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An Integrated Maximizing Consistency and Multi-Choice Goal Programming Approach for Hybrid Multiple Criteria Group Decision Making Based on Interval-Valued Intuitionistic Fuzzy Number

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Abstract. Interval-valued intuitionistic fuzzy numbers (IVIFNs) characterized by a membership function and a non-membership function with values that are intervals, have strong ability to handle imprecise and ambiguous information in real-world applications. This paper proposes an integrated maximizing consistency and multi-choice goal programming (MCGP) approach to handle hybrid multi-criteria group decision making problems based on IVIFNs. Firstly, the hybrid decision information (including crisp numbers, intervals, intuitionistic fuzzy numbers and linguistic variables) are normalized into the IVIFNs. Then, an ordinal consistency index and a cardinal consistency index are proposed to measure the consistency between the individual opinion and the group opinion, respectively. And an optimal model based on maximizing consistency is constructed to derive the weights of experts. Afterwards, the comprehensive ratings and the ranking values of alternatives are obtained by the hybrid weighted aggregation operator and the proposed ratio function of IVIFNs, respectively. Furthermore, a MCGP model based on the ranking values is constructed to identify the optimal alternatives and their optimum quantities. At length, an illustrative case is provided to verify the proposed approach.

Key words: hybrid multiple criteria group decision making, interval-valued intuitionistic fuzzy number, consistency, multi-choice goal programming.

1. Introduction

Multiple criteria group decision making (MCGDM) usually refers to the decision process that several experts make evaluations with their respective knowledge, experience and preference for a set of alternatives over multiple criteria to give the criteria values of alternatives, and then the decision results from all experts are aggregated to form an overall ranking order of alternatives, which is an important research topic in decision theory (Zhang and Xu, 2014a). In classical MCGDM processes, all decision data are known precisely or given as real numbers (Sadeghi *et al.*, 2012; Stanujkic *et al.*, 2013). However, due to the inherent vagueness of human preferences as well as the objects being fuzzy and uncertain, the criteria values involved in MCGDM problems are not always expressed by real numbers, but some are better suited to be denoted by fuzzy numbers, such as intervals (Zadeh, 1965; Stanujkic *et al.*, 2012), hesitant fuzzy elements (Xu and Zhang, 2009; Zhang and Xu, 2015), intuitionistic fuzzy numbers (IFNs) (Zeng *et al.*, 2013; Liu and Liu, 2014), intuitionistic linguistic numbers (Liu, 2013a, 2013b; Liu and Wang, 2014) and the interval-valued IFNs (IVIFNs) (Xu and Yager, 2009; Park *et al.*, 2011b; Li, 2011), etc. IVIFNs with the membership and non-membership functions denoted by intervals have received increasing attentions because of their ability to handle imprecise and ambiguous information in real-world applications. Many scholars investigated the basic operators of interval-valued intuitionistic fuzzy sets (IVIFSs) and their properties (Atanassov, 1994; Lakshmana Gomathi Nayagam and Sivaraman, 2011), aggregating operators (Xu, 2007b; Yu *et al.*, 2012; Liu, 2014), correlation coefficient (Bustince and Burillo, 1995; Park *et al.*, 2009), topological properties (Kumar Mondal and Samanta, 2001) and their distance and similarity measures (Xu, 2010; Zhang *et al.*, 2010; Wei *et al.*, 2011). These research works have made great contributions to enrich IVIFS theory. The IVIFNs have also been used in a wide range of applications, such as game theory filed (Li, 2010) and decision making fields (Chen and Li, 2013; Tan, 2011; Park *et al.*, 2011a; Razavi Hajiagha *et al.*, 2013).

In recent decades, a considerable number of studies have reported decision-making models and methods within IVIFN environments. For instance, Tan (2011) presented a multi-criteria interval-valued intuitionistic fuzzy group decision making method using Choquet integral-based TOPSIS (*Technique for Order Preference by Similarity to Ideal Solution*) approach. Park *et al.* (2011a) extended the TOPSIS method to solve MCGDM problems with IVIFNs. Yue and Jia (2013) investigated the MCGDM problems with IVIFNs in which the weights of experts are unknown in advance and developed a method based on the TOPSIS and the optimistic coefficient to obtain the weights of experts. Yue (2011) also proposed a method based on distance measure for determining experts' weights in MCGDM problems with IVIFNs. Wang and Li (2012), Chen (2013) respectively developed the interval-valued intuitionistic fuzzy LINMAP *(linear programming technique for multidimensional analysis of preference)* methods to handle MCGDM problems with IVIFNs. Depending on the likelihood of fuzzy preference relations between IVIFNs, Chen (2014) proposed an interval-valued intuitionistic fuzzy QUALIFLEX (*qualitative flexible multiple criteria method*) outranking method with a likelihood-based comparison approach for handling the decision problems within a decision environment of IVIFNs.

However, due to the complex structures of IVIFNs, it may be difficult for the experts to directly collect the IVIFN decision data. Yu *et al.* (2011) developed a method based on preference degrees to transform the linguistic variables into IVIFNs. That is to say, the linguistic variables can be used to collect the IVIFN decision data. Therefore, this paper establishes a decision environment based on IVIFNs for the hybrid MCGDM problems in which the ratings of alternatives on the criteria take the forms of real numbers, interval numbers, IFNs and linguistic variables. Furthermore, an integrated consistency maximization model and multi-choice goal programming (MCGP) approach is proposed to solve the hybrid MCGDM problem based on IVIFNs. This approach first normalizes the hybrid decision information into the IVIFNs, and then constructs a consistency maximization model to calculate the weights of experts, and further calculates the comprehensive ratings and the ranking values of alternatives, and finally a MCGP model on the basis of the ranking values is constructed to determine the optimal order quantities from the best alternatives being subjected to some resource constraints.

The remainder of this paper is organized as follows: in Section 2, some basic concepts related to IVIFNs are briefly reviewed and a hybrid MCGDM problem based on IVIFNs is described. In Section 3, a method based on maximizing consistency and MCGP is developed. In Section 4, an illustrative example is provided to demonstrate the applicability and implementation process of the proposed method and the paper finishes with some concluding remarks in Section 5.

2. Multiple Criteria Group Decision Environment with IVIFNs

2.1. *Preliminaries on IVIFSs and IVIFNs*

Definition 1. (See Atanassov and Gargov, 1989.) Let a set X be a universe of discourse. An IVIFS is an object having the form:

$$\tilde{A} = \left\{ \langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) \rangle | x \in X \right\} \tag{2.1}$$

where $\mu_{\tilde{A}}(x) = [\mu^L_{\tilde{A}}(x), \mu^R_{\tilde{A}}(x)] \subseteq [0, 1]$ and $\upsilon_{\tilde{A}}(x) = [\upsilon^L_{\tilde{A}}(x), \upsilon^R_{\tilde{A}}(x)] \subseteq [0, 1]$ are intervals, $\mu^L_{\tilde{A}}(x) = \inf \mu_{\tilde{A}}(x)$, $\mu^R_{\tilde{A}}(x) = \sup \mu_{\tilde{A}}(x)$, $\upsilon^L_{\tilde{A}}(x) = \inf \upsilon_{\tilde{A}}(x)$, $\upsilon^R_{\tilde{A}}(x) = \sup \upsilon_{\tilde{A}}(x)$ and $\mu^R_{\tilde{A}}(x) + \upsilon^R_{\tilde{A}}(x) \leqslant 1$.

For each element $x \in X$, its uncertainty interval relative to $\tilde{A}$ is given as:

$$\pi_{\tilde{A}}(x) = \left[\pi^L_{\tilde{A}}(x), \pi^R_{\tilde{A}}(x)\right] = \left[1 - \mu^R_{\tilde{A}}(x) - \upsilon^R_{\tilde{A}}(x), 1 - \mu^L_{\tilde{A}}(x) - \upsilon^L_{\tilde{A}}(x)\right] \subseteq [0, 1]. \tag{2.2}$$

For convenience, Xu (2007a) called $\tilde{\alpha} = (\mu_{\tilde{\alpha}}, \nu_{\tilde{\alpha}})$ an IVIFN and denoted it by $\tilde{\alpha} = ([a, b], [c, d])$, where $[a, b] \subseteq [0, 1]$, $[c, d] \subseteq [0, 1]$ and $b + d \leqslant 1$.

Remark 1. It is noted that if $a = b$ and $c = d$, then the IVIFN $\tilde{\alpha}$ is reduced to an IFN $\tilde{\alpha} = (a, c)$. Furthermore, if $a + c = 1$, then the IFN $\tilde{\alpha}$ is reduced to a real number $\tilde{\alpha} = a$. On the other hand, the IFN $\tilde{\alpha}$ is equivalent to the interval number $\tilde{\alpha} = [a, 1 - c]$. That is to say, all the real numbers, interval numbers and IFNs are the special cases of IVIFNs.

