INFORMATICA, 2014, Vol. 25, No. 2, 185–208

Comparative Analysis of Normalization Procedures in TOPSIS Method: With an Application to Turkish Deposit Banking Market

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Received: October 2012; accepted: January 2013

Abstract. In this study, we evaluated the effects of the normalization procedures on decision outcomes of a given MADM method. For this aim, using the weights of a number of attributes calculated from FAHP method, we applied TOPSIS method to evaluate the financial performances of 13 Turkish deposit banks. In doing this, we used the most popular four normalization procedures. Our study revealed that vector normalization procedure, which is mostly used in the TOPSIS method by default, generated the most consistent results. Among the linear normalization procedures, max-min and max methods appeared as the possible alternatives to the vector normalization procedure.

Key words: MADM, TOPSIS, normalization, consistency, performance evaluation.

1. Introduction

Multi-attribute decision making (MADM) is the most popular branch of decision making. MADM refers to making preference decisions (e.g., evaluation, prioritization, and selection) over finite number of alternatives which are characterized by multiple, often conflicting, attributes (Hwang and Yoon, 1981; Zavadskas and Turskis, 2011). In MADM models, each alternative has a performance rating for each attribute, and performance ratings for different attributes are usually measured by different units. Thus, normalization procedures are used in MADM models to convert the different measurement units of the performance ratings into a comparable unit. Several normalization procedures are available in literature to eliminate computation problems caused by different measurement units. MADM methods generally use one of these normalization procedures without considering the suitability of other available procedures (Chakraborty and Yeh, 2007). Among the linear normalization procedures, for example, *sum* method is used in Fuzzy Analytic Hierarchy Process (FAHP), Complex Proportional Assessment (COPRAS) and Additive Ratio Assessment (ARAS) applications (Gumus, 2009; Seçme *et al.*, 2009; Ertugrul and Karakaşoğlu, 2009; Chatterjee *et al.*, 2011; Antuchevičienė *et al.*, 2011; Kaklauskas *et al.*, 2007; Ginevičius and Podvezko, 2008; Viteikiene and Zavadskas, 2007; Zavadskas and Turskis, 2010), while *max* method is used with Simple Additive Weighting

(SAW), Fuzzy Technique for Order Preference by Similarity to Ideal Solution (FTOPSIS), Weighted Sum Model (WSM) and Weighted Aggregated Sum Product Assessment (WASPAS) methods (Hwang and Yoon, 1981; Yeh, 2003; Wang and Chang, 2007; Sun and Lin, 2009; Sun, 2010; Zeydan *et al*., 2011; Zavadskas *et al*., 2012). Another linear procedure, *max-min* method, is preferred in VlseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) and extended PROMETHEE II (EXPROM2) applications (Opricovic and Tzeng, 2004; Ginevičius, 2008; Yalçın *et al*., 2012; Antuchevičienė *et al*., 2011; Chatterjee and Chakraborty, 2012). Vector normalization, which is a non-linear procedure, is very popular in TOPSIS, Elimination and Choice Translating Reality (ELECTRE) and Multi-Objective Optimization by Ratio Analysis (MOORA) applications (Antuchevičienė *et al*., 2010; Ginevičius and Podvezko, 2008; Ertugrul and Karakaşoğlu, 2009; Özcan *et al*., 2011; Yalçın *et al*., 2012; Chakraborty, 2011; Brauers *et al*., 2007).[1]

In this study, we aimed to contribute to the literature comparing the effects of the normalization procedures on decision outcomes of MADM problems. To be more specific, using the weights of a number of attributes calculated from FAHP method, we applied TOPSIS method to evaluate the financial performances of 13 Turkish deposit banks. In doing this, following Chakraborty and Yeh (2007, 2009), we focused on the most popular four normalization procedures. In evaluating and comparing the results of the alternative TOPSIS models based on different normalization procedures, we benefited from the consistency conditions set by Bauer *et al*. (1998).

The remainder of the paper is organized as follows: Section 2 presents a literature review on the impact of the normalization procedures on the decision outcome. In Section 3, research methodology including FAHP, TOPSIS and alternative normalization procedures are explained. Empirical results including those of both application of the alternative models and consistency search are presented in Section 4. And finally in Section 5, the results of the study are discussed.

2. Literature Review on the Impact of the Normalization Procedures on the Decision Outcomes

In literature, the impact of the different normalization procedures on the decision results of a given MADM method has been examined by several studies. These comparative studies are reviewed in this section.

Pavlicic (2001) examined the effects of three popular normalization procedures on three different MADM methods (SAW, TOPSIS and ELECTRE). Pavlicic (2001) concluded that the normalization procedure used affected the final choices. This study also witnessed that MADM methods violated certain conditions of consistent choice and that this violation could be attributed to the normalization procedures used.

[1] In fact, we are aware of the several studies which are exceptions to our generalization. For example, Lai and Hwang (1994) and Wu *et al*. (2009) used max-min method in a TOPSIS application, Turskis *et al*. (2006) used max-min method in a SAW application, while Torlak *et al*. (2011) utilized vector normalization method in a fuzzy TOPSIS application.

Using a new program called LEVI 3.0, Zavadskas *et al.* (2003) compared the result of a non-linear normalization procedure (proposed by Peldschus *et al.*, 1983) with those of four linear ones (proposed by Stopp, 1975; Weitendorf, 1976; Körth, 1969; Jüttler, 1966). The results showed that the non-linear normalization procedure proposed by Peldschus *et al.* (1983) improves the quality of transformation and solves technological and organizational problems more precisely.

Milani *et al.* (2005) evaluated the effect of five different normalization procedures by applying TOPSIS method to the problem of gear material selection for power transmission. Milani *et al.* (2005) concluded that different normalization procedures generated rather different closeness coefficients. However, this was not enough for linear normalization procedures to change the ranking of the alternative gear materials while non-linear normalization procedure produced somewhat different ranking.

Zavadskas *et al.* (2006) developed a methodology for measuring the accuracy of the relative significance of the alternatives as a function of the attribute values. Zavadskas *et al.* (2006) employ this methodology in a TOPSIS application by normalizing attribute values with both non-linear vector and linear normalization (proposed by Lai and Hwang, 1994) procedures. In this study, it is shown that the accuracy of results is influenced not only by errors of the initial attribute values but also depends on solution techniques and normalization methods of the initial attribute values used. In addition, the study witnesses that the relative closeness of the alternatives to the ideal solution is approximately 2.3 times less accurate in linear normalization than in vector normalization.

Brauers and Zavadskas (2006) discussed the normalization procedures by proposing a new MADM method called Multi-Objective Optimization on the basis of Ratio Analysis (MOORA). In this method, a ratio system is developed in which each performance of an alternative on an attribute is compared to a denominator which is a representative for all the alternatives concerning that attribute. Then, these ratios, taking values between zero and one, are summed in the case of maximization or subtracted in case of minimization. Finally, all alternatives are ranked according to the obtained sums. Brauers and Zavadskas (2006) considers various ratio systems, such as total ratio, Weitendorf (1976) ratio, Jüttler (1966) ratio, Stopp (1975) ratio, Körth (1969) ratio and concluded that for this denominator, the best choice is the square root of the sum of squares of each alternative per attribute, which is indeed the vector normalization.

In selecting effective construction alternatives, Migilinskas and Ustinovichius (2007) studied twelve attributes by help of eight methods of normalization separated into four groups. This paper concludes that normalization method must be chosen according to the objectives so as to meet special requirements, with regard to possible inaccuracy or uncertainty threats and effects on the final decision about the ranking of the alternatives.

Peldschus (2007) examined several normalization formulae and showed that the solution in a MADM problem varies depending on the normalization method used. It is also shown that the stability of the solution for maximization or minimization problems is not ensured using a linear normalization.

Chakraborty and Yeh (2007), generating several alternative environments by simulation, compared four commonly known normalization procedures when used with SAW.

