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A Unifying Survey of Agent-Based Approaches for Equality-Generalized Traveling Salesman Problem

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Abstract. The *Generalized Traveling Salesman Problem* is one of a well known complex combinatorial optimization problems. *Equality-Generalized Traveling Salesman Problem* is a particular case of it. The main objective of the problem it is to find a minimum cost tour passing through exactly one node from each cluster of a large-scale undirected graph. Multi-agent approaches are successfully used nowadays for solving real life complex problems. The aim of the current paper is to illustrate some agent-based algorithms, including particular ant-based models and virtual robots-agents with specific properties for solving *Equality-Generalized Traveling Salesman Problem*.

Key words: combinatorial optimization, multi-agent system, pattern recognition, ant colony optimization, complex systems.

1. Introduction

A large number of combinatorial optimization problems are *NP*-hard. Today, approximation and heuristic algorithms are used widely in order to find near optimal solutions of difficult problems, within reasonable running time. Heuristics are among the best strategies in terms of efficiency and solution quality for complex problems (Pop and Zelina, 2004).

The Generalized Traveling Salesman Problem (*GTSP*) introduced in Laporte and Nobert (1983) and Noon and Bean (1991) is also a difficult complex problem. One of its variants, *E-GTSP* where *E* means "equality", it is named generally just *GTSP*, as in the current paper. In *E-GTSP*, in a partitioned complex graph, exactly one node from a cluster is visited.

Several approaches for solving the *GTSP* in a relatively short period of time, were considered. In Fischetti *et al.* (2002a) a branch-and-cut algorithm for *Symmetric GTSP* is described and analyzed and in Cacchiani *et al.* (2011) it is proposed a multistart heuristic with a decomposition approach combined with improvement procedures. In Renaud and Boctor (1998) it is proposed an efficient composite heuristic for the *Symmetric GTSP* including insertion of a node from each non-visited node-subset. A random-key genetic algorithm for the *GTSP* is described in Snyder and Daskin (2006) were genetic algorithm includes a local tour improvement heuristic and the solutions are encoded using random keys. The memetic algorithm, proposed in Gutin and Karapetyan (2010) exploited a strong local search procedure together with a well tuned genetic framework.

There are significant achievements in the area of local search algorithms for the *GTSP*. In Karapetyan and Gutin (2012) is provided an exhaustive survey of *GTSP* local search neighborhoods and proposed efficient exploration algorithms for each of them. A hybrid *ACS* approach using an effective combination of two local search heuristics of different classes is introduced in Reihaneh and Karapetyan (2012).

Hybrid heuristics are some of nowadays valuable instruments for finding good results on complex real-life problems. Some successful hybrid techniques involve agent-based algorithms and also in particular ant systems and virtual robot agents. The specific agents features as the level of sensibility, direct communications, the capability to learn and stigmergy have a direct and in many cases a positive impact on the solution of a difficult, complex problem. The current survey paper illustrates and finally makes a comparison on the several agent-based approaches stated below.

In Pintea *et al.* (2006) was used for the first time the *Ant Colony System* (*ACS*) for solving *GTSP*. Ant-based optimization was introduced by Marco Dorigo in his PhD thesis, as stated in Dorigo and Gambardella (1996) and used for solving the classical *Traveling Salesman Problem*. Based on newly updating rules and some *MAX–MIN* Ant System's features stated in Stützle and Hoos (1997), a reinforced *ACS* algorithm for *GTSP* was introduced in Pintea *et al.* (2006). The reinforced algorithm was competitive with the already proposed heuristics for the *GTSP*.

Several new heuristics involving agents properties were also introduced: *Sensitive Ant Colony System* (*SACS*), *Sensitive Robot Metaheuristic* (*SRM*) and *Sensitive Stigmergic Agent System* (*SSAS*). *Sensitive ACS* (*SACS*) (Chira *et al.*, 2007a) heuristic uses the sensitive reactions of ants to pheromone trails. Numerical experiments illustrated in Chira *et al.* (2007a) shows the potential of the *SACS* model. *Sensitive Robot Metaheuristic* (*SRM*) (Pintea *et al.*, 2008) uses virtual autonomous robots in order to obtain improved solutions. In *SSAS* (Chira *et al.*, 2007b) the agents adopt a stigmergic behavior in order to identify problem solutions and have the possibility to share information about dynamic environments improving the quality of the search process. Using an Agent Communication Language (ACL) (Wooldridge and Dunne, 2005) the agents communicate by exchanging messages.

