INFORMATICA, 2015, Vol. 26, No. 2, 335–355

DOI: http://dx.doi.org/10.15388/Informatica.2015.51

Extension of the ARAS Method for Decision-Making Problems with Interval-Valued Triangular Fuzzy Numbers

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Received: April 2013; accepted: October 2013

Abstract. This paper proposes an extension of the ARAS method which, due to the use of interval-valued fuzzy numbers, can be more appropriate for solving real-world problems. In order to overcome the complexity of real-world decision-making problems, the proposed extension also includes the use of linguistic variables and a group decision making approach. In order to highlight the proposed methodology an example of a faculty websites evaluation is considered.

Key words: MCDM, Multiple criteria decision making, ARAS, interval-valued triangular fuzzy numbers, linguistic variables, uncertainty.

1. Introduction

Decision making is often associated with the selection of an alternative from a set of alternatives. During the evaluation of alternatives, not so rare, it is also necessary to take into account the impact of multiple, often conflicting, criteria. In due course, for that purpose, numerous Multiple Criteria Decision Making (MCDM) methods have been formed.

Out of many, here are mentioned only a few, such as: SAW (MacCrimmon, 1968), AHP (Saaty, 1980), TOPSIS (Hwang and Yoon, 1981), PROMETHEE (Brans and Vincke, 1985), ELECTRE (Roy, 1991), COPRAS (Zavadskas *et al.*, 1994), VIKOR (Opricovic, 1998), MULTIMOORA (Brauers and Zavadskas, 2010, 2012), WASPAS (Zavadskas *et al.*, 2012; Chakraborty and Zavadskas, 2014), and so on.

These methods have been based mainly on the use of crisp numbers, and they are called ordinary MCDM methods. However, in the real world, many decision-making problems take place in the environment that is characterized by the absence of precise and reliable information, or they are associated with some kinds of predictions, uncertainties and ambiguities. Therefore, ordinary MCDM methods have not provided the adequate ability to solve such kinds of problems.

A significant progress in solving real-world decision-making problems appeared after Zadeh (1965) introduced the Fuzzy sets theory. As part of the Fuzzy set theory there are also introduced fuzzy numbers, usually based on triangular or trapezoidal shapes, which are much more adequate for modeling and solving a number of complex decision-making problems.

Based on Fuzzy sets theory, Bellman and Zadeh (1970) introduced the fuzzy multiple criteria decision making methodology, which was subsequently widely accepted and used for solving many decision-making problems.

Another important approach was introduced by Deng (1982). Deng (1982, 1989) proposed the Grey system theory which, similar to the Fuzzy set theory, also provide an adequate approach for modeling and solving a number of complex decision making problems.

Based on the Fuzzy set theory and the Grey system theory, many ordinary MCDM methods have been extended to enable the use of fuzzy numbers or interval grey numbers. From a really large number of proposed extensions here are mentioned just some prominent, such as: Grey TOPSIS (Sadeghi *et al.*, 2013; Zavadskas *et al.*, 2010b; Chen and Tsao, 2008), COPRAS-G (Zavadskas *et al.*, 2008), SAW-G (Zavadskas *et al.*, 2010b; Medineckiene *et al.*, 2010) VIKOR-F (Opricovic, 2007), MULTIMOORA-FG (Balezentis *et al.*, 2012a, 2012b) and many extensions of Fuzzy TOPSIS method (Wang and Elhag, 2006; Yang and Hung, 2007; Saremi *et al.*, 2009).

Although fuzzy and grey numbers have allowed solving a larger number of real-world decision-making problems, they can not fully meet all the requirements that can occur when such kind of problems are being solved.

The interval-valued fuzzy numbers, as a special form of fuzzy numbers, provide significantly more opportunities for solving the real-world decision-making problems. Therefore, some of the prominent MCDM methods, in addition to the fuzzy and grey extensions, also have got their interval-valued fuzzy extensions.

In addition to the fuzzy and grey extensions, solving of some real-world problems sometimes require the use of a group decision making approach, and often the use of linguistic variables, too.

Extensions, that allow the use of interval-valued fuzzy numbers, are proposed for many prominent MCDM methods, such as TOPSIS (Vahdani *et al.*, 2013; Park *et al.*, 2011; Ye, 2010; Ashtiani *et al.*, 2009; Chen, 2000), VIKOR (Samantra *et al.*, 2013; Datta *et al.*, 2012; Liu and Wang, 2011; Vahdani *et al.*, 2010), MULTIMOORA (Balezentis and Zeng, 2012), and so on.

These extensions typically support the group decision making approach and the use of linguistic variables.

The Additive Ratio ASsessment (ARAS) method was proposed by Zavadskas and Turskis (2010), and it can be specified as a newly proposed MCDM method. Even so, the ARAS method has been applied to solve various decision making problems. So, for example, Zavadskas and Turskis (2010) used the ARAS method to evaluate the microclimate in office rooms, Zavadskas *et al.* (2010a) used the ARAS method to select the most appropriate foundation installment alternative for a building which stands on the aquiferous soil, Tupenaite *et al.* (2010), Bakshi and Sinharay (2011), Bakshi and Sarkar (2011) used the ARAS method for a project selection.

In order to enable the use of fuzzy and grey numbers, Turskis and Zavadskas proposed a fuzzy extension (Turskis and Zavadskas, 2010a), named ARAS-F, and a grey extension (Turskis and Zavadskas, 2010b), named ARAS-G, of the ARAS method.

Although the mentioned extensions significantly influenced the increase in a number of decision-making problems that can be successfully solved using ARAS methods, the use of interval-valued fuzzy numbers can further increase the use of the ARAS method.

Therefore, in this paper the extension of the ARAS method is proposed which allows the use of interval-valued triangular fuzzy numbers. In order to highlight the applicability of the proposed extensions, in this paper it is also presented an example of a faculty website evaluation.

