.25),(s_5,-0.053)] & [(s_3,-0.032),(s_5,-0.203)] & [(s_6,-0.469),(s_6,-0.235)] & [(s_2,-0.223),(s_3,-0.026)] \\ [(s_3,0.169),(s_3,0.082)] & [(s_4,-0.026),(s_6,-0.249)] & [(s_4,0.448),(s_5,0.448)] & [(s_3,0.448),(s_5,-0.04)] \end{pmatrix}.$$

Step 6: From the comprehensive interval-valued 2-tuple linguistic matrix R, we have

$$D = (d_{ij})_{5\times 4} = \begin{pmatrix} 0.619 & 0.615 & 0.607 & 0.58 \\ 0.603 & 0.558 & 0.687 & 0.586 \\ 0.567 & 0.613 & 0.555 & 0.588 \\ 0.606 & 0.605 & 0.702 & 0.551 \\ 0.577 & 0.623 & 0.684 & 0.588 \end{pmatrix}.$$

According to model (6), the following linear programming model for the optimal fuzzy measure v on C is built.

$$\begin{aligned} &\max -0.0187\big(v(c_1) - v(c_2,c_3,c_4)\big) - 0.0046\big(v(c_2) - v(c_1,c_3,c_4)\big) \\ &\quad + 0.0686\big(v(c_3) - v(c_1,c_2,c_4)\big) - 0.0453\big(v(c_4) - v(c_1,c_2,c_3)\big) \\ &\quad - 0.0117\big(v(c_1,c_2) - v(c_3,c_4)\big) + 0.025\big(v(c_1,c_3) - v(c_2,c_4)\big) \\ &\quad - 0.032\big(v(c_1,c_4) - v(c_2,c_3)\big) + 3.0284, \\ &\text{s.t.} \begin{cases} v(c_1,c_2,c_3,c_4) = 1, \\ v(S) \leqslant v(T), \quad \forall S, T \subseteq \{c_1,c_2,c_3,c_4\} \text{ s.t. } S \subseteq T, \\ v(c_1) \in [0.1, 0.3], \quad v(c_2) \in [0.1, 0.2], \\ v(c_3) \in [0.2, 0.4], \quad v(c_4) \in [0.3, 0.5]. \end{cases} \end{aligned}$$

Solving the above model, we get

$$\begin{aligned} &v(c_1) = v(c_2) = v(c_1,c_2) = 0.1, \qquad v(c_3) = 0.4, \\ &v(c_4) = v(c_1,c_4) = v(c_2,c_4) = v(c_1,c_2,c_4) = 0.3, \\ &v(c_1,c_3) = v(c_2,c_3) = v(c_3,c_4) = v(c_1,c_2,c_3) = v(c_1,c_3,c_4) = v(c_2,c_3,c_4) \\ &\qquad = v(c_1,c_2,c_3,c_4) = 1, \end{aligned}$$

and the attributes' Shapley values are $\varphi_{c_1}(v,C) = \varphi_{c_2}(v,C) = 0.075$, $\varphi_{c_3}(v,C) = 0.675$, $\varphi_{c_4}(v,C) = 0.175$.

Step 7: From the matrix D in step 6 and model (8), the following linear programming model for the optimal fuzzy measure η on N is built.

$$\begin{aligned} &\max -0.0693\big(\eta(1) - \eta(2,3,4)\big) - 0.0255\big(\eta(2) - \eta(1,3,4)\big) \\ &\quad + 0.0024\big(\eta(3) - \eta(1,2,4)\big) + 0.0924\big(\eta(4) - \eta(1,2,3)\big) \end{aligned}$$

$$
\begin{aligned}
&- 0.0474\big(\eta(1,2) - \eta(3,4)\big) - 0.0335\big(\eta(1,3) - \eta(2,4)\big) \\
&+ 0.0115\big(\eta(1,4) - \eta(2,3)\big) + 3.0284, \\
&\text{s.t.} \begin{cases} \eta(1,2,3,4) = 1, \\ \eta(S) \leqslant \eta(T), \quad \forall S, T \subseteq \{1,2,3,4\} \text{ s.t. } S \subseteq T, \\ \eta(1) \in [0.1, 0.2], \quad \eta(2) \in [0.15, 0.25], \\ \eta(3) \in [0.2, 0.3], \quad \eta(4) \in [0.25, 0.35]. \end{cases}
\end{aligned}
$$