Generalized Traveling Salesman Problem has many applications, as location and telecommunication problems (Fischetti *et al.*, 2002a; Laporte and Nobert, 1983) or in routing problems (Pintea *et al.*, 2011; Pop *et al.*, 2009).

The paper is organized as follows. Section 2 provides a description and a mathematical model of the *Generalized Traveling Salesman Problem*. The proposed agent-based models are illustrated in Section 3. Several discussions and analysis of the agent-based techniques involved for solving *GTSP* are illustrated in Section 4. The paper concludes with further research directions.

2. The Generalized Traveling Salesman Problem

The current section includes the description of the *Generalized Traveling Salesman Problem* including a mathematical model and its complexity. For the mathematical model of *GTSP* it is considered a complete undirected graph $G = (V, E)$ with n nodes. The graphs edges are associated with non-negative costs. The cost of an edge $e = \{i, j\} \in E$ is denoted by c_{ij}.

The generalization of *TSP* implies an existing partition of set V. The subsets of V are called *clusters*. Let $V_1, \dots, V_p$ be a partition of V into p *clusters*: $V = V_1 \cup V_2 \cup \dots \cup V_p$ and $V_l \cap V_k = \emptyset$ for all $l, k \in \{1, \dots, p\}$. The objective of the *Generalized Traveling Salesman Problem* is to find a minimum-cost tour. In other words, it has to find a minimum-cost tour, a subset H, with exactly one node from each cluster V_i, $i \in \{1, \dots, p\}$.

A *tour* is a subset of nodes such that the subset contains exactly one node from each cluster of the graph partition. There are involved the following decisions: (i) choose a node subset $S \subseteq V$, such that $|S \cap V_k| = 1$, for all $k = 1, \dots, p$; (ii) find a minimum cost *Hamiltonian* cycle H in the subgraph of G induced by S.

DEFINITION 1. The *Generalized Traveling Salesman Problem* is called *symmetric* if and only if the equality $c(i, j) = c(j, i)$ holds for every $i, j \in V$, where c is the cost function associated to the edges of G.

The time complexity for an exact algorithm is $|V_{k_1}|O(m + n \log n)$ and in the worst case the complexity is $O(nm + n \log n)$ (Pop, 2007). An accurate discussion about time complexity for the *GTSP* is given in Karapetyan and Gutin (2012).

3. Agent-Based Approaches for Solving the Generalized Traveling Salesman Problem

The following subsections will describe in detail the *Ant Colony System*, the reinforced, sensitive, multi-agent hybrid sensitive and stigmergic agent-based approaches for solving *Generalized Traveling Salesman Problem*.

3.1. *Ant Colony System for Generalized Traveling Salesman Problem*

The first *Ant Colony Optimization* (*ACO*) heuristic was *Ant System* (*AS*). The algorithm was proposed in Dorigo (2011). *ACO* it is a particular multi-agent approach introduced for solving complex combinatorial optimization problems. The ant-based algorithms, as the entire *ACO* framework, was inspired by the observation of real ant colonies.

In ant-based models an artificial ant can find shortest paths between ‘food’ sources and a ‘nest’. While walking the ants deposit on the ground a substance called *pheromone*. In real life ants smells pheromone when choosing their paths and the trails with the largest amount of pheromone is choose. This behavior leads to the emergence of shortest paths and after a while the entire ant colony uses the shortest path. These features are used also in the algorithmic mechanism of artificial ant systems (Dorigo, 2011; Crisan, 2007). The artificial agents called artificial ants iteratively construct candidate solution to an optimization problem. The solution construction is guided by pheromone trails and the specific information of the problem, in particular *GTSP*.

Ant Colony System (*ACS*) (Dorigo and Gambardella, 1996) was developed to improve *Ant System* making it more efficient and robust. The model was developed and extended for the *GTSP* by Pintea *et al.* (2006). Based on the mathematical model of *GTSP* from Section 2, let $V_k(y)$ be the node y from the cluster V_k. The number of clusters is denoted with *nc*. The extended model works as follows.