This study suggested that vector normalization and linear scale transformation (*max* method) outperforms other normalization procedures. Chakraborty and Yeh (2009) compared the same normalization procedures for the TOPSIS method. This study supported the use of vector normalization with the TOPSIS method.

Liping *et al*. (2009) searched the most appropriate normalization procedure (among nine alternative procedures) for multiple attribute evaluation (MAE).[2] This study, reaching the conclusion that the evaluation result is greatly affected by different data normalization methods, recommended two different linear normalization methods.

With the help of LEVI 3.1. program, Zavadskas and Turskis (2008) proposed a new logarithmic normalization method and compares its results with those of two non-linear normalization methods (vector normalization and Peldschus *et al*., 1983 method) and two linear normalization methods (Weitendorf, 1976; Körth, 1969). According to the results, the proposed logarithmic normalization procedure yields more stable results in solving multi-attribute decision problems. It is also shown that logarithmic normalization may be used in the cases when the values of the attributes differ considerably. Another study making comparisons among these normalization procedures by utilizing LEVI program is Turskis *et al*. (2009). This study also supports the new proposed logarithmic normalization procedure.

Peldschus (2009) analyzed the linear functions and non-linear functions (the hyperbolic function, the quadratic and cubic function, the square root and the logarithmic function) with respect to their normalization features. This study concluded that linear functions present a good mapping to the interval [1; 0]. However, for the minimization, when characteristic values, which exceed the double minimal value, are included in the description of the variants, non-linear functions should be used. According to the results, if maximization and minimization are jointly required for the solution of the decision problem, there should not be large differences in the deformation between both cases.

3. Research Methodology

3.1. *Fuzzy Analytic Hierarchy Process (FAHP)*

Analytic Hierarchy Process (AHP), firstly proposed by Saaty (1980), has a wide range of applications in multi-attribute decision making (MADM) methods. AHP uses hierarchical structures to represent a problem and then calculate weights for alternatives according to the judgments of the decision makers in a pair-wise comparison framework. The conventional version of AHP method is often criticized owing to using the exact and crisp judgments of the decision makers. On the other hand, decision makers are more confident about interval judgments than fixed value judgments. Because of the vagueness and ambiguous are inherent of the human judgments and preferences, real life situations can

[2] MAE and MADM are used for different purposes although they are rather similar to each other. MAE usually focuses on the evaluation of all objects involved, while MADM deals with the selection of the optimal decision alternative (Liping *et al*., 2009).

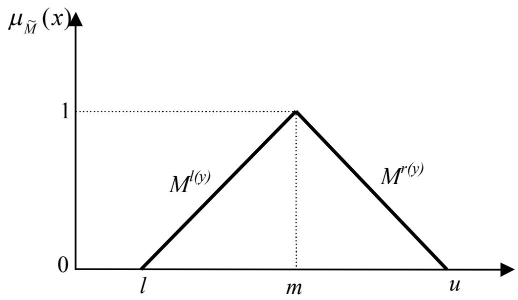

Fig. 1. A triangular membership function, $\mu_{\tilde{M}}(x)$.

be modeled more adequately by using fuzzy values than the exact numerical values. Another handicap of the AHP method is that the preferences in AHP are essentially human judgments based on their subjective perceptions. Therefore, a fuzzy version of the AHP method, called fuzzy analytic hierarchy process (FAHP), has been introduced in order to take into consideration subjective uncertainty of the variables.

A fuzzy number is a special fuzzy set $A = \{x \in R \mid \mu_A(x)\}$, where x takes its values on the real line $\Re^1$: $-\infty < x < +\infty$ and $\mu_A(x)$ is a continuous mapping from to the closed interval [0, 1]. Triangular fuzzy numbers and trapezoidal fuzzy numbers are the most popular fuzzy numbers thanks to their computational simplicity. Triangular fuzzy numbers are preferred for representing the linguistic variables in this study.

A triangular fuzzy number can be donated as $\tilde{M} = (l, m, u)$ and its membership function $\mu_{\tilde{M}}(x) : \Re^1 \to [0, 1]$ can be given as

$$\mu_{\tilde{M}}(x) = \begin{cases} 0, & x < l \text{ or } x > u, \\ (x-l)/(m-l), & l \leqslant x \leqslant m, \\ (x-u)/(m-u), & m \leqslant x \leqslant u, \end{cases} \tag{1}$$

where $l \leqslant m \leqslant u$ and l, m, and u describe the smallest possible value, the most promising value, and the largest possible value of a fuzzy event, respectively. Membership function of a triangular fuzzy number $\tilde{M}$ is illustrated in Fig. 1 (Deng, 1999).

Let $\tilde{M}_1 = (l_1, m_1, u)$, $\tilde{M}_2 = (l_2, m_2, u_2)$ be two triangular fuzzy numbers, the basic operations of triangular fuzzy numbers used in this study are defined as follows (Kaufmann and Gupta, 1991):

$$\begin{aligned} &\tilde{M}_1 \oplus \tilde{M}_2 = (l_1 + l_2, m_1 + m_2, u_1 + u_2), \\ &\tilde{M}_1 \otimes \tilde{M}_2 \approx (l_1 l_2, m_1 m_2, u_1 u_2), \\ &\lambda \otimes \tilde{M}_1 = (\lambda l_1, \lambda m_1 \lambda u_1), \quad \lambda > 0,\ \lambda \in R, \\ &\tilde{M}_1^{-1} \approx (1/u_1, 1/m_1, 1/l_1), \end{aligned} \tag{2}$$

where $l_1, m_1, u_1, l_2, m_2, u_2 > 0$.

In this study, attribute weights of the performance measures are calculated by using extent analysis of Chang (1966). To describe the extent analysis of Chang (1966), firstly let $X = \{x_1, x_2, \ldots, x_n\}$ an object set, and $G = \{g_1, g_2, \ldots, g_n\}$ be a goal set. According to the method, extent analysis for each goal is performed respectively. Therefore, m extent analysis values for each object can be obtained:

$$M_{gi}^1, M_{gi}^2, \ldots, M_{gi}^m, \quad i = 1, 2, \ldots, n, \tag{3}$$

where all M_{gi}^j $(j = 1, 2, \ldots, m)$ are triangular fuzzy numbers. In this framework, Chang's extent analysis can be given as follows (Ertugrul and Karakaşoğlu, 2009):

Step 1: The value of fuzzy synthetic extent with respect to the ith object is defined as follows:

$$S_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1}. \tag{4}$$

To obtain $\sum_{j=1}^{m} M_{gi}^j$, the fuzzy addition operation of m extent analysis values for a particular matrix is performed as follows:

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{5}$$

and to obtain $[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j]^{-1}$, the fuzzy addition operation of M_{gi}^j $(j = 1, 2, \ldots, m)$ values is performed as follows:

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \tag{6}$$

and then the inverse of the vector above is computed as follows:

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} = \left(1 \Big/ \sum_{i=1}^{n} u_i, 1 \Big/ \sum_{i=1}^{n} m_i, 1 \Big/ \sum_{i=1}^{n} l_i \right). \tag{7}$$

Step 2: When $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility $M_2 = (l_2, m_2, u_2) \geqslant M_1 = (l_1, m_1, u_1)$ is defined as

$$V(M_2 \geqslant M_1) = \sup_{y \geqslant x} \left(\min \left(\mu_{M_1}(x), \mu_{M_2}(y) \right) \right) \tag{8}$$

and can be equivalently stated as:

$$V(M_2 \geqslant M_1) = hgt(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \geqslant m_1, \\ 0, & \text{if } l_1 \geqslant u_2, \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise.} \end{cases} \tag{9}$$

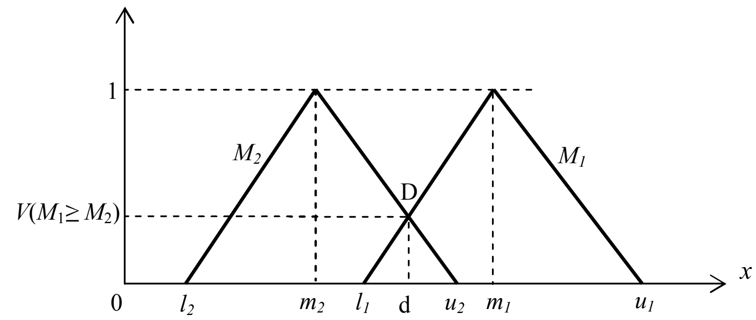

Fig. 2. The intersection between M_1 and M_2.