Definition 2. (See Xu, 2007a.) Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, then the score function of $\tilde{\alpha}$ can be defined as:

$$s(\tilde{\alpha}) = \frac{1}{2}(a - c + b - d) \tag{2.3}$$

and the accuracy function of $\tilde{\alpha}$ as:

$$h(\tilde{\alpha}) = \frac{1}{2}(a + b + c + d). \tag{2.4}$$

Thus, for two IVIFNs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, the ranking law of IVIFNs is introduced as follows (Xu, 2007a):

(1) If $s(\tilde{\alpha}_1) < s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \prec_s \tilde{\alpha}_2$;

(2) If $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, then $\begin{cases} h(\tilde{\alpha}_1) < h(\tilde{\alpha}_2) \Rightarrow \tilde{\alpha}_1 \prec_{sh} \tilde{\alpha}_2, \\ h(\tilde{\alpha}_1) = h(\tilde{\alpha}_2) \Rightarrow \tilde{\alpha}_1 \sim_{sh} \tilde{\alpha}_2. \end{cases}$

Example 1. Let $\tilde{a}_1 = ([0.3, 0.4], [0.1, 0.2])$ and $\tilde{a}_2 = ([0.1, 0.6], [0.1, 0.2])$ be two IVIFNs. According to the score and accuracy functions of IVIFNs developed by Xu (2007a), it can be obtained: $s(\tilde{a}_1) = 0.2$, $h(\tilde{a}_1) = 0.5$, $s(\tilde{a}_2) = 0.2$, $h(\tilde{a}_2) = 0.5$. Using the ranking law of IVIFNs developed by Xu (2007a), it is easy to know that $\tilde{a}_1 \sim_{sh} \tilde{a}_2$.

It is observed from Example 1 that there may yield many indistinguishable pairs of IVIFNs when using the score and accuracy functions based-ranking method developed by Xu (2007a). In other words, this kind of the ranking method of IVIFNs is invalid in many situations like Example 1 and therefore should be improved. Bearing this fact in mind, the novel score function and the new accuracy function are developed for measuring IVIFNs.

Definition 3. Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, then the improved score function of $\tilde{\alpha}$ is defined as follows:

$$\mathbb{S}_\delta(\tilde{\alpha}) = \frac{1}{2}\left(a^\delta - c^\delta + b^\delta - d^\delta\right) \tag{2.5}$$

and the improved accuracy function of the IVIFN $\tilde{\alpha}$ as:

$$\mathbb{F}_\delta(\tilde{\alpha}) = \frac{1}{4}\left(a^\delta + b^\delta + c^\delta + d^\delta\right) \tag{2.6}$$

where δ $(0 < \delta \leqslant 1)$ is a parameter determined by the decision maker, which can be tuned according to the decision making problem at hand.

Remark 2. It is worth pointing out that if $\delta = 1$, the $\mathbb{S}_\delta(\tilde{\alpha})$ is reduced to the score function $s(\tilde{\alpha})$ developed by Xu (2007a). In the practical application, the value of the parameter δ usually equates 0.1.

Thus, for two IVIFNs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, the novel ranking method for IVIFNs is presented as follows:

(1) If $\mathbb{S}_\delta(\tilde{\alpha}_1) < \mathbb{S}_\delta(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \prec_{\mathbb{S}} \tilde{\alpha}_2$;

(2) If $\mathbb{S}_\delta(\tilde{\alpha}_1) = \mathbb{S}_\delta(\tilde{\alpha}_2)$, then $\begin{cases} \mathbb{F}_\delta(\tilde{\alpha}_1) < \mathbb{F}_\delta(\tilde{\alpha}_2) \Rightarrow \tilde{\alpha}_1 \prec_{\mathbb{SF}} \tilde{\alpha}_2, \\ \mathbb{F}_\delta(\tilde{\alpha}_1) = \mathbb{F}_\delta(\tilde{\alpha}_2) \Rightarrow \tilde{\alpha}_1 \sim_{\mathbb{SF}} \tilde{\alpha}_2. \end{cases}$

Following Example 1, using Definition 3 the corresponding calculation results can be obtained and showed in Table 1.

Table 1
The results obtained by the novel ranking approach of IVIFNs.

	$\mathbb{S}_\delta(\tilde{\alpha}_1)$	$\mathbb{F}_\delta(\tilde{\alpha}_1)$	$\mathbb{S}_\delta(\tilde{\alpha}_2)$	$\mathbb{F}_\delta(\tilde{\alpha}_2)$	Ranking orders
$\delta = 0.001$	0.0009	0.9985	0.0005	0.9983	$\tilde{a}_2 \prec_{\mathbb{SF}} \tilde{a}_1$
$\delta = 0.01$	0.0088	0.9851	0.0054	0.9832	$\tilde{a}_2 \prec_{\mathbb{SF}} \tilde{a}_1$
$\delta = 0.1$	0.0767	0.8611	0.0494	0.8475	$\tilde{a}_2 \prec_{\mathbb{SF}} \tilde{a}_1$

Table 2
The results obtained by the ratio function-based ranking method.

	$\mathbb{Q}_\delta(\tilde{\alpha}_1)$	$\mathbb{Q}_\delta(\tilde{\alpha}_2)$	Ranking orders
$\delta = 0.01$	0.5022	0.5014	$\tilde{a}_2 \prec_{\mathbb{Q}} \tilde{a}_1$
$\delta = 0.1$	0.5223	0.5146	$\tilde{a}_2 \prec_{\mathbb{Q}} \tilde{a}_1$
$\delta = 0.5$	0.6072	0.5883	$\tilde{a}_2 \prec_{\mathbb{Q}} \tilde{a}_1$

With the help of Example 1, it is easy to see that the proposed ranking approach for IVIFNs (Definition 3) is superior to Xu's approach (Definition 2). Although the new ranking approach (Definition 3) seems to be effective, using this approach makes the process of decision making more time-consuming. Because the decision process is required to be divided into several steps and it is necessary to add other rules for obtaining the best alternative when utilizing this approach to deal with the MCGDM problems (Zhang and Xu, 2014b). Therefore, we next develop a direct ranking method based on the ratio function to compare the magnitude of IVIFNs. The concept of the ratio function is introduced as follows:

Definition 4. Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN, then a ratio function of $\tilde{\alpha}$ is defined as follows:

$$\mathbb{Q}_\delta(\tilde{\alpha}) = \frac{a^\delta + b^\delta}{a^\delta + b^\delta + c^\delta + d^\delta} \tag{2.7}$$

where δ $(0 < \delta \leqslant 1)$ is a parameter determined by the decision maker, which can be tuned according to the decision making problem at hand.

Obviously, the ratio function $\mathbb{Q}_\delta(\tilde{\alpha}) \in [0, 1]$. In the practical application, the value of the parameter δ usually equates 0.5. Thus, for two IVIFNs $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, the ratio function-based ranking method for IVIFNs is presented as follows:

(1) if $\mathbb{Q}_\delta(\tilde{\alpha}_1) < \mathbb{Q}_\delta(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \prec_{\mathbb{Q}} \tilde{\alpha}_2$;
(2) if $\mathbb{Q}_\delta(\tilde{\alpha}_1) = \mathbb{Q}_\delta(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \sim_{\mathbb{Q}} \tilde{\alpha}_2$;
(3) if $\mathbb{Q}_\delta(\tilde{\alpha}_1) > \mathbb{Q}_\delta(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1 \succ_{\mathbb{Q}} \tilde{\alpha}_2$.

Following the Example 1, according to the ratio function-based ranking method we can calculate the corresponding ranking results which are showed in Table 2.

All ranking results obtained by these three methods in Example 1 are listed in Table 3.

It is observed from Table 3 that the ranking order of IVIFNs obtained by Definition 4 is the same as the result obtained by Definition 3, but is not consistent with the result obtained by Definition 2. The main reason is that in Definition 2 (Xu, 2007a) both the

Table 3
The comparison results of the ranking orders of IVIFNs.

The ranking approach	Ranking orders
Definition 2	$\tilde{a}_2 \sim_{sh} \tilde{a}_1$
Definition 3	$\tilde{a}_2 \prec_{\mathbb{SF}} \tilde{a}_1$
Definition 4	$\tilde{a}_2 \prec_{\mathbb{Q}} \tilde{a}_1$

score function and the accuracy function are the functions being linear in their arguments and are just the special cases of the corresponding functions in Definition 3, which cannot distinguish these two IVIFNs in Example 1. Although the proposed ranking method in Definition 3 is consistent with the ratio function-based ranking method in Definition 4, using this ranking method in Definition 3 makes the process of decision making more time-consuming. Therefore, the ranking method in Definition 4 is much superior to the ranking approach in Definition 2 and the proposed ranking method in Definition 3.