Initialization procedure. Initially the ants are placed in the nodes of the graph, choosing randomly the *clusters* and also a random node from a chosen cluster. Each edge has a label with a trail intensity: $\tau_{ij}(t)$ the trail intensity of the edge (i, j) at time t. Each trail has a given initial pheromone value, τ_0. Build an initial tour T using a Greedy algorithm.

Construction of a tour. At each iteration every ant moves to a new node from an unvisited *cluster* and the parameters controlling the algorithm are updated. An ant decides which node is the next move with a probability that is based on the distance to that node, or the cost of the edge, and the amount of trail intensity on the connecting edge. The inverse of distance from a node to the next node is known as the *visibility*, η_{ij}. At each time unit evaporation takes place in order to stop the intensity of pheromone on the trails. The rate evaporation is $\rho \in (0, 1)$.

A tabu list is maintained with the purpose to forbid ants visiting the same *cluster* in the same tour. The ant tabu list is cleared after each completed tour. In order to favor the selection of an edge that has a high pheromone value, τ, and high visibility value, η a probability function ${p^k}_{iu}$ is considered. ${J^k}_i$ are the unvisited neighbors of node i by ant k and $u \in {J^k}_i, u = V_k(y)$, being the node y from the unvisited cluster V_k. The probability function is defined as follows, where β is a parameter used for tuning the relative importance of edge cost in selecting the next node.

$$ {p^k}_{iu}(t) = \frac{[\tau_{iu}(t)][\eta_{iu}(t)]^\beta}{\Sigma_{o \in {J^k}_i}[\tau_{io}(t)] \cdot [\eta_{io}(t)]^\beta}, \tag{1} $$

${p^k}_{iu}$ is the probability of choosing $j = u$, where $u = V_k(y)$ is the next node, if $q > q_0$, when the current node is i; else the next node j is chosen as follows, where q is a random variable uniformly distributed over $[0, 1]$ and q_0 is a parameter similar to the temperature in simulated annealing, $q_0 \in [0, 1]$

$$ j = \underset{u \in J_i^k}{\operatorname{argmax}} \left\{ \tau_{iu}(t) \left[\eta_{iu}(t) \right]^\beta \right\}. \tag{2} $$

The ants guides the local search by constructing promising solutions based on good locally optimal solutions. A local update rule for pheromone trails is based on *ACS* (Dorigo and Gambardella, 1996) local rule:

$$ \tau_{ij}(t+1) = (1 - \rho) \cdot \tau_{ij}(t) + \rho \cdot \tau_0. \tag{3} $$

Compute a solution. After each step is computed the local best tour length. The length of each tour is computed and compared with the length of the already known best tour. If it is found an improved tour, the best tour T^+ and its length L^+ are updated.
Global update procedure. The global update procedure for the generalized problem is based on *Ant Colony System* for *Traveling Salesman Problem* only the ant that generate the best tour is allowed to *globally* update the pheromone. The current update rule is applied to the edges of the *best tour*. The correction rule follows where $\Delta\tau_{ij}(t)$ is the inverse cost of the best tour.

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \rho\Delta\tau_{ij}(t). \tag{4}$$

The algorithm concludes with finding an optimal cost tour, a subset of nodes with exactly one node from each partition of the *GTSP*. The main steps of *Ant Colony System for GTSP* are summarized in Algorithm 1.

Algorithm 1: Ant colony system for GTSP

1: Initialization procedure
2: **repeat**
3: Construction of a tour
4: Compute a solution (3.1)
5: Global update procedure (4)
6: **until** end condition
7: **return** the shortest tour and its length

3.2. *Reinforcing Ant Colony System for Generalized Traveling Salesman Problem*

An *Ant Colony System* for the *Generalized Traveling Salesman Problem* it is introduced and detailed in Pintea *et al.* (2006), Pintea (2010). In order to enforces the construction of a valid solution used in *ACS* a new algorithm called *Reinforcing Ant Colony System* (*RACS*) it is elaborated with a new pheromone rule as in Pintea and Dumitrescu (2005) and pheromone evaporation technique as in Stützle and Hoos (1997). The main improvements of the introduced algorithm are further described.
Reinforced construction of tours. The construction of a tour Section 3.1 from Algorithm 1 is changed as follows. After each transition the trail intensity is updated using the inner correction rule from Pintea and Dumitrescu (2005). The *inner rule* updates the pheromone trail for the neighbors u of a potential candidate node j, where L^+ is the cost of the current known best tour

$$\tau_{iu}(t+1) = (1-\rho)\cdot\tau_{iu}(t) + \rho\cdot\frac{1}{n\cdot L^+}. \tag{5}$$