Solving the above model, we derive

$$
\begin{aligned}
&\eta(1) = 0.1, \qquad \eta(2) = \eta(1,2) = 0.15, \\
&\eta(3) = \eta(1,3) = \eta(2,3) = \eta(1,2,3) = 0.2, \\
&\eta(4) = 0.35, \eta(1,4) = \eta(2,4) = \eta(3,4) = \eta(1,2,4) = \eta(1,3,4) = \eta(2,3,4) \\
&\qquad = \eta(1,2,3,4) = 1,
\end{aligned}
$$

and the ordered positions' generalized Shapley values are $\Phi_{\{1,2,3,4\}}(\eta, N) = 1$, $\Phi_{\{2,3,4\}}(\eta, N) = 0.95$, $\Phi_{\{3,4\}}(\eta, N) = 0.91$, $\Phi_4(\eta, N) = 0.704$.

Step 8: If $\lambda = 1$, by the GIVTLNSCWA operator the comprehensive interval-valued 2-tuple linguistic values of the alternatives a_i, $i = 1, 2, 3, 4, 5$, are obtained as follows:

$$
\begin{aligned}
&x_1 = \big[(s_3, 0.474), (s_5, 0.096)\big], \qquad x_2 = \big[(s_4, 0.488), (s_6, 0.323)\big], \\
&x_3 = \big[(s_3, 0.209), (s_4, 0.237)\big], \qquad x_4 = \big[(s_5, 0.395), (s_6, 0.314)\big], \\
&x_5 = \big[(s_4, 0.407), (s_5, 0.433)\big].
\end{aligned}
$$

Step 9: From the comprehensive interval-valued 2-tuple linguistic values x_i, $i = 1, 2, 3, 4, 5$, the scores are

$$
\begin{aligned}
&S(x_1) = 0.698, \qquad S(x_2) = 0.847, \qquad S(x_3) = 0.586, \\
&S(x_4) = 0.923, \qquad S(x_5) = 0.82.
\end{aligned}
$$

Thus, the ranking result is $a_4 > a_2 > a_5 > a_1 > a_3$. Namely, the alternative a_4 is the best choice.

In the above example, we only give the ranking order according to the GLHFHSWA operator with $\lambda = 1$. With respect to the different values of λ, ranking order is obtained as shown in Table 1.

From Table 1, we see that the ranking orders are different for the different values of λ. The ranking orders are stable with the increase of the value of λ, and the alternative a_4 changes from the worst to the best.

In this example, if we assume that there are no interactions, by the GIVTLHWA operator ranking orders are obtained as shown in Table 2.

Table 1
Ranking order based on the GIVTLNSCWA operator.

	$S(x_1)$	$S(x_2)$	$S(x_3)$	$S(x_4)$	$S(x_5)$	Ranking order
$\lambda = -2$	0.6386	0.6607	0.6431	0.5875	0.6643	$a_5 > a_2 > a_3 > a_1 > a_4$
$\lambda = -1$	0.689	0.6325	0.6723	0.641	0.724	$a_5 > a_1 > a_3 > a_4 > a_2$
$\lambda \to 0$	0.7359	0.7421	0.6614	0.7899	0.8036	$a_5 > a_4 > a_2 > a_1 > a_3$
$\lambda = 1$	0.6982	0.847	0.5856	0.9234	0.82	$a_4 > a_2 > a_5 > a_1 > a_3$
$\lambda = 2$	0.5772	0.891	0.4666	0.9728	0.8038	$a_4 > a_2 > a_5 > a_1 > a_3$
$\lambda = 5$	0.5003	0.9153	0.3732	0.9985	0.7643	$a_4 > a_2 > a_5 > a_1 > a_3$

Table 2
Ranking order based on the GIVTLHWA operator.