Definition 5. (See Xu and Chen, 2008.) Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be two IVIFNs, then the interval-valued intuitionistic fuzzy Euclidean distance is defined as follows:

$$d(\tilde{\alpha}_1, \tilde{\alpha}_2) = \sqrt{\frac{1}{4}\big((a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2 + (d_1 - d_2)^2\big)}. \tag{2.8}$$

2.2. *Description of the Hybrid MCGDM Problem Based on IVIFNs*

This section establishes a decision environment based on IVIFNs for the hybrid MCGDM problems in which the criteria values take the forms of real numbers, intervals, IFNs and linguistic variables.

Consider the following hybrid MCGDM problem: let $\boldsymbol{A} = \{A_1, A_2, \ldots, A_m\}$ $(m \geqslant 2)$ be a discrete set of m feasible alternatives, $\mathbb{C} = \{C_1, C_2, \ldots, C_n\}$ be a finite set of criteria, and $E = \{e_1, e_2, \ldots, e_g\}$ be a group of experts. Let $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)^{\mathrm{T}}$ be the weight vector of criteria, which satisfies the condition that $0 \leqslant w_j \leqslant 1$ $(j = 1, 2, \ldots, n)$, $\sum_{j=1}^{n} w_j = 1$; and $\boldsymbol{\lambda} = \{\lambda_1, \lambda_2, \ldots, \lambda_g\}$ be the weight vector of experts, where $0 \leqslant \lambda_k \leqslant 1$ $(k = 1, 2, \ldots, g)$ and $\sum_{k=1}^{g} \lambda_k = 1$. Without loss of generality, this study assumes that the information about criteria weights is completely known in advance, while the information involving the experts' weights is partially known. Owing to the fact that the experts may provide different forms of the ratings of alternatives according to different evaluation criteria, this paper considers four distinct forms of the evaluation information, i.e., real numbers, interval numbers, IFNs and linguistic variables. The criterion C_j in the criterion set $\mathbb{C}$ is evaluated using only one of the aforementioned four distinct forms. Thus, the criterion set $\mathbb{C}$ is divided into four subsets $\mathbb{C}_l$ $(l = 1, 2, 3, 4)$ in which the criteria values are expressed as real numbers, interval numbers, IFNs and linguistic variables, respectively. Let $\mathbb{C}_1 = \{C_1, C_2, \ldots, C_{j_1}\}$, $\mathbb{C}_2 = \{C_{j_1+1}, C_{j_1+2}, \ldots, C_{j_2}\}$, $\mathbb{C}_3 = \{C_{j_2+1}, C_{j_2+2}, \ldots, C_{j_3}\}$, $\mathbb{C}_4 = \{C_{j_3+1}, C_{j_3+2}, \ldots, C_n\}$ and $1 \leqslant j_1 \leqslant j_2 \leqslant j_3 \leqslant n$. Thus, $\mathbb{C}_l \bigcap \mathbb{C}_\eta = \emptyset$ $(l, \eta = 1, 2, 3, 4; l \neq \eta)$ and $\bigcup_{l=1}^{4} \mathbb{C}_l = \mathbb{C}$, where $\emptyset$ is an empty set.

Let the ratings of the alternative $A_i \in \boldsymbol{A}$ on the criterion $C_j \in \mathbb{C}$ given by the expert $e_k \in E$ be expressed by r_{ij}^k, it is noted that:

(1) if $j = 1, 2, \dots, j_1$, then $r_{ij}^k = a_{ij}^k$ is expressed as a real number;
(2) if $j = j_1+1, j_1+2, \dots, j_2$, then $r_{ij}^k = [a_{ij}^k, b_{ij}^k]$ is expressed as an interval number;
(3) if $j = j_2+1, j_2+2, \dots, j_3$, then $r_{ij}^k = (a_{ij}^k, c_{ij}^k)$ is expressed as an IFN;
(4) if $j = j_3+1, j_3+2, \dots, n$, then $r_{ij}^k = s_{ij}^k \in S$ is a linguistic variable which can be captured by an IVIFN $s_{ij}^k = ([a_{ij}^k, b_{ij}^k], [c_{ij}^k, d_{ij}^k])$ (Yu *et al.*, 2011).

For convenience of understanding, the four cases of r_{ij}^k are rewritten as the form in Eq. (2.9).

$$r_{ij}^k = \begin{cases} a_{ij}^k, & \text{if } j = 1, 2, \dots, j_1, \\ [a_{ij}^k, b_{ij}^k], & \text{if } j = j_1+1, j_1+2, \dots, j_2, \\ (a_{ij}^k, c_{ij}^k), & \text{if } j = j_2+1, j_2+2, \dots, j_3, \\ ([a_{ij}^k, b_{ij}^k], [c_{ij}^k, d_{ij}^k]), & \text{if } j = j_3+1, j_3+2, \dots, n. \end{cases} \tag{2.9}$$

Thus, this hybrid MCGDM problem is concisely expressed in the matrix format $\Re^k = (r_{ij}^k)_{m\times n}$ $(k = 1, 2, \dots, g)$. Meanwhile, the criterion subset $\mathbb{C}_l$ can be further divided into two subsets $\mathbb{C}_l^b$ and $\mathbb{C}_l^c$, where $\mathbb{C}_l^b$ and $\mathbb{C}_l^c$ are respectively the sets of benefit (the bigger the better) and cost (the smaller the better) criteria, $\mathbb{C}_l = \mathbb{C}_l^b \bigcup \mathbb{C}_l^c$ and $\mathbb{C}_l^b \bigcap \mathbb{C}_l^c = \emptyset$ $(l = 1, 2, 3, 4)$. Moreover, the dimensions and measurements of criteria values are usually different because the natures of these criteria are different. To this end, the criteria values should be normalized to ensure their compatibility. According to the normalized method introduced by Yu *et al.* (2011), the criterion value r_{ij}^k is normalized to $\bar{r}_{ij}^k$ as below:

$$\bar{r}_{ij}^k = \begin{cases} (a_{ij}^k - \min_i(a_{ij}^k))/U_1, \quad \text{if } j = 1, 2, \dots, j_1^b, \\ (\max_i(a_{ij}^k) - a_{ij}^k)/U_1, \quad \text{if } j = j_1^b+1, j_1^b+2, \dots, j_1, \\ ((a_{ij}^k - \min_i(a_{ij}^k))/U_2, (\max_i(b_{ij}^k) - b_{ij}^k)/U_2), \\ \quad \text{if } j = j_1+1, j_1+2, \dots, j_2^b, \\ ((\max_i(b_{ij}^k) - b_{ij}^k)/U_2, (a_{ij}^k - \min_i(a_{ij}^k))/U_2), \\ \quad \text{if } j = j_2^b+1, j_2^b+2, \dots, j_2, \\ ((a_{ij}^k - \min_i(a_{ij}^k))/U_3, (c_{ij}^k - \min_i(c_{ij}^k))/U_3), \\ \quad \text{if } j = j_2+1, j_2+2, \dots, j_3^b, \\ ((c_{ij}^k - \min_i(c_{ij}^k))/U_3, (a_{ij}^k - \min_i(a_{ij}^k))/U_3), \\ \quad \text{if } j = j_3^b+1, j_3^b+2, \dots, j_3, \\ \left(\begin{array}{l} [(a_{ij}^k - \min_i(a_{ij}^k))/U_4, (b_{ij}^k - \min_i(a_{ij}^k))/U_4], \\ [(c_{ij}^k - \min_i(c_{ij}^k))/U_4, (d_{ij}^k - \min_i(c_{ij}^k))/U_4] \end{array}\right), \\ \quad \text{if } j = j_3+1, j_3+2, \dots, j_4^b, \\ \left(\begin{array}{l} [(c_{ij}^k - \min_i(c_{ij}^k))/U_4, (d_{ij}^k - \min_i(c_{ij}^k))/U_4], \\ [(a_{ij}^k - \min_i(a_{ij}^k))/U_4, (b_{ij}^k - \min_i(a_{ij}^k))d/U_4] \end{array}\right), \\ \quad \text{if } j = j_4^b+1, j_4^b+2, \dots, n. \end{cases} \tag{2.10}$$

where $U_1 = \max_i(a_{ij}^k) - \min_i(a_{ij}^k)$ $(j = 1, 2, \dots, j_1;\ k = 1, 2, \dots, g)$, $U_2 = \max_i(b_{ij}^k) - \min_i(a_{ij}^k)$ $(j = j_1 + 1, j_1 + 2, \dots, j_2;\ k = 1, 2, \dots, g)$, $U_3 = 1 - \min_i(a_{ij}^k) - \min_i(c_{ij}^k)$ $(j = j_2 + 1, j_2 + 2, \dots, j_3;\ k = 1, 2, \dots, g)$, $U_4 = 1 - \min_i(a_{ij}^k) - \min_i(c_{ij}^k)$ $(j = j_3 + 1, j_3 + 2, \dots, n;\ k = 1, 2, \dots, g)$.

