INFORMATICA, 2015, Vol. 26, No. 2, 259–282

DOI: http://dx.doi.org/10.15388/Informatica.2015.48

Selection of Material Handling Equipment Using Fuzzy Axiomatic Design Principles

Anant V. KHANDEKAR, Shankar CHAKRABORTY*
Department of Production Engineering, Jadavpur University
Kolkata, 700 032, West Bengal, India
e-mail: s_chakraborty00@yahoo.co.in

Received: January 2014; accepted: April 2014

Abstract. Effective movement of materials plays an important role in successful operation of any organization. Proper methods adopted for material movement are also crucial for the overall safety of the personnel involved in the manufacturing processes. Selection of the appropriate material handling equipment (MHE) is a vital task for improving productivity of an organization. In today's technological era, varieties of MHEs are available to carry out a desired task. Depending on the type of material to be moved, there are many quantitative and qualitative factors influencing the selection decision of a suitable MHE. The problem of selecting the right type of MHE for a given purpose can be solved using multi-criteria decision-making (MCDM) methods which are capable of dealing with the combination of crisp and fuzzy data. In this paper, an MCDM method employing fuzzy axiomatic design principles is applied for selecting the most appropriate MHE for the given task. As a measure of suitability, the total information content is calculated for each MHE and the MHE alternative with the least total information content is regarded as the best choice. Two real time problems from the literature, i.e. selection of an automated guided vehicle, and selection of loading and hauling equipments in surface mines, are solved to validate the applicability, flexibility and potentiality of the adopted approach.

Key words: material handling equipment, MCDM, fuzzy set theory, axiomatic design principles, information axiom.

1. Introduction

The role of MHEs is very important in all service and manufacturing organizations. They are used for transporting semi-finished and finished products from one workstation to other without causing obstruction to the processes and act like basic integrators in real sense (Sujono and Lashkari, 2007). In addition to this main function of transporting products, MHEs also perform other auxiliary but equally important functions as positioning, unit formation and storage (Karande and Chakraborty, 2013). Usually, all these functions are carried out in various combination modes or separately depending on the type of MHE used. Equipments in transport category include conveyors, cranes, industrial trucks etc. Generally after transportation, positioning of the object or workpiece is carried out to

*Corresponding author.

make it convenient for machining, and the equipments, like rotary tables, robots and indexing units mainly do this task. Equipments under unit formation category, like bins, pallets and containers carry materials in standardized quantities. Finally, storage equipments are used for holding or buffering materials over a period of time. Typical equipments performing this function are pallet racks and shelves.

Application of a suitable MHE can decrease manufacturing lead times, increase efficiency of material flow and improve facility utilization. Ultimately, it leads to the optimal use of labour force and system flexibility. As a result, effective use of MHEs significantly contributes towards increased productivity of the manufacturing organizations (Onut *et al.*, 2009). Over the last few years, there is a phenomenal increase in the types of MHEs available in the commercial market, to be used for a certain industrial application. At the same time, the capital cost investment is huge while designing and implementing a material handling system in a plant, making the right type of MHE selection to be a strategic decision. Material handling activities constitute for about 30 to 75% of the cost of final product and use of efficient material handling systems results in 15 to 30% reduction in manufacturing operations cost (Kulak, 2005). These figures vehemently stress the role played by MHEs while manufacturing cost-effective products.

It is thus obvious that the decision maker (DM) has to select the appropriate type of MHE from various alternatives available because implementation of the selected MHE should be beneficial to the manufacturing organization with respect to its production objectives. This selection process is to be justified against some criteria which can be classified as technical, economical and environmental. Technical criteria include specifications of the MHE as load carrying capacity, horizontal distance moved and so forth. Economical criteria are capital investment cost, operational and maintenance cost. Environmental criteria include factors, like safety standard and environment friendliness. Looking at the gamut of all these criteria, it is obvious that some of them are subjective in nature, while others, like load capacity, energy consumption and cost are of crisp quantifiable type. Fuzzy set theory is a convenient mathematical tool to deal with those criteria of subjective nature. Besides this, some selection criteria are of beneficial type, i.e. higher values are desired and others are of non-beneficial type, i.e. lower values are preferred. In addition, factors contributing to the complexity of MHE selection process are the constraints imposed by the layout of the existing facility, type of the material to be handled, variety of process requirements, wide range of MHEs available for a specific purpose and uncertainty in the operational environment. The selection problem is again compounded by the frequent changes in facility design and rapid changes in automation technology. Different manufacturing organizations, like automobile, food processing, pharmaceutical, mining, textile, steel, oil and gas etc. require different types of MHEs in their operations. Obviously, some special characteristics are needed to be identified in each MHE to carry out its intended functions. Therefore, to arrive at the optimal solution of MHE selection problem against the backdrop of mutually conflicting criteria, is a challenging and complicated task. It is observed that there are about 50 different types of MHEs and they are characterized by about 30 different attributes, which can be grouped into four categories, i.e. (a) move type, (b) material type, (c) operation requirement, and (d) area constraints (Park, 1996). So, in order to rationally decide about the most feasible MHE alternative, all these conflicting criteria need to be brought on a single platform for analysis and further decision. Such a complicated problem can only be efficiently solved using multi-criteria decision-making (MCDM) methods. Thus, in this paper, the application of fuzzy axiomatic design (FAD) principles is demonstrated while solving two MHE selection problems having vague and imprecise information. The derived results are largely in agreement with those obtained by the earlier researchers employing other MCDM methodologies, which vehemently establishes the potentiality of FAD principles as an efficient MCDM tool. As it involves less mathematical steps, its low computation time favors for its application. It has also a systematic and scientific base which helps to obtain accurate ranking results.

2. Review of the Literature

The past researchers already worked in this field of MHE selection while applying various mathematical tools and techniques. Park (1996) developed an expert system for MHE selection and evaluation, consisting of four modules, i.e. (a) a knowledge base to select an appropriate equipment type, (b) a database to store the list of commercial equipment types with their specifications, (c) an MCDM procedure to select the optimal commercial model of the selected equipment type, and (d) a simulator to evaluate the performance of the selected equipment model. Chittratanawat and Noble (1999) presented a Tabu search meta-heuristic procedure for solving facility layout, pick-up/drop-off locations and MHE selection problems. Yaman (2001) devised a system in which an inference engine would study the features of the products and processes from the knowledge base, and select an appropriate MHE. Deb *et al.* (2002) used a fuzzy MCDM method to aggregate rating attitudes of the decision makers (DMs) and trade-off various selection criteria to find out values of fuzzy suitability indices for final ranking of MHE alternatives. Lashkari *et al.* (2004) presented an integrated model of operation allocation and MHE selection in cellular manufacturing systems. Kulak (2005) developed a decision support system to identify the most appropriate equipment among the alternatives of the same type using fuzzy information axiom of axiomatic design (AD) principles. Chakraborty and Banik (2006) applied analytic hierarchy process (AHP) for selecting the best MHE under a specific material handling environment. To identify the most critical and robust criteria in MHE selection process, sensitivity analysis was also performed. Sujono and Lashkari (2007) proposed a method for simultaneously allocating operation and selecting MHE in a flexible manufacturing environment with multiple performance objectives as minimizing cost of the processes and maximising the part-equipment compatibility. Onut *et al.* (2009) proposed a combined MCDM methodology for evaluation and selection of MHE types for a steel construction company in Turkey. Fuzzy analytic network process (FANP) was utilized for assigning criteria weights and fuzzy technique for order preference by similarity to ideal solution (FTOPSIS) was applied for deciding the best. Mirhosseyni and Webb (2009) developed a hybrid method for selection and assignment of the most appropriate MHE for a given operation. At first, the developed system would select the most appropriate type of

MHE for every material handling operation using an expert system consisting of crisp and fuzzy rules, and in the second phase, a genetic algorithm would search throughout the feasible solution space, constituting of all possible combinations of feasible equipments, in order to discover the optimal choice. Bazzazi *et al.* (2009) adopted fuzzy set theory along with AHP and TOPSIS methods to select the loading and hauling equipments used in open pit mines in Iran. Ulubeyli and Kazaz (2009) applied ELECTRE III (ELimination and Et Choice Translating REality) method as an MCDM tool for selection of concrete pumps while collecting and analyzing the real time data from about 70 civil engineering firms having various construction equipments. Sawant and Mohite (2009) applied fuzzy TOPSIS method for assessing and ranking of automated guided vehicles (AGVs), and studied the effect of varying the impreciseness of criteria values on the suitability ranking of the alternatives. Tuzkaya *et al.* (2010) applied an integrated MCDM methodology consisting of FANP and fuzzy preference ranking organization method for enrichment evaluation (FPROMETHEE) for solving MHE selection problems. Momani and Ahmed (2011) concluded that a more confidence in the results of MHE selection problems could be attained using a combined approach of Monte Carlo simulation and AHP method. Bazzazi *et al.* (2011) proposed a new fuzzy MCDM model for selecting the optimal open pit mining equipment while considering objective, critical and subjective factors as encountered in real time situations. Lashgari *et al.* (2012) selected the optimal fleet of loading and hauling equipments to be used in an open pit iron mine at Gole Gohar, Iran, while employing a hybrid approach of fuzzy AHP and TOPSIS methods. Yazdani-Chamzini and Yakhchali (2012) presented an integrated model based on fuzzy AHP and TOPSIS methods for selection of MHEs in surface mines. Karande and Chakraborty (2013) applied weighted utility additive method to solve an industrial conveyor belt selection problem. Sawant and Mohite (2013) developed a decision support system for selection of AGVs, while integrating objective weights of importance of the attributes as well as subjective preferences of the DMs to decide the composite attribute weights. Mousavi *et al.* (2013) presented a fuzzy MCDM method for MHE selection while combining the concept of compromise solution and grey relational model under the condition of uncertain information.

From this extensive review of literature, it can be noted that amongst the adopted methods used for MHE selection, about 75% of them considered purely quantitative information, whereas, only about 25% of the methods dealt with both quantitative and qualitative information. But it is quite evident that MHE selection problems involve many criteria. Some of them are stated exclusively in linguistic terms, as it is difficult to express them quantitatively. For example, the criteria related to safety and environmental concerns need to be stated in linguistic terms as high, medium or low and so forth. So in order to have equitable selection of MHE for a given task, it is necessary to consider the dependent criteria which are expressed qualitatively in addition to quantitative ones. Hence, it is the ardent need of the DMs to consider fuzzy MCDM methods to solve the MHE selection problems in real time manufacturing environment.