Reinforced global update procedure. It is used the global update procedure 4 and when the pheromone trail is over an upper bound $\tau_{\max}$, the pheromone trail is re-initialized. In order to avoid stagnation it is used the pheromone evaporation technique introduced in *MAX–MIN Ant System* (Stützle and Hoos, 1997) (6).

$$\text{if } (\tau_{ij}(t) > \tau_{ij}(t)) \text{ then } \tau_{ij}(t) = \tau_0. \tag{6}$$

The reinforcement technique, as illustrated in Algorithm 2, gives good results for *GTSP* as are stated in Pintea *et al.* (2006) and as follows in Section 4, *Evaluations of Agent-Based Algorithms for E-GTSP*.

Algorithm 2: Reinforcing ant colony system for GTSP

1: Initialization procedure.
2: **repeat**
3: Reinforced construction of tours.
4: Compute a solution (3.1)
5: Reinforced global update procedures (4), (6).
6: **until** end condition
7: **return** the shortest tour and its length

3.3. *Sensitive Ant Colony System for Generalized Traveling Salesman Problem*

The *Sensitive Ant Colony System (SACS)* for *GTSP* is based on the *Heterogeneous Sensitive Ant Model for Combinatorial Optimization* (Chira *et al.*, 2008). In sensitive ant-based models there are used a set of heterogeneous agents (sensitive ants) able to communicate in a stigmergic manner and take individual decisions based on changes of the environment and on pheromone sensitivity levels specific to each agent. The sensitivity variable induce various types of reactions to a changing environment.

A good balance between search diversification and exploitation can be achieved by combining stigmergic communication with heterogeneous agent behavior. Each agent is characterized by a pheromone sensitivity level, PSL expressed by a real number from [0, 1]. The transition probabilities from *ACS* model (Dorigo and Gambardella, 1996) are changed using the *PSL* values in a re-normalization process. The *ACS* transition probability is reduced proportionally with the *PSL* value of each agent in the sensitive ant-based approach (Chira *et al.*, 2008). Extreme situations for pheromone sensitivity level values are when an ant is 'pheromone blind', $PSL = 0$, with a null sensitivity level the ant will ignore completely the stigmergic information and when an ant has maximum pheromone sensitivity, $PSL = 1$.

Low *PSL* values indicate that a sensitive ant will choose very high pheromone levels moves. These ants are more independent and can be considered environment explorers and have the potential to discover in an autonomous way new promising regions. The ants with high *PSL* values are able to intensively exploit the promising search regions already identified. The *PSL* value can increase or decrease according to the search space encoded in the ant's experience. In the *SACS* model for solving *GTSP* two ant colonies are involved. Each ant is endowed with a pheromone sensitivity level. In the first colony

the ants have *small PSL* values ($sPSL$) and the second colony with *high PSL* values *hPSL*. The *sPSL* ants autonomously discover new promising regions of the solution space to sustain search diversification. The sensitive-exploiter *hPSL* ants normally choose any pheromone marked move. *SACS* for solving *GTSP* (Chira *et al.*, 2007a) works as follows.
Initialization procedure for sensitive ants. The ants are placed randomly in the nodes of the graph and the parameters are initialized. The sensitivity level is denoted by s and its value is randomly generated in $(0; 1)$. For *sPSL* ants the sensitivity parameter s is considered between 0 and s_0 and for *hPSL* ants s values are randomly chosen in $(s_0; 1)$, where $s_0 \in [0, 1]$
Construction of a tour for sensitive ants. At each iteration every *sPSL*-ant moves to a new node and the parameters are updated. When an ant decides which node from a cluster is the next move it does so with a probability that is based on the distance to that node and the amount of trail intensity on the connecting edge (Section 3.1).
New Local Updating Rule. The trail intensity is updated (Chira *et al.*, 2007a), using the local rule as following, where n is the total number of the nodes

$$\tau_{ij}(t+1) = s^2 \cdot \tau_{ij}(t) + (1-s)^2 \Delta\tau(t) \cdot \frac{1}{n}. \tag{7}$$

These steps are reconsidered by the *hPSL*-ant using the information of the *sPSL* ants. *Compute a solution* and *Global update procedure* are performed as in Section 3.1. The shortest tour found by sensitive ants is the result of the algorithm. Algorithm 3 illustrates the *Sensitive Ant Colony System* model for *GTSP*.