It is noted that real numbers, interval numbers and the IFNs are the special cases of IVIFNs. In other words, these hybrid information can be unified into the expressing form of IVIFNs.

Definition 6. (See Yu *et al.*, 2011.) Given the real number $\tilde{\alpha}_j = a_j$ $(j = 1, 2, \dots, j_1)$, the IFNs $\tilde{\alpha}_j = (a_j, c_j)$ $(j = j_1 + 1, j_1 + 2, \dots, j_3)$ and the IVIFNs $\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])$ $(j = j_3 + 1, j_3 + 2, \dots, n)$, then the hybrid weighted averaging (*HWA*) operator can be defined as follows:

$$HWA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \sum_{j=1}^{n} w_j \tilde{\alpha}_j = \left(\left[\sum_{j=1}^{n} w_j a_j, \sum_{j=1}^{j_3} w_j a_j + \sum_{j=j_3+1}^{n} w_j b_j \right], \left[\sum_{j=1}^{j_1} w_j (1 - a_j) + \sum_{j=j_1+1}^{n} w_j c_j, \sum_{j=1}^{j_1} w_j (1 - a_j) + \sum_{j=j_1+1}^{j_3} w_j c_j + \sum_{j=j_3+1}^{n} w_j d_j \right] \right), \tag{2.11}$$

where $\boldsymbol{w} = (w_1, w_2, \dots, w_n)^{\mathrm{T}}$ be the weight vector of $\tilde{\alpha}_j$ $(j = 1, 2, \dots, n)$.

3. The Proposed Approach

In the proposed method, a consistency maximization model is established to calculate the weights of experts. Then, the comprehensive ratings of alternatives are obtained by the *HWA* operator and the corresponding ratio function values are further calculated. Finally, a MCGP model is constructed to determine the optimal order quantity from the optimal alternatives being subjected to some resource constraints.

3.1. *The Maximizing Consistency Model for Deriving the Experts' Weights*

The estimation of the experts' weights plays an important role in MCGDM processes. Owing to the fact that in practical decision process, the criteria values provided by different experts for the same alternative under a criterion usually have various differences (i.e., exist the inconsistency), we next construct a maximizing consistency model to determine the experts' weights.

On the one hand, motivated by the idea from Pang and Liang (2012) the concept of the ordinal consistency (abbreviated as *OC*) index is defined from the perspective of the ranking of hybrid decision information to measure the consistency between the individual expert's opinion and the common opinion of the group. For the hybrid MCGDM problem described in Section 2.2, it can be said for the expert $e_k \in E$ that the alternative $A_\xi \in \boldsymbol{A}$ dominates $\boldsymbol{A}_\zeta \in \boldsymbol{A}$ with respect to the criterion $C_j \in \mathbb{C}$ if $f_j(r^k_{\xi j}) \geqslant f_j(r^k_{\zeta j})$ ($1 \leqslant \xi, \zeta \leqslant m$, $\xi \neq \zeta$) where

$$f_j\left(r^k_{\xi j}\right) = \begin{cases} a^k_{\xi j}, & \text{if } j = 1, 2, \ldots, j_1, \\ \frac{1}{2}(a^k_{\xi j} + b^k_{\xi j}), & \text{if } j = j_1 + 1, j_1 + 2, \ldots, j_2, \\ a^k_{\xi j} - c^k_{\xi j}, & \text{if } j = j_2 + 1, j_2 + 2, \ldots, j_3, \\ \frac{(a^k_{\xi j})^\delta + (b^k_{\xi j})^\delta}{(a^k_{\xi j})^\delta + (b^k_{\xi j})^\delta + (c^k_{\xi j})^\delta + (d^k_{\xi j})^\delta}, & \text{if } j = j_3 + 1, j_3 + 2, \ldots, n. \end{cases} \tag{3.1}$$

Thus, for the expert $e_k \in E$, the dominance class of the alternative $A_\xi \in \boldsymbol{A}$ on the criterion $C_j \in \mathbb{C}$ is defined as follows:

$$[A_\xi]^{e_k \geqslant}_{C_j} = \left\{A_\zeta \in \boldsymbol{A} \,\middle|\, f_j\left(r^k_{\xi j}\right) \geqslant f_j\left(r^k_{\zeta j}\right)\right\}. \tag{3.2}$$

Clearly, the individual expert's opinion should be consistent with the common opinion of the group to the greatest extent. For convenience of description, the common opinion of the group is assumed to be provided by the ideal expert e^*. Analogously, for the expert e^*, the dominance class of the alternative $A_\xi \in \boldsymbol{A}$ with respect to the criterion $C_j \in \mathbb{C}$ is obtained as follows:

$$[A_\xi]^{e* \geqslant}_{C_j} = \left\{A_\zeta \in \boldsymbol{A} \,\middle|\, f_j\left(r^*_{\xi j}\right) \geqslant f_j\left(r^*_{\zeta j}\right)\right\}. \tag{3.3}$$

Hence, the ordinal consistency with respect to the criterion $C_j \in \mathbb{C}$ between the expert e_k and the ideal expert e^* is defined as:

$$OC^*_k(C_j) = \frac{1}{m} \sum_{\xi=1}^{m} \left(\left|[A_\xi]^{e_k \geqslant}_{C_j} \bigcap [A_\xi]^{e* \geqslant}_{C_j}\right| / \left|[A_\xi]^{e_k \geqslant}_{C_j} \bigcup [A_\xi]^{e* \geqslant}_{C_j}\right|\right). \tag{3.4}$$

Furthermore, the weighted ordinal consistency index between the expert e_k and the ideal expert e^* is calculated by the following equation:

$$OC^*_k = \frac{1}{m} \sum_{j=1}^{n} \sum_{\xi=1}^{m} w_j \left(\left|[A_\xi]^{e_k \geqslant}_{C_j} \bigcap [A_\xi]^{e* \geqslant}_{C_j}\right| / \left|[A_\xi]^{e_k \geqslant}_{C_j} \bigcup [A_\xi]^{e* \geqslant}_{C_j}\right|\right). \tag{3.5}$$

The sum of OC^*_k is normalized into a unit as below:

$$O\bar{C}^*_k = \frac{\sum_{j=1}^{n} \sum_{\xi=1}^{m} w_j (|[A_\xi]^{e_k \geqslant}_{C_j} \bigcap [A_\xi]^{e* \geqslant}_{C_j}| / |[A_\xi]^{e_k \geqslant}_{C_j} \bigcup [A_\xi]^{e* \geqslant}_{C_j}|)}{\sum_{k=1}^{g} \sum_{j=1}^{n} \sum_{\xi=1}^{m} w_j (|[A_\xi]^{e_k \geqslant}_{C_j} \bigcap [A_\xi]^{e* \geqslant}_{C_j}| / |[A_\xi]^{e_k \geqslant}_{C_j} \bigcup [A_\xi]^{e* \geqslant}_{C_j}|)}. \tag{3.6}$$

Thus, the overall ordinal consistency is defined as:

$$OC = \frac{\sum_{k=1}^{g}\sum_{j=1}^{n}\sum_{\xi=1}^{m}\lambda_k w_j(|[A_\xi]_{C_j}^{e_k\geqslant}\bigcap[A_\xi]_{C_j}^{e*\geqslant}|/|[A_\xi]_{C_j}^{e_k\geqslant}\bigcup[A_\xi]_{C_j}^{e*\geqslant}|)}{\sum_{k=1}^{g}\sum_{j=1}^{n}\sum_{\xi=1}^{m} w_j(|[A_\xi]_{C_j}^{e_k\geqslant}\bigcap[A_\xi]_{C_j}^{e*\geqslant}|/|[A_\xi]_{C_j}^{e_k\geqslant}\bigcup[A_\xi]_{C_j}^{e*\geqslant}|)}. \tag{3.7}$$

On the other hand, from the perspective of the magnitude of decision information the cardinal consistency (abbreviated as CC) index is defined for measuring the consistency between individual expert's opinions and the common opinion of group. This study employs the similarity measure to calculate the cardinal consistency index.

