

# Missing Data Restoration Algorithm

Kazys KAZLAUSKAS\*, Rimantas PUPEIKIS

*Institute of Mathematics and Informatics, Vilnius University*

*Akademijos 4, LT-08663 Vilnius, Lithuania*

*e-mail: kazys.kazlauskas@mii.vu.lt, rimantas.pupeikis@mii.vu.lt*

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**Abstract.** The paper presents a novel algorithm for restoration of the missing samples in additive Gaussian noise based on the forward–backward autoregressive (AR) parameter estimation approach and the extrapolation technique. The proposed algorithm is implemented in two consecutive steps. In the first step, the forward–backward approach is used to estimate the parameters of the given neighbouring segments, while in the second step the extrapolation technique for the segments is applied to restore the samples of the missing segment. The experimental results demonstrate that the restoration error of the samples of the missing segment using the proposed algorithm is reduced as compared with the Burg algorithm.

**Key words:** missing data, restoration, forward–backward parameter estimation, extrapolation.

## 1. Introduction

Time series in econometrics, biometrics, techniques and other applications in many cases have missing data. Missing data are obtained by a variety of reasons. The missing data can be caused by faulty equipment or as a result of outliers removal, or they may follow a deterministic pattern due to inaccessibility of the data during certain times. In radar measurements of the Moon surface, it is observed a time series which represents the echo of a radar signal transmitted to the Moon. In meteorology, the weather conditions may disturb the equidistant sampling scheme. In paleodimatic data, the relation between the chronological time and the physical depth causes an observed time series with missing observations (Broersen *et al.*, 2004). Signal losses may occur due to the presence of spurious noises such as clicks, pops, and crackles which are associated with the reproduction of old disk records (Esquef *et al.*, 2003).

Restoration of missing data is the estimation of the lost samples of a signal using a known samples at the neighbouring segments. Restoration of discrete missing data has been approached through various methods. The finite interval likelihood minimization algorithm ARFIL is a numerically stable method for estimating AR models from incomplete data and was proposed by Broersen *et al.* (2004). Bos *et al.* (2008) introduced a new estimator that applies the Burg algorithm for autoregressive spectral estimation to unevenly spaced data. The paper of Etter (1996) presents an algorithm for the interpolation of a

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\* Corresponding author.

missing signal segment on the assumption that the signal can be modeled as an autoregressive process. Janssen *et al.* (1986) investigate an adaptive algorithm for the restoration of lost sample values in discrete-time signals that can be described by means of autoregressive processor. The work of Esquef *et al.* (2003) addresses the reconstruction of missing samples in audio signals via model-based schemes that employs a frequency-warped version of Burg's method and is advantageous for interpolation of long duration signal gaps. In Paulikas and Navakauskas (2006) the problem of discrimination of homographs when a lengthy segment of an uttered word is missing. The article of Dahimene *et al.* (2008) deals with the problem of peak clipped speech with assumption that the clipped speech is voiced and can be linearly predicted with high accuracy. In the paper of Rosen and Porat (1989) the problem of spectral estimation through the autoregressive moving-average modeling of stationary processes with missing observations based on the sample covariances is presented. In the paper of Porat and Friendlander (1984) a new ARMA method for spectral estimation problem with missing samples based on nonlinear optimization of a weighted squared error criterion is proposed. The Burg algorithm for segments was applied by Waele and Broersen (2000). Investigation of Zgheib *et al.* (2008) deals with the problem of adaptive reconstruction and identification of nonstationary AR process with randomly missing observations. In the paper of Wolfe and Godsill (2005) the problem of missing data interpolation over repeated short gaps in audio signals is analyzed.

*Problem statement:* consider a problem of restoration of a missing data segment in relatively short data sequence on the assumption that the data can be modeled as a finite order autoregressive process contaminated with the additive Gaussian noise. The restoration must be done in such a way that the restored data fits the assumed model as well as possible in the least square sense.