Figure 2 illustrates Eq. (9) where d is the ordinate of the highest intersection point D between $\mu_{M_1}(x)$ and μ_{M_2} (Zhu *et al.*, 1999). The values of both $V(M_1 \geqslant M_2)$ and $V(M_2 \geqslant M_1)$ are needed to compare M_1 and M_2.

Step 3: The degree of possibility for a fuzzy number to be greater than k fuzzy numbers M_i $(i = 1, 2, \ldots, k)$ can be defined by

$$V(M \geqslant M_1, M_2, \ldots, M_k) = V\big[(M \geqslant M_1) \text{ and } (M \geqslant M_2) \text{ and } (M \geqslant M_k)\big] \quad (10)$$

$$= \min V(M \geqslant M_i), \quad i = 1, 2, \ldots, k. \quad (11)$$

Assume that $d'(A_i) = \min V(S_i \geqslant S_k)$ for $k = 1, 2, \ldots, n$; $k \neq i$. Then the weight vector is given by

$$W' = \big(d'(A_1), d'(A_2), \ldots, d'(A_n)\big)^{\mathrm{T}}, \quad (12)$$

where A_i $(i = 1, 2, \ldots, n)$ are n elements.

Step 4: The normalized weight vectors are obtained by normalization as

$$W = \big(d(A_1), d(A_2), \ldots, d(A_n)\big)^{\mathrm{T}}. \quad (13)$$

3.2. *Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)*

TOPSIS is originally proposed by Hwang and Yoon (1981), and became one of the classical MADM methods. According to this method, alternatives to be evaluated by n attributes are presented as points in an n-dimensional space. A fundamental assumption of TOPSIS is that each attribute has a tendency towards monotonically increasing or decreasing utility. In this method, firstly positive ideal solutions (PIS) and negative ideal solutions (NIS) are determined. The positive ideal solution is a solution that maximizes the benefit attributes and minimizes the cost attributes, whereas the negative ideal solution maximizes the cost attributes and minimizes the benefit attributes (Wang and Elhag, 2006). In short, the positive ideal solution is composed of all best values attainable of attributes, whereas the negative ideal solution consists of all worst values attainable of attributes (Wang, 2008). TOPSIS method considers the distances to both the PIS and the NIS simultaneously by defining "relative closeness to ideal solution". The alternative which is the closest to positive ideal solution and farthest from the negative ideal solution is selected as the best alternative.

To explain the algorithm of TOPSIS, suppose we have m alternatives $(A_1, A_2, \dots, A_m)$, and n decision attributes – criteria $(C_1, C_2, \dots, C_n)$. Each alternative is evaluated with respect to the n attributes. All the rating scores assigned to the alternatives with respect to each attribute form a decision matrix denoted by $X = (x_{ij})_{mxn}$. Let $W = (w_1, w_2, \dots, w_n)$ be the relative weight vector about attributes, satisfying $\sum_{j=1}^{n} w_j = 1$. The algorithm of TOPSIS is as follows (Ertugrul and Karakaşoğlu, 2009):

Step 1: Decision matrix $X = (x_{ij})_{mxn}$ is normalized according to one of the normalization methods described in Section 3.3.

Step 2: Weighted normalized decision matrix $V = (v_{ij})_{mxn}$ is obtained by multiplying normalized matrix with the weights of the attributes:

$$v_{ij} = r_{ij} \cdot w_j, \tag{14}$$

where $i = 1, 2, \dots, m$ and $j = 1, 2, \dots n$.

Step 3: Positive ideal solution (PIS) and negative ideal solution (NIS) are determined:

$$\text{PIS} = \{v_1^+, v_2^+, \dots, v_n^+\} \quad \text{where } v_j^+ = \max_i(v_{ij}), \tag{15}$$

$$\text{NIS} = \{v_1^-, v_2^-, \dots, v_n^-\} \quad \text{where } v_j^- = \min_i(v_{ij}). \tag{16}$$

Step 4: The distance of each alternative from PIS and NIS are calculated:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, \tag{17}$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \tag{18}$$

where $i = 1, 2, \dots, m$.

Step 5: The closeness coefficient of each alternative (CC_i) is calculated:

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad \text{where } i = 1, 2, \dots, m. \tag{19}$$

Step 6: The ranking of the alternatives are determined according to CC_i values: The bigger CC_i, the better the relevant alternative. In other words, the alternative with the highest closeness coefficient is determined as the best alternative.

3.3. *Normalization Procedures*

In this study, the four well known normalization procedures used in MADM are applied separately to the Turkish deposit banking sector. These normalization procedures are (i) vector normalization, (ii) linear scale transformation (*max–min*), (iii) linear scale transformation (*max*) and (iv) linear scale transformation (*sum*). These procedures, denoted by N1, N2, N3 and N4 respectively, are briefly described below. Meanwhile, we called the alternative TOPSIS applications which are based on these normalization procedures as model N1, model N2, model N3 and model N4, respectively.

3.3.1. *Vector Normalization [N1]*

In this procedure, each performance rating of the decision matrix is divided by its norm.

For benefit attributes, the normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}. \tag{20}$$

For cost attributes, r_{ij} is computed as

$$r_{ij} = \frac{(1/x_{ij})}{\sqrt{\sum_{i=1}^{m} (1/x_{ij}^2)}}, \tag{21}$$

where x_{ij} is the performance rating of i-th alternative for attribute C_j.

This procedure has the advantage of converting all attributes into dimensionless measurement unit, thus making inter-attribute comparison easier. But it has the drawback of having non-equal scale length leading to difficulties in straightforward comparison (Chakraborty and Yeh, 2007, 2009).

3.3.2. *Linear Scale Transformation (Max–Min) [N2]*

This method considers both the maximum and minimum performance ratings of attributes during calculation.

For benefit attributes, the normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij} - x_j^{\min}}{x_j^{\max} - x_j^{\min}}. \tag{22}$$

For cost attributes, r_{ij} is computed as

$$r_{ij} = \frac{x_j^{\max} - x_{ij}}{x_j^{\max} - x_j^{\min}}, \tag{23}$$

where x_{ij} is the performance rating of ith alternative for attribute C_j, $x_j^{\max}$ is the maximum performance rating among alternatives for attribute C_j and $x_j^{\min}$ is the minimum performance rating among alternatives for attribute C_j.

This procedure has the advantage that the scale measurement is precisely between 0 and 1 for each attribute. The drawback is that the scale transformation is not proportional to outcome (Chakraborty and Yeh, 2007, 2009).

3.3.3. *Linear Scale Transformation (Max) [N3]*

This method divides the performance ratings of each attribute by the maximum performance rating for that attribute.

For benefit attributes, the normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij}}{x_j^{\max}}. \tag{24}$$

For cost attributes, r_{ij} is computed as

$$r_{ij} = 1 - \frac{x_{ij}}{x_j^{\max}}, \tag{25}$$

where x_{ij} is the performance rating of ith alternative for attribute C_j and $x_j^{\max}$ is the maximum performance rating among alternatives for attribute C_j.

Advantage of this procedure is that outcomes are transformed in a linear way (Chakraborty and Yeh, 2007, 2009).

3.3.4. *Linear Scale Transformation (Sum) [N4]*

This method divides the performance ratings of each attribute by the sum of performance ratings for that attribute.