Hence, for the alternative $A_\xi \in \boldsymbol{A}$, the cardinal consistency with respect to the criterion $C_j \in \mathbb{C}$ between the expert e_k and the ideal expert e^* is defined as:

$$CC_k^*(A_\xi, C_j) = 1 - d_j\big(r_{\xi j}^k, r_{\xi j}^*\big) \tag{3.8}$$

where

$$d_j(r_{\xi j}^k, r_{\xi j}^*) = \begin{cases} |a_{\xi j}^k - a_{\xi j}^*|, \quad \text{if } j = 1, 2, \dots, j_1, \\ \sqrt{\frac{1}{2}((a_{\xi j}^k - a_{\xi j}^*)^2 + (b_{\xi j}^k - b_{\xi j}^*)^2)}, \\ \quad \text{if } j = j_1+1, j_1+2, \dots, j_2, \\ \sqrt{\frac{1}{2}\begin{pmatrix}(a_{\xi j}^k - a_{\xi j}^*)^2 + (c_{\xi j}^k - c_{\xi j}^*)^2 \\ + (a_{\xi j}^* + c_{\xi j}^* - a_{\xi j}^k - c_{\xi j}^k)^2\end{pmatrix}}, \\ \quad \text{if } j = j_2+1, j_2+2, \dots, j_3, \\ \sqrt{\frac{1}{4}\begin{pmatrix}(a_{\xi j}^k - a_{\xi j}^*)^2 + (b_{\xi j}^k - b_{\xi j}^*)^2 \\ + (c_{\xi j}^k - c_{\xi j}^*)^2 + (d_{\xi j}^k - d_{\xi j}^*)^2\end{pmatrix}}, \\ \quad \text{if } j = j_3+1, j_3+2, \dots, n. \end{cases} \tag{3.9}$$

Then, the weighted cardinal consistency between the expert e_k and the ideal expert e^* is defined as follows:

$$CC_k^* = \frac{1}{m}\sum_{\xi=1}^{m}\sum_{j=1}^{n}\big(w_j\big(1 - d_j\big(r_{\xi j}^k, r_{\xi j}^*\big)\big)\big) \tag{3.10}$$

and the sum of CC_k^* is normalized into a unit as below:

$$C\bar{C}_k^* = \frac{1}{m}\sum_{\xi=1}^{m}\sum_{j=1}^{n}\big(w_j\big(1 - d_j\big(r_{\xi j}^k, r_{\xi j}^*\big)\big)\big)\Big/ \sum_{k=1}^{g}\left(\frac{1}{m}\sum_{\xi=1}^{m}\sum_{j=1}^{n}\big(w_j\big(1 - d_j\big(r_{\xi j}^k, r_{\xi j}^*\big)\big)\big)\right). \tag{3.11}$$

Thus, the overall cardinal consistency is obtained as:

$$CC = \sum_{k=1}^{g}\Big(\lambda_k\Big(\frac{1}{m}\sum_{\xi=1}^{m}\sum_{j=1}^{n}\big(w_j\big(1-d_j\big(r_{\xi j}^k, r_{\xi j}^*\big)\big)\big)\Big)\Big)\Big/ \sum_{k=1}^{g}\Big(\frac{1}{m}\sum_{\xi=1}^{m}\sum_{j=1}^{n}\big(w_j\big(1-d_j\big(r_{\xi j}^k, r_{\xi j}^*\big)\big)\big)\Big)\Big). \tag{3.12}$$

According to the definitions of the ordinal consistency and the cardinal consistency, we construct a maximizing consistency model to determine the optimal weights of experts. Thus, a bi-objective programming model which maximizes simultaneously the ordinal consistency and the cardinal consistency is established as follows:

$$\text{(M-1)}\quad \max\left\{\begin{array}{l} \dfrac{\sum_{k=1}^{g}\sum_{j=1}^{n}\sum_{\xi=1}^{m}\lambda_k w_j(|[A_\xi]_{C_j}^{e_k\geqslant}\bigcap[A_\xi]_{C_j}^{e*\geqslant}|/|[A_\xi]_{C_j}^{e_k\geqslant}\bigcup[A_\xi]_{C_j}^{e*\geqslant}|)}{\sum_{k=1}^{g}\sum_{j=1}^{n}\sum_{\xi=1}^{m} w_j(|[A_\xi]_{C_j}^{e_k\geqslant}\bigcap[A_\xi]_{C_j}^{e*\geqslant}|/|[A_\xi]_{C_j}^{e_k\geqslant}\bigcup[A_\xi]_{C_j}^{e*\geqslant}|)}, \\ \sum_{k=1}^{g}(\lambda_k\sum_{\xi=1}^{m}\sum_{j=1}^{n}(w_j(1-d_j(r_{\xi j}^k, r_{\xi j}^*))))\Big/ \\ \quad\sum_{k=1}^{g}(\sum_{\xi=1}^{m}\sum_{j=1}^{n}(w_j(1-d_j(r_{\xi j}^k, r_{\xi j}^*)))) \end{array}\right\}$$
$$\text{s.t.}\quad (\lambda_1,\lambda_2,\cdots,\lambda_g)^T\in\Theta$$

where Θ expresses the set of the known weighted information of experts. In general, the Θ consists of several sets of the following five basic sets or may contain all the five basic sets, which depends on the characteristic and need of the real-world decision problems: (1) A weak ranking $\Theta_1=\{\lambda_i\geqslant\lambda_j\}$; (2) A strict ranking: $\Theta_2=\{\lambda_i-\lambda_j\geqslant\beta_i\}$ ($\beta_i>0$); (3) A ranking of differences: $\Theta_3=\{\lambda_i-\lambda_j\geqslant\lambda_k-\lambda_l\}$ ($i\neq j\neq k\neq l$); (4) A ranking with multiples: $\Theta_4=\{\lambda_i\geqslant\beta_i\lambda_j\}$ ($0\leqslant\beta_i\leqslant 1$); (5) An interval form: $\Theta_5=\{\beta_i\leqslant\lambda_i\leqslant\gamma_i\}$ ($0\leqslant\beta_i\leqslant\gamma_i\leqslant 1$).

Using the weighted average approach, the bi-objective mathematical programming model (M-1) can be aggregated into the following linear programming model:

$$\text{(M-2)}\quad \max\left\{\begin{array}{l} (1-\theta)\dfrac{\sum_{k=1}^{g}\sum_{j=1}^{n}\sum_{\xi=1}^{m}\lambda_k w_j(|[A_\xi]_{C_j}^{e_k\geqslant}\bigcap[A_\xi]_{C_j}^{e*\geqslant}|/|[A_\xi]_{C_j}^{e_k\geqslant}\bigcup[A_\xi]_{C_j}^{e*\geqslant}|)}{\sum_{k=1}^{g}\sum_{j=1}^{n}\sum_{\xi=1}^{m} w_j(|[A_\xi]_{C_j}^{e_k\geqslant}\bigcap[A_\xi]_{C_j}^{e*\geqslant}|/|[A_\xi]_{C_j}^{e_k\geqslant}\bigcup[A_\xi]_{C_j}^{e*\geqslant}|)} \\ \quad+\theta\sum_{k=1}^{g}(\lambda_k\sum_{\xi=1}^{m}\sum_{j=1}^{n}(w_j(1-d_j(r_{\xi j}^k, r_{\xi j}^*))))\Big/ \\ \quad\sum_{k=1}^{g}(\sum_{\xi=1}^{m}\sum_{j=1}^{n}(w_j(1-d_j(r_{\xi j}^k, r_{\xi j}^*)))) \end{array}\right\}$$
$$\text{s.t.}\quad (\lambda_1,\lambda_2,\cdots,\lambda_g)^T\in\Theta$$

where $\theta\in[0,1]$ is a parameter which can be tuned according to the practical decision problem at hand.