*The aim of this paper* is to present a new algorithm to restore the missing observations of the signals in Gaussian additive noise using forward–backward approach and extrapolation technique. The organization of the paper is as follows. Section 2 provides a description of the forward–backward autoregressive parameter estimation approach. In Section 3, we propose the missing data restoration algorithm. We evaluate the performance of the proposed algorithm in Section 4. Conclusions are given in Section 5.

## 2. A Forward–Backward Autoregressive Parameter Estimation

Linear prediction plays an important role in computational and practical areas of signal processing and deals with the problem of predicting the value  $x(n)$  of signal at the time instant  $n$  by using a set of the samples from the same signal. The forward prediction involves the prediction of the value  $x(n)$ ,  $n = 0, 1, \dots, N - 1$  of a stochastic process by using a linear combination of the past values  $x(n - 1), \dots, x(n - p)$  (Proakis and Manolakis, 1996)

$$\hat{x}(n) = - \sum_{k=1}^p a_k x(n - k), \quad (1)$$

and the forward prediction error is

$$e^f(n) = x(n) - \hat{x}(n) = \sum_{k=0}^p a_k x(n-k) = \mathbf{a}^T \mathbf{x}(n), \quad (2)$$

where  $\mathbf{a}^T = [1, a_1, \dots, a_p]$ ,  $\mathbf{x}^T(n) = [x(n), \dots, x(n-p)]$ .

Thus the forward prediction error over the range  $p \leq n \leq N-1$  can be expressed as a vector

$$\mathbf{e}^f = \mathbf{X}\mathbf{a}, \quad (3)$$

where  $\mathbf{e}^f = [e(p), \dots, e(N-1)]^T$  and  $\mathbf{X}$  is the data matrix

$$\mathbf{X} = \begin{pmatrix} x(p) & x(p-1) & \dots & x(0) \\ x(p+1) & x(p) & \dots & x(1) \\ \vdots & \vdots & \vdots & \vdots \\ x(N-1) & x(N-2) & \dots & x(N-p-1) \end{pmatrix}. \quad (4)$$

The backward prediction error is predicting the sample  $x(n-p)$  from the samples  $\{x(n-p+1), \dots, x(n)\}$

$$x(n-p) = - \sum_{k=1}^p b_k x(n+k-p), \quad (5)$$

where  $b_k$  are the coefficients of the backward prediction filter.

The backward prediction error is

$$e^b(n) = x(n-p) - \hat{x}(n-p) = \sum_{k=0}^p b_k x(n+k-p) = \mathbf{b}^T \mathbf{x}(n), \quad (6)$$

where  $\mathbf{b}^T = [1, b_1, \dots, b_p]$ .

The backward prediction error over the range  $p \leq n \leq N-1$  can be expressed as a vector

$$\mathbf{e}^b = \hat{\mathbf{X}}\mathbf{b}, \quad (7)$$

where  $\hat{\mathbf{X}}$  is the data matrix

$$\hat{\mathbf{X}} = \begin{pmatrix} x(N-p-1) & \dots & x(N-1) \\ x(N-p-2) & \dots & x(N-2) \\ \vdots & \vdots & \vdots \\ x(0) & \dots & x(p) \end{pmatrix}. \quad (8)$$

The forward and backward estimates of parameters are different because the data read forward and backward are different. We could improve performance by minimizing the total forward and backward squared errors

$$E^{fb} = \sum_{n=p}^{N-1} (|e^f(n)|^2 + |e^b(n)|^2) = \|\mathbf{e}^f\|^2 + \|\mathbf{e}^b\|^2, \quad (9)$$

where

$$\|\mathbf{e}^f\|^2 = \mathbf{a}^T \mathbf{X}^T \mathbf{X} \mathbf{a} \quad (10)$$

and

$$\|\mathbf{e}^b\|^2 = \mathbf{b}^T \hat{\mathbf{X}}^T \hat{\mathbf{X}} \mathbf{b}. \quad (11)$$