As envisaged from the literature review, the most frequently adopted fuzzy MCDM methods for solving MHEs include AHP, ANP, TOPSIS, ELECTRE and VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje). Each of these methods has its own advantages and limitations. AHP method can measure the consistency of the DM's judgment, but at the same time, it is dependent on the DM's knowledge of criteria and their preferences. It is also not suitable for large number of alternatives and criteria. ANP method is quite capable of dealing with complex interrelationships among the decision levels and attributes as it is based on a strong feedback system. But it involves lengthy and time consuming calculations, and found to be less suitable for handling uncertainties. The advantages of TOPSIS method include its simple computation procedure, requirement of no pair-wise comparison, capacity to deal with large number of alternatives and criteria, and ability to provide performance scores to the considered alternatives with respect to the ideal solution. But it lacks in controlling the consistency of the final solution. ELECTRE, based on outranking principle, is a more time consuming method as individual concordance and discordance matrices need to be developed for each alternative-criteria pair. VIKOR method provides a compromised solution considering the advantages and acceptability effects of the decision. The final solution in VIKOR method is often affected by the value of an additional parameter. Against all the popular MCDM techniques, the present method based on FAD principles can assess large number of alternatives and criteria, with their ratings expressed quantitatively and semantically too. In this approach, the alternative which is unable to satisfy even a single criterion is considered as unsuitable and only the promising alternatives are shortlisted as potential candidates, thus reducing its computation procedure a lot. The DM can derive the final scores in terms of information content for the purpose of ranking the suitability of the alternatives.

From 2005 onwards, FAD principles are being used in manufacturing decision-making. But in the field of MHE selection, the utility of FAD principles as an MCDM tool is not fully explored. In this paper, two real time problems from the published literature, i.e. a) selection of an AGV, and b) selection of loading and hauling equipments in an open surface iron mine are considered and subsequently solved applying FAD principles. The derived results are quite in unanimity with those obtained by the past researchers.

In this paper, a literature review on selection of MHEs is provided in Section 2 and AD principles are explained in Section 3. A brief outline on fuzzy set theory and MCDM methodology based on FAD principles are presented in Section 4. To foster better understanding of the adopted methodology, two real time problems are presented in Section 5. Section 6 contains the concluding remarks.

3. Axiomatic Design Principles

The theory of AD was basically developed for the purpose of designing products with logical thinking (Suh, 1990). Till that time, the process of designing products was carried out simply by trial and error method, and was not supported by much scientific and engineering concepts. There used to be too many iterations before getting the final design. The concept of AD was thus proposed as a systematic, scientific approach for the design of products. It takes into consideration the customer needs related to a product to be incorporated in terms of functional requirements (FRs) and establishes the relation with the

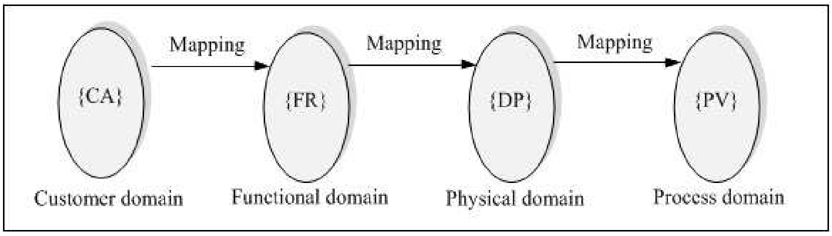

Fig. 1. Different design domains.

final design parameters (DPs) of the product. While designing a product, the customer needs/attributes related to the product are determined first. The FRs represent those features of the product which are able to satisfy the customer needs. In the present context of decision-making, FRs stand for different criteria with respect to which an alternative is judged suitable for its intended function. According to AD theory, the design process consists of the following steps:

Step 1: Establishment of design goals to satisfy a given set of customer needs.
Step 2: Conceptualization of design solutions.
Step 3: Analysis of the proposed design solutions.
Step 4: Selection of the best design solution from among those proposed.
Step 5: Implementation of the selected solution.

These steps occur in and between different design domains, such as customer domain, functional domain, physical domain and process domain, as illustrated in Fig. 1. Customer domain consists of customer needs or attributes (CA) that the customer is looking for in a product or system to be designed. These customer needs are translated into a set of FRs and constraints in the functional domain. These FRs are then mapped into physical domain, where the DPs are conceived to satisfy the FRs. The DPs represent physical properties that define the design solution in the physical domain. The DPs are then mapped into process variables (PVs) in process domain. The PVs can generate the specified DPs. The process of mapping is the systematic way of synthesis and transformation of factors from previous to the next domain.

In AD theory, the DPs are expressed in terms of range of values. Usually, this range is fixed by the designer or DM for a certain DP and is known as design range (DR). The range of values of DPs for different available alternatives is known as system range (SR). Naturally, in order to satisfy a certain set of FRs, there can be different combinations of DPs. In other words, the best combination, i.e. the design solution from various alternatives available needs to be chosen. So here comes the utility of AD theory as a decision-making tool. It is based on two axioms, stated as below.

3.1. *Independence Axiom*

It stresses on maintaining the independence of FRs. The meaning of independence axiom is that a particular FR should be fulfilled independently by a certain DP without affecting the other FRs (Kulak, 2005). In real time problems too, a given complex design or a decision task is decomposed into smaller components and the independent solution for each of them is sought. So, the independence axiom supports this analogy. The relationship between FRs and DPs is reflected by the type of design matrix connecting them. As a result, uncoupled, decoupled and coupled designs are obtained respectively for diagonal matrix, triangular matrix and for rest of the cases. Uncoupled design is the most preferred one as in this case, each FR is independently satisfied by the corresponding single DP. So, the independence axiom distinguishes between good and bad designs, or acceptable and unacceptable decisions. The relationship between FRs and DPs is defined using Eq. (1).

$$(FR) = [A](DP),$$
$$A = [A_{ij}]_{m\times n} = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1n} \\ A_{21} & A_{22} & \dots & A_{2n} \\ \dots & \dots & \dots & \dots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}, \tag{1}$$

where A is the relation matrix between FRs and DPs. The elements of $[A]$ are either 1 or 0, representing either 'relation' or 'no relation at all' respectively between FR and DP. Depending on the relative number of FRs and DPs, the types of designs in AD methodology are as (a) if the number of DPs is higher than the number of FRs, the design is termed as coupled; (b) the design is redundant if FRs outnumbers DPs; and (c) if the number of DPs is equal to that of FRs, the types of design are defined according to the relationships between FRs and DPs. If the relation matrix is diagonal, as shown in Eq. (2), the design is uncoupled.

$$A = \begin{bmatrix} A_{11} & 0 & \dots & 0 \\ 0 & A_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & A_{mn} \end{bmatrix}. \tag{2}$$

If the design matrix is triangular, as shown in Eq. (3), the design solution is termed as decoupled. Otherwise, the design solution is called as coupled (Suh, 1990).

$$A = \begin{bmatrix} A_{11} & 0 & \dots & 0 \\ A_{21} & A_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A_{m1} & A_{m2} & \dots & A_{mn} \end{bmatrix}. \tag{3}$$

3.2. *Information Axiom*

The design solutions satisfying the first axiom, i.e. the independence axiom are further analyzed by the second axiom, i.e. the information axiom. According to this axiom, the information content (IC) of each alternative design solution is determined and the alternative with the minimum IC value is treated as the optimal choice (Suh, 2001;

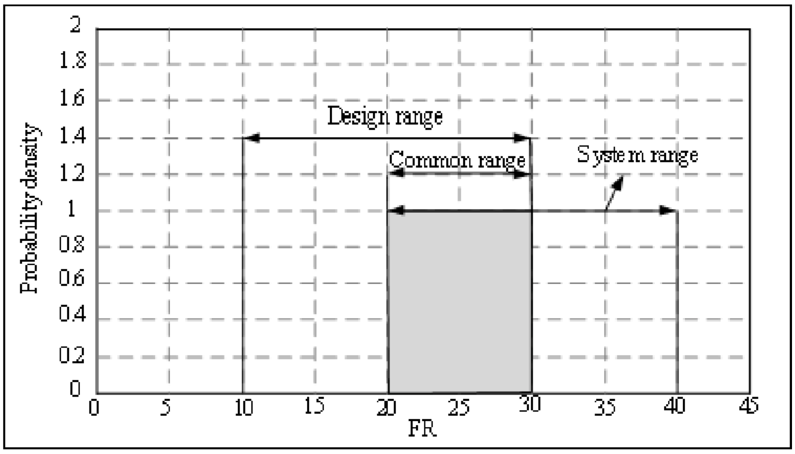

Fig. 2. Design range, system range and common range for FR.

Kulak, 2005). The IC is related in its simplest form to the probability of satisfying a given FR. It determines that the design with the highest probability of success is the best design. The IC_i value for a given FR_i is defined using the following equation:

$$IC_i = \log_2\left(\frac{1}{p_i}\right), \tag{4}$$

where p_i is the probability of satisfying the functional requirement FR_i. The information is expressed in units of bits. The logarithmic function is chosen so that the IC values will be additive when there are many FRs that must be satisfied simultaneously (Suh, 2001) and the logarithm is based on 2 which is the unit of bits.

The DR is decided by the designer or DM and it is the ideal range of values to be tried to achieve in the design process. The SR denotes the capability of the available manufacturing system. As shown in Fig. 2, the overlap between the designer-specified DR and the system capability range is known as 'common range' (CR), where the acceptable solutions exist.