Obviously, the model (M-2) can be easily solved by MATLAB or LINGO software package and the weights of experts can be obtained. Afterwards, we can calculate the overall weighted assessment value of each alternative by using the following expression:

$$\mathbb{Z}(A_i) = \sum_{k=1}^{g} \lambda_k \sum_{j=1}^{n} w_j r_{ij}^k$$
$$= \begin{pmatrix} \Big[\sum_{k=1}^{g} \lambda_k \sum_{j=1}^{n} w_j a_{ij}^k, \\ \quad \sum_{k=1}^{g} \lambda_k \sum_{j=1}^{j_3} w_j a_{ij}^k + \sum_{k=1}^{g} \lambda_k \sum_{j=j_3+1}^{n} w_j b_{ij}^k \Big], \\ \begin{bmatrix} \sum_{k=1}^{g} \lambda_k \sum_{j=1}^{j_1} w_j (1 - a_{ij}^k) + \sum_{k=1}^{g} \lambda_k \sum_{j=j_1+1}^{n} w_j c_{ij}^k, \\ \sum_{k=1}^{g} \lambda_k \sum_{j=1}^{j_1} w_j (1 - a_{ij}^k) + \sum_{k=1}^{g} \lambda_k \sum_{j=j_1+1}^{j_3} w_j c_{ij}^k \\ \quad + \sum_{k=1}^{g} \lambda_k \sum_{j=j_3+1}^{n} w_j d_{ij}^k \end{bmatrix} \end{pmatrix}. \quad (3.13)$$

According to Definition 4, the ratio function values of alternatives can be obtained. By comparing the magnitude of the ratio function values, the decision can be made.

In several real-world decision problems, the decision makers may not only need to select the optimal alternatives but also need to determine the corresponding optimum quantities for the optimal alternatives under some tangible constraints. To this end, we next construct a MCGP model to identify the optimal alternatives and their optimum order quantities.

3.2. *Establish the MCGP Model for Determining the Order Quantities of Optimal Suppliers*

The MCGP originally introduced by Chang (2007) is mainly used to address the situation in which decision makers set multi-choice aspiration levels for each goal to achieve. The most characteristic of the MCGP model is that it can effectively avoid underestimation or overestimation of the decision. Afterwards, Chang (2008) proposed a revised MCGP method which does not involve multiplicative terms of binary variables to solve the MCGP model. In this paper, we take the supplier selection problems for example, and borrowing the idea of Liao and Kao (2011) we construct a MCGP model for deriving the optimum order quantities from the optimal alternatives.

In order to formulate the MCGP model, the following notations are first defined:
Parameters:

m is the number of suppliers;

ϖ_i is the ranking values (priority values) of the ith supplier A_i (obtained by the *HWA* operator and the ratio function of IVIFNs);

p_i is the sale price of the ith supplier A_i;

g_0 represents the total demand for the company;

gc_i is the capacity of the ith supplier A_i;

Objective functions:

Total value of purchasing (TVP): the ranking values of the suppliers are used as coefficients of the total value of purchasing to allocate order quantities among the optimal suppliers. The TVP should be less than $g_{1,\max}$ but more than $g_{1,\min}$, and the more the better; this objective function is formed as follows:

$$\sum_{i=1}^{m} \varpi_i x_i \leqslant g_{1,\max} \quad \text{and} \quad \sum_{i=1}^{m} \varpi_i x_i \geqslant g_{1,\min}. \tag{3.14}$$

Total cost of purchasing: the total cost of purchasing should be less than $g_{2,\max}$ but more than $g_{2,\min}$, and the less the better; thus this objective function can be expressed as below:

$$\sum_{i=1}^{m} p_i x_i \geqslant g_{2,\min} \quad \text{and} \quad \sum_{i=1}^{m} p_i x_i \leqslant g_{2,\max}. \tag{3.15}$$

Constraints:

Total order quantity: the total order quantity from all suppliers should be not more than a constant g_0, thus this constraint can be formed as below:

$$\sum_{i=1}^{m} x_i \leqslant g_0. \tag{3.16}$$

Suppliers' capacity: the order quantity from the ith supplier cannot exceed the corresponding supplier's capacity gc_i:

$$0 \leqslant x_i \leqslant gc_i. \tag{3.17}$$

In terms of the revised MCGP model developed by Chang (2008), the above problem can be formulated as follows:

$$\text{(M-3)} \quad \begin{aligned} &\text{Min } \textstyle\sum_{i=1}^{2} \big(\varphi_i\big(\varepsilon_i^+ + \varepsilon_i^-\big) + \phi_i\big(\kappa_i^+ + \kappa_i^-\big)\big) \\ &\text{s.t.} \quad \begin{cases} \sum_{i=1}^{m} \varpi_i x_i - \varepsilon_1^+ + \varepsilon_1^- = y_1 \\ y_1 - \kappa_1^+ + \kappa_1^- = g_{1,\max} \\ g_{1,\min} \leqslant y_1 \leqslant g_{1,\max} \\ \sum_{i=1}^{m} p_i x_i - \varepsilon_2^+ + \varepsilon_2^- = y_2 \\ y_2 - \kappa_2^+ + \kappa_2^- = g_{2,\min} \\ g_{2,\min} \leqslant y_2 \leqslant g_{2,\max} \\ \sum_{i=1}^{m} x_i \leqslant g_0 \\ 0 \leqslant x_i \leqslant gc_i, \quad i = 1, 2, \ldots, m \\ \varepsilon_1^+, \varepsilon_1^-, \varepsilon_2^+, \varepsilon_2^-, \kappa_1^+, \kappa_1^-, \kappa_2^+, \kappa_2^- \geqslant 0 \end{cases} \end{aligned}$$

where ε_i^+ (κ_i^+) and ε_i^- (κ_i^-) are the positive and negative deviations, respectively, φ_i and ϕ_i are the weights of deviations.

The model (M-3) can be easily solved by MATLAB or LINGO software, and the optimal order quantities from the corresponding optimal suppliers can be obtained.

3.3. *The Proposed Algorithm*

On the basis of the above models and analysis, the algorithm of the proposed method for solving the hybrid MCGDM problems based on IVIFNs is given as follows:

Step 1. For a hybrid MCGDM problem, the hybrid decision matrix is first identified, and then the hybrid decision matrix is normalized by using Eq. (2.10).
Step 2. Use Eq. (3.7) and Eq. (3.12) to calculate the overall ordinal consistency index and the overall cardinal consistency index, respectively.
Step 3. Determine the optimal weight vector $\boldsymbol{\lambda}^* = (\lambda_1^*, \lambda_2^*, \dots, \lambda_g^*)^T$ of all experts by the model (M-2).
Step 4. Utilize Eq. (3.13) to aggregate all elements in the hybrid decision matrix in order to obtain the comprehensive values of alternatives.
Step 5. Calculate the ratio function values of alternatives using the Eq. (2.7).
Step 6. Employ the model (M-3) to establish the MCGP model in order to determine the optimal order quantities.

4. An Illustrative Case Based on the Supplier Selection Problem

This section employs the supplier selection problem as an illustrative case to demonstrate the applicability and the implementation process of the proposed method.

With the increase of public awareness of the need to protect the environment, it is urgent for businesses to introduce and promote business practices that help ease the negative impacts of their actions on the environment (Wang and Chan, 2013). In the automobile manufacturing industries, the manufacturers want to improve their environmental management practices, not only internally, but also with their suppliers. To this end, the automobile manufacturing company plans to find the environmentally and economically powerful suppliers as strategic partners, with whom the company intends to build long-term collaborative relationships. There are four qualified suppliers which are named as A_1, A_2, A_3, A_4. The decision organization including four experts (e_1, e_2, e_3, e_4) from the purchasing department, the management department, the environmental department and the production department, respectively, is invited to evaluate these four suppliers. The weight vector of the experts is given as below:

$$\Theta = \left\{ \begin{array}{l} \lambda_2 - \lambda_1 \leqslant 0.1,\ 0.1 \leqslant \lambda_3 \leqslant 0.3,\ \lambda_4 - \lambda_2 \leqslant \lambda_3 - \lambda_1,\ \lambda_4 \geqslant 1.5\lambda_1 \\ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1,\ \lambda_k \geqslant 0,\ k = 1, 2, 3, 4 \end{array} \right\}.$$

The supplier selection criteria are identified by the experts as follows: (1) C_1 is the quality of product; (2) C_2 is the delivery time; (3) C_3 is the score of credibility; (4) C_4 is the environmental performance. The weight vector of criteria is given as $(w_1, w_2, w_3, w_4)^{\mathrm{T}} =$

Table 4
The linguistic terms and their corresponding IVIFNs (Yu *et al.*, 2011).

Linguistic variables	Abbreviation	IVIFNs
Definitely low	DL	([0, 0], [0.75, 0.95])
Very low	VL	([0, 0.2], [0.5, 0.7])
Low	L	([0.25, 0.45], [0.25, 0.45])
Medium	M	([0.5, 0.7], [0.125, 0.22])
High	H	([0.75, 0.85], [0.0625, 0.11])
Very high	VH	([0.875, 0.92], [0, 0.05])
Definitely high	DH	([0.9375, 0.98], [0, 0])

Table 5
The criteria values of suppliers provided by experts under various criteria.