Substituting (10) and (11) in (9), we have

$$E^{fb} = \mathbf{a}^T \mathbf{X}^T \mathbf{X} \mathbf{a} + \mathbf{b}^T \hat{\mathbf{X}}^T \hat{\mathbf{X}} \mathbf{b}. \quad (12)$$

The total error  $E^{fb}$  is minimized under the constraint

$$\mathbf{a}^{fb} = \mathbf{a} = J\mathbf{b}, \quad (13)$$

where  $J$  is the vector reversing operator, i.e., we have  $J\mathbf{x} = J[x(1), \dots, x(M)] = [x(M), \dots, x(1)]$ .

Substituting (13) into (12), we obtain

$$E^{fb} = \mathbf{a}^T (\mathbf{X}^T \mathbf{X} + \hat{\mathbf{X}}^T \hat{\mathbf{X}}) \mathbf{a}. \quad (14)$$

We minimize (14) subject to the constraint that  $a_0 = 1$ .

The problem can be solved by differentiating (14) according to  $\mathbf{a}$ , i.e., we get (Manolakis *et al.*, 2005)

$$(\mathbf{X}^T \mathbf{X} + \hat{\mathbf{X}}^T \hat{\mathbf{X}}) \mathbf{a} = \begin{pmatrix} E^{fb} \\ 0 \end{pmatrix}. \quad (15)$$

The time-averaged forward–backward correlation matrix

$$\mathbf{R}^{fb} = \mathbf{X}^T \mathbf{X} + \hat{\mathbf{X}}^T \hat{\mathbf{X}} \quad (16)$$

is symmetric about both main diagonals elements  $r_{ij}^{fb} = r_{ij} + r_{p-i, p-j}$ ,  $0 \leq i, j \leq p$ .

### 3. Proposed Missing Data Restoration Algorithm

In this section, we propose the missing data restoration algorithm. The restoration algorithm is a two stage process: in the first stage, the AR model coefficients are estimated using forward–backward autoregressive parameter estimation, and in the second stage the estimates of the model coefficients are used to extrapolate the missing samples. The extrapolation order depends on the number of extrapolated samples.

Consider a signal  $x(n) = \{x_1(n), x_2(n), x_3(n)\}$ ,  $n = 0, 1, \dots, n - 1$  where  $x_1(n)$ ,  $n = 0, 1, \dots, N_1 - 1$  is the first given segment of the signal  $x(n)$ ;  $x_2(n)$ ,  $n = N_1, N_1 + 1, \dots, N_1 + N_2 - 1$  is the missing segment of the signal  $x(n)$ , and  $x_3(n)$ ,  $n = M, M + 1, \dots, n - 1$ , where  $M = N_1 + N_2$ , is the second given segment of the signal  $x(n)$ .

For the first given segment  $x_1(n)$ , using forward–backward approach (15), we have estimated the parameters  $c_k$ ,  $k = 1, 2, \dots, p_1$  ( $c_0 = 1$ ) of the first AR model. The model order  $p_1 = \text{floor}(\frac{2}{3}N_1 - 1)$ , where the function  $\text{floor}(x)$  maps a real number  $x$  to the largest previous integer, i.e.,  $\text{floor}(x) = \max\{m \in \mathbb{Z}!, m \leq x\}$ .

Similarly, for the second given segment  $x_3(n)$ ,  $n = M, M + 1, \dots, n - 1$ , using forward–backward approach (15), we have estimated the parameters  $d_k$ ,  $k = 1, 2, \dots, p_3$  ( $d_0 = 1$ ), of the second AR model. The model order  $p_3 = \text{floor}(\frac{2}{3}N_3 - 1)$ .