Therefore, in the case of uniform probability distribution function, the value of p_i is given as follows:

$$p_i = \left(\frac{CR}{SR}\right). \tag{5}$$

So, the value of IC can now be expressed as below:

$$IC_i = \log_2\left(\frac{SR}{CR}\right). \tag{6}$$

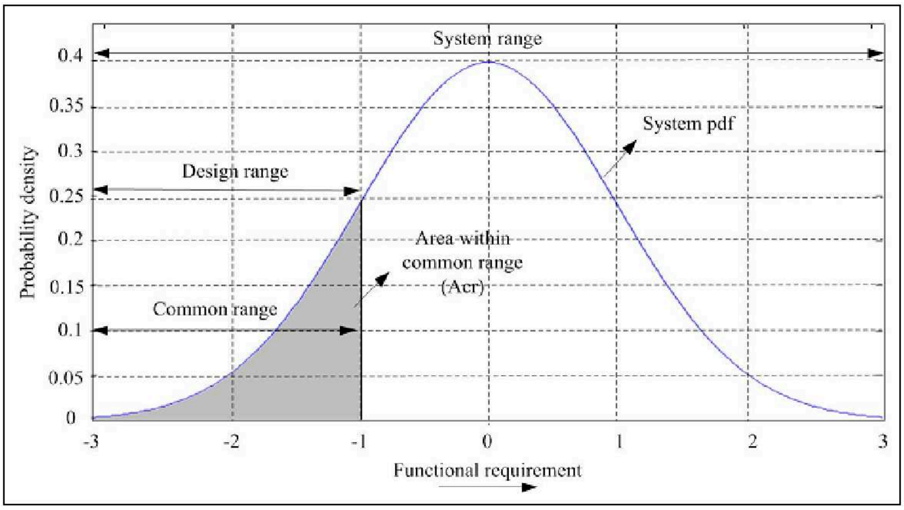

Fig. 3. Design range, system range, common range and system pdf for a FR (Celik *et al.*, 2008).

If FR_i is a continuous random variable (Fig. 3), then the probability of achieving FRi in DR is given as:

$$p_i = \int_{dr^l}^{dr^u} p_s(FR_i)\, dFR_i, \tag{7}$$

where $p_s(FR_i)$ is the system probability density function (pdf) of FR_i. dr^l and dr^u are the lower and upper bounds of DR. The probability of success is calculated by integrating the system pdf over the complete DR. In Fig. 3, the area of system pdf over the common range (A_{cr}) is equal to the probability of success p_i (Suh, 1990). Therefore, the IC can be expressed as follows:

$$IC_i = \log_2\left(\frac{1}{A_{cr}}\right). \tag{8}$$

4. Application of Fuzzy Axiomatic Principles for MHE Selection

In this paper, an integrated approach consisting of AD principles coupled with fuzzy set theory is used for solving two real time problems of MHE selection.

4.1. *Fuzzy Set Theory*

In general, understanding of all the physical processes by human beings is based on either precise or imprecise reasoning. It can be easily traced that in day to day life, human beings are exposed to such type of imprecise information more regularly than the quantum of precise information while taking real time decisions. Fuzzy set theory (Zadeh, 1965) was

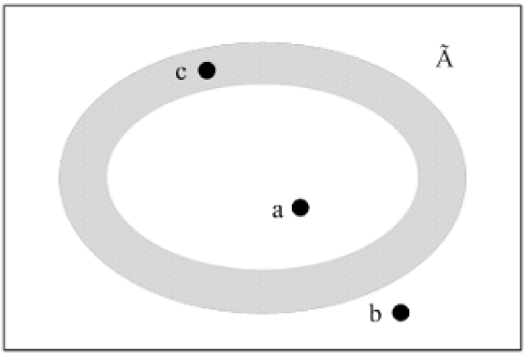

Fig. 4. Fuzzy set with ambiguous boundary.

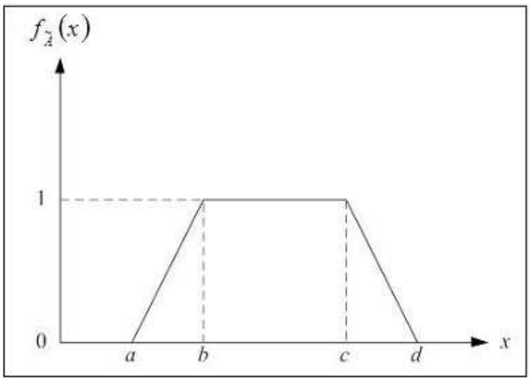

Fig. 5. Membership function of trapezoidal fuzzy number.

developed to handle such type of imprecise information in an efficient manner to arrive at the logical conclusions in a more scientific manner. It is used to convert such imprecise linguistic terms into numerical values using triangular or trapezoidal fuzzy numbers. Figure 4 shows the vague, ambiguous boundary of fuzzy set $\tilde{A}$. The shaded boundary represents the boundary region of $\tilde{A}$ In the central (un-shaded) region of the fuzzy set, point 'a' is clearly a full member of the set and a membership value of 1 can be assigned to it. Point 'b" is outside the boundary region and is clearly not a member of the fuzzy set. Its value of membership function is 0. However, the membership of point 'c', which is on the boundary region, is ambiguous. So, the point 'c' must have some intermediate value of membership in the interval [0, 1]. Naturally, the membership of point 'c' approaches a value of 1 as it moves closer to the central (un-shaded) region, and its membership approaches a value of 0 as it moves closer to leaving the boundary region of fuzzy set (Ross, 2004). Figure 5 exhibits a trapezoidal fuzzy membership function. Some basic definitions about fuzzy sets and fuzzy numbers are given as below.

Definition 1. Fuzzy set theory states that, in a universe of discourse X, a fuzzy subset $\tilde{A}$ of X is defined by a membership function $f_{\tilde{A}}(x)$, which maps each element x in X to a real number R in unit interval of [0, 1]. The function value $f_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$. The larger the value of $f_{\tilde{A}}(x)$, the stronger is the grade of membership for x in $\tilde{A}$.

Definition 2. A fuzzy number is a trapezoidal fuzzy number (TrFN) if its membership function is expressed as follows:

$$\begin{aligned} f_{\tilde{A}}(x) &= \frac{x-\alpha}{\beta-\alpha}, \quad \alpha \leqslant x \leqslant \beta \\ &= 1, \quad \beta \leqslant x \leqslant \tau \\ &= \frac{x-\delta}{\tau-\delta}, \quad \tau \leqslant x \leqslant \delta \\ &= 0, \quad \text{otherwise} \end{aligned} \tag{9}$$

with $\alpha \leqslant \beta \leqslant \tau \leqslant \delta$. The TrFN, as given above, can be denoted as $(\alpha, \beta, \tau, \delta)$.

Definition 3. A fuzzy number is a triangular fuzzy number (TFN) if its membership function is as follows:

$$\begin{aligned} f_{\tilde{A}}(x) &= \frac{x-\alpha}{\beta-\alpha}, \quad \alpha \leqslant x \leqslant \beta \\ &= \frac{x-\delta}{\beta-\delta}, \quad \beta \leqslant x \leqslant \delta \end{aligned} \tag{10}$$

with $\alpha \leqslant \beta \leqslant \delta$. A TFN is a special case of a TrFN. Thus, a TFN can also be represented as a TrFN and is usually denoted as $(\alpha, \beta, \beta, \delta)$.

So, as discussed above, when a quantitative property is vaguely or imprecisely given, it is expressed by TrFN or TFN according to fuzzy set theory. For example, if it is stated that the horizontal distance traveled by an MHE is approximately equal to 300 units, it becomes difficult for the DM to ascertain the exactness of the data for further processing. Therefore, this imprecise value is represented by a TrFN as (270, 300, 300, 330), taking into consideration 10% fuzziness. Similarly, another MHE property with a value approximately between 360 and 400 can be denoted by a TrFN as (324, 360, 400, 440), again considering 10% fuzzification. A desired MHE cost smaller or equal to about USD 20K can be represented as (0, 0, 20, 22) as the minimum cost is zero. A qualitative property is a linguistic variable expressed in words or sentences. For example, the value of an MHE property, like 'position accuracy' is expressed by the linguistic variables as 'low' ($\widetilde{L}$), 'medium' ($\widetilde{M}$) or 'high' ($\widetilde{H}$).

4.2. *Fuzzy Axiomatic Design Principles*

In many decision-making situations, criteria values are more comfortably expressed in qualitative terms, such as low, medium, high or good, very good, excellent etc. Such type

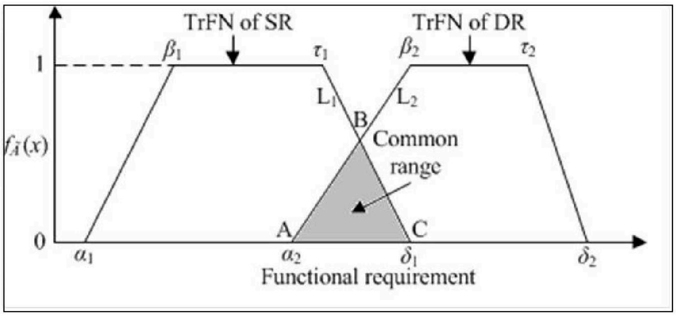

Fig. 6. Common range between system and design ranges.

of unquantifiable data can be well handled using fuzzy set theory. FAD methodology was developed by Kulak and Kahraman (2005) to use AD principles under fuzzy environment. This method is used as an efficient tool to solve MCDM problems involving imprecise, linguistic type of information. As discussed earlier, according to independence axiom of AD theory, the relationships between FRs and DPs are signed by 1 and 0, where 1 represents a relation, and 0 represents no relation between FRs and DPs. However, these numbers do not depict the degrees of relation between FRs and DPs. Under unpredicted environment, the relations between FRs and DPs are not known precisely. Therefore, under such type of circumstances, fuzzy set theory along with AD principles is used for rational decision-making in manufacturing environment. Let the TrFN of SR and DR be $(\alpha_1, \beta_1, \tau_1, \delta_1)$ and $(\alpha_2, \beta_2, \tau_2, \delta_2)$ respectively for a certain combination of alternative and criteria. The membership functions of these two TrFNs are shown in Fig. 6. In this figure, the area of triangle ABC is the CR value obtained from the intersection of two TrFNs representing SR and DR. Point B is the apex of the triangle, representing the maximum of both the membership functions in the CR. Referring to Eq. (9), the values of the corresponding membership functions, and for lines L_1 and L_2 can be derived as follows:

$$f_1(x) = \frac{x - \delta_1}{\tau_1 - \delta_1}, \tag{11}$$

$$f_2(x) = \frac{x - \alpha_2}{\beta_2 - \alpha_2}. \tag{12}$$

Solving these two membership functions yields the x-coordinate of vertex B as:

$$x = \frac{(\tau_1 - \delta_1) \times \alpha_2 - (\beta_2 - \alpha_2) \times \delta_1}{(\tau_1 - \delta_1) - (\beta_2 - \alpha_2)}. \tag{13}$$