Algorithm 3: Sensitive ant colony system for GTSP

1: Initialization procedure for sensitive ants
2: **repeat**
3: Construction of a tour for sensitive ants; a local update rule (7) for each sPSL-ant and hPSL-ant
4: Compute a solution (3.1)
5: Global update procedure (4)
6: **until** end condition
7: **return** the shortest tour and its length

3.4. *Sensitive Robot Metaheuristic for Generalized Traveling Salesman Problem*

A particular technique, inspired from both *SACS* and involving autonomous robots is *Sensitive Robot Metaheuristic* (*SRM*) (Pintea *et al.*, 2008). The model relies on the reaction of virtual sensitive autonomous robots to different stigmergic variables. Each robot is endowed with a distinct stigmergic sensitivity level. *SRM* ensures a balance between search diversification and intensification. As it is detailed in Pintea *et al.* (2008), a stigmergic robot action is determined by “the environmental modifications caused by prior actions of other robots”. *Sensitive robots* are artificial entities with a *Stigmergic Sensitivity Level* (*SSL*) which is expressed by a real number in the unit interval [0, 1].

As it is in general for agents, here, in particular, robots with small *SSL* values are considered explorers of the search space and are considered independent sustaining diversification. The robots with high *SSL* values are exploiting the promising search regions already identified by explorers. The *SSL* values in *SRM* model increase or decrease based on the search space topology encoded in the robot experience.

Now something about the stigmergic robots involved in the process of solving a combinatorial optimization problem (Zelina, 2008), including *GTSP*. Qualitative stigmergy means a different action and quantitative stigmergy is interpreted as a continuous variable which change the intensity or probability of future actions (Bonabeau *et al.*, 1999). Because the robots have not the capability of ants to deposit chemical substances on their trail, a qualitative stigmergic mechanism is involved in *Sensitive Robot Metaheuristic*. These robots communicate using the local environmental modifications that can trigger specific actions. There is a set of so called "micro-rules" defining the action-stimuli pairs for a homogeneous group of stigmergic robots. These rules define the robots particular behavior and find the type of structure the robots will create (Bonabeau *et al.*, 1999).

In Pintea *et al.* (2008) the algorithm is used to solve a large drilling problem, a particular *GTSP*. It follows a detailed description of the *Sensitive Robot Metaheuristic* for *GTSP*.

Initialization procedure for robots. Initially the robots are placed randomly in the search space. A robot moves at each iteration to a new node. The parameters controlling the algorithm are set.

Construction of a tour with sensitive robots.The next move of a robot is probabilistically based on the distance to the candidate node and the stigmergic intensity on the connecting edge. In order to stop increasing stigmergic intensity, evaporation process is invoked. Also, is maintained a tabu list preventing robots to visit a cluster twice in the same tour. The stigmergic value of an edge is τ and the visibility value is η. As already mentioned in Section 3.1, ${J^k}_i$ is the unvisited successors of node i by robot k and $u \in {J^k}_i$. The *sSSL* robots probabilistically choose the next node. i is the current robot position. The probability of choosing u as the next node is given by (1).

An autonomous robot could be in the team with high or in the team with low stigmergic sensitivity on the basis of a random variable uniformly distributed over [0, 1]. Let q be a realization of this random variable and q_0 a constant, $q_0 \in [0, 1]$. The robots with small stigmergic sensitivity *sSSL* are characterized by the inequality $q > q_0$ while for the robots with high stigmergic sensitivity *hSSL* robots $q_0 >= q$ holds. A *hSSL-robot* uses the information given by the *sSSL* robots. *hSSL* robots choose the new node j in a deterministic manner according to (2). The trail stigmergic intensity is updated using the local stigmergic correction rule:

$$\tau_{ij}(t+1) = q_0^2 \tau_{ij}(t) + (1-q_0)^2 \cdot \tau_0. \tag{8}$$

Global update procedure. Global updating (Eq. (9)) the stigmergic value is the role of the elitist robot that generates the best intermediate solution. These elitist robots are the only robots having the opportunity to know the best tour found and reinforce this tour in order to focus future searches more effectively.