Due to all above mentioned reasons, the rest of this manuscript is organized as follows: In Section 2, the basic elements of the fuzzy set theory, fuzzy numbers and interval-fuzzy numbers are considered. In Section 3, after a detailed consideration of the computational procedure of the ordinary ARAS method, an extension of the ARAS method, which allows the use of interval-valued fuzzy numbers, is proposed. To highlight the proposed methodology, in Section 4, an example of a faculty website evaluation is considered. The conclusions are given in the final section.

2. Preliminaries

In this section are discussed some significant parts of the fuzzy set theory, ordered weighted averaging operators and group decision making approach, which have been necessary to propose an extension of the ARAS method suitable for dealing with interval-valued triangular fuzzy numbers.

2.1. *The Fuzzy Set Theory and Fuzzy Numbers*

The classical MCDM methods are based on the use of the classical set theory, where an element can belong or does not belong to the set. Let A be a classical set of objects, called the universe, whose generic elements are denoted by x. The belonging to the set A can be represented by a membership function μ_A, which has the following form:

$$\mu_A(x) = \begin{cases} 1, & x \in A, \\ 0, & x \notin A. \end{cases} \tag{1}$$

Unfortunately, many real-world decision making problems are often very complex and related to the impact of uncertainty, which can not be easily expressed using the classical sets.

As it has been previously mentioned, Zadeh (1965) introduced the Fuzzy sets theory, which allows a partial membership in a set. As a result, instead of the exclusive use of crisp numbers, the fuzzy set theory allows the use of other forms of numbers, such as triangular, trapezoidal, and bell-shaped numbers. In addition, an approach for the formalization of natural language specification, called computation with words, was established as an extension of the fuzzy set theory.

2.1.1. *Generalized Fuzzy Numbers*

A generalized fuzzy number $\tilde{A} = (a, b, c, d; \omega)$, $0 \leqslant a \leqslant b \leqslant c \leqslant d \leqslant 1$ and $0 \leqslant \omega \leqslant 1$, is a fuzzy subset of the real line $\Re$ with the membership function $\mu_{\tilde{A}}$ which has the following

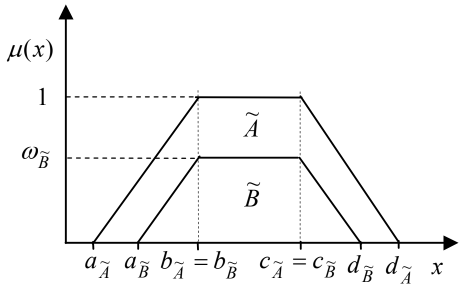

Fig. 1. The generalized and normalized fuzzy numbers.

properties (Chen and Chen, 2003):

(i) $\mu_{\tilde{A}}$ is a continuous mapping from $\Re$ to the closed interval $[0, \omega]$.
(ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a]$.
(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$.
(iv) $\mu_{\tilde{A}}(x) = \omega$ for all $x \in [b, c]$, where ω is a constant and $0 \leqslant w \leqslant 1$.
(v) $\mu_{\tilde{A}}(x)$ is strictly decreasing on $[c, d]$.
(vi) $\mu_{\tilde{A}}(x) = 0$ for all $x \in [d, +\infty)$.

If $\mu_{\tilde{A}}$ is linear in $[a, b]$ and $[c, d]$, then a generalized fuzzy number is called a generalized trapezoidal fuzzy number.

Figure 1 shows a relationship between the generalized $\tilde{B}$ and the normalized $\tilde{A}$ trapezoidal fuzzy number. From Fig. 1, it can be also concluded that the normalized (traditional) trapezoidal fuzzy numbers are particular cases of generalized fuzzy numbers, where $\omega = 1$. Also, when $b = c$ the trapezoidal fuzzy number becomes a triangular fuzzy number.

2.1.2. *Interval-Valued Fuzzy Numbers*

The interval-valued fuzzy numbers are a special form of generalized fuzzy numbers. Similarly to generalized fuzzy numbers, the interval-valued fuzzy numbers can have a trapezoidal shape, interval-valued trapezoidal fuzzy numbers, and triangular shape, interval-valued triangular fuzzy numbers.

A graphical representation of an interval-valued triangular fuzzy number is shown in Fig. 2.

According to Yao and Lin (2002) an interval-valued triangular fuzzy number can be represented as follows:

$$\tilde{A} = \left[\tilde{A}^L, \tilde{A}^U\right] = \left[\left(a'_l, a'_m, a'_u; \omega'_A\right), (a_l, a_m, a_u; \omega_A)\right], \tag{2}$$

where $\tilde{A}^L$ and $\tilde{A}^U$ denote the lower and upper triangular fuzzy numbers, $\tilde{A}^L \subset \tilde{A}^U$; $\mu_{\tilde{A}}(x)$ is the membership function, and it denotes the degree in which an event may be a member of $\tilde{A}$; $\mu_{\tilde{A}^L}(x) = \omega'_A$ and $\mu_{\tilde{A}^U}(x) = \omega_A$ are the lower and upper membership functions, respectively.

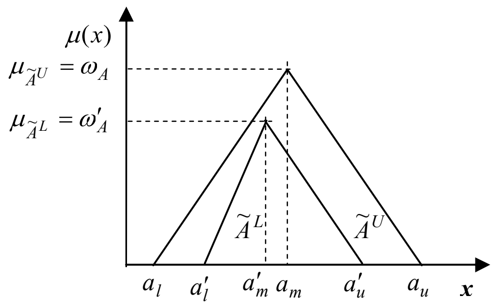

Fig. 2. The interval-valued triangular fuzzy number.