The membership function at B is obtained as follows:

$$y = \frac{x - \delta_1}{\tau_1 - \delta_1}. \tag{14}$$

Now putting the value of x in Eq. (14), the y-coordinate of vertex B is computed as below:

$$y = \frac{\alpha_2 - \delta_1}{\tau_1 - \delta_1 - \beta_2 + \alpha_2}. \tag{15}$$

Thus, the value of CR becomes as follows:

$$CR = \frac{(\delta_1 - \alpha_2) \times y}{2} = \frac{(\delta_1 - \alpha_2)^2}{2 \times (\delta_1 - \tau_1 + \beta_2 - \alpha_2)}. \tag{16}$$

Similarly, for two TFNs expressed as $(\alpha_1, \beta_1, \delta_1)$ and $(\alpha_2, \beta_2, \delta_2)$, the value of CR becomes as follows:

$$CR = \frac{(\delta_1 - \alpha_2)^2}{2 \times (\delta_1 - \beta_1 + \beta_2 - \alpha_2)}. \tag{17}$$

Actually, a TFN expressed as $(\alpha_1, \beta_1, \beta_1, \delta_1)$ is the special case of a TrFN considered as $(\alpha_1, \beta_1, \tau_1, \delta_1)$. Therefore, Eq. (17) can simply be obtained by putting $\tau_1 = \beta_1$ in Eq. (16). The above-developed equation for estimating the CR value is applicable for the intersection of two TrFNs as presented in Fig. 6.

Finally, the IC value as estimated applying FAD principles is expressed as follows:

$$IC = \log_2\left(\frac{\text{TrFN or TFN of system area}}{\text{Area of common range between SR and DR}}\right). \tag{18}$$

5. Illustrative Examples

In order to prove the applicability and appropriateness of FAD principles in MHE selection process, two real time problems are analyzed and solved.

5.1. *Example 1*

Nowadays, AGVs are increasingly being used in all manufacturing sectors, like automobile, pharmaceutical, printing, painting, food processing, aerospace and so on. This is because of their superior built-in qualities, such as programming capability for path selection and positioning, quick response to frequently changing transport patterns, and ability to integrate with fully automated manufacturing systems. The considered AGV selection problem (Sawant and Mohite, 2013) consists of 16 different alternatives to be evaluated with respect to nine criteria. The expected design values of those criteria according to the industrial requirements are given in Table 1. The feasible set of AGV alternatives, with their specifications, is shown in Table 2.

From Table 1, it is observed that amongst the nine criteria, six criteria, e.g. length of AGV, width of AGV, height of AGV, maximum travel speed, position accuracy and lift

Table 1
Design values of criteria.

Sl. No.	Criteria	Value
1	Length of AGV (C_1)	High
2	Width of AGV (C_2)	High
3	Height of AGV (C_3)	High
4	Maximum load to be carried (C_4)	3600 kg
5	Maximum travel speed (C_5)	High
6	Battery should not get discharged before (C_6)	4 hr
7	Position accuracy (C_7)	High
8	Maximum lift height (C_8)	150 mm
9	Lift speed (C_9)	High

Table 2
Specifications of AGV alternatives.

Model	C_1	C_2	C_3	C_4 (in ton)	C_5	C_6 (in min)	C_7	C_8 (in mm)	C_9
HK40/O	2	0.9	1.5	3.63	91.4	345	6.4	610	45
F150	2.6	1.8	2.3	3.63	67.1	345	9.5	180	17.5
P330	3	1.5	2.5	3.63	61	560	9.5	180	16.5
P325	4.6	1.9	2.5	18.14	61	240	9.5	180	11
C530	1	1.6	0.5	18.14	45.7	300	12.7	180	12
DT-40	1.3	0.9	1.6	3.63	61	345	25.4	180	12
DT-60	1.7	2.5	1.4	3.63	61	345	25.4	180	12
RLV/N	2.8	2	1.7	6.1	119.8	300	6.4	300	30
AD100	4	3.6	3.3	11.34	54.9	300	12.7	180	12
AD130	3.7	3	2.7	11.34	54.9	300	12.7	180	12
T-20	2.9	1.7	1.8	4.54	41.2	350	6.4	180	12
T-40	4.5	2.4	2.5	9.07	30.5	350	6.4	180	12
T-60	5.1	2.8	2.6	13.61	24.4	350	6.4	180	12
T-100	5.8	3.2	3.3	22.68	18.3	350	6.4	180	12
UV-200	3.9	3.4	4.6	9.07	45.7	345	9.5	150	6
UV-600	5.6	4.9	5.8	27.22	15.2	345	9.5	150	2.5

Table 3
Fuzzy scale for criteria.

Sl. No.	Linguistic variable	TrFN
1	Low	$(0, 0.2, 0.4, 0.6)$
2	Medium	$(0.2, 0.4, 0.6, 0.8)$
3	High	$(0.4, 0.6, 0.8, 1.0)$

speed are expressed in linguistic terms, whereas, the remaining three criteria, i.e. maximum load capacity, battery capacity and maximum lift height are expressed quantitatively. A three level fuzzy scale for transforming the linguistic criteria values into corresponding TrFNs is given in Table 3.

The crisp values of other three criteria are also converted into corresponding TrFNs. The ranges of these TrFNs are chosen in such a way to cover all the specifications of the considered AGVs. Thus, it becomes possible to check the extent of suitability of all the

Table 4
Design range of criteria.

Criteria	Design specification	TrFN
Length (C_1)	High	$(0.4, 0.6, 0.8, 1.0)$
Width (C_2)	High	$(0.4, 0.6, 0.8, 1.0)$
Height (C_3)	High	$(0.4, 0.6, 0.8, 1.0)$
Load capacity (C_4)	Maximum load to be carried in tons from 3.6 to 27.5	$(3.6, 11.5, 19.5, 27.5)$
Speed (C_5)	High	$(0.4, 0.6, 0.8, 1.0)$
Battery capacity (C_6)	Battery capacity in minutes from 240 to 600	$(240, 360, 480, 600)$
Position accuracy (C_7)	High	$(0.4, 0.6, 0.8, 1.0)$
Lift height (C_8)	Maximum lift height in mm from 150 to 650	$(150, 300, 500, 650)$
Lift speed (C_9)	High	$(0.4, 0.6, 0.8, 1.0)$

alternatives with respect to the desired performance. Therefore, the DRs of both types of criteria are expressed in terms of TrFNs, as shown in Table 4.

To compute the IC values of all the alternatives for each FR, the SR values also need to be expressed in terms of TrFNs. For converting the crisp values of SR into TrFNs, i.e. fuzzification of the crisp values, the following procedure is adopted. The set of FRs is denoted as C_i $(i = 1, 2, \dots, 9)$ and the set of alternatives is expressed as A_j $(j = 1, 2, \dots, 16)$. For a particular FR, the set of values corresponding to all the alternatives is denoted by C_{ij} (a set of 16 values). The maximum of these values is C^*_{ij}. Therefore, the normalized value $\widehat{C}_{ij}$ is obtained as follows (Chen, 2008):

$$\widehat{C}_{ij} = \left(\frac{C_{ij}}{C^*_{ij}}\right). \tag{19}$$

Now, the TrFN denoted by $(\alpha, \beta, \tau, \delta)$ corresponding to a given crisp value of FR is obtained as $\alpha = 0$, $\delta = \mathrm{Min}[(2 \times \widehat{C}_{ij}), 1]$, $\beta = 0.33 \times \delta$ and $\tau = \mathrm{Min}[(0.67 \times 2 \times \widehat{C}_{ij}), 1]$.

The above procedure of fuzzification for a crisp value (equal to 2) corresponding to C_{11}, i.e. SR value of length of AGV model HK40/O, is illustrated as $\widehat{C}_{11} = (\frac{C_{11}}{C^*_{1j}}) = 2/5.8 = 0.34$, $\alpha = 0$, $\delta = \mathrm{Min}[(2 \times \widehat{C}_{11}), 1] = \mathrm{Min}[(2 \times 0.34), 1] = 0.68$, $\beta = 0.33 \times \delta = 0.33 \times 0.68 = 0.22$ and $\tau = \mathrm{Min}[(0.67 \times 2 \times \widehat{C}_{11}), 1] = \mathrm{Min}[(0.67 \times 2 \times 0.34), 1] = 0.46$. Thus, the TrFN of crisp value 2 is $(0, 0.22, 0.46, 0.68)$. This procedure is adopted for converting the crisp SR values of all AGV alternatives for FRs of length (C_1), width (C_2), height (C_3), speed (C_5), position accuracy (C_7) and lift speed (C_9). The DRs for these FRs are stated in linguistic terms and are already represented by TrFNs having range between 0 and 1. In this way, the compatibility between SR and DR values of the above-mentioned FRs (C_1, C_2, C_3, C_5, C_7, C_9) is analyzed.

The DR values of the remaining three FRs, i.e. load capacity (C_4), battery capacity (C_6) and lift speed (C_8) are given in crisp form, and they are already represented by TrFNs, covering the entire range of the corresponding set of SR values. In order to convert the given crisp SR values (C_{ij}) of these FRs into TrFNs, the following formulation is adopted.

$$\alpha = 0.8 \times C_{ij}, \qquad \beta = 0.9 \times C_{ij}, \qquad \tau = 1.1 \times C_{ij}, \qquad \delta = 1.2 \times C_{ij}. \tag{20}$$

Table 5
System range data in terms of trapezoidal fuzzy numbers.