In the proposed algorithm, the first given segment  $x_1(n)$ ,  $n = 0, 1, \dots, N_1 - 1$  is forward extrapolated to get the forward estimates of missing samples. The forward extrapolated estimates  $x_2^f(N_1), \dots, x_2^f(N_1 + N_2 - 1)$  of missing samples, we have obtained as follows:

$$x_2^f(N_1 + j) = - \sum_{k=1}^{p_1(j)} c_k x_1(N_1 + j - k), \quad j = 0, 1, \dots, N_2 - 1, \quad (17)$$

where the growing extrapolation order  $p_1(j) = \text{floor}(\frac{2}{3}(N_1 + j))$ ,  $j = 0, 1, \dots, N_2 - 1$ .

Similarly, the second given segment  $x_3(n)$ ,  $n = M, M + 1, \dots, n - 1$  is backward extrapolated to get the backward estimates of missing samples. The backward extrapolated estimates  $x_2^b(N_1 + N_2 - 1), \dots, x_2^b(N_1)$  of missing samples, we have computed as follows:

$$x_2^b(M - j) = - \sum_{k=1}^{p_3(j)} d_k x_3(M - j + k), \quad j = 1, 2, \dots, N_2, \quad (18)$$

where the growing extrapolation order  $p_3(j) = \text{floor}(\frac{2}{3}(N_3 + j - 1))$ ,  $j = 1, 2, \dots, N_2$ .

The parameters, which were calculated by the forward–backward approach, are updated each time to find the next estimate of missing sample, and every the extrapolated missing sample is used to re-estimate the AR parameters  $c_k$  and  $d_k$ .

We can solve the missing data restoration task by weighting the forward extrapolated and backward extrapolated estimates of missing samples. The restored missing segment  $\hat{x}_2(n)$ ,  $n = N_1, N_1 + 1, \dots, N_1 + N_2 - 1$  can be written as a weighted sum (19).

$$\hat{x}_2(n) = w^f(n)x_2^f(n) + w^b(n)Jx_2^b(n), \quad (19)$$

where the forward weighting sequence

$$w^f(n) = 0.5 - 0.5 \cos\left(\pi\left(1 + \frac{n}{N_2}\right)\right), \quad (20)$$

and the backward weighting sequence

$$w^b(n) = 0.5 - 0.5 \cos\left(\pi\frac{n}{N_2}\right). \quad (21)$$

#### 4. Simulation Results

In this section, we examine the performance of the proposed algorithm and compare the results with that of the Burg algorithm. To investigate the abilities of the proposed approach, we have generated a signal from the signal generator comprised of four sinusoids ( $M = 4$ ) embedded in the noise

$$x(n) = s(n) + w(n) = \sum_{i=1}^M \cos(2\pi f_i n) + w(n), \quad n = 0, \dots, 199 \quad (22)$$

with normalized frequencies  $f_1 = 0.15$ ,  $f_2 = 0.3$ ,  $f_3 = 0.32$ ,  $f_4 = 0.38$ , and  $w(n)$  is a zero-mean white Gaussian noise with the unit variance  $\sigma_w^2 = 1$ . To get the desired signal-to-noise ratio (SNR) from the signal generator, the output signal is defined by

$$x(n) = s(n) + kw(n), \quad (23)$$

in which the coefficient  $k$  is computed such that

$$SNR = 10 \log \frac{P_s}{k^2 P_w}, \quad (24)$$

where  $P_s = \frac{1}{N} \sum_{n=1}^N s^2(n)$ ,  $P_w = \frac{1}{N} \sum_{n=1}^N w^2(n)$ , and  $N$  is the length of the  $s(n)$  and  $w(n)$ .

From (24) we obtain that for desired SNR, the coefficient  $k$  is calculated as follows

$$k = \frac{\sqrt{P_s}}{\sqrt{P_w}} 10^{-\frac{SNR}{20}}. \quad (25)$$

To evaluate the accuracy of the algorithm, we define the mean absolute error (MAE) of missing samples restoration as follows:

$$MAE = \frac{1}{N_2} \sum_{n=N_1}^{N_1+N_2-1} |x_2(n) - \hat{x}_2(n)|, \quad (26)$$

where  $N_2$  is the number of missing samples,  $x_2(n)$  are true samples, and  $\hat{x}_2(n)$  are restored missing samples.