Suppose that $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a'_l, a'_m, a'_u; \omega'_A), (a_l, a_m, a_u; \omega_A)]$ and $\tilde{B} = [\tilde{B}^L, \tilde{B}^U] = [(b'_l, b'_m, b'_u; \omega'_B), (b_l, b_m, b_u; \omega_B)]$ are two interval-valued triangular fuzzy numbers. Then, the basic arithmetic operations on these fuzzy numbers are defined as follows:

(i) Interval-Valued Fuzzy Numbers Addition:

$$\begin{aligned} \tilde{A} + \tilde{B} = {} & \big[\big(a'_l + b'_l, a'_m + b'_m, a'_u + b'_u; \min\big(\omega'_A, \omega'_B\big)\big), \\ & \big(a_l + b_l, a_m + b_m, a_u + b_u; \min(\omega_A, \omega_B)\big)\big]. \end{aligned} \tag{3}$$

(ii) Interval-Valued Fuzzy Numbers Subtraction:

$$\begin{aligned} \tilde{A} - \tilde{B} = {} & \big[\big(a'_l - b'_u, a'_m - b'_m, a'_u - b'_l; \min\big(\omega'_A, \omega'_B\big)\big), \\ & \big(a_l - b_u, a_m - b_m, a_u - b_l; \min(\omega_A, \omega_B)\big)\big]. \end{aligned} \tag{4}$$

(iii) Interval-Valued Fuzzy Numbers Multiplication:

$$\begin{aligned} \tilde{A} \times \tilde{B} = {} & \big[\big(a'_l \times b'_l, a'_m \times b'_m, a'_u \times b'_u; \min\big(\omega'_A, \omega'_B\big)\big), \\ & \big(a_l \times b_l, a_m \times b_m, a_u \times b_u; \min(\omega_A, \omega_B)\big)\big]. \end{aligned} \tag{5}$$

(iv) Interval-Valued Fuzzy Numbers Division:

$$\begin{aligned} \tilde{A} \div \tilde{B} = {} & \big[\big(a'_l \div b'_u, a'_m \div b'_m, a'_u \div b'_l; \min\big(\omega'_A, \omega'_B\big)\big), \\ & \big(a_l \div b_u, a_m \div b_m, a_u \div b_l; \min(\omega_A, \omega_B)\big)\big]. \end{aligned} \tag{6}$$

The particular case of generalized interval-valued fuzzy numbers, shown in Fig. 3., is a normalized ($\omega'_A = \omega_A = 1$) interval-valued triangular fuzzy number with the same mode ($a'_m = a_m$), and it can be denoted as follows:

$$\tilde{A} = \big[\tilde{A}^L, \tilde{A}^U\big] = \big[\big(a_l, a'_l\big), a_m, \big(a'_u, a_u\big)\big]. \tag{7}$$

Suppose that $\tilde{A} = [\tilde{A}^L, \tilde{A}^U] = [(a_l, a'_l), a_m, (a'_u, a_u)]$ and $\tilde{B} = [\tilde{B}^L, \tilde{B}^U] = [(b_l, b'_l), b_m, (b'_u, b_u)]$ are two normalized interval-valued triangular fuzzy numbers with the same

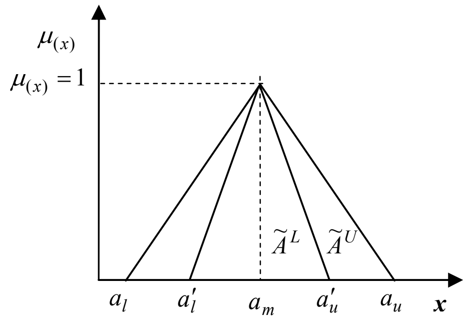

Fig. 3. The normalized interval-valued triangular fuzzy number with the same mode.

mode. Then, the basic arithmetic operations on these fuzzy numbers (Chen, 1997; Chen and Chen, 2008) are defined as follows:

$$\text{Addition: } \tilde{A} + \tilde{B} = \big[\big(a_l + b_l, a_l' + b_l'\big), a_m + b_m, \big(a_u' + b_u', a_u + b_u\big)\big], \tag{8}$$

$$\text{Subtraction: } \tilde{A} - \tilde{B} = \big[\big(a_l - b_u, a_l' - b_u'\big), a_m - b_m, \big(a_u' - b_l', a_u - b_l\big)\big], \tag{9}$$

$$\text{Multiplication: } \tilde{A} \times \tilde{B} = \big[\big(a_l \times b_l, a_l' \times b_l'\big), a_m \times b_m, \big(a_u' \times b_u', a_u \times b_u\big)\big], \tag{10}$$

$$\text{Division: } \tilde{A} \div \tilde{B} = \big[\big(a_l \div b_u, a_l' \div b_u'\big), a_m \div b_m, \big(a_u' \div b_l', a_u \div b_l\big)\big]. \tag{11}$$

The following unary operation on interval-valued triangular fuzzy numbers is also important:

$$\frac{1}{k} \times \tilde{A} = \left[\left(\frac{1}{k} \times a_l, \frac{1}{k} \times a_l'\right), \frac{1}{k} \times a_m, \left(\frac{1}{k} \times a_u', \frac{1}{k} \times a_u\right)\right]. \tag{12}$$

2.2. *Linguistic Variables*

In a series of papers, Zadeh (1975a, 1975b, 1975c) introduced the concept of linguistic variables. According to Zadeh, the linguistic variables are defined as variables whose values are words or sentences in a natural or artificial language.

The concept of a linguistic variable is very suitable for dealing with many real-world decision making problems, which are usually complex, slightly defined and related to uncertainties. Therefore, many authors, in their published papers, proposed different linguistic variables (linguistic terms scales), usually based on the use of triangular or trapezoidal fuzzy numbers, such as Wang and Chang (1995), Chen (2000), Wang and Elhag (2006), Mahdavi *et al.* (2008).

Tables 1 and 2 show the linguistic variables for the weights of criteria and performance ratings, based on the use of triangular fuzzy numbers (Saremi *et al.*, 2009).

In literature also are proposed linguistic variables based on the use of interval-valued fuzzy numbers. Wei and Chen (2009) proposed the nine level linguistic terms scale based on the use of interval-valued trapezoidal fuzzy numbers. Ashtiani *et al.* (2009), Kuo (2011), Kuo and Liang (2012) proposed the seven level linguistic terms scale based on the use of interval-valued triangular fuzzy numbers.