Sl. No.	Model	C_1	C_2	C_3	C_4	C_5
A_1	HK40/O	(0, 0.22, 0.46, 0.68)	(0, 0.12, 0, 24, 0.36)	(0, 0.17, 0.35, 0.52)	(2.9, 3.27, 3.99, 4.36)	(0, 0.33, 1.0, 1.0)
A_2	F150	(0, 0.3, 0.6, 0.9)	(0, 0.24, 0.5, 0.74)	(0, 0.26, 0.54, 0.8)	(2.9, 3.27, 3.99, 4.36)	(0, 0.33, 0.75, 1.0)
A_3	P330	(0, 0.33, 0.7, 1.0)	(0, 0.2, 0.42, 0.62)	(0, 0.28, 0.58, 0.86)	(2.9, 3.27, 3.99, 4.36)	(0, 0.33, 0.68, 1.0)
A_4	P325	(0, 0.33, 1.0, 1.0)	(0, 0.26, 0.52, 0.78)	(0, 0.28, 0.58, 0.86)	(14.51, 16.33, 19.95, 21.77)	(0, 0.33, 0.68, 1.0)
A_5	C530	(0, 0.11, 0.23, 0.34)	(0, 0.22, 0.44, 0.66)	(0, 0.06, 0.12, 0.18)	(14.51, 16.33, 19.95, 21.77)	(0, 0.25, 0.51, 0.76)
A_6	DT-40	(0, 0.15, 0.29, 0.44)	(0, 0.12, 0.24, 0.36)	(0, 0.18, 0.38, 0.56)	(2.9, 3.27, 3.99, 4.36)	(0, 0.33, 0.68, 1.0)
A_7	DT-60	(0, 0.19, 0.39, 0.58)	(0, 0.33, 0.68, 1.0)	(0, 0.16, 0.32, 0.48)	(2.9, 3.27, 3.99, 4.36)	(0, 0.33, 0.68, 1.0)
A_8	RLV/N	(0, 0.32, 0.64, 0.96)	(0, 0.27, 0.55, 0.82)	(0, 0.19, 0.39, 0.58)	(4.88, 5.49, 6.71, 7.32)	(0, 0.33, 1.0, 1.0)
A_9	AD100	(0, 0.33, 0.92, 1.0)	(0, 0.33, 0.98, 1.0)	(0, 0.33, 0.76, 1.0)	(9.07, 10.21, 12.47, 13.61)	(0, 0.3, 0.62, 0.92)
A_{10}	AD130	(0, 0.33, 0.86, 1.0)	(0, 0.33, 0.82, 1.0)	(0, 0.31, 0.63, 0.94)	(9.07, 10.21, 12.47, 13.61)	(0, 0.3, 0.62, 0.92)
A_{11}	T-20	(0, 0.33, 0.67, 1.0)	(0, 0.23, 0.47, 0.7)	(0, 0.2, 0.42, 0.62)	(3.63, 4.09, 4.99, 5.45)	(0, 0.22, 0.46, 0.68)
A_{12}	T-40	(0, 0.33, 1.0, 1.0)	(0, 0.32, 0.66, 0.98)	(0, 0.28, 0.58, 0.86)	(7.26, 8.16, 9.98, 10.88)	(0, 0.17, 0.34, 0.5)
A_{13}	T-60	(0, 0.33, 1.0, 1.0)	(0, 0.33, 0.76, 1.0)	(0, 0.3, 0.6, 0.9)	(10.89, 12.25, 14.97, 16.33)	(0, 0.13, 0.27, 0.4)
A_{14}	T-100	(0, 0.33, 1.0, 1.0)	(0, 0.33, 0.87, 1.0)	(0, 0.33, 0.76, 1.0)	(18.14, 20.41, 24.95, 27.22)	(0, 0.1, 0.2, 0.3)
A_{15}	UV-200	(0, 0.33, 0.9, 1.0)	(0, 0.33, 0.92, 1.0)	(0, 0.33, 1.0, 1.0)	(7.26, 8.16, 9.98, 10.88)	(0, 0.25, 0.51, 0.76)
A_{16}	UV-600	(0, 0.33, 1.0, 1.0)	(0, 0.33, 1.0, 1.0)	(0, 0.33, 1.0, 1.0)	(21.78, 24.5, 29.94, 32.66)	(0, 0.09, 0.17, 0.26)

Table 6
System range data in terms of trapezoidal fuzzy numbers.

Sl. No.	Model	C_6	C_7	C_8	C_9
A_1	HK40/O	(276, 310.5, 379.5, 414)	(0, 0.33, 1.0, 1.0)	(488, 549, 671, 732)	(0, 0.33, 1.0, 1.0)
A_2	F150	(276, 310.5, 379.5, 414)	(0, 0.33, 0.9, 1.0)	(144, 162, 198, 216)	(0, 0.26, 0.52, 0.78)
A_3	P330	(448, 504, 616, 672)	(0, 0.33, 0.9, 1.0)	(144, 162, 198, 216)	(0, 0.24, 0.5, 0.74)
A_4	P325	(192, 216, 264, 288)	(0, 0.33, 0.9, 1.0)	(144, 162, 198, 216)	(0, 0.16, 0.32, 0.48)
A_5	C530	(240, 270, 330, 360)	(0, 0.33, 0.67, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_6	DT-40	(276, 310.5, 379.5, 414)	(0, 0.17, 0.34, 0.5)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_7	DT-60	(276, 310.5, 379.5, 414)	(0, 0.17, 0.34, 0.5)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_8	RLV/N	(240, 270, 330, 360)	(0, 0.33, 1.0, 1.0)	(240, 270, 330, 360)	(0, 0.33, 0.9, 1.0)
A_9	AD100	(240, 270, 330, 360)	(0, 0.33, 0.67, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_{10}	AD130	(240, 270, 330, 360)	(0, 0.33, 0.67, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_{11}	T-20	(280, 315, 385, 420)	(0, 0.33, 1.0, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_{12}	T-40	(280, 315, 385, 420)	(0, 0.33, 1.0, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_{13}	T-60	(280, 315, 385, 420)	(0, 0.33, 1.0, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_{14}	T-100	(280, 315, 385, 420)	(0, 0.33, 1.0, 1.0)	(144, 162, 198, 216)	(0, 0.18, 0.36, 0.54)
A_{15}	UV-200	(276, 310.5, 379.5, 414)	(0, 0.33, 0.9, 1.0)	(120, 135, 165, 180)	(0, 0.09, 0.17, 0.26)
A_{16}	UV-600	(276, 310.5, 379.5, 414)	(0, 0.33, 0.9, 1.0)	(120, 135, 165, 180)	(0, 0.04, 0.08, 0.12)

For example, the crisp value of $C_{41} = 3.63$ is converted into its corresponding TrFN as $\alpha = 0.8 \times 3.63 = 2.9$, $\beta = 0.9 \times 3.63 = 3.27$, $\tau = 1.1 \times 3.63 = 3.99$ and $\delta = 1.2 \times 3.63 = 4.36$. So, the TrFN of C_{41} is $(2.9, 3.27, 3.99, 4.36)$. In this way, all the crisp SR values of Table 2 are converted into corresponding TrFNs, as given in Tables 5 and 6.

For calculation of IC value, the FR of length of AGV (C_1) is considered here. Its DR is represented as $(0.4, 0.6, 0.8, 1.0)$. The TrFNs representing SR values of all AGV alternatives are reproduced in Table 7. The IC value of length for the second alternative A_2, i.e., ($IC_{C_{12}}$) is now calculated. The TrFNs representing SR and DR values for the combination of C_1 and A_2 are plotted in Fig. 7.

The system area (SA) is the area of trapezium ABCD representing the SR. Therefore, $\text{SA} = 0.5 \times [\text{Sum of the parallel sides}] \times \text{height} = 0.5 \times [(0.9 - 0) + (0.6 - 0.3)] \times 1.0 = 0.6$.

Now employing Eq. (16), the CR between the intersection of TrFNs for SR and DR for C_1–A_2 combination is calculated as 0.25.

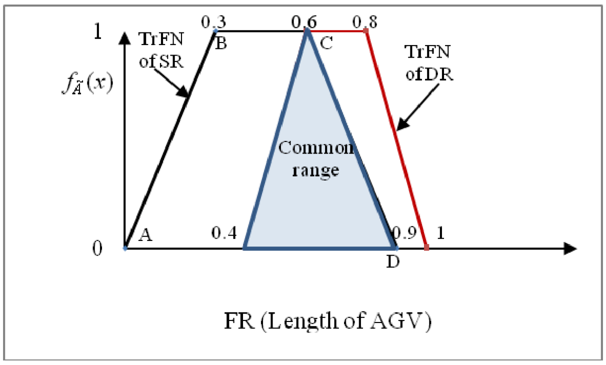

Fig. 7. System, design and common ranges for (C_1–A_2) combination.

Table 7
Information content for length of AGVs.

Sl. No.	Model	SR of length (C_1)	SA	CR	IC_{C_1}
A_1	HK40/O	(0, 0.22, 0.46, 0.68)	0.46	0.0933	2.3012
A_2	F150	(0, 0.3, 0.6, 0.9)	0.6	0.25	1.2630
A_3	P330	(0, 0.33, 0.7, 1.0)	0.685	0.35	0.9687
A_4	P325	(0, 0.33, 1.0, 1.0)	0.835	0.4	1.0618
A_5	C530	(0, 0.11, 0.23, 0.34)	0.23	0	Infinite
A_6	DT-40	(0, 0.15, 0.29, 0.44)	0.29	0.0023	6.9873
A_7	DT-60	(0, 0.19, 0.39, 0.58)	0.39	0.0415	3.2309
A_8	RLV/N	(0, 0.32, 0.64, 0.96)	0.64	0.3	1.0931
A_9	AD100	(0, 0.33, 0.92, 1.0)	0.795	0.4	0.9909
A_{10}	AD130	(0, 0.33, 0.86, 1.0)	0.765	0.4	0.9355
A_{11}	T-20	(0, 0.33, 0.67, 1.0)	0.67	0.335	1
A_{12}	T-40	(0, 0.33, 1.0, 1.0)	0.835	0.4	1.0618
A_{13}	T-60	(0, 0.33, 1.0, 1.0)	0.835	0.4	1.0618
A_{14}	T-100	(0, 0.33, 1.0, 1.0)	0.835	0.4	1.0618
A_{15}	UV-200	(0, 0.33, 0.9, 1.0)	0.785	0.4	0.9727
A_{16}	UV-600	(0, 0.33, 1.0, 1.0)	0.835	0.4	1.0618

The value of IC is then calculated as $\text{IC}_{C12} = \log_2$(TrFN of system area/Area of common range between SR and DR) $= \log_2(0.6/0.25) = 1.2630$.