$$\tau_{ij}(t+1) = q_0^2 \tau_{ij}(t) + (1-q_0)^2 \cdot \Delta\tau_{ij}(t), \tag{9}$$

where $\Delta\tau_{ij}(t)$ is the inverse value of the best tour length. Furthermore q_0 is used as the evaporation rate factor. An execution of the algorithm returns the shortest tour found. The description of the *Sensitive Robot Metaheuristic* for solving the *GTSP* is illustrated further in Algorithm 4.

Algorithm 4: Sensitive robot algorithm for GTSP

```
1: Initialization procedure-for robots.
2: repeat
3:    Construction of a tour with sensitive robots
4:    Compute a solution (3.1)
5:    Global update procedure (9)
6: until end condition
7: return the shortest tour and its length
```

3.5. *Sensitive Stigmergic Agent System for Generalized Traveling Salesman Problem*

The *Sensitive Stigmergic Agent System for GTSP (SSAS)* introduced in Chira *et al.* (2007b) is based on the *Sensitive Ant Colony System* (*SACS*) (Chira *et al.*, 2007a) and *Stigmergic Agent System* (*SAS*) (Chira *et al.*, 2006). In Chira *et al.* (2006) was introduced the concept of stigmergic agents, involving a direct communication and also a communication based on stigmergy using artificial pheromone trails similar with some biological systems. The novelty of *SSAS* is that the agents are endowed with sensitivity. The advantage is that agents with both sensitivity and stigmergy have an increased capability to solve complex static and dynamic real life problems.

A multi-agent system (*MAS*) approach to developing complex systems involves the employment of several agents capable of interacting with each other to achieve objectives. Some benefits of multi-agents are the ability to solve complex problems, their interconnection and inter-operation with multiple systems and are capable to handle distributed areas (Wooldridge and Dunne, 2005).

The *SSAS* model inherits also agent properties: autonomy, reactivity, learning (Pchelkin, 2003), mobility and pro-activeness (Iantovics and Enachescu, 2009). They are able to cooperate, to exchange information, to learn and are capable to communicate through an agent communication language (*ACL*). Sensitivity is considered a plus for the agents; with this new feature, stronger artificial pheromone trails are preferred and the most promising paths receive a greater pheromone trail after some time. Within the *SSAS* model each agent is characterized by a pheromone sensitivity level *PSL* (Section 3.3).

Based on *SACS*, two sets of sensitive stigmergic agents are performing in *SSAS*: with small and high sensitivity *PSL* values. The role of sensitive ants from *SACS* is taken now, more generally, by sensitive-explorer agents, with small *PSL–sPSL* agents and sensitive exploiter agents with high *PSL–hPSL* agents. The *sPSL* agents discover new promising regions of the solution space in an autonomous way, sustaining search diversification. The *hPSL* agents exploit the promising search regions already identified by the *sPSL* agents. Each *PSL* agent deposit pheromone on the followed path. Evaporation takes place each cycle preventing unbounded intensity trail increasing.

Algorithm 5 illustrates the *Sensitive Stigmergic Agent System for GTSP*. A specific initialization procedure is shown in *Algorithm 5.1: Initialization procedure for agents*. *Algorithm 5.2: Construction of a tour for agents.* describes the main mechanism of the current model. There is also employed the *Reinforced global update procedure* with the update from Section 3.2. A run of the algorithm returns the shortest tour found.

Algorithm 5.1: Initialization procedure for agents

forall edges (i, j) **do** $\tau_{ij}(0) = \tau_0$ **end for**
Activate a set of agents with various PSL
for $k = 1$ to m **do**
Place randomly an agent in a node from a random cluster.
end for
Initialize knowledge base.
Build an initial tour T using a Greedy algorithm.