Experts	Suppliers	Criteria			
		C_1	C_2	C_3	C_4
e_1	A_1	(0.5, 0.3)	[15.5, 16.8]	2.35	M
	A_2	(0.6, 0.2)	[16.6, 18.35]	2.48	H
	A_3	(0.4, 0.4)	[14.34, 16.42]	1.53	VH
	A_4	(0.3, 0.6)	[12.67, 15.32]	4.50	DH
e_2	A_1	(0.4, 0.5)	[12.5, 15.8]	4.35	H
	A_2	(0.7, 0.2)	[11.6, 14.65]	2.28	L
	A_3	(0.6, 0.4)	[13.34, 16.42]	1.53	VH
	A_4	(0.3, 0.7)	[14.67, 15.32]	2.50	M
e_3	A_1	(0.4, 0.3)	[13.25, 15.48]	1.55	H
	A_2	(0.6, 0.2)	[11.6, 15.45]	4.48	H
	A_3	(0.8, 0.1)	[13.34, 15.42]	1.53	VH
	A_4	(0.6, 0.2)	[16.67, 18.32]	3.50	DH
e_4	A_1	(0.7, 0.3)	[12.5, 14.85]	2.35	L
	A_2	(0.6, 0.2)	[16.6, 18.62]	1.48	H
	A_3	(0.5, 0.2)	[14.34, 16.42]	3.53	M
	A_4	(0.4, 0.6)	[12.67, 15.32]	2.56	M

$(0.2, 0.3, 0.35, 0.15)^{\mathrm{T}}$. The criteria values of suppliers for the quality of product (C_1) can be divided into two parts: satisfaction degree and dissatisfaction degree, which just consist in the membership degree and non-membership degree of IFNs. Thus, IFNs are used to express the criteria values of suppliers for the quality of product (C_1). Due to the uncertainty of the delivery time, it is better to use interval numbers to represent the criteria values of suppliers for the delivery time (C_2). The criteria values of suppliers for the score of credibility (C_3) are represented by real numbers. While the environmental performance (C_4) is a qualitative criterion, the criteria values of suppliers for the criterion C_4 can be represented by linguistic terms. It is noted that the criterion C_2 is the cost criterion and the others are the benefit criteria. The linguistic terms used here and their corresponding IVIFNs are shown in Table 4. The decision data provided by experts is presented in Table 5.

Table 6
The normalized hybrid decision matrix.

Experts	Suppliers	Criteria			
		C_1	C_2	C_3	C_4
e_1	A_1	(0.4, 0.2)	(0.2729, 0.4982)	0.2761	([0.5, 0.7], [0.125, 0.22])
	A_2	(0.6, 0.0)	(0, 0.6919)	0.3199	([0.75, 0.85], [0.0625, 0.11])
	A_3	(0.2, 0.4)	(0.3398, 0.294)	0.0	([0.875, 0.92], [0, 0.05])
	A_4	(0.0, 0.8)	(0.5335, 0.0)	1.0	([0.9375, 0.98], [0, 0])
e_2	A_1	(0.2, 0.6)	(0.1286, 0.1867)	1.0	([0.75, 0.85], [0.0625, 0.11])
	A_2	(0.8, 0.0)	(0.3672, 0.0)	0.266	([0.25, 0.45], [0.25, 0.45])
	A_3	(0.6, 0.4)	(0.0, 0.361)	0.0	([0.875, 0.92], [0, 0.05])
	A_4	(0.0, 1.0)	(0.2282, 0.6369)	0.344	([0.5, 0.7], [0.125, 0.22])
e_3	A_1	(0.0, 0.4)	(0.4226, 0.2455)	0.0068	([0.75, 0.85], [0.0625, 0.11])
	A_2	(0.4, 0.2)	(0.4271, 0.0)	1.0	([0.75, 0.85], [0.0625, 0.11])
	A_3	(0.8, 0.0)	(0.4315, 0.2589)	0.0	([0.875, 0.92], [0, 0.05])
	A_4	(0.4, 0.2)	(0.0, 0.7545)	0.6678	([0.9375, 0.98], [0, 0])
e_4	A_1	(0.7, 0.25)	(0.616, 0.0)	0.4244	([0.25, 0.45], [0.25, 0.45])
	A_2	(0.5, 0.0)	(0, 0.6699)	0.0	([0.75, 0.85], [0.0625, 0.11])
	A_3	(0.25, 0.25)	(0.3595, 0.3007)	1.0	([0.5, 0.7], [0.125, 0.22])
	A_4	(0.0, 1.0)	(0.5392, 0.0278)	0.5268	([0.5, 0.7], [0.125, 0.22])
e^*	A_1	(0.3375, 0.3635)	(0.36, 0.2326)	0.4268	([0.5625, 0.7125], [0.125, 0.2225])
	A_2	(0.575, 0.05)	(0.1986, 0.3405)	0.3965	([0.625, 0.75], [0.1094, 0.195])
	A_3	(0.4625, 0.2625)	(0.2827, 0.3036)	0.25	([0.7813, 0.865], [0.0313, 0.0925])
	A_4	(0.1, 0.75)	(0.3252, 0.3548)	0.6346	([0.7188, 0.84], [0.0625, 0.11])

4.1. *Determine Weights of Experts by Constructing the Maximization Consistency Model*

Firstly, the proposed approach employs the Eq. (2.10) to normalize the hybrid decision matrix, and the normalized results are showed in Table 6. The common opinion of the group in this paper, in a reason, should be the mean of group decision, and the *HWA* operator is used to obtain the common opinion of the group, listed in Table 6.

Using Eq. (3.1) and let $\delta = 0.5$, the dominance classes of the alternatives A_i $(i = 1, 2, 3, 4)$ with respect to the criterion C_1 under the expert e_1 can be easily derived from Table 6 as follows:

$$[A_1]_{C_1}^{e_1 \geqslant} = \{A_1, A_3, A_4\}, \qquad [A_2]_{C_1}^{e_1 \geqslant} = \{A_1, A_2, A_3, A_4\},$$
$$[A_3]_{C_1}^{e_1 \geqslant} = \{A_3, A_4\}, [A_4]_{C_1}^{e_1 \geqslant} = \{A_4\},$$

and the dominance classes under the expert e^* for the alternative set A with respect to the criterion C_1 are obtained as follows:

$$[A_1]_{C_1}^{e* \geqslant} = \{A_1, A_4\}, \qquad [A_2]_{C_1}^{e* \geqslant} = \{A_1, A_2, A_3, A_4\},$$
$$[A_3]_{C_1}^{e* \geqslant} = \{A_1, A_3, A_4\}, [A_4]_{C_1}^{e* \geqslant} = \{A_4\}.$$

Hence, according to Eq. (3.4), the ordinal consistency $OC_1^*(C_1)$ of alternatives with respect to the criterion C_1 between the expert e_1 and the ideal expert e^* can be calculated as:

$$OC_1^*(C_1) = \frac{1}{4}\sum_{i=1}^{4}\frac{|[A_i]_{C_1}^{e_1\geqslant} \bigcap [A_i]_{C_1}^{e*\geqslant}|}{|[A_i]_{C_1}^{e_1\geqslant} \bigcup [A_i]_{C_1}^{e*\geqslant}|} = \frac{1}{4}\left(\frac{2}{3}+\frac{4}{4}+\frac{2}{3}+\frac{1}{1}\right) = 0.8333.$$

Analogously, the following calculation results can be obtained:

$$OC_1^*(C_2) = 0.6875, \qquad OC_1^*(C_3) = 0.7083, \qquad OC_1^*(C_4) = 0.875.$$

According to Eq. (3.5), the weighted ordinal consistency OC_1^* between the expert e_1 and the ideal expert e^* can be calculated as:

$$OC_1^* = \sum_{j=1}^{4} w_j OC_1^*(C_j) = 0.7521.$$

Analogously, the other weighted ordinal consistency indices can be obtained:

$$OC_2^* = 0.7, \qquad OC_3^* = 0.6594, \qquad OC_4^* = 0.7781.$$

Furthermore, using Eq. (3.6) the normalized ordinal consistency indices can be calculated:

$$O\bar{C}_1^* = 0.2603, \qquad O\bar{C}_2^* = 0.2422, \qquad O\bar{C}_3^* = 0.2282, \qquad O\bar{C}_4^* = 0.2693.$$

Afterwards, the proposed approach needs to calculate the cardinal consistency index between the individual expert's opinion and the common opinion of group from the perspective of the magnitude of decision data. Using Eq. (3.10) the weighted cardinal consistency indices between the experts e_k $(k = 1, 2, 3, 4)$ and the ideal expert e^* can be calculated, respectively:

$$CC_1^* = 0.8089, \qquad CC_2^* = 0.7219, \qquad CC_3^* = 0.7246, \qquad CC_4^* = 0.7275.$$