Table 1
Linguistic variables for the weights of criteria.

Linguistic variables	Triangular fuzzy numbers
Very low (VL)	(0.0, 0.0, 0.1)
Low (L)	(0.0, 0.1, 0.3)
Medium low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium high (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.7, 1.0)
Very high (VH)	(0.9, 1.0, 1.0)

Table 2
Linguistic variables for the performance ratings.

Linguistic variables	Triangular fuzzy numbers
Very poor (VP)	(0.0, 0.0, 0.1)
Poor (P)	(0.0, 0.1, 0.3)
Medium poor (MP)	(0.1, 0.3, 0.5)
Fair (F)	(0.3, 0.5, 0.7)
Medium good (MG)	(0.5, 0.7, 0.9)
Good (G)	(0.7, 0.7, 1.0)
Very good (VG)	(0.9, 1.0, 1.0)

Table 3
Linguistic variables for the weights of criteria.

Linguistic variables	Interval-valued triangular fuzzy numbers
Very low (VL)	[(0.00, 0.00), 0.0, (0.10, 0.15)]
Low (L)	[(0.00, 0.50), 0.1, (0.25, 0.35)]
Medium low (ML)	[(0.00, 0.15), 0.3, (0.45, 0.55)]
Medium (M)	[(0.25, 0.35), 0.5, (0.65, 0.75)]
Medium high (MH)	[(0.45, 0.55), 0.7, (0.80, 0.95)]
High (H)	[(0.55, 0.75), 0.9, (0.95, 1.00)]
Very high (VH)	[(0.85, 0.95), 1.0, (1.00, 1.00)]

Table 4
Linguistic variables for the ratings.

Linguistic variables	Interval-valued triangular fuzzy numbers
Very poor (VP)	[(0.00, 0.00), 0.0, (0.10, 0.15)]
Poor (P)	[(0.00, 0.50), 0.1, (0.25, 0.35)]
Medium poor (MP)	[(0.00, 0.15), 0.3, (0.45, 0.55)]
Fair (F)	[(0.25, 0.35), 0.5, (0.65, 0.75)]
Medium good (MG)	[(0.45, 0.55), 0.7, (0.80, 0.95)]
Good (G)	[(0.55, 0.75), 0.9, (0.95, 1.00)]
Very good (VG)	[(0.85, 0.95), 1.0, (1.00, 1.00)]

Tables 3 and 4 show the linguistic variables for the weights of criteria and performance ratings, based on the use of interval-valued triangular fuzzy numbers (Ashtiani *et al.*, 2009; Kuo, 2011).

Some advantages can be also achieved by combining the use of fuzzy and interval-fuzzy numbers. If it is considered that it is true that:

(i) the results obtained by using linguistic variables may be better if the participants are more familiar with their meaning and usage, and

(ii) interval-valued fuzzy numbers are more complex than ordinary fuzzy numbers,

then it can be said that some benefits may be also achieved using linguistic variables based on ordinary fuzzy numbers and by their further transformation into interval-valued fuzzy numbers.

The weights and performance ratings obtained using linguistic variables, which are based on the use of triangular fuzzy numbers, further can be transformed into the corresponding interval-valued triangular fuzzy numbers using the follows formulae:

$$l = \min_k \left(l^k\right), \tag{13}$$

$$l' = \left(\prod_{k=1}^{K} l^k\right)^{1/K}, \tag{14}$$

$$m = \left(\prod_{k=1}^{K} m^k\right)^{1/K}, \tag{15}$$

$$u' = \left(\prod_{k=1}^{K} u^k\right)^{1/K}, \tag{16}$$

$$u = \max_k \left(u^k\right), \tag{17}$$

where $\tilde{x} = [(l, l'), m, (u', u)]$ denotes the corresponding interval-valued triangular fuzzy number, $\tilde{x}^k = (l^k, m^k, u^k)$ denotes the triangular fuzzy number obtained on the basis of opinion of k-th participant (decision maker), $k = 1 \dots K$; and K is the number of participants.

The parameters l and u represent the smallest and the greatest performance ratings among all stakeholders, and they reflect the extreme attitudes provided by the stakeholders involved in the evaluation.

Unlike them, other parameters of the interval-valued triangular fuzzy number much more realistically reflect the attitudes of all stakeholders, where l', m and u' denote the smallest performance rating, the most promising performance rating, and the largest performance rating that describe a fuzzy event, obtained as the geometric mean of attitudes of all stakeholders.

2.3. *Defuzzification of Interval-Valued Triangular Fuzzy Numbers*

As a result of performing an arithmetic operation on fuzzy numbers, the obtained results are also fuzzy numbers. Therefore, in order to rank alternatives in fuzzy environment using MCDM methods:

(i) these methods must be able to perform the ranking based on fuzzy overall performance ratings, or
(ii) fuzzy overall performance ratings must be transformed into crisp overall performance ratings before ranking of alternatives has been made.

In due course, a number of different procedures for ranking fuzzy numbers and/or for their defuzzification are proposed, but these procedures have been mainly proposed for the trapezoidal fuzzy and the triangular fuzzy numbers.

However, a number of procedures, that are intended for defuzzification of interval-valued fuzzy numbers, is much more modest. Therefore, starting from the following formulae:

$$gm(\tilde{A}) = \frac{1}{2}\big[(1-\lambda)l + m + \lambda u\big], \tag{18}$$

$$gm(\tilde{A}) = \frac{l+m+u}{3}, \tag{19}$$

that have been proposed for defuzzification of triangular fuzzy numbers, the following formulae are proposed for defuzzification of interval-valued triangular fuzzy numbers:

$$gm(\tilde{B}) = \frac{l + l' + m + u' + u}{5}, \tag{20}$$

$$gm(\tilde{B}) = \frac{(1-\lambda)l + \lambda l' + m + \lambda u' + (1-\lambda)u}{3}, \tag{21}$$

where $\tilde{A}$ denotes ordinary triangular fuzzy numbers, $\tilde{B}$ denotes interval-valued fuzzy numbers, λ is a coefficient, and $\lambda \in [0, 1]$.