Algorithm 5.2: Construction of a tour for agents.

repeat
Move to a new node each hPSL-agent (1), (2)
An agent send an ACL message about latter edge formed
until all hPSL-agents have built a complete solution
repeat
Each sPSL-agent receive and use the information send by hPSL-agents or the information available in the knowledge base
Apply a local pheromone update rule (5)
until all sPSL-agents have built a complete solution

Algorithm 5: Sensitive stigmergic agent system for GTSP

Algorithm 5.1: Initialization procedure for agents
repeat
Algorithm 5.2: Construction of a tour for agents
Compute a solution (3.1)
Reinforced global update procedures (4), (6)
until end condition
return the shortest tour and its length

4. Discussions and Analysis of Agent-Based Algorithms for E-Generalized Traveling Salesman Problem

The benefits of the reinforced, sensitive and stigmergic agent-based algorithms for *E-GTSP* discussed in the following are based on the many experimental results from previous works (Pintea *et al.*, 2006; Chira *et al.*, 2007a, 2007b; Pintea, 2010, 2014). At first something about the parameters involved in the compared algorithms. It was noted that a larger number of ants does not improve the performance of the algorithms. The particular *sensitivity parameter* s_0 was considered 0.5, the *hPSL* sensitivity level of ants was considered to be distributed between s_0 and 1, while the ants with small sensitivity pheromone level in the interval $(0, s_0)$; for the virtual robots of *SRM* the sensitive stigmergic level is considered random in [0, 1] at each trial. For using *SSAS* with good results, in Chira *et al.* (2007b) were tested several sensitive parameters values; the best results for considered instances were obtained by assigning low pheromone sensitive level *0.01* values for most of the agents.

Ant Colony System shows once again the stability of the model introduced by Dorigo (2011) and developed for *GTSP* in Pintea *et al.* (2006), Pintea (2010). *Reinforced ACS* performs well on the small instances, obtaining for several instances the optimal solution. Sensitivity involved in ant-based models have the ability to identify good solutions with *SACS* for medium and large size instances. The autonomous stigmergic robots from *SRM* seems to have good results overall. *SSAS* reports the best solutions when compared to the other models suggesting the benefits of the model heterogeneity in the search process, but the execution time should be improved. The time could be reduce with better parameter settings, highest hardware performance and/or better local search heuristics. The presented agent-based algorithms could be improved using hybrid models or parallel algorithms.

The studied techniques for solving *GTSP* could be also involved with success for solving other real-life problem based on large-scale graph representation, complex network or for example on improving classification techniques (Parpinelli *et al.*, 2002; Stoean *et al.*, 2009), on large graphs representations (Jancauskas *et al.*, 2012). They are already used for vehicle routing problem (Pintea *et al.*, 2011; Pop *et al.*, 2009), the *Generalized Covering Salesman Problem* and compared with other existing techniques (Cacchiani *et al.*, 2011; Karapetyan and Gutin, 2012).

5. Conclusions

The purpose of the current survey paper is to describe several agent-based algorithms involved for solving a *NP*-hard problem, the *Equality-Generalized Traveling Salesman Problem*. The properties of agents as sensitivity, cooperation, learning capacities along with virtual robots autonomy and artificial ants stigmergic features are strongly implied in the process of finding good solutions for the specified problem. The multiple parameters used for the algorithms and the running time are not in the favor of the studied algorithms. Based on these models could be hopefully developed hybrid techniques for solving complex real-life problems.

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Agentais grindžiamų metodų lygybinio apibendrinto keliaujančio pirklio uždaviniui spręsti apžvalga

Camelia-M. PINTEA

Apibendrintas keliaujančio pirklio uždavinys yra vienas iš gerai žinomų sudėtingų kombinatorinio optimizavimo uždavinių. Vienas jų atvejų yra vadinamasis lygybinis apibendrintas keliaujančio pirklio uždavinys. Jo pagrindinis tikslas – rasti minimalias kelionės išlaidas aplankant tik vieną tašką iš kiekvieno didelės apimties nekryptinio grafo klasterio. Šiuo metu sprendžiant praktinius sudėtingus uždavinius dažnai sėkmingai taikomi daugelio agentų metodai. Šio straipsnio tikslas – apžvelgti algoritmus lygybiniam apibendrintam keliaujančio pirklio uždaviniui spręsti, įskaitant tam tikrus skruzdėlėmis grįstus modelius bei virtualius robotus-agentus, turinčius specifinių savybių.