Considering the restoration error as a noise, the signal to noise ratio (STNR) is defined as

$$\text{STNR} = 10 \log \frac{\sum_{n=N_1}^{N_1+N_2-1} x_2^2(n)}{\sum_{n=N_1}^{N_1+N_2-1} (x_2(n) - \hat{x}_2(n))^2}. \quad (27)$$

Tables 1–3 illustrate the error estimates MAE and STNR averaged by  $L = 200$  experiments and their confidence intervals  $\Delta = \pm t_{\alpha/2; L-1} \frac{\hat{\sigma}}{\sqrt{L}}$ , in which  $\hat{\sigma}$  is the estimate of the standard deviation and  $\alpha$  is the significance level. The value  $t_{\alpha/2; L-1}$  is the point

Table 1  
Mean absolute error (MAE), and signal to noise ratio (STNR) with confidence intervals  $\Delta$  versus SNR.

SNR (dB)	MAE	STNR (dB)
0	1.4701 $\pm$ 0.1770	0.8481 $\pm$ 0.2573
10	0.5916 $\pm$ 0.0988	5.9846 $\pm$ 1.1879
30	0.1398 $\pm$ 0.0368	18.3454 $\pm$ 1.3559
50	0.0755 $\pm$ 0.0013	23.2654 $\pm$ 1.2970
100	0.0312 $\pm$ 0.0009	30.6215 $\pm$ 0.3759

The signal  $x(n)$  length  $n = 200$ ; the predictive filter order  $p = 16$ ; missing samples are in the interval (75, 125); Monte Carlo runs are equal to 200.

Table 2  
Mean absolute error (MAE), and signal to noise ratio (STNR) with confidence intervals  $\Delta$  versus the number of missing samples  $N_2$ .

$N_2$	MAE	STNR (dB)
10	0.5563 $\pm$ 0.1229	6.6404 $\pm$ 1.5183
30	0.7042 $\pm$ 0.1140	5.4176 $\pm$ 1.2039
50	0.8472 $\pm$ 0.1346	3.2107 $\pm$ 1.2111
100	0.8133 $\pm$ 0.1340	3.2634 $\pm$ 1.0968
130	1.0156 $\pm$ 0.1512	1.7054 $\pm$ 1.5933

The signal  $x(n)$  length  $n = 200$ ; the predictive filter order  $p = 16$ ; SNR = 10 dB; Monte Carlo runs are equal to 200.

Table 3  
Mean absolute error (MAE), and signal to noise ratio (STNR) with confidence intervals  $\Delta$  versus the predictive filter order  $p$ .

$p$	MAE	STNR (dB)
5	1.2484 $\pm$ 0.0774	0.0251 $\pm$ 0.0192
15	0.9587 $\pm$ 0.1035	2.1748 $\pm$ 0.6442
20	0.6953 $\pm$ 0.0831	4.8401 $\pm$ 0.7119
30	0.5049 $\pm$ 0.0641	7.4029 $\pm$ 0.8896
40	0.5545 $\pm$ 0.1043	6.7874 $\pm$ 1.1484

The signal  $x(n)$  length  $n = 200$ ; missing samples are in the interval (75, 125); SNR = 10 dB; Monte Carlo runs are equal to 200.