Formula (20) is a simple extension of the formula (19), and it provides a simple and effective determination of the Best Nonfuzzy Performance (BNP) value of an interval-valued fuzzy number.

In comparison to the formula (20), formula (21) is slightly more complex, but it also has some advantages. Varying the coefficient λ the greater importance can be given to parameters l' and u' compared to the l and u of an interval-valued triangular fuzzy number, and vice versa.

2.4. *Multiple Criteria Group Decision Making*

Ordinary MCDM models, are usually based on the opinion of a single decision maker, and they can be precisely shown in the following form:

$$D = [x_{ij}]_{m\times n}, \tag{22}$$

where D is a decision matrix, x_{ij} is the performance rating of i-th alternative to the j-th criterion, $i = 1, 2, \dots, m$; m is a number of alternatives, $j = 1, 2, \dots, n$; n is a number of criteria.

In the MCDM, the evaluation criteria usually have a different importance. To express the importance of each criterion the MCDM models also include criteria weights, as follows:

$$W = [w_j], \tag{23}$$

where W is a weight vector, w_j is the weight of j-th criterion, $j = 1, 2, \ldots, n$; n is a number of criteria.

For solving a number of complex decision making problems, it is necessary to take into account opinions of more decision makers, i.e. usually of relevant experts. In such cases, the Multiple Criteria Group Decision Making (MCGDM) approach is commonly used, and it can be precisely shown in the following form:

$$D = [x_{ij}^k]_{m\times n\times K}, \tag{24}$$

$$W = [w_j^k]_{n\times K}, \tag{25}$$

where x_{ij}^k is the performance rating of i-th alternative to the j-th criterion given by k-th decision maker; $k = 1, 2, \ldots, K$; K is a number of decision makers and/or experts involved in MCGDM.

In order to evaluate alternatives MCGDM problems are usually transformed into a MCDM problems, in one of the stages of problem solving procedure, and further solving as MCDM problems.

3. Additive Ratio Assessment Method

As previously mentioned, the ARAS method was proposed by Zavadskas and Turskis (2010). The process of solving decision making problems using ARAS method, similarly to the other methods of MCDM, starts with forming the decision matrix and determining weights of criteria. After these initial steps, the remaining part of solving MCDM problem using ARAS method can be precisely expressed using the following steps:

Step 1. *Determine the optimal performance rating for each criterion.* In this step the decision maker sets the optimal performance rating for each criterion. If the decision maker does not have a preferences, the optimal performance ratings are calculated as:

$$x_{0j} = \begin{cases} \max_i x_{ij}; & j \in \Omega_{\max}, \\ \min_i x_{ij}; & j \in \Omega_{\min}, \end{cases} \tag{26}$$

where x_{0j} denotes the optimal performance rating of j-th criterion, $\Omega_{\max}$ denotes the benefit criteria, i.e. the higher the values are, the better it is; and $\Omega_{\min}$ denotes the set of cost criteria, i.e. the lower the values are, the better it is.

Step 2. *Calculate the normalized decision matrix*. The normalized performance ratings are calculated using the following formula:

$$r_{ij} = \begin{cases} \frac{x_{ij}}{\sum_{i=0}^{m} x_{ij}}, & j \in \Omega_{\max}, \\ \frac{1/x_{ij}}{\sum_{i=0}^{m} 1/x_{ij}}, & j \in \Omega_{\min}, \end{cases} \tag{27}$$

where r_{ij} denotes the normalized performance rating of i-th alternative in relation to the j-th criterion, $i = 0, 1, \dots, m$.

Step 3. *Calculate the weighted normalized decision matrix*. The weighted normalized performance ratings are calculated using the following formula:

$$v_{ij} = w_j r_{ij}, \tag{28}$$

where v_{ij} denotes the weighted normalized performance rating of i-th alternative in relation to the j-th criterion, $i = 0, 1, \dots, m$.

Step 4. *Calculate the overall performance rating, for each alternative*. The overall performance ratings can be calculated using the following formula:

$$S_i = \sum_{j=1}^{n} v_{ij}, \tag{29}$$

where S_i denotes the overall performance rating of i-th alternative, $i = 0, 1, \dots, m$.

Step 5. *Calculate the degree of utility for each alternative*. When evaluating alternatives, it is not only important to determine the best ranked alternative. It is also important to determine relative performances of considered alternatives, in relation to the optimal alternative. For this purpose the degree of utility is used, and it can be calculated using the following formula:

$$Q_i = \frac{S_i}{S_0}, \tag{30}$$

where Q_i denotes the degree of utility of i-th alternative, and S_0 is the overall performance index of optimal alternative, $i = 1, 2, \dots, m$.

Step 6. *Rank alternatives and/or select the most efficient one*. The considered alternatives are ranked by ascending Q_i, i.e. the alternative with the largest value of Q_i is the best placed. Therefore, the most acceptable alternative can be determined using the following formula:

$$A^* = \Big\{ A_i \,\big|\, \max_i Q_i \Big\}, \tag{31}$$

where A^* denotes the most acceptable alternative, $i = 1, 2, \dots, m$.