For benefit attributes, the normalized value r_{ij} is obtained by

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}. \tag{26}$$

For cost attributes, r_{ij} is computed as

$$r_{ij} = \frac{(1/x_{ij})}{\sum_{i=1}^{m}(1/x_{ij})}, \tag{27}$$

where x_{ij} is the performance rating of i-th alternative for attribute C_j.

4. Emprical Results

4.1. *Data*

Turkey suffered from a severe financial crisis in 2001. Since then, the regulations surrounding the financial institutions have been expanded in order to provide resilience to

Table 1
Number of banks, branches, employees and total asset in Turkish banking sector.

	Bank	Branch	Employee	Total asset
Deposit banks	31	9798	176 032	605 570
State-owned banks	3	2894	49 218	185 958
Private banks	11	4996	89 154	334 696
Banks in SDIF	1	1	243	445
Foreign banks	16	1907	37 417	84 472
Dev't. and inv. banks	13	43	5 245	20 472
Total	44	9841	181 277	626 043

Notes: (1) Source: BAT. (2) Total assets are in million USD.

both domestic and external financial fluctuations. Thanks to these heavy regulations, Turkish banks are able to remain well-capitalized, sturdy and profitable with strong balance sheets during the current global financial crisis. In Turkey, Banking Regulation and Supervision Agency (BRSA), the regulatory body of the banking sector, is responsible from preserve the rights and benefits of depositors. The Banks Association of Turkey (BAT), the representative body of the banking sector, protects and promotes the professional interests of banks.

Turkish financial sector shows annual growth of 20% between 2002 and 2010. Although Turkish insurance sector grows more rapidly with 25% during the same period, Turkish financial sector is still dominated by banks: According to the asset size, 77% of the assets belong to the banks. As can be seen from Table 1, as of February 2012, Turkey has 44 banks in total, 31 of them being deposit and 13 development and investment banks. Amongst deposit banks, there are 3 state-owned banks, 11 privately-owned banks and 16 foreign banks. The Saving Deposits Insurance Fund (SDIF) owns 1 bank. As parallel to the growth in the financial market in Turkey, the number of branches and employees of banks increase continuously. As of February 2012, the number of branches and employees reach to 9 841 and 181 277, respectively. Total asset of the banking sector is approximately 606 billion USD. Almost all of this total is owned by the deposit banks. Indeed, the deposit banks dominate not only banking sector, but also all financial sector.

In this study, we aimed to measure the financial performances of Turkish deposit banks. Among the 31 deposit banks, we selected the largest 13 banks from three segments (state-owned, private and foreign) of the sector. The deposit banks studied are; (i) state-owned: TC Ziraat Bankası, Türkiye Halk Bankası, Türkiye Vakıflar Bankası; (ii) private: Akbank, Şekerbank, Türk Ekonomi Bankası, Türkiye Garanti Bankası, Türkiye İş Bankası, Yapı ve Kredi Bankası; (iii) foreign: Denizbank, Finans Bank, HSBC Bank and ING Bank. Selecting these banks, according to the asset size, we can study 100% of the state-owned deposit banks, 56% of private banks and 81% foreign banks. On the whole, 73% of deposit banks and 71% of all banking sector is covered in our study. Meanwhile, the data used in this study belongs to year 2010, and is obtained from BAT.

4.2. *Application*

This study analyzes the financial performances of Turkish deposit banks by using financial ratios of the banks. For this aim, FAHP and TOPSIS methods are integrated. While FAHP

Table 2
Scales for pair-wise comparison.

Preferences in linguistic variables	Preferences in numeric variables
Equal importance	1
Moderate importance	3
Strong importance	5
Very strong importance	7
Extreme importance	9
Intermediate values if necessary	2, 4, 6, 8

is used for determining the weights of main and sub-attributes in the light of opinions of an expert group, the TOPSIS method is used for evaluating the performances of the banks. To convert the different financial ratios into a comparable measurement unit, four different normalization procedures which are described in Section 3.3 are used.

Although there are many types of financial ratios in the evaluation of banks' performances, evaluation results can vary according to the different ratios. A bank indicating a high performance according to one ratio may have a very low performance according to another ratio (Seçme *et al.*, 2009). For this reason, we tried to obtain the evaluations of the expert group regarding the relative importance of all available financial ratios.[3] To determine the relative importance of two attributes, Saaty' 1–9 scale (Saaty, 1980), illustrated in Table 2, is employed.[4]

Financial ratios which are 29 in total are grouped under 6 main attributes. These main attributes are Capital Ratios, Balance-Sheet Ratios, Assets Quality, Liquidity Ratios, Profitability Ratios and Income-Expenditure Structure. The abbreviations denoting financial attributes and their meanings are presented in Table 3. This table also includes the calculated weights for all main and sub-attributes in parentheses.

In constructing the triangular fuzzy numbers from the decision makers' pair-wise comparison grades, we used respectively the minimum and maximum grades given by decision makers for the lower (l) and upper (u) bound of the relevant fuzzy number. As for the most promising value of the fuzzy number, we used the arithmetic mean of the grades given by decision makers. The pair-wise comparison matrix including the fuzzy numbers calculated is presented in Table 4.

Then using the fuzzy numbers in comparison matrix, synthesis values respect to main attributes calculated as in Eq. (4):

$$\begin{aligned} S_{CR} &= (15.33, 28.15, 42) \otimes (1/206.29, 1/97.48, 1/28.51) \\ &= (0.0743, 0.2888, 1.4733), \end{aligned}$$

[3]The expert group consists of three decision makers. The first decision maker is selected from a state-owned deposit bank while the second one is from private deposit bank. And the third one is an academician with a considerable experience on banking.

[4]Here we only explained how the weights for main criteria are calculated. The explanations regarding sub-criteria were not presented here, but may be provided upon request.

Table 3
Hierarchy of the attributes set.

Attribute	Explanation and calculated weight
CR	**Capital ratios (0.20)**
CR1	Shareholders' equity/(amount subject to credit + market + operational risk) (0.20)
CR2	Shareholders' equity/total assets (0.21)
CR3	(Shareholders' equity – permanent assets)/total assets (0.21)
CR4	Net on balance sheet position/total Shareholders' equity (0.18)
CR5	Net on and off balance sheet position/total Shareholders' equity (0.20)
BR	**Balance-sheet ratios (0.06)**
BR1	TC assets/total assets (0.16)
BR2	TC liabilities/total liabilities (0.18)
BR3	FC assets/FC liabilities (0.17)
BR4	TC deposits/total deposits (0.15)
BR5	Total deposits/total assets (0.17)
BR6	Funds borrowed/total assets (0.17)
AQ	**Assets quality (0.20)**
AQ1	Financial assets (net)/total assets (0.16)
AQ2	Total loans and receivables/total assets (0.17)
AQ3	Total loans and receivables/total deposits (0.17)
AQ4	Loans under follow-up (net)/total loans and receivables (0.16)
AQ5	Specific provisions/loans under follow-up (0.17)
AQ6	Permanent assets/total assets (0.17)
LR	**Liquidity ratios (0.19)**
LR1	Liquid assets/total assets (0.43)
LR2	Liquid assets/short-term liabilities (0.28)
LR3	TC Liquid assets/total assets (0.29)
PR	**Profitability ratios (0.20)**
PR1	Net profit/losses/total assets (0.18)
PR2	Net profit/losses/total Shareholders' equity (0.44)
PR3	Profit/losses before taxes after continuing operations/total assets (0.38)
IE	**Income-expenditure structure (0.16)**
IE1	Net interest income after specific provisions/total assets (0.19)
IE2	Net interest income after specific provisions/total operating income (0.20)
IE3	Non-interest income (net)/total assets (0.16)
IE4	Other operating expenses/total assets (0.12)
IE5	Personnel expenses/other operating expenses (0.16)
IE6	Non-interest income (net)/other operating expenses (0.17)

Table 4
Fuzzy pair-wise comparison matrix.