Similarly, the IC values for length of AGVs corresponding to all 16 alternatives are calculated in Table 7. It can be seen from this table that there is no common range between TrFNs representing SR and DR for A_5. As a result, the value of IC becomes infinite, making AGV alternative A_5 (model C530) to be unsuitable.

Similarly, the IC values of all the nine FRs corresponding to each of the 16 AGV alternatives are determined and are shown in Table 8. The value of total IC (IC_{TOTAL}) for each alternative is then obtained by adding up the individual IC values for all the FRs. The AGV alternatives are subsequently arranged in ascending order of their IC_{TOTAL} values, as shown in Table 9. According to FAD principles, the alternative with the minimum IC_{TOTAL} value is adjudged as the most preferable choice.

Sawant and Mohite (2013) solved this AGV selection problem using three different methods, i.e. TOPSIS, block-TOPSIS and modified-TOPSIS, while using various sets of

Table 8
Total information content.

Sl. No.	Model	IC_{C_1}	IC_{C_2}	IC_{C_3}	IC_{C_4}	IC_{C_5}	IC_{C_6}	IC_{C_7}	IC_{C_8}	IC_{C_9}	IC_{TOTAL}
A_1	HK40/O	2.3012	Infinite	4.1688	4.9641	1.0618	0.2150	1.0618	1.5571	1.0618	Infinite
A_2	F150	1.2630	1.9284	1.6346	4.9641	0.9209	0.2150	0.9727	2.0584	1.7281	15.6852
A_3	P330	0.9688	2.7954	1.3958	4.9641	0.9893	1.3559	0.9727	2.0584	1.9284	17.4287
A_4	P325	1.0618	1.72814	1.3958	0.0042	0.9893	3.1699	0.9727	2.0584	5.1699	16.5502
A_5	C530	Infinite	2.4509	Infinite	0.0042	1.8244	0.9069	1	2.0584	3.8031	Infinite
A_6	DT-40	6.9873	Infinite	3.4958	4.9641	0.9893	0.2150	4.5921	2.0584	3.8031	Infinite
A_7	DT-60	3.2309	0.9893	5.1699	4.9641	0.9893	0.2150	4.5921	2.0584	3.8031	26.0124
A_8	RLV/N	1.0931	1.5513	3.2309	1.3864	1.0618	0.9069	1.0618	0.0614	0.9727	11.3263
A_9	AD100	0.9909	1.0444	0.9119	0.0518	1.1993	0.9069	1	2.0584	3.8031	11.9669
A_{10}	AD130	0.9355	0.8972	1.1444	0.0518	1.1993	0.9069	1	2.0584	3.8031	11.9967
A_{11}	T-20	1	2.1671	2.7954	2.7320	2.3012	3.4798	1.0618	2.0584	3.8031	21.3988
A_{12}	T-40	1.0618	1.0444	1.3958	0.4048	4.5921	3.4798	1.0618	2.0584	3.8031	18.9021
A_{13}	T-60	1.0618	0.9119	1.2630	0	Infinite	3.4798	1.0618	2.0584	3.8031	Infinite
A_{14}	T-100	1.0618	0.9448	0.9119	0.6750	Infinite	3.4798	1.0618	2.0584	3.8031	Infinite
$A15$	UV-200	0.9727	0.9909	1.0618	0.4048	1.8244	0.2150	0.9727	4.0444	Infinite	Infinite
$A16$	UV-600	1.0618	1.0618	1.0618	2.4188	Infinite	0.2150	0.9727	4.0444	Infinite	Infinite

Table 9
Ranking of AGV alternatives.

Sl. No.	Model	IC_{TOTAL}	Rank/Remark
A_8	RLV/N	11.3263	1
A_9	AD100	11.9669	2
A_{10}	AD130	11.9967	3
A_2	F150	15.6852	4
A_4	P325	16.5502	5
A_3	P330	17.4287	6
A_{12}	T-40	18.9021	7
A_{11}	T-20	21.3988	8
A_7	DT-60	26.0124	9
A_1	HK40/O	Infinite	Rejected
A_5	C530	Infinite	Rejected
A_6	DT-40	Infinite	Rejected
A_{13}	T-60	Infinite	Rejected
A_{14}	T-100	Infinite	Rejected
A_{15}	UV-200	Infinite	Rejected
A_{16}	UV-600	Infinite	Rejected

subjective and objective weights. While applying each method, the rankings of AGVs were first derived using subjective and objective weights, and then, their rank orderings were determined for various combinations of subjective and objective weights. The combinations of subjective and objective weights used were compromised weight (CW), integrated weight (IW), combined weight (CombineW) and proposed weight (PW). The rank orderings of the alternative AGVs obtained using FAD principles are compared with those derived by Sawant and Mohite (2013). From Table 9, it is observed that amongst 16 AGVs, seven alternatives, i.e. A_1, A_5, A_6, A_{13}, A_{14}, A_{15}, A_{16} are found unsuitable, because they do not satisfy the specified DRs for one or more criteria. These rejected alternatives were

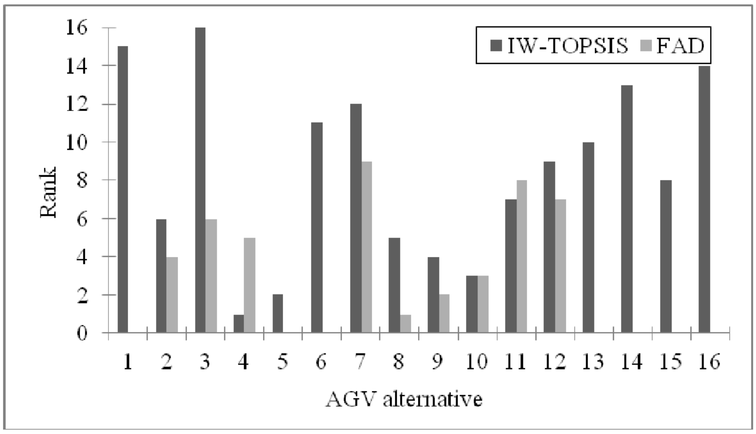

Fig. 8. Comparison of ranking results for AGV selection problem.

also adjudged to have the worst rankings by Sawant and Mohite (2013) for the IW-based approach. The rankings of the remaining nine alternatives are fairly in agreement with those obtained by Sawant and Mohite (2013), as shown in Fig. 8.

5.2. *Example 2*

In open pit iron mines, loading and hauling equipments used for handling raw materials significantly contribute towards the total mining cost. In order to minimize the cost per ton of iron ore mined and fulfil the production needs, it is essential to select a suitable combination of loading and hauling equipments. Selection of the optimal system becomes complicated at times due to availability of many competitive and viable options to be judged with respect to numerous conflicting evaluation criteria. The problem of selecting the most suitable loading and hauling equipment system for Gole Gohar iron mine (Lashgari *et al.*, 2012) is considered here. After the preliminary survey of the operational requirements, five MHEs, e.g. hydraulic shovel, cable shovel, dragline, wheel loader and backhoe loader were shortlisted for further scrutiny. According to expert knowledge from the field of surface mining, a total of 28 evaluation criteria consisting of operational parameters, technical specifications and cost factors were considered to arrive at the optimal decision. Those selection criteria, with their expected DRs, are expressed in terms of TFNs in Table 10.

The performance ratings of all MHEs for each selection criterion are shown in terms of TFNs in Table 11, which can be treated as SR values. Now, the IC values of all MHE alternatives are calculated for each selection criterion (functional requirement) and are shown in Table 12, 13, and 14. As an illustration, the IC value of dragline type of MHE for FR 'operator skill' is calculated with the help of Fig. 9.

In Fig. 9, area of triangle ABR $=$ SA $= 0.5 \times (0.776 - 0) \times 1.0 = 0.388$. Applying Eq. (17), the CR value is calculated as 0.0139. Now, the value of IC is calculated as $\mathrm{IC}_{C14-3} = \log_2(\text{TFN of system area/Area of common range}) = \log_2(0.388/0.0139) = 4.8028$.

Table 10
Design range of selection criteria

Criteria	TFN of DR
Daily production rate (C_1)	(0.52, 0.79, 0.91)
Assay and blending (C_2)	(0.001, 0.31, 0.57)
Breaking size of blasting C_3)	(0.002, 0.45, 0.72)
Rolling resistance (C_4)	(0.001, 0.4, 0.66)
Bench height (C_5)	(0.66, 0.91, 1.0)
Inflation factor (C_6)	(0.003, 0.45, 0.72)
Ground condition (C_7)	(0.57, 0.83, 1.0)
Weather condition (C_8)	(0.001, 0.45, 0.72)
Moisture (C_9)	(0.001, 0.45, 0.72)
Environment (C_{10})	(0.25, 0.5, 0.75)
Fill factor (C_{11})	(0.002, 0.52, 0.79)
Matching with truck (C_{12})	(0.31, 0.57, 0.83)
Flexibility (C_{13})	(0.5, 0.75, 1.0)
Operator skill (C_{14})	(0.66, 0.91, 1.0)
Maintenance (C_{15})	(0.4, 0.66, 0.91)
Utilization (C_{16})	(0.75, 1.0, 1.0)
Mobility (C_{17})	(0.57, 0.83, 1.0)
Availability (C_{18})	(0.45, 0.72, 0.91)
Continuous working (C_{19})	(0.4, 0.66, 0.91)
Working stability (C_{20})	(0.5, 0.75, 1.0)
Useful life (C_{21})	(0.003, 0.57, 0.79)
Working space (C_{22})	(0.001, 0.63, 0.79)
Time cycle (C_{23})	(0.001, 0.57, 0.79)
Automation (C_{24})	(0.36, 0.63, 0.83)
Operational parameters (C_{25})	(0.45, 0.72, 0.91)
Technical parameters (C_{26})	(0.36, 0.63, 0.83)
Operating cost (C_{27})	(0.52, 0.79, 0.91)
Capital cost (C_{28})	(0.45, 0.72, 0.91)

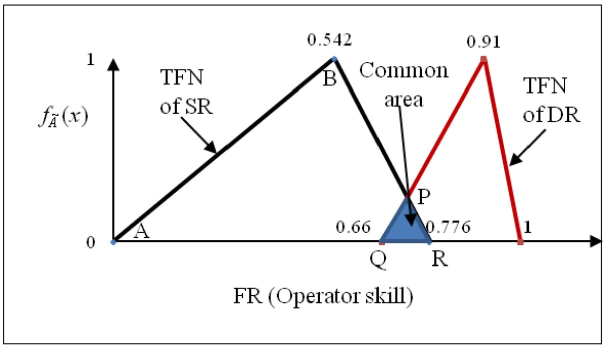

Fig. 9. SR, DR and common area of dragline for operator skill.