Using Eq. (3.11) the normalized cardinal consistency indices are obtained as follows:

$$C\bar{C}_1^* = 0.2712, \qquad C\bar{C}_2^* = 0.2420, \qquad C\bar{C}_3^* = 0.2429, \qquad C\bar{C}_4^* = 0.2439.$$

Then, according to the model (M-2), the following objective programming model is established:

$$\text{(M-4)} \quad \begin{array}{ll} \max & \begin{pmatrix} (1-\theta)\,(0.2603\lambda_1 + 0.2422\lambda_2 + 0.2282\lambda_3 + 0.2693\lambda_4) \\ +\,\theta\,(0.2712\lambda_1 + 0.242\lambda_2 + 0.2429\lambda_3 + 0.2439\lambda_4) \end{pmatrix} \\ \text{s.t.} & \begin{cases} \lambda_2 - \lambda_1 \leqslant 0.1,\ 0.1 \leqslant \lambda_3 \leqslant 0.3,\ \lambda_4 - \lambda_2 \leqslant \lambda_3 - \lambda_1 \\ \lambda_4 \geqslant 1.5\lambda_1,\ \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1,\ \lambda_k \geqslant 0,\ k = 1, 2, 3, 4. \end{cases} \end{array}$$

Table 7
The coefficients of variables in model.

Suppliers	Unit price	Capacity	Weights
A_1	\$10	700	0.5636
A_2	\$12	500	0.5213
A_3	\$16	800	0.4942
A_3	\$18	600	0.4825

Let $\theta = 0.5$ and solve the model (M-4), the corresponding optimal weight vector of experts can be obtained as follows:

$$\left(\lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*\right)^T = (0.2, 0.3, 0.2, 0.3)^T.$$

Using the Eq. (3.13) to aggregate the elements of the normalized hybrid decision matrix, and the comprehensive ratings values of alternatives are acquired as follows:

$$\mathbb{Z}(A_1) = \big([0.4336, 0.4561], [0.3368, 0.3522]\big),$$
$$\mathbb{Z}(A_2) = \big([0.3870, 0.4065], [0.3573, 0.3713]\big),$$
$$\mathbb{Z}(A_3) = \big([0.4021, 0.4158], [0.4138, 0.4233]\big),$$
$$\mathbb{Z}(A_4) = \big([0.3244, 0.3449], [0.3546, 0.3632]\big).$$

Furthermore, using the Eq. (2.7) the ranking values of alternatives are obtained as follows:

$$\mathbb{Q}_{\delta=1}\big(\mathbb{Z}(A_1)\big) = 0.5636, \qquad \mathbb{Q}_{\delta=1}\big(\mathbb{Z}(A_2)\big) = 0.5213,$$
$$\mathbb{Q}_{\delta=1}\big(\mathbb{Z}(A_3)\big) = 0.4942, \qquad \mathbb{Q}_{\delta=1}\big(\mathbb{Z}(A_4)\big) = 0.4825.$$

4.2. *MCGP Model for the Order Quantities*

As mentioned previously, the automobile manufacturing company may wish to identify the quantities of the product buying from the optimal suppliers. The unit price of product buying from the four suppliers and the capacities for these suppliers are given in Table 7. Similar to Liao and Kao (2011), the ranking values of the optimal suppliers are regarded as the weights of suppliers, which are also shown in Table 7.

The manager of the company sets the following three goals as below:

(1) The TVP should be less than 1000 and more than 600, and the more the better, i.e.

$$0.5636x_1 + 0.5213x_2 + 0.4942x_3 + 0.4825x_4 \geqslant 600 \text{ and } \leqslant 1000;$$

(2) The total cost of procurement should be less than 29,000 and more than 18,000, and the less the better, i.e.

$$10x_1 + 12x_2 + 16x_3 + 18x_4 \geqslant 18000 \text{ and } \leqslant 29000;$$

(3) The total order quantity from all suppliers should be not more than 1800 units, i.e.

$$x_1 + x_2 + x_3 + x_4 \leqslant 1800.$$

On the basis of the model (M-3), the above decision problem can be formulated as the following MCGP model:

$$\text{(M-5)} \quad \begin{aligned} &\text{Min} \quad z = \varepsilon_1^+ + \varepsilon_1^- + \varepsilon_2^+ + \varepsilon_2^- + \kappa_1^+ + \kappa_1^- + \kappa_2^+ + \kappa_2^- \\ &\text{s.t.} \quad \begin{cases} 0.5636x_1 + 0.5213x_2 + 0.4942x_3 + 0.4825x_4 - \varepsilon_1^+ + \varepsilon_1^- = y_1 \\ y_1 - \kappa_1^+ + \kappa_1^- = 1000 \\ 600 \leqslant y_1 \leqslant 1000 \\ 10x_1 + 12x_2 + 16x_3 + 18x_4 - \varepsilon_2^+ + \varepsilon_2^- = y_2 \\ y_2 - \kappa_2^+ + \kappa_2^- = 18000 \\ 18000 \leqslant y_2 \leqslant 29000 \\ x_1 + x_2 + x_3 + x_4 \leqslant 1800 \\ x_1 \leqslant 700, \ x_2 \leqslant 500, \ x_3 \leqslant 800, \ x_4 \leqslant 600 \\ x_1, x_2, x_3, x_4 \geqslant 0 \\ \varepsilon_1^+, \varepsilon_1^-, \varepsilon_2^+, \varepsilon_2^-, \kappa_1^+, \kappa_1^-, \kappa_2^+, \kappa_2^- \geqslant 0. \end{cases} \end{aligned}$$

By solving the model (M-5), the optimal solutions, i.e., the optimal suppliers and their optimum quantities are obtained as below:

$$A_1(x_1 = 700), \qquad A_2(x_2 = 500), \qquad A_3(x_3 = 31\bar{3}),$$
$$A_4(x_4 = 0) \quad \text{and} \quad TVP = 809.8546.$$

5. Conclusions

This paper has established a decision environment based on IVIFNs for the hybrid MCGDM problems in which the ratings of alternatives on the criteria take the forms of real numbers, interval numbers, IFNs and linguistic variables. An integrated consistency maximization model and MCGP approach has been proposed to solve the hybrid MCGDM problem based on IVIFNs. The main contribution of his paper is fivefold: (1) new score function, new accuracy function and a ratio function for IVIFNs have been developed, and two new kinds of ranking methods for IVIFNs have also been proposed; (2) from the perspectives of the ranking and the magnitude of hybrid decision information, an ordinal consistency index and a cardinal consistency index have been defined to measure the consistency between the individual opinion and the common opinion of group, respectively; (3) a consistency maximization model has been established to objectively determine the weights of experts; (4) the comprehensive ratings of alternatives have been determined by aggregating the hybrid decision information and the ranking of alternatives have been obtained by the proposed ratio function; (5) a MCGP model on the basis of the ranking values has been constructed to determine the optimal order quantities from the best alternatives being subjected to some resource constraints.

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Integruotas suderintumą maksimizuojantis ir daugelio pasirinkimų tikslinio programavimo požiūris hibridiniam daugiakriteriniam grupiniam sprendimų priėmimui taikant intervalinius intuityviuosius neraiškiuosius skaičius

Xiaolu ZHANG

Intervaliniai intuityvieji neraiškieji skaičiai (IINS), aprašomi priklausomumo ir nepriklausomumo funkcijomis su intervalinėmis reikšmėmis, gali būti taikomi sprendžiant problemas, susijusias su netikslia ir abejotina informacija. Šiame straipsnyje siūlomas integruotas suderintumą maksimizuojantis ir daugelio pasirinkimų tikslinio programavimo (DPTP) požiūris hibridiniam daugiakriteriniam grupiniam sprendimų priėmimui taikant IINS. Pirmajame etape hibridinė informacija (realieji skaičiai, intervaliniai skaičiai, intuityvieji skaičiai ir lingvistiniai kintamieji) yra normalizuojami ir konvertuojami į IINS. Tuomet ordinalusis suderintumo indeksas ir kardinalusis suderintumo indeksas yra pasiūlomi matuoti individualių ir grupės vertinimų suderintumą. Pasiūlytas optimizavimo modelis, maksimizuojantis suderintumą, ekspertų reikšmingumo nustatymui. Apibendrinti įvertinimai ir rangavimo reikšmės apskaičiuojamos, atitinkamai, taikant hibridinį svertinį operatorių ir santykių funkciją, skirtą IINS. Be to, DPTP modelis pritaikytas nustatant optimalųjį sprendinį (kiekius). Pateikiamas pasiūlyto metodo taikymo pavyzdys.