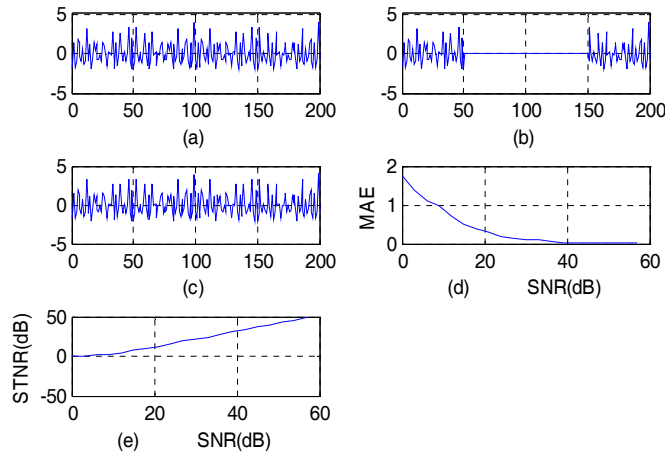


Fig. 1. Missing data estimates: (a) signal  $x(n)$  generated according to (23) (SNR = 57 dB); (b) signal  $x(n)$  with missing samples in the interval (50, 150); (c) restored signal  $x(n)$ ; (d) MAE versus SNR; (e) STNR versus SNR; model order  $p = 18$ .

of Student's distribution with  $L - 1$  degrees of freedom which cuts the  $\alpha/2$  part of the distribution. In case  $\alpha = 0.05$  and  $L = 200$ , we find from Student's distribution table that  $t_{0.025;199} = 1.9720$ .

Table 1 shows the mean absolute error (MAE) and the signal to noise ratio (STNR) as a function of SNR. The MAE decreases and STNR increases with an increase of SNR. From Table 2, it follows that MAE increases and STNR decreases if missing samples  $N_2$  increase. As we can see from Table 3, the MAE decreases and STNR increases with an increase of model order  $p$ .

In the first experiment, we have generated the signal  $x(n)$ ,  $n = 0, 1, \dots, 199$  with SNR = 57 dB (Fig. 1(a)). The signal  $x(n)$  with missing samples in the interval (50, 150) is shown in Fig. 1(b). The restored signal is shown in Fig. 1(c). Figure 1(d) shows MAE dependence on SNR, and in Fig. 1(e), we show STNR dependence on SNR. Model order  $p = 18$ .

In the second experiment, the same signal  $x(n)$  was generated with SNR = 10 dB (Fig. 2(a)). The signal  $x(n)$  with missing samples in the interval (30, 145) is shown in Fig. 2(b), and restored signal is shown in the Fig. 2(c). In Fig. 2(d), the relation between STNR and the number of missing samples is shown. Model order  $p = 16$ .

In the third experiment, we have analyzed how performance of the proposed algorithm depends on model order  $p$ . We generated the same signal  $x(n)$  with SNR = 0 dB (Fig. 3(a)). In Fig. 3(b), we show signal  $x(n)$  with missing samples in the interval (75, 125). Figure 3(c) shows the restored signal. In Fig. 3(d), (e), the relation between restoration errors MAE and STNR, and the model order  $p$  is shown.

In Fig. 4, the relation between STNR and SNR for the proposed algorithm ( $\circ$ ), and the Burg algorithm ( $*$ ) is presented in case when model order  $p = 18$ , and missing samples are in the interval (50, 150).

In Fig. 5, we show performance criterion STNR versus missing data for Burg algorithm ( $*$ ), and the proposed algorithm ( $\circ$ ) in case when SNR = 10 dB. Model order  $p = 18$ .



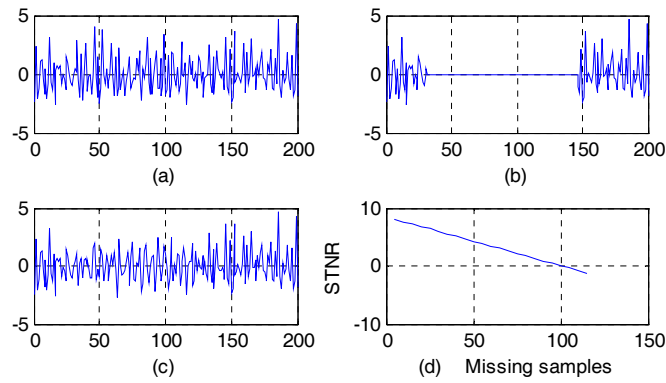


Fig. 2. Missing data estimates: (a) signal  $x(n)$  generated according to (23) (SNR = 10 dB); (b) signal  $x(n)$  with missing samples in the interval (30, 145); (c) restored signal  $x(n)$ ; (d) MAE versus the number of missing samples from 5 to 115; model order  $p = 16$ .