	CR	BR	AQ	LR	PR	IE
CR	(1, 1, 1)	(7, 8.33, 9)	(0.11, 2.7, 7)	(0.11, 5.37, 9)	(0.11, 2.41, 7)	(7, 8.33, 9)
BR	(0.11, 0.12, 0.14)	(1, 1, 1)	(0.11, 0.41, 1)	(0.11, 0.41, 1)	(0.11, 0.41,1)	(0.11, 0.41, 1)
AQ	(0.14, 3.38, 9)	(1, 6.33, 9)	(1, 1, 1)	(0.11, 4.7, 9)	(0.11, 4.7, 9)	(1, 5, 9)
LR	(0.11, 3.08, 9)	(1, 6.33, 9)	(0.11, 3.1, 9)	(1, 1, 1)	(0.11, 3.37, 9)	(0.11, 3.37, 9)
PR	(0.14, 6.05, 9)	(1, 6.33, 9)	(0.11, 3.1, 9)	(0.11, 3.37, 9)	(1, 1, 1)	(1, 6.33, 9)
IE	(0.11, 0.12, 0.14)	(1, 6.33, 9)	(0.11, 0.44, 1)	(0.11, 3.37, 9)	(0.11, 0.41,1)	(1, 1, 1)

$$
\begin{aligned}
S_{BR} &= (1.56, 2.75, 5.14) \otimes (1/206.29, 1/97.48, 1/28.51) \\
&= (0.0075, 0.0282, 0.1804), \\
S_{AQ} &= (3.37, 25.12, 46) \otimes (1/206.29, 1/97.48, 1/28.51) \\
&= (0.0163, 0.2577, 1.6136), \\
S_{LR} &= (2.44, 20.26, 46) \otimes (1/206.29, 1/97.48, 1/28.51) \\
&= (0.0118, 0.2079, 1.6136), \\
S_{PR} &= (3.37, 26.19, 46) \otimes (1/206.29, 1/97.48, 1/28.51) \\
&= (0.0163, 0.2687, 1.6136), \\
S_{IE} &= (2.44, 11.67, 21.14) \otimes (1/206.29, 1/97.48, 1/28.51) \\
&= (0.0118, 0.1197, 0.7416).
\end{aligned}
$$

By using Eq. (9), fuzzy numbers are compared:

$$
\begin{aligned}
&V(S_{CR} \geqslant S_{BR}) = 1, \quad V(S_{CR} \geqslant S_{AQ}) = 1, \quad V(S_{CR} \geqslant S_{LR}) = 1, \\
&V(S_{CR} \geqslant S_{PR}) = 1, \quad V(S_{CR} \geqslant S_{IE}) = 1, \quad V(S_{BR} \geqslant S_{CR}) = 0.29, \\
&V(S_{BR} \geqslant S_{AQ}) = 0.42, \quad V(S_{CR} \geqslant S_{LR}) = 0.48, \quad V(S_{CR} \geqslant S_{PR}) = 0.41, \\
&V(S_{CR} \geqslant S_{IE}) = 0.65, \quad V(S_{AQ} \geqslant S_{CR}) = 0.98, \quad V(S_{AQ} \geqslant S_{BR}) = 1, \\
&V(S_{AQ} \geqslant S_{LR}) = 1, \quad V(S_{AQ} \geqslant S_{PR}) = 0.99, \quad V(S_{AQ} \geqslant S_{IE}) = 1, \\
&V(S_{LR} \geqslant S_{CR}) = 0.95, \quad V(S_{LR} \geqslant S_{BR}) = 1, \quad V(S_{LR} \geqslant S_{AQ}) = 0.97, \\
&V(S_{LR} \geqslant S_{PR}) = 0.96, \quad V(S_{LR} \geqslant S_{IE}) = 1, \quad V(S_{PR} \geqslant S_{CR}) = 0.99, \\
&V(S_{PR} \geqslant S_{BR}) = 1, \quad V(S_{PR} \geqslant S_{AQ}) = 1, \quad V(S_{PR} \geqslant S_{LR}) = 1, \\
&V(S_{PR} \geqslant S_{IE}) = 1, \quad V(S_{IE} \geqslant S_{CR}) = 0.8, \quad V(S_{IE} \geqslant S_{BR}) = 1, \\
&V(S_{IE} \geqslant S_{AQ}) = 0.84, \quad V(S_{IE} \geqslant S_{LR}) = 0.89, \quad V(S_{IE} \geqslant S_{PR}) = 0.83.
\end{aligned}
$$

Then, according to Eq. (11), priority weights are calculated:

$$
\begin{aligned}
&d'(CR) = \min(1, 1, 1, 1, 1) = 1, \\
&d'(BR) = \min(0.29, 0.42, 0.48, 0.41, 0.65) = 0.29, \\
&d'(AQ) = \min(0.98, 1, 1, 0.99, 1) = 0.98, \\
&d'(LR) = \min(0.95, 1, 0.97, 0.96, 1) = 0.95, \\
&d'(PR) = \min(0.99, 1, 1, 1, 1) = 0.99, \\
&d'(IE) = \min(0.8, 1, 0.84, 0.89, 0.83) = 0.8.
\end{aligned}
$$

Table 5
Total values of main attributes for model N1.

Banks	CR	BR	AQ	LR	PR	IE
TC Ziraat Bankası	0.158	0.443	0.259	0.304	0.372	0.269
Türkiye Halk Bankası	0.085	0.253	0.218	0.149	0.396	0.285
Türkiye Vakıflar Bankası	0.182	0.243	0.231	0.258	0.213	0.259
Akbank	0.211	0.223	0.387	0.405	0.305	0.293
Şekerbank	0.090	0.240	0.206	0.220	0.198	0.255
Türk Ekonomi Bankası	0.145	0.231	0.226	0.277	0.216	0.246
Türkiye Garanti Bankası	0.192	0.225	0.223	0.340	0.326	0.290
Tr̈kiye İş Bankası	0.220	0.237	0.321	0.271	0.288	0.273
Yapı ve Kredi Bankası	0.117	0.218	0.197	0.132	0.320	0.291
Denizbank	0.181	0.201	0.210	0.226	0.228	0.238
Finans Bank	0.025	0.224	0.228	0.271	0.303	0.260
HSBC Bank	0.049	0.198	0.202	0.375	0.167	0.253
ING Bank	0.075	0.212	0.223	0.234	0.088	0.206

When we normalize these priority weights of main attributes, we obtain the weight of 0.20 for Capital Ratios, 0.06 for Balance-Sheet Ratios, 0.20 for Assets Quality, 0.19 for Liquidity Ratios, 0.20 for Profitability Ratios and 0.16 for Income-Expenditure Structure. Accordingly, Capital Ratios, Assets Quality and Profitability Ratios are seen almost equally as the most important attribute while Balance-Sheet Ratios is evaluated as the least important attribute.

After determining the weights of the all main and sub-attributes (financial performance attributes), we proceed to the application of TOPSIS method. Financial performance attributes are normalized by using the normalization procedures explained in Section 3.3.[5] In doing this, all sub-attributes – except BR6, AQ4, AQ6, IE4 and IE5, are considered as benefit attributes rather than cost attributes. After getting the normalized matrix, we multiply each normalized value of sub-attributes with their weights according to Eq. (14). Then, these weighted normalized values of sub-attributes under each main attribute are aggregated, and Table 5 is obtained. At the end of application, the total values of main attributes are multiplied by the weights of the main attributes $(0.20, 0.06, 0.20, 0.19, 0.20, 0.16)$, and total weighted values of main attributes (Table 6) are obtained.

After calculating total weighted values of main attributes, positive ideal solution (PIS) and negative ideal solution (NIS) are determined by taking the maximum and minimum values for each attribute according to Eqs. (15) and (16):

$$\text{PIS} = \{0.044, 0.026, 0.076, 0.077, 0.078, 0.047\} \text{ maximum values,}$$

$$\text{NIS} = \{0.005, 0.011, 0.039, 0.025, 0.017, 0.033\} \text{ minimum values.}$$

Then, the distance of each bank from the positive ideal solution and negative ideal solution with respect to each attribute is calculated by using Eqs. (17) and (18). Distances from positive ideal solution and negative ideal solution are presented in Table 7.