As shown in Table 14, the MHE alternative with the least total IC value is given the top rank and that with the maximum score is adjudged as the worst choice. For FR of ‘capital cost’ criterion, the TFNs of SR and DR values for alternative A_3 (dragline) have no common range between them. Therefore, the IC value of dragline becomes infinite

Table 11
System range data.

Criteria	Hydraulic shovel	Cable shovel	Dragline	Wheel loader	Backhoe loader
Daily production rate (C_1)	(0.638, 0.891, 1.0)	(0.588, 0.841, 1.0)	(0, 0.379, 0.638)	(0, 0.411, 0.675)	(0, 0.411, 0.660)
Assay and blending (C_2)	(0, 0.715, 0.871)	(0, 0.638, 0.871)	(0, 0.435, 0.675)	(0, 0.623, 0.822)	(0, 0.623, 0.822)
Breaking size of blasting (C_3)	(0.588, 0.841, 1.0)	(0.588, 0.841, 1.0)	(0, 0.623, 0.822)	(0, 0.588, 0.822)	(0, 0.623, 0.822)
Rolling resistance (C_4)	(0.483, 0.758, 0.891)	(0.555, 0.822, 0.944)	(0, 0.588, 0.822)	(0, 0.555, 0.822)	(0, 0.588, 0.822)
Bench height (C_5)	(0.512, 0.776, 0.944)	(0.692, 0.944, 1.0)	(0, 0.588, 0.822)	(0, 0.411, 0.7)	(0, 0.472, 0.715)
Inflation factor (C_6)	(0.588, 0.841, 1.0)	(0.512, 0.776, 0.944)	(0, 0.715, 0.871)	(0, 0.512, 0.758)	(0.542, 0.794, 1.0)
Ground condition (C_7)	(0.588, 0.841, 1.0)	(0.542, 0.794, 1.0)	(0.446, 0.715, 0.891)	(0.588, 0.841, 1.0)	(0.638, 0.891, 1.0)
Weather condition (C_8)	(0.472, 0.733, 0.944)	(0.512, 0.776, 0.944)	(0.358, 0.623, 0.841)	(0.388, 0.660, 0.841)	(0.446, 0.715, 0.891)
Moisture (C_9)	(0.588, 0.841, 1.0)	(0.588, 0.841, 1.0)	(0.358, 0.623, 0.841)	(0.411, 0.675, 0.891)	(0.358, 0.623, 0.841)
Environment (C_{10})	(0, 0.472, 0.715)	(0, 0.446, 0.715)	(0, 0, 0.555)	(0, 0, 0.588)	(0, 0.435, 0.675)
Fill factor (C_{11})	(0.638, 0.891, 1.0)	(0.638, 0.891, 1.0)	(0.411, 0.675, 0.891)	(0.472, 0.733, 0.944)	(0.411, 0.675, 0.891)
Matching with truck (C_{12})	(0.5, 0.75, 1.0)	(0.50, 0.75, 1.0)	(0.250, 0.500, 0.750)	(0.75, 1.0, 1.0)	(0.75, 1.0, 1.0)
Flexibility (C_{13})	(0, 0.675, 0.871)	(0.435, 0.692, 0.944)	(0, 0.500, 0.733)	(0.411, 0.675, 0.891)	(0, 0.574, 0.776)
Operator skill (C_{14})	(0.542, 0.794, 1.0)	(0.472, 0.733, 0.944)	(0, 0.542, 0.776)	(0.472, 0.733, 0.944)	(0.588, 0.841, 1.0)
Maintenance (C_{15})	(0.472, 0.733, 0.944)	(0.512, 0.776, 0.944)	(0.379, 0.638, 0.891)	(0.472, 0.733, 0.944)	(0.50, 0.75, 1.0)
Utilization (C_{16})	(0.5, 0.75, 1.0)	(0.692, 0.944, 1.0)	(0.388, 0.660, 0.841)	(0.411, 0.675, 0.891)	(0.411, 0.675, 0.891)
Mobility (C_{17})	(0, 0.638, 0.871)	(0, 0.588, 0.822)	(0, 0.512, 0.776)	(0.542, 0.794, 1.0)	(0, 0.638, 0.871)
Availability (C_{18})	(0.435, 0.692, 0.944)	(0.411, 0.675, 0.891)	(0, 0.542, 0.776)	(0.411, 0.675, 0.891)	(0, 0.588, 0.822)
Continuous working (C_{19})	(0.512, 0.776, 0.944)	(0.435, 0.692, 0.944)	(0.512, 0.776, 0.944)	(0, 0.512, 0.776)	(0, 0.542, 0.776)
Working stability (C_{20})	(0.411, 0.675, 0.891)	(0.542, 0.794, 1.0)	(0.411, 0.675, 0.891)	(0.330, 0.588, 0.841)	(0, 0.542, 0.776)
Useful life (C_{21})	(0.435, 0.692, 0.944)	(0.588, 0.841, 1.0)	(0.555, 0.822, 0.944)	(0.330, 0.588, 0.841)	(0, 0.472, 0.733)
Working space (C_{22})	(0, 0.555, 0.822)	(0.411, 0.675, 0.891)	(0, 0.623, 0.822)	(0.512, 0.776, 0.944)	(0.555, 0.822, 0.944)
Time cycle (C_{23})	(0.435, 0.692, 0.944)	(0.555, 0.822, 0.944)	(0.358, 0.623, 0.841)	(0, 0.512, 0.776)	(0, 0.472, 0.715)
Automation (C_{24})	(0, 0.483, 0.758)	(0, 0.574, 0.758)	(0, 0, 0.675)	(0, 0.446, 0.715)	(0, 0.379, 0.623)
Operational parameters (C_{25})	(0.542, 0.794, 1.0)	(0.542, 0.794, 1.0)	(0, 0, 0.638)	(0, 0.555, 0.822)	(0.411, 0.675, 0.891)
Technical parameters (C_{26})	(0.588, 0.841, 1.0)	(0.638, 0.891, 1.0)	(0, 0.512, 0.776)	(0.435, 0.692, 0.944)	(0.358, 0.623, 0.841)
Operating cost (C_{27})	(0.411, 0.675, 0.891)	(0.602, 0.871, 0.944)	(0, 0.435, 0.675)	(0.446, 0.715, 0.891)	(0.330, 0.588, 0.841)
Capital cost (C_{28})	(0, 0.588, 0.822)	(0, 0.512, 0.776)	(0, 0, 0.411)	(0, 0.660, 0.822)	(0.483, 0.758, 0.891)

Table 12
Total IC values for loading and hauling equipments.

Alternative	IC_{C1}	IC_{C_2}	IC_{C_3}	IC_{C_4}	IC_{C_5}	IC_{C_6}	IC_{C_7}	IC_{C_8}	IC_{C_9}	$IC_{C_{10}}$	$IC_{C_{11}}$
Hydraulic shovel (A_1)	0.8679	1.3861	3.6284	2.8006	1.1627	3.6284	0.0380	2.0268	3.6284	0.7051	3.0346
Cable shovel (A_2)	0.5677	1.2675	3.6284	4.2168	0.1511	2.4147	0.2070	2.4147	3.6284	0.7792	3.0346
Dragline (A_3)	4.5992	0.5300	0.5018	0.6783	3.9221	0.7268	0.9130	0.9796	0.9796	2.2639	0.8355
Wheel loader (A_4)	3.9072	1.1596	0.4441	0.6210	7.8815	0.1934	0.0380	1.1554	1.4247	2.1087	1.3094
Backhoe loader (A_5)	4.1273	1.1596	0.5018	0.6783	6.8645	2.9156	0.2247	1.6757	0.9796	0.8727	0.8355

Table 13
Total IC values for loading and hauling equipments.

Alternative	$IC_{C_{12}}$	$IC_{C_{13}}$	$IC_{C_{14}}$	$IC_{C_{15}}$	$IC_{C_{16}}$	$IC_{C_{17}}$	$IC_{C_{18}}$	$IC_{C_{19}}$	$IC_{C_{20}}$	$IC_{C_{21}}$	$IC_{C_{22}}$
Hydraulic shovel (A_1)	1.2275	1.4969	0.8533	0.3302	2	2.2447	0.1814	0.4873	0.5490	0.9460	0.1463
Cable shovel (A_2)	1.2275	0.3742	1.4318	0.4873	0.5926	2.6768	0.2624	0.1938	0.1323	2.2558	0.5027
Dragline (A_3)	0.3974	2.7052	4.8028	0.1237	4.5593	3.2603	1.8797	0.4873	0.5490	1.7784	0.0613
Wheel loader (A_4)	4.3163	0.5490	1.4318	0.3302	3.4920	0.2070	0.2624	1.5241	1.1443	0.2071	1.2449
Backhoe loader (A_5)	4.3163	2.2030	0.5437	0.5726	3.4920	2.2447	1.5819	1.4391	2.3017	0.1876	1.5887

and it occupies the last position (rejected). The ranking order of the loading and hauling equipments derived using FAD principles closely agrees with that derived by Lashgari *et al.* (2012) using fuzzy TOPSIS method. In FAD method, hydraulic shovel (A_1) with the total IC value of 40.2941 is the best choice, followed by cable shovel (A_2) with a total IC value of 40.5293. As there is a negligible difference between the total IC values for these two MHE alternatives, they may be treated as nearly equal to each other on their performance. A comparison of rank orderings of MHE alternatives between those derived employing FAD principles and by Lashgari *et al.* (2012) is provided in Table 15.

Table 14
Total IC values for loading and hauling equipments.