3.1. *An Extension of the ARAS Method Based on the Use of Interval-Valued Fuzzy Numbers*

To enable the use of interval-valued fuzzy numbers, some modifications have to be done in the ARAS method. Therefore, the computational procedure for determining the most acceptable alternative using the ARAS method, based on the use of interval-valued fuzzy numbers, can be precisely expressed using the following steps:

Step 1. *Determine the optimal performance rating for each criterion*. The first modification is necessary in the first step, when determining the optimal performance rating for each criterion. Instead of the crisp number, the optimal performance rating of each criterion should be an interval-valued fuzzy number, and these optimal interval-valued fuzzy performance ratings are calculated as follows:

$$\tilde{x}_{0j} = \left[\left(l_{0j}, l'_{0j}\right), m_{0j}, \left(u'_{0j}, u_{0j}\right)\right], \tag{32}$$

where $\tilde{x}_{0j}$ denotes the interval-valued fuzzy optimal performance rating of j-th criterion, and

$$l_{0j} = \begin{cases} \max_i l_{ij}; & j \in \Omega_{\max}, \\ \min_i l_{ij}; & j \in \Omega_{\min}, \end{cases} \tag{33}$$

$$l'_{0j} = \begin{cases} \max_i l'_{ij}; & j \in \Omega_{\max}, \\ \min_i l'_{ij}; & j \in \Omega_{\min}, \end{cases} \tag{34}$$

$$m_{0j} = \begin{cases} \max_i m_{ij}; & j \in \Omega_{\max}, \\ \min_i m_{ij}; & j \in \Omega_{\min}, \end{cases} \tag{35}$$

$$u'_{0j} = \begin{cases} \max_i u'_{ij}; & j \in \Omega_{\max}, \\ \min_i u'_{ij}; & j \in \Omega_{\min} \end{cases} \tag{36}$$

$$u_{0j} = \begin{cases} \max_i u_{ij}; & j \in \Omega_{\max}, \\ \min_i u_{ij}; & j \in \Omega_{\min}. \end{cases} \tag{37}$$

Step 2. *Calculate the normalized decision matrix*. In order to use interval-valued fuzzy numbers, some modifications are also necessary in the normalization process. Therefore, instead of formula (29) the following formula can be used:

$$\tilde{r}_{ij} = \begin{cases} \left[\left(\frac{a_{ij}}{c_j^+}, \frac{a'_{ij}}{c_j^+}\right), \frac{b_{ij}}{c_j^+}, \left(\frac{c'_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right)\right]; & j \in \Omega_{\max}, \\ \left[\left(\frac{1/a_{ij}}{a_j^-}, \frac{1/a'_{ij}}{a_j^-}\right), \frac{1/b_{ij}}{a_j^-}, \left(\frac{1/c'_{ij}}{a_j^-}, \frac{1/c_{ij}}{a_j^-}\right)\right]; & j \in \Omega_{\min}, \end{cases} \tag{38}$$

where $\tilde{r}_{ij}$ denotes the normalized interval-valued fuzzy performance rating of i-th alternative in relation to the j-th criterion, $i = 0, 1, \ldots, m$; $c_j^+ = \sum_{i=0}^{m} c_{ij}$, and $a_j^- = \sum_{i=0}^{m} 1/a_{ij}$.

Step 3. *Calculate the weighted interval-valued normalized fuzzy decision matrix*. This step is very similar to the third step in the ordinary ARAS method, but instead of crisp numbers multiplication is performed on interval-valued triangular fuzzy numbers. Therefore, this step can be expressed using the following formula:

$$\tilde{v}_{ij} = \tilde{w}_j \cdot \tilde{r}_{ij}, \tag{39}$$

where $\tilde{v}_{ij}$ denotes the weighted normalized interval-valued fuzzy performance rating of i-th alternative in relation to the j-th criterion, $i = 0, 2, \dots, m$.

Step 4. *Calculate the overall interval-valued fuzzy performance ratings*. The overall interval-valued fuzzy performance ratings can be calculated using the following formula:

$$\tilde{S}_i = \sum_{j=1}^{n} \tilde{v}_{ij}, \tag{40}$$

where $\tilde{S}_i$ denotes overall interval-valued fuzzy performance rating of i-th alternative, $i = 0, 1, \dots, m$.

Step 5. *Calculate the degree of utility, for each alternative*. As a result of performing the previous steps, the obtained overall performance ratings are interval-valued fuzzy numbers. Therefore, the calculation of the overall degree of utility is somewhat more complex.

It is clear that defuzzification must be performed, but here arises one important question: "When is it supposed to perform defuzzification, before or after determining the degree of utility ?". Another question arises too: "Does it make any impact on the effects of ranking?"

In the first case, the degree of utility can be determined as a ratio between the degree of utility of considered alternative and the degree of utility of the optimal alternative, as follows:

$$\tilde{Q}_i = \frac{\tilde{S}_i}{\tilde{S}_0}. \tag{41}$$

However, the results obtained by using the formula (41) are still interval-valued fuzzy numbers, and they must also be defuzzified.

In the second case, the obtained overall performance ratings should be transformed into the exact values before degree of utility is determined.

It is also known that the use of various defuzzification procedures may have an impact on the obtained results, and therefore, except for the stage of the problem solving process in which defuzzification should be done, the choice of appropriate defuzzifiation procedure may be also important.

Step 6. *Rank alternatives and/or select the most efficient one*. This step remains the same as in the original ARAS method.

4. A Numerical Example

In order to highlight the proposed extension of the ARAS method, in this section an example of the faculty websites evaluation, adopted from Stanujkic and Jovanovic (2012), has been considered. This evaluation is based on the criteria proposed by Kapoun (1998) referring to measuring the quality of the information presented on websites.

According to Kapoun (1998), the following criteria are important for determining quality of websites:

- accuracy;
- authority;
- objectivity;
- currency;
- coverage.

Some initial research, carried out in order to make this manuscript, pointed out that the faculty websites have a relatively uniform Accuracy and Authority. Therefore, but also for the sake of making a clearer presentation of the proposed extension, these criteria have been omitted.

After that, the set of chosen criteria for the evaluation of the faculty websites contains the following criteria:

- C_1 – objectivity of the website (O),
- C_2 – currency of the website (Cu), and
- C_3 – coverage of the website (Co).

The set of chosen evaluation criteria contains a small number of criteria, but it is intentionally formed in that way. In order to evaluate the quality of university websites, it is very important to precisely determine the weights of the evaluation criteria, and sometimes is not so easy. The use of a larger number of criteria can significantly affect to the more precise determination of the website quality. However, the use of a larger number of criteria can sometimes make it difficult to determine the weights of the criteria.