[5]In the remaining of this section, we present explanations regarding only the application of the vector normalization (N1) to save space. The explanations about the application of other normalization procedures may be provided upon request.

Table 6
Total weighted values of main attributes for model N1.

Banks	CR	BR	AQ	LR	PR	IE
TC Ziraat Bankası	0.032	0.026	0.051	0.058	0.073	0.043
Türkiye Halk Bankası	0.017	0.015	0.043	0.028	0.078	0.045
Türkiye Vakıflar Bankası	0.036	0.014	0.045	0.049	0.042	0.041
Akbank	0.042	0.013	0.076	0.077	0.060	0.047
Şekerbank	0.018	0.014	0.040	0.042	0.039	0.041
Türk Ekonomi Bankası	0.029	0.013	0.044	0.053	0.043	0.039
Türkiye Garanti Bankası	0.038	0.013	0.044	0.065	0.064	0.046
Türkiye İş Bankası	0.044	0.014	0.063	0.051	0.057	0.043
Yapı ve Kredi Bankası	0.023	0.013	0.039	0.025	0.063	0.046
Denizbank	0.036	0.012	0.041	0.043	0.045	0.038
Finans Bank	0.005	0.013	0.045	0.051	0.060	0.041
HSBC Bank	0.010	0.011	0.039	0.071	0.033	0.040
ING Bank	0.015	0.012	0.044	0.044	0.017	0.033

Table 7
Distances from positive ideal solution and negative ideal solution for model N1.

Banks	Distance from PIS	Distance from NIS
TC Ziraat Bankası	0.034	0.073
Türkiye Halk Bankası	0.066	0.064
Türkiye Vakıflar Bankası	0.057	0.048
Akbank	0.022	0.086
Şekerbank	0.070	0.032
Türk Ekonomi Bankası	0.057	0.045
Türkiye Garanti Bankası	0.040	0.071
Türkiye İş Bankası	0.038	0.067
Yapı ve Kredi Bankası	0.070	0.051
Denizbank	0.061	0.046
Finans Bank	0.060	0.051
HSBC Bank	0.069	0.049
ING Bank	0.084	0.022

Once the distances from positive ideal solution and negative ideal solution are determined, the closeness coefficients of utilities (CC_i) are calculated by Eq. (19). And finally, according to the closeness coefficient values, the rankings of the banks are determined, as presented in Table 8.

4.3. *Consistency Search*

In the way of searching consistency between financial performance results of our models, we were inspired from Bauer *et al.* (1998) setting the conditions of the consistency between different performance estimation methods. According to Bauer *et al.* (1998) the performance estimates from different approaches should be consistent in their efficiency levels, rankings, and identification of best and worst firms, consistent over time and with competitive conditions in the markets and consistent with standard non-frontier measures of performance. Among these conditions, only four are found to be important with respect to our study. These conditions can be expressed as follows:

Table 8
Rankings of banks according to closeness coefficient values for model N1.

Ranking	Banks	Closeness coefficient
1	Akbank	0.796
2	TC Ziraat Bankası	0.682
3	Türkiye Garanti Bankası	0.644
4	Türkiye İş Bankası	0.640
5	Türkiye Halk Bankası	0.492
6	Finans Bank	0.459
7	Türkiye Vakıflar Bankası	0.458
8	Türk Ekonomi Bankası	0.442
9	Denizbank	0.427
10	Yapı ve Kredi Bankası	0.423
11	HSBC Bank	0.417
12	Şekerbank	0.312
13	ING Bank	0.212

Table 9
Statistics of closeness coefficient values.

Statistic	Model N1	Model N2	Model N3	Model N4
Mean	0.493	0.482	0.546	0.529
Standard deviation	0.159	0.163	0.224	0.217
Minimum	0.212	0.218	0.149	0.065
Maximum	0.796	0.805	0.825	0.858

Condition 1: Alternative models should generate performance measures which have similar distributional properties such as means, standard deviations, minimum and maximum values.
Condition 2: Alternative models should identify mostly the same banks as the “best performers” and as the “worst performers”.
Condition 3: Alternative models should rank the banks mostly in the same order.
Condition 4: Alternative models should generate the same performance scores for banks.

We ordered these consistency conditions according to their easiness to be fulfilled. In other words, Condition 1 can be seen as the easiest condition while Condition 4 seems to be the most difficult one.

Condition 1: Statistics of performance measures generated from different models are tabulated in Table 9. By just examining the relevant figures in this table, one cannot draw healthy conclusions whether the performance measures from different models have similar distributional properties. To test this condition statistically, we applied the Kolmogorov–Smirnov test to the performance measures. According to the Kolmogorov–Smirnov test statistics, illustrated in Table 10, performance measures generated from different models are not statistically different from each other. In other words, all models seem to satisfy the first consistency condition.

Table 10
Kolmogorov–Smirnov test statistics.

Models	D-value	P-value
Model N1 – Model N2	0.31	0.57
Model N1 – Model N3	0.38	0.29
Model N1 – Model N4	0.31	0.57
Model N2 – Model N3	0.38	0.29
Model N2 – Model N4	0.31	0.57
Model N3 – Model N4	0.23	0.88

Table 11
The best and the worst performers.

Ranking	Model N1	Model N2	Model N3	Model N4
1	Akbank	Akbank	Türkiye Garanti Bankası	Finans Bank
2	TC Ziraat Bankası	Türkiye Garanti Bankası	Akbank	HSBC Bank
3	Türkiye Garanti Bankası	TC Ziraat Bankası	Türkiye İş Bankası	Türkiye Halk Bankası
11	HSBC Bank	Denizbank	HSBC Bank	Türkiye Vakıflar Bankası
12	Şekerbank	Şekerbank	Denizbank	Türkiye İş Bankası
13	ING Bank	ING Bank	ING Bank	Denizbank

Condition 2: To evaluate whether the same banks can be determined as the "best performers" and as the "worst performers" in different models, we examined the banks having the highest three and the lowest three performance scores, which are illustrated in Table 11. The most striking observation from this table is that model N4 identified completely different banks as best and worst performers in comparison to the other models. As for this condition, the other three models (model N1, model N2 and model N3) can generate rather consistent results. For example, ING Bank is identified as the worst performer in all three models, while Akbank and Türkiye Garanti Bankası are located among the best three performers according to these models. Therefore, all alternative models – except model N4 seem to satisfy the second condition.

Condition 3: To see whether alternative models should rank the banks in the same order, we firstly ranked all banks according to their performance scores. The rankings of the banks in different models are presented in Table 12. Then, Pearson correlations test is applied to the rankings generated from different models. The result of the correlations test can be seen from Table 13. Accordingly, the correlations among the first three alternative models (model N1, model N2 and model N3) are high. Especially mutual consistency between model N1 and model N2 is found to be very high. In contrast, no correlation can be detected between model N4 and the remaining models. Thus, one may safely claim that the model N1, model N2 and model N3 rank the banks in a similar order and fulfill the third condition, while the model N4 fails to fulfill this condition.

Table 12
Rankings according to alternative normalization methods.

Banks	Model N1	Model N2	Model N3	Model N4
TC Ziraat Bankası	2	3	4	9
Türkiye Halk Bankası	5	7	8	3
Türkiye Vakıflar Bankası	7	8	5	11
Akbank	1	1	2	8
Şekerbank	12	12	9	4
Türk Ekonomi Bankası	8	9	7	10
Türkiye Garanti Bankası	3	2	1	6
Türkiye İş Bankası	4	4	3	12
Yapı ve Kredi Bankası	10	10	6	5
Denizbank	9	11	12	13
Finans Bank	6	5	10	1
HSBC Bank	11	6	11	2
ING Bank	13	13	13	7

Table 13
Correlations between rankings of alternative models.