Alternative	$IC_{C_{23}}$	$IC_{C_{24}}$	$IC_{C_{25}}$	$IC_{C_{26}}$	$IC_{C_{27}}$	$IC_{C_{28}}$	IC_{TOTAL}	Rank
Hydraulic shovel (A_1)	0.9460	1.3829	0.5800	1.6721	0.7611	1.5819	40.2941	1
Cable shovel (A_2)	1.7784	1.1193	0.5800	2.1533	0.4879	1.9631	40.5293	2
Dragline (A_3)	0.3279	2.6845	4.0348	1.2597	3.8408	Infinite	Infinite	5
Wheel loader (A_4)	0.1060	1.6126	1.6734	0.5761	0.5280	1.3596	40.8081	3
Backhoe loader (A_5)	0.1876	2.2109	0.2624	0.0474	1.3750	0.0808	45.4710	4

Table 15
Comparison of results for example 2.

Alternative	F-TOPSIS		FAD	
	Score	Rank	Score	Rank
Hydraulic shovel (A_1)	0.3800	2	40.2941	1
Cable shovel (A_2)	0.3838	1	40.5293	2
Dragline (A_3)	0.3121	5	Infinite	5
Wheel loader (A_4)	0.3476	3	40.8081	3
Backhoe loader (A_5)	0.3431	4	45.4710	4

6. Conclusions

An MCDM method employing FAD principles is applied here for solving two MHE selection problems. From the results of both these MHE selection problems, it can be concluded that the adopted methodology is quite efficient in dealing with imprecise, vague information coupled with crisp numerical data. The main advantage of this method is that it involves comparatively less number of computational steps as against the other MCDM methods. It has also the capability of dealing with any number of design criteria and MHE alternatives as encountered in many real time decision-making situations. The use of fuzzy membership functions provides flexibility to this method as fuzzy logic permits transformation of qualitative information into processable numerical data. Similarly, it can also be used for other management and strategic decision-making problems, such as supplier evaluation, personnel selection, project selection etc. to provide more acceptable and accurate results. As a future scope, a software prototype may be developed for automatically plotting the TFNs or TrFNs and computing the required common range to increase the computational speed and solution accuracy of this adopted methodology.

References

Bazzazi, A.A., Osanloo, M., Karimi, B. (2009). Optimal open pit mining equipment selection using fuzzy multiple attribute decision making approach. *Archive of Mining Science*, 54(2), 301–320.

Bazzazi, A.A., Osanloo, M., Karimi, B. (2011). A new fuzzy multi-criteria decision making model for open pit mines equipment selection. *Asia-Pacific Journal of Operational Research*, 28(3), 279–300.

Celik, M., Kahraman,C., Cebi, S., Er, I.D. (2009). Fuzzy axiomatic design-based performance evaluation model for docking facilities in shipbuilding industry: the case of Turkish shipyards. *Expert Systems with Applications*, 36(1), 599–615.

Chakraborty, S., Banik, D. (2006). Design of a material handling equipment selection model using analytic hierarchy process. *International Journal of Advanced Manufacturing Technology*, 28(11–12), 1237–1245.

Chen, C.-T. (2000). A fuzzy MCDM method based on interval analysis for solving plant location selection problem. *Journal of the Chinese Institute of Industrial Engineers*, 17(1), 111–120.

Chittratanawat, S., Noble, J.S. (1999). An integrated approach for facility layout, P/D location and material handling system design. *International Journal of Production Research*, 37(3), 683–706.

Deb, S.K., Bhattacharyya, B., Sorhkel, S.K. (2002). Material handling equipment selection by fuzzy multi-criteria decision-making methods. In: *Proceedings of AFSS 2002, International Conference on Fuzzy Systems*, India, pp. 99–105.

Karande, P., Chakraborty, S. (2013). Material handling equipment selection using weighted utility additive theory. *Journal of Industrial Engineering* Article ID 268708, 9 pp. DOI:10.1155/2013/268708.

Kulak, O. (2005). A decision support system for fuzzy multi-attribute selection of material handling equipments. *Expert Systems with Applications*, 29(2), 310–319.

Kulak, O, Kahraman, C. (2005). Multi-attribute comparison of advanced manufacturing systems using fuzzy vs. crisp axiomatic design approach. *International Journal of Production Economics*, 95(3), 415–424.

Lashkari, R.S., Boparai, R., Paulo, J. (2004). Towards an integrated model of operation allocation and material handling selection in cellular manufacturing systems. *International Journal of Production Economics*, 87(2), 115–139.

Lashgari, A., Abdolreza, Y.-C., Fouladgar, M.M., Zavadskas, E.K., Shafiee, S., Abbate, N. (2012). Equipment selection using fuzzy multi-criteria decision-making model: key study of Gole Gohar iron mine. *Inzinerine Ekonomika – Engineering Economics*, 23(2), 125–136.

Mirhosseyni, S.H.L., Webb, P. (2009). A hybrid fuzzy knowledge-based expert system and genetic algorithm for efficient selection and assignment of material handling equipment. *Expert Systems with Applications*, 36(9), 11875–11887.

Momani, A. M. and Ahmed, A. A. (2011). Material handling equipment selection using hybrid Monte Carlo simulation and analytic hierarchy process. *World Academy of Science, Engineering and Technology*, 5(11), 953–958.

Mousavi, S.M., Vahdani, B., Tavakkoli-Moghaddam, R., Tajik, N. (2013). Soft computing based on a fuzzy grey group compromise solution approach with an application to the selection problem of material handling equipment. *International Journal of Computer Integrated Manufacturing*, 26(1–2), 1–23.

Onut, S., Kara, S.S., Mert, S. (2009). Selecting the suitable material handling equipment in the presence of vagueness. *International Journal of Advanced Manufacturing Technology*, 44(7–8), 818–828.

Park, Y., (1996). ICMESE: intelligent consultant system for material handling equipment selection and evaluation. *Journal of Manufacturing Systems*, 15(5), 325–333.

Ross, T.J. (2004). *Fuzzy Logic with Engineering Applications*. Wiley, New Delhi.

Sawant, V.B., Mohite, S.S. (2009). Investigations on benefits generated by using fuzzy numbers in a TOPSIS model developed for automated guided vehicle selection problem. In: Sakai, H., et al. (Eds.), *Rough Sets, Fuzzy Sets, Data Mining and Granular Computing*. Springer, Berlin/Heidelberg, pp. 295-302.

Sawant, V.B., Mohite, S.S. (2013). A composite weight based multiple attribute decision support system for the selection of automated guided vehicles. *International Journal of Computer Applications*, 70(21), 8–16.

Suh, N.P. (1990). *The Principles of Design*. Oxford University Press, New York.

Suh, N.P. (2001). *Axiomatic Design: Advances and Applications*. Oxford University Press, New York.

Sujono, S., Lashkari, R.S. (2007). A multi-objective model of operation allocation and material handling system selection in FMS design. *International Journal of Production Economics*, 105(1), 116–133.

Tuzkaya, G., Gulsun, B., Kahraman, C., Ozgen, D. (2010). An integrated fuzzy multi-criteria decision-making methodology for material handling equipment selection problem and an application. *Expert Systems with Applications*, 37(4), 2853–2863.

Ulubeyli, S., Kazaz, A. (2009). A multiple criteria decision-making approach to the selection of concrete pumps. *Journal of Civil Engineering and Management*, 15(4), 369–376.

Yaman, R., (2001). A knowledge-based approach for selection of material handling equipment and material handling system pre-design. *Turkish Journal of Engineering and Environmental Sciences*, 25(4), 267–278.

Yazdani-Chamzini, A., Yakhchali, S.H. (2012). Handling equipment selection in open pit mines by using an integrated model based on group decision making. *International Journal of Industrial Engineering Computations*, 3(5), 907–924.

Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8(3), 338–353.

S. Chakraborty is an Associate Professor in Production Engineering Department of Jadavpur University. He graduated in 1986 from University of Calcutta and obtained his post-graduate degree from Jadavpur University in 1989. He was awarded with his PhD (Engineering) degree from Jadavpur University in 1994. His research interests are in the areas of applications of different multi-criteria decision-making methods in manufacturing environment, control chart pattern recognition, and development of MIS and ERP systems for diverse engineering applications. He has guided several ME and PhD (Engineering) theses, and published numerous papers in international journals. He is also a regular reviewer of several journals of international repute.

A.V. Khandekar is working as a Senior lecturer at Government Polytechnic, Mumbai (India). He graduated in Mechanical Engineering from Government College of Engineering, Amravati, Maharashtra (India) in 1989. He obtained his Masters in Manufacturing Technology from Thapar Institute of Engineering and Technology, Patiala (India) in 2002. He has published few papers in journals of international repute as well as presented his work in some conferences. At present, he is pursuing his PhD (Engg) research work at Jadavpur University, Kolkata (India) and his research interests include the applications of fuzzy multi-criteria decision-making techniques in manufacturing environment.

Medžiagų tvarkymo įrangos pasirinkimas taikant neraiškiojo aksiomatinio planavimo principus

Anant V. KHANDEKAR, Shankar CHAKRABORTY

Veiksmingas medžiagų judėjimas yra svarbus kiekvienos organizacijos veikloje. Tinkamų metodų, skirtų medžiagų judėjimui, taikymas leidžia užtikrinti bendrąjį gamybos procesuose dalyvaujančio personalo saugumą. Medžiagų tvarkymo įrangos (MTĮ) pasirinkimas yra svarbi priemonė organizacijos produktyvumo didinimui. Šiuolaikinėje technologijų eroje įvairios MTĮ gali būti taikomos, atliekant reikalingas užduotis. Atsižvelgiant, į nagrinėjamos medžiagos tipą, egzistuoja daugybė kiekybinių ir kokybinių veiksnių, lemiančių tinkamiausios MTĮ pasirinkimą. Tinkamiausios MTĮ pasirinkimas gali būti atliekamas, naudojant daugiakriterinio vertinimo (MCDM) metodus, leidžiančius atsižvelgti tiek į įprastus, tiek į neraiškiuosius duomenis. Šiame straipsnyje taikomas MCDM metodas su neraiškiojo aksiomatinio planavimo principu. Bendrasis informacijos turinys naudojamas, kaip MTĮ tinkamumo matas. Nagrinėjamos dvi realaus laiko problemos: automatizuotos valdomos transporto priemonės pasirinkimas, ir pakrovimo bei iškrovimo įrangos paviršinėse kasyklose pasirinkimas.