The next important question is how to determine the weights of criteria. In literature there has been proposed several procedures for determining criteria weights. The use of linguistic variables can be singled out as one of the most frequently used approaches, especially when dealing with complex problems, which also include the use of the group decision making approach.

In the traditional decision making linguistic variables are often used to express performance ratings and criteria weights, and they are usually converted into the interval numbers, triangular fuzzy numbers, or trapezoidal fuzzy numbers (Liu, 2009; Liu and Zhang, 2010).

Compared to the above mentioned forms of fuzzy numbers, interval-valued fuzzy numbers provide significantly more possibilities when dealing with vague information and risk. However, when solving real-world decision-making problems, it is very difficult to get the interval-valued fuzzy numbers for the weights and ratings directly by the decision makers (Liu *et al.*, 2011).

Table 5
Weights of criteria obtained from three students.

Criteria	E_1	E_2	E_3
C_1	MH	M	M
C_2	H	VH	H
C_3	VH	VH	H

Table 6
Weights of criteria obtained from three students.

	E_1	E_2	E_3	Interval-valued fuzzy weights
C_1	(0.5, 0.7, 0.9)	(0.3, 0.5, 0.7)	(0.3, 0.5, 0.7)	[(0.3, 0.36), 0.56, (0.76, 0.9)]
C_2	(0.7, 0.9, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	[(0.70, 0.76), 0.93, (1.0, 1.0)]
C_3	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	[(0.70, 0.83), 0.97, (1.0, 1.0)]

Due to the above mentioned reasons, as for this example, linguistic variables are used; however, they are not transformed directly into interval-valued fuzzy numbers. In this example they are transformed into ordinary triangular fuzzy numbers. This transformation is done because it is estimated that this approach can better exploit the some possibilities which interval-valued fuzzy numbers provide.

For ordinary users, including students, the meaning of the interval-valued fuzzy numbers can be somewhat confusing. Therefore, using ordinary fuzzy numbers, in such cases, some advantages can be achieved.

Using linguistic variables from Table 1, the surveyed students, after a short introduction to the meaning of the used criteria, meaning of fuzzy numbers and linguistic variables, have performed the evaluation of the chosen evaluation criteria.

In order to make a clear presentation, in this example the characteristic results obtained from the three students are used. These results are shown in Table 5.

The obtained linguistic variables, from Table 5, are transformed into fuzzy numbers, and then into the interval-valued fuzzy numbers, as proposed in Section 2.2.

The results of these transformations, obtained using the formulae (13)–(17), are shown in Table 6.

In a similar way, performance ratings of the faculty websites are evaluated, using linguistic variables from Table 2. Obtained results are shown in Table 7.

After that, these linguistic variables are transformed into triangular fuzzy numbers, as shown in Table 8.

The resulting performance ratings, expressed using interval-valued fuzzy numbers, are shown in Table 9.

After the above preparatory activities, the evaluation of the quality of the faculty websites has been performed using the proposed extensions of the ARAS method.

As shown in Section 3.2., the first steep in using the proposed extension of the ARAS method is the determination of the optimal performance ratings, and this is done by using the formula (32). The obtained optimal performance ratings are shown in Table 10.

The next step, in the proposed methodology, is the normalization, and it is done by using the formula (38). The result of normalization are shown in Table 11.

Table 7
Ratings of evaluation criteria.

Criteria	Alternatives	E_1	E_2	E_3
C_1	A_1	VG	VG	VG
	A_2	VG	VG	G
	A_3	G	G	MG
C_2	A_1	MG	G	MG
	A_2	G	VG	VG
	A_3	VG	G	MG
C_3	A_1	F	G	F
	A_2	VG	VG	MG
	A_3	F	MG	G

Table 8
Ratings of evaluation criteria.

Criteria	Alternatives	E_1	E_2	E_3
C_1	A_1	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)
	A_2	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)
	A_3	(0.7, 0.9, 1.0)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
C_2	A_1	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
	A_2	(0.7, 0.9, 1.0)	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)
	A_3	(0.9, 1.0, 1.0)	(0.7, 0.9, 1.0)	(0.5, 0.7, 0.9)
C_3	A_1	(0.3, 0.5, 0.7)	(0.7, 0.9, 1.0)	(0.3, 0.5, 0.7)
	A_2	(0.9, 1.0, 1.0)	(0.9, 1.0, 1.0)	(0.5, 0.7, 0.9)
	A_3	(0.3, 0.5, 0.7)	(0.5, 0.7, 0.9)	(0.7, 0.9, 1.0)

Table 9
Interval-valued performance ratings of websites.

	C_1	C_2	C_3
A_1	[(0.90, 0.90), 1.00, (1.00, 1.00)]	[(0.50, 0.56), 0.76, (0.93, 1.00)]	[(0.30, 0.40), 0.61, (0.79, 1.00)]
A_2	[(0.70, 0.83), 0.97, (1.00, 1.00)]	[(0.70, 0.83), 0.97, (1.00, 1.00)]	[(0.50, 0.74), 0.89, (0.97, 1.00)]
A_3	[(0.50, 0.63), 0.83, (0.97, 1.00)]	[(0.50, 0.68), 0.86, (0.97, 1.00)]	[(0.30, 0.47), 0.68, (0.86, 1.00)]

Table 10
The optimal interval-valued triangular fuzzy performance ratings.

	C_1	C_2	C_3
A_0	[(0.90, 0.90), 1.00, (1.00, 1.00)]	[(0.70, 0.83), 0.97, (1.00, 1.00)]	[(0.5, 0.74), 0.89, (0.97, 1.00)]

Table 11
The normalized interval-valued triangular fuzzy performance rating.