	Model N1	Model N2	Model N3	Model N4
Model N1	1.00	0.90	0.80	−0.21
Model N2		1.00	0.74	0.03
Model N3			1.00	−0.27
Model N4				1.00

Condition 4: This condition requires that alternative models should generate the same performance scores. Similar to the previous ranking consistency, using Pearson correlations test, we examined the correlations between performance scores of the banks in different models. Performance scores of the alternative models are tabulated in Table 14, while the relevant correlation statistics are illustrated in Table 15. As can be seen from Table 15, the consistency among the first three models (model N1, model N2 and model N3) continues with respect to the fourth condition. In addition, we observe an almost perfect correlation between performance scores of the model N1 and model N2. In contrast, model N4 generates almost irrelevant performance scores for banks. As a result, the fourth consistency condition is fulfilled by all models – except model N4.

5. Conclusion

In this study, we aimed to determine the effects of different normalization procedures on decision outcomes of a given MADM method. In other words, the suitability of a specific normalization procedure for a given MADM method was searched. For this aim, we applied FAHP and TOPSIS methods to assess the financial performances of 13 Turkish deposit banks. In FAHP, three decision makers selected made pair-wise comparisons for main (6 in total) and sub-attributes (29 in total). Then, by taking into account the triangular fuzzy numbers generated from these pair-wise comparisons, the weights of main

Table 14
Closeness coefficient values according to alternative normalization methods.

Banks	Model N1	Model N2	Model N3	Model N4
TC Ziraat Bankası	0.682	0.630	0.732	0.446
Türkiye Halk Bankası	0.492	0.476	0.595	0.696
Türkiye Vakıflar Bankası	0.458	0.409	0.669	0.379
Akbank	0.796	0.805	0.786	0.545
Şekerbank	0.312	0.302	0.506	0.681
Türk Ekonomi Bankası	0.442	0.397	0.603	0.417
Türkiye Garanti Bankası	0.644	0.692	0.825	0.572
Türkiye İş Bankası	0.640	0.555	0.750	0.257
Yapı ve Kredi Bankası	0.423	0.394	0.618	0.632
Denizbank	0.427	0.357	0.230	0.065
Finans Bank	0.459	0.539	0.363	0.858
HSBC Bank	0.417	0.496	0.275	0.776
ING Bank	0.212	0.218	0.149	0.547

Table 15
Correlations between closeness coefficient values of alternative models.

	Model N1	Model N2	Model N3	Model N4
Model N1	1.00	0.95	0.78	−0.18
Model N2		1.00	0.68	0.10
Model N3			1.00	−0.10
Model N4				1.00

and sub-attributes are calculated. In the TOPSIS stage, first of all, the decision matrix was formed, and then four different TOPSIS models were generated by help of four different normalization procedures (1 non-linear normalization procedure (vector normalization) and 3 linear normalization procedures (*max–min*, *max* and *sum*)). Then, positive ideal solution and negative ideal solution were defined, and the distance of each bank from positive ideal solution and negative ideal solution was calculated. According to the closeness coefficients which are calculated from distances from positive ideal solution and negative ideal solution, the financial performance ranking of the banks was determined in each alternative TOPSIS model. And finally, we compared the financial performance results of the alternative models based on different normalization procedures by using the consistency conditions set by Bauer *et al*. (1998).

According to the results of FAHP method, Capital Ratios, Assets Quality and Profitability Ratios were seen almost equally as the most important attributes. On the other hand, Balance-Sheet Ratios was evaluated as the least important attribute. As for the consistency between financial performance results of alternative models, the non-linear normalization procedure (vector normalization) and two of the linear normalization procedures (*max–min* and *max*) generated rather consistent results. The consistency conditions of Bauer *et al*. (1998) are satisfied by the models generated from these three normalization procedures. In contrast, the remaining linear normalization procedure (*sum*) generated rather irrelevant performance results, and thus failed to satisfy the consistency conditions. In other words, our study justified the use of vector normalization procedure with the

TOPSIS method. This finding is line with Chakraborty and Yeh (2009). It has been also shown that two linear normalization procedures (*max–min* and *max*) are possible alternatives to the vector normalization procedure.

It is certainly true that result of a given MADM method will be more reliable when its decision outcome does not vary significantly depending on the normalization procedure used. In the light of the limited number of comparative studies in the literature, normalization procedures may affect the decision outcome of a MADM method. Thus, we strongly suggest the application of a given MADM method with different normalization procedures instead of relying on just one normalization procedure by default.

References

Antuchevičienė, J., Zakarevičius, A., Zavadskas, E.K. (2011). Measuring congruence of ranking results applying particular MCDM methods. *Informatica*, 22(3), 319–338.

Antuchevičienė, J., Zavadskas, E.K., Zakarevičius, A. (2010). Multiple criteria construction management decisions considering relations between criteria. *Technological and Economic Development of Economy*, 16(1), 109–125.

Bauer, P.W., Allen, N.B., Gary, D.F., Humphrey, D.B. (1998). Consistency conditions for regulatory analysis of financial institutions: a comparison of frontier efficiency methods. *Journal of Economics and Business*, 50, 85–114.

Brauers, W.K., Zavadskas, E.K. (2006). The MOORA method and its application to privatization in a transition economy. *Control and Cybernetics*, 35(2), 443–468.

Brauers, W.K., Ginevičius, M.R., Zavadskas, E.K., Antuchevičienė, J. (2007). The European Union in a transition economy. *Transformations in Business and Economics*, 6(2), 21–37.

Chakraborty, S. (2011). Applications of the MOORA method for decision making in manufacturing environment. *The International Journal of Advanced Manufacturing Technology*, 54, 1155–1166.

Chakraborty, S., Yeh, C.H. (2007). A simulation based comparative study of normalization procedures in multiattribute decision making. In: *Proceedings of the 6th WSEAS International Conference on Artificial Intelligence, Knowledge Engineering and Data Bases*, Corfu Island, Greece.

Chakraborty, S., Yeh, C.H. (2009). A simulation comparison of normalization procedures for TOPSIS. In: *Computers Industrial Engineering CIE 2009 International Conference*.

Chang, D.Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research*, 95, 649–655.

Chatterjee, P., Chakraborty, S. (2012). Material selection using preferential ranking methods. *Materials and Design*, 35, 384–393.

Chatterjee, P., Athawale, V.M., Chakraborty, S. (2011). Materials selection using complex proportional assessment and evaluation of mixed data methods. *Materials and Design*, 32, 851–860.

Deng, H., (1999). Multicriteria analysis with fuzzy pair-wise comparison. *International Journal of Approximate Reasoning*, 21, 215–231.

Ertugrul, I., Karakaşoğlu, N. (2009). Performance evaluation of Turkish cement firms with fuzzy analytic hierarchy process and TOPSIS methods. *Expert Systems with Applications*, 36(1), 702–715.

Ginevičius, R. (2008). Normalization of quantities of various dimensions. *Journal of Business Economics and Management*, 9(1), 79–86.

Ginevičius, R., Podvezko, V. (2008). Multicriteria evaluation of Lithuanian banks from the perspective of their reliability for clients. *Journal of Business Economics and Management*, 9(4), 257–267.

Gumus, A.T. (2009). Evaluation of hazardous waste transportation firms by using a two step fuzzy-AHP and TOPSIS methodology. *Expert Systems with Applications*, 36(2), 4067–4074.

Hwang, C.L., Yoon, K. (1981). *Multiple Attributes Decision Making Methods and Applications*. Springer, Berlin.

Jüttler, H. (1966). *Untersuchungen zur Fragen der Operationsforschung und ihrer Anwendungsmöglichkeiten auf ökonomische Problemstellungen unter besonderer Berücksichtigung der Spieltheorie*. Dissertation A an der Wirtschaftswissenschaftlichen Fakultät der Humboldt-Universität Berlin.