	$S(x_1)$	$S(x_2)$	$S(x_3)$	$S(x_4)$	$S(x_5)$	Ranking order
$\lambda = -2$	0.6631	0.601	0.4219	0.6202	0.5478	$a_1 > a_4 > a_2 > a_5 > a_3$
$\lambda = -1$	0.6565	0.6148	0.4786	0.6317	0.6451	$a_1 > a_5 > a_4 > a_2 > a_3$
$\lambda \to 0$	0.634	0.6512	0.5452	0.6613	0.7257	$a_5 > a_4 > a_2 > a_1 > a_3$
$\lambda = 1$	0.6095	0.7013	0.5833	0.7176	0.7778	$a_5 > a_4 > a_2 > a_1 > a_3$
$\lambda = 2$	0.5916	0.7409	0.5935	0.775	0.8148	$a_5 > a_4 > a_2 > a_3 > a_1$
$\lambda = 5$	0.5677	0.7895	0.5799	0.8491	0.8791	$a_5 > a_4 > a_2 > a_3 > a_1$

From Table 2, we also obtain the different ranking orders for the different values of λ. The ranking orders are stable with the increase of the value of λ, but the alternative as changes from the worse to the best.

From this example, we know that the different ranking orders are obtained using the different aggregation operators as well as the different values of λ. Thus, before making a decision, the experts need to decide the using aggregation operator and λ value. If there is no special explanation, we suggest that the experts use the GLHFHSWA operator. For the value of λ, the pessimistic experts could use the smaller λ value and the optimistic experts may apply the larger λ value. Meanwhile, the neutral experts could use $\lambda = 1$.

In this example, we use the introduced interval-valued 2-tuple linguistic representation model to make decision, whereas we can also apply models given in the literature (Lin *et al.*, 2009; Zhang, 2012) to obtain the best choice.

6. Conclusion

With respect to multi-attribute decision making with uncertain linguistic variables, the paper introduces an interval-valued 2-tuple linguistic representation model, by which any interval-valued 2-tuple linguistic variable can be transformed into an interval in $[0, 1]$. To address the interactions between elements, some interval-valued 2-tuple linguistic aggregation operators are defined. Because of various reasons, such as the complexity and uncertainty of real world decision making problems and the inherent subjective nature of human thinking, the weight information is usually incompletely known. To obtain the weight vector, some models based on the correlation coefficient and the Shapley function are established. Then, an approach to uncertain linguistic multi-attribute group decision

making is developed. It is worth pointing out that the new defined Choquet operators can process the interval-valued 2-tuple linguistic representation models introduced by Lin *et al.* (2009) and Zhang (2012).

In addition, when the domain of uncertain linguistic variables is restricted in the setting of linguistic variables, the introduced method can be directly used in linguistic multi-attribute decision making. Besides the application in the decision making, the defined aggregation operators and models for the optimal weight vectors can also be used in some other fields, such as human resources management, pattern recognition, clustering analysis, and social science.

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Kai kurie apibendrintieji koreliuojantieji intervaluose vertinami kortežiniai lingvistiniai apjungimo operatoriai ir jų taikymas priimant sprendimus

Fanyong MENG, Mingxun ZHU, Xiaohong CHEN

Pristatytas naujas sprendimų priėmimo intervaluose vertinimų daugelio rodiklių (daugiatikslėje) neraiškioje kalbinėje aplinkoje kortežininis lingvistinis išreiškimo modelis. Uždaviniams spręsti, kai aibės elementai yra tarpusavyje priklausomi, yra apibrėžti apibendrintieji koreliuojantieji intervaluose vertinami kortežiniai lingvistiniai apjungimo operatoriai. Verta pažymėti, kad intervaluose vertinami lingvistiniai kortežų operatoriai, grindžiami suminiais matais, yra ypatingieji mūsų operatorių atvejai. Aptariami kai kurie atvejai ir jų norimos savybės. Sudaryti modeliai, grindžiami koreliacijos koeficientu. Jais galima gauti optimalias reikšmingumų (svorių) vektoriaus reikšmes. Sudarytas grupinis daugiarodiklis (daugiatikslis) sprendimų priėmimo modelis, kuris apibrėžiamas neraiškia lingvistine informacija. Galiausiai, pasirinktas pavyzdys, sukurtos procedūros efektyvumui ir tikslingumui parodyti.