	C_1	C_2	C_3
A_0	[(0.23, 0.23), 0.25, (0.25, 0.25)]	[(0.18, 0.21), 0.24, (0.25, 0.25)]	[(0.13, 0.18), 0.22, (0.24, 0.25)]
A_1	[(0.23, 0.23), 0.25, (0.25, 0.25)]	[(0.13, 0.14), 0.19, (0.23, 0.25)]	[(0.08, 0.10), 0.15, (0.20, 0.25)]
A_2	[(0.18, 0.21), 0.24, (0.25, 0.25)]	[(0.18, 0.21), 0.24, (0.25, 0.25)]	[(0.13, 0.18), 0.22, (0.24, 0.25)]
A_3	[(0.13, 0.16), 0.21, (0.24, 0.25)]	[(0.13, 0.17), 0.21, (0.24, 0.25)]	[(0.08, 0.12), 0.17, (0.21, 0.25)]

Table 12
The weighted interval-valued f triangular fuzzy performance ratings.

	C_1	C_2	C_3
w_j	[(0.3, 0.36), 0.56, (0.76, 0.90)]	[(0.70, 0.76), 0.93, (1.00, 1.00)]	[(0.70, 0.83), 0.97, (1.00, 1.00)]
A_0	[(0.07, 0.08), 0.14, (0.19, 0.23)]	[(0.12, 0.16), 0.23, (0.25, 0.25)]	[(0.09, 0.15), 0.21, (0.24, 0.25)]
A_1	[(0.07, 0.08), 0.14, (0.19, 0.23)]	[(0.09, 0.11), 0.18, (0.23, 0.25)]	[(0.05, 0.08), 0.15, (0.20, 0.25)]
A_2	[(0.05, 0.07), 0.14, (0.19, 0.23)]	[(0.12, 0.16), 0.23, (0.25, 0.25)]	[(0.09, 0.15), 0.21, (0.24, 0.25)]
A_3	[(0.04, 0.06), 0.12, (0.18, 0.23)]	[(0.09, 0.13), 0.20, (0.24, 0.25)]	[(0.05, 0.10), 0.16, (0.21, 0.25)]

Table 13
The overall interval-valued triangular fuzzy performance ratings.

A_0	[(0.28, 0.39), 0.58, (0.68, 0.73)]
A_1	[(0.21, 0.27), 0.46, (0.62, 0.73)]
A_2	[(0.26, 0.38), 0.57, (0.68, 0.73)]
A_3	[(0.18, 0.28), 0.48, (0.64, 0.73)]

Table 14
The degree of utility and ranking order of analyzed faculty websites.

Alternatives	BNP	Q_i	Rank
A_0	0.60		
A_1	0.51	0.84	3
A_2	0.60	1.00	1
A_3	0.53	0.88	2

Table 15
The degree of utility and ranking order of analyzed websites, for some characteristic values of λ.

Alternatives	$\lambda = 0$			$\lambda = 0.5$			$\lambda = 1$		
	S_i	Q_i	Rank	S_i	Q_i	Rank	S_i	Q_i	Rank
A_0	0.53			0.63			0.55		
A_1	0.47	0.88	2	0.55	0.87	3	0.45	0.82	3
A_2	0.52	0.99	1	0.62	0.99	1	0.55	0.99	1
A_3	0.46	0.87	3	0.55	0.88	2	0.47	0.85	2

Weighted normalized interval-valued fuzzy performance ratings, obtained by the formula (39), are shown in Table 12.

Finally, the interval-valued fuzzy performance rating, obtained by the formula (40), are shown in Table 13.

In order to determine the quality of the analyzed websites, these values must be defuzzified using some of the procedures discussed in Section 2.3.

Results obtained using the simplest of all of the considered defuzzification procedures, i.e. using the formula (20), are shown in Table 14. The relative quality, i.e. degree of utility, of analyzed websites as well as their ranking orders are also shown in Table 14.

The formula (21) provides greater opportunities compared to the formula (20). By varying the coefficient λ, greater importance can be given to l and u in relation to l' and u', and vice versa. The results obtained using the formula (21) for some characteristic values of the coefficient λ are shown in Table 15.

Results of ranking alternatives presented in Table 18 indicate that the proposed extension of the ARAS method can be very effective tool for analyzing different decision-making scenarios, as well as selection of the most appropriate alternative also.

5. Conclusion

A number of earlier published papers have shown the applicability of ordinary MCDM methods, but they also pointed to some of their limitations that arise when solving complex decision-making problems, such as one associated with the lack of precise or reliable information, as well as decision-making problems that are associated with some kinds of predictions.

The use of fuzzy numbers instead of crisp numbers has brought significant advantages in solving complex decision-making problems, and this is why many ordinary MCDM methods have also been extended to allow their use. As a result of the recent research, it was also observed that the use of interval-valued fuzzy numbers has some advantages compared to the ordinary fuzzy numbers in the case of solving complex problems, especially problems that are associated with some kind of prediction.

Therefore, in this paper, the use of effective but also easy to use procedure of ARAS method is considered, as well as its extension, which has been formed with the aim to provide the use of interval-valued triangular fuzzy numbers.

In addition, on the basis of the considered examples, it can be concluded that the proposed methodology provides significant opportunities in the case of solving the complex decision-making problems, especially problems that are associated with some types of predictions.

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ARAS metodo išplėtimas sprendimo priėmimo problemoms spręsti taikant intervalais matuojamus trikampius neraiškiuosius skaičius

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Straipsnyje siūlomas ARAS metodo išplėtimas, kuris dėl intervalinio neraiškiųjų skaičių panaudojimo, gali būti taikomas, sprendžiant realias pasaulio problemas. Siekiant, įveikti realaus pasaulio sprendimo priėmimo problemų sudėtingumą, siūlomas duomenų aprėpties išplėtimas, naudojant lingvistinius kintamuosius bei grupinį sprendimų priėmimą. Siūlomos metodologijos taikymas, iliustruojamas fakultetų svetainių tinklalapių vertinimo uždaviniu.