Kaklauskas, A., Zavadskas, E.K., Banaitis, A., Šatkauskas, G. (2007). Defining the utility and market value of real estate: a multiple criteria approach. *International Journal of Strategic Property Management*, 11(2), 107–120.

Kaufmann, A., Gupta, M.M. (1991). *Introduction to Fuzzy Arithmetic: Theory and Applications*. Van Nostrand–Reinhold, New York.

Körth, H. (1969). Zur Berücksichtigung mehrer Zielfunktionen bei der Optimierung von Produktionsplanen. *Mathematik und Wirtschaft*, 6, 184–201.

Lai, Y.J., Hwang, C.L. (1994). *Fuzzy Multiple Objective Decision Making: Methods and Applications*. Springer, Berlin.

Liping Y., Yuntao, P., Yishan, W. (2009). Research on data normalization methods in multi-attribute evaluation. In: *International Conference on Computational Intelligence and Software Engineering*, pp. 1–5.

Migilinskas, D., Ustinovichius, L. (2007). Normalisation in the selection of construction alternatives. *International Journal of Management and Decision Making*, 8(5–6), 623–639.

Milani, A.S., Shanian, R., Madoliat, R., Nemes, J.A. (2005). The effect of normalization norms in multiple attribute decision making models: a case study in gear material selection. *Structural and Multidisciplinary Optimization*, 29(4), 312–318.

Opricovic, S., Tzeng, G.H. (2004). Compromise solution by MCDM methods: comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, 156(2), 445–455.

Özcan, T., Çelebi, N., Esnaf, Ş. (2011). Comparative analysis of multi-criteria decision making methodologies and implementation of a warehouse location selection problem. *Expert Systems with Applications*, 38(8), 9773–9779.

Pavlicic, D. (2001). Normalization affects the results of MADM methods. *Yugoslav Journal of Operations Research*, 11(2), 251–265.

Peldschus, F. (2007). The effectiveness of assessment in multiple criteria decisions. *International Journal of Management and Decision Making*, 8(5–6), 519–526.

Peldschus, F. (2009). The analysis of the quality of the results obtained with the methods of multi-criteria decisions. *Technological and Economic Development of Economy*, 15(4), 580–592.

Peldschus, F., Vaigauskas, E., Zavadskas, E.K. (1983). Technologische Entscheidungen bei der Berücksichtigung mehrerer Ziehle. *Bauplanung–Bautechnik*, 37(4), 173–175.

Saaty, T.L. (1980). *The Analytic Hierarchy Process*. McGraw-Hill, New York.

Seçme, N.Y., Bayrakdaroğlu, A., Kahraman, C. (2009). Fuzzy performance evaluation in Turkish banking sector using analytic hierarchy process and TOPSIS. *Experts Systems with Applications*, 36(9), 11699–11709.

Stopp, F. (1975). Variantenvergleich durch Matrixspiele. *Wissenschaftliche Zeitschrift der Hochschule für Bauwesen Leipzig*, 2, 117.

Sun, C.C. (2010). A performance evaluation model by integrating fuzzy AHP and fuzzy TOPSIS methods. *Expert Systems with Applications*, 37(12), 7745–7754.

Sun, C.C., Lin, G.T.R. (2009). Using fuzzy TOPSIS method for evaluating the competitive advantages of shopping websites. *Expert Systems with Applications*, 36(9), 11764–11771.

Torlak, G., Sevkli, M., Sanal, M., Zaim, S. (2011). Analyzing business competition by using fuzzy TOPSIS method: an example of Turkish domestic airline industry. *Expert Systems with Applications*, 38(4), 3396–3406.

Turskis, Z., Zavadskas, E.K., Zagorskas, J. (2006). Sustainable city compactness evaluation of the basis of GIS and Bayes rule. *International Journal of Strategic Property Management*, 10(3), 185–207.

Turskis, Z., Zavadskas, E.K., Peldschus, F. (2009). Multi-criteria optimization system for decision making in construction design and management. *Economics of Engineering Decisions*, 1(61) 7–17.

Viteikiene, M., Zavadskas, E.K. (2007). Evaluating the sustainability of Vilnius city residential areas. *Journal of Civil Engineering and Management*, 13(2), 149–155.

Wang, T.C., Chang, T.H. (2007). Application of TOPSIS in evaluating initial training aircraft under a fuzzy environment. *Expert Systems with Applications*, 33(4), 870–880.

Wang, Y.M., Elhag, T.M.S. (2006). Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment. *Expert Systems with Applications*, 31(2), 309–319.

Wang, Y.J. (2008). Applying FMCDM to evaluate financial performance of domestic airlines in Taiwan. *Expert Systems with Applications*, 34(3), 1837–1845.

Weitendorf, D. (1976). *Beitrag zur Optimierung der räumlichen Struktur eines Gebäudes*. Dissertation A, Hochschule für Architektur und Bauwesen, Weimar.

Wu, H.Y., Tzeng, G.H., Chen, Y.H. (2009). A fuzzy MCDM approach for evaluating banking performance based on Balanced Scorecard. *Expert Systems with Applications*, 36(6), 10135–10147.

Yalçın, N., Bayrakdaroğlu, A., Kahraman, C. (2012). Application of fuzzy multi-criteria decision making methods for financial performance evaluation of Turkish manufacturing industries. *Experts Systems with Applications*, 39(1), 350–364.

Yeh, C.H. (2003). The selection of multiattribute decision making methods for scholarship student selection. *International Journal of Selection and Assessment*, 11(4), 289–296.

Zavadskas, E.K., Turskis, Z. (2008). A new logarithmic normalization method in games theory, *Informatica*, 19(2), 303–314.

Zavadskas, E. K., Turskis, Z. (2010). A new additive ratio assessment (ARAS) method in multicriteria decision-making. *Technological and Economic Development of Economy*, 16(2), 159–172.

Zavadskas, E.K., Turskis, Z. (2011). Multiple criteria decision making (MCDM) methods in economics: an overview. *Technological and Economic Development of Economy*, 17(2) 397–427.

Zavadskas, E.K., Peldschus, F., Ustinovichius, L. (2003). Development of software for multiple criteria evaluation. *Informatica*, 14(2), 259–272.

Zavadskas, E.K., Zakarevicius, A., Antucheviciene, J. (2006). Evaluation of ranking accuracy in multi-criteria decisions. *Informatica*, 17(4), 601–617.

Zavadskas, E. K., Turskis, Z., Antuchevičienė, J., Zakarevicius, A. (2012). Optimization of weighted aggregated sum product assessment. *Elektronics and Electrical Engineering*, 122(6), 3–6.

Zeydan, M., Çolpan, C., Çobanoğlu, C. (2011). A combined methodology for supplier selection and performance evaluation. *Experts Systems with Applications*, 38(3), 2741–2751.

Zhu, K., Jing, Y., Chang, D. (1999). A discussion on extent analysis method and applications of fuzzy AHP. *European Journal of Operational Research*, 116(2), 450–456.

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Normalizavimo metodų lyginamoji analizė TOPSIS metodu: Turkijos banko indėlių rinkos tyrimas

Aydın ÇELEN

Tyrime įvertintas normalizavimo metodo efektas sprendimo rezultatui, taikant parinktą MADM metodą. Šiam tikslui pasiekti, rodiklių reikšmingumų nustatymui taikytas FAHP metodas. 13 Turkijos bankų indėlių finansinių charakteristikų vertinimui pasirinktas TOPSIS metodas. Tikslui pasiekti taikyti vieni iš populiariausių keturi normalizavimo metodai. Tyrimai parodė, kad vektorinio normalizavimo metodas, kuris yra daugiausiai taikomas kartu su TOPSIS metodu, pateikia nuosekliausius rezultatus. Tiesinio normalizavimo metodai maks–min ir max yra alternatyvūs metodai vektoriniam normalizavimo metodui.