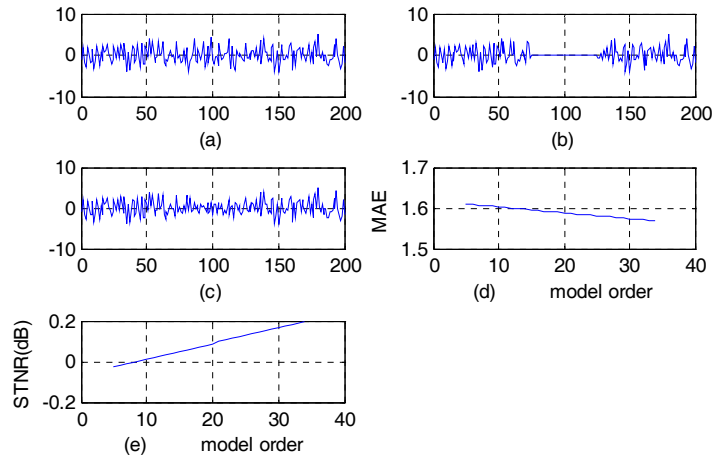


Fig. 3. Missing data estimates: (a) signal  $x(n)$  generated according to (23) (SNR = 0 dB); (b) signal  $x(n)$  with missing samples in the interval (75, 125); (c) restored signal  $x(n)$ ; (d) MAE versus model order  $p$ ; (e) STNR versus model order  $p$ .

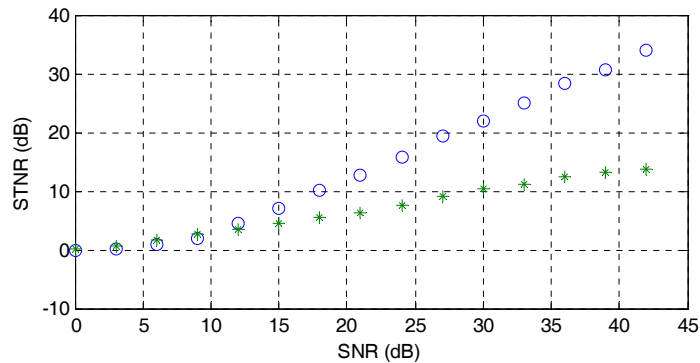


Fig. 4. STNR versus SNR: Results for comparative study between the proposed algorithm (o) and the Burg algorithm (\*). Missing samples are in the interval (50, 150). Model order  $p = 18$ .

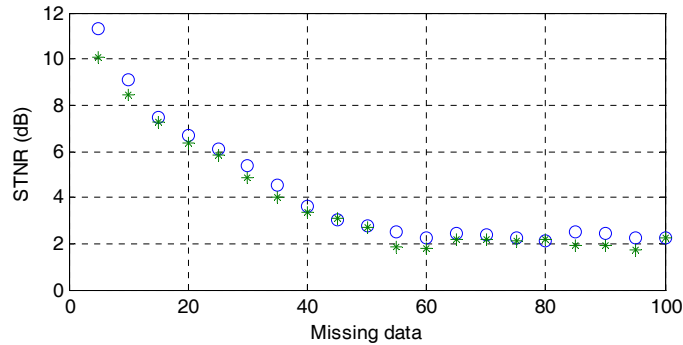


Fig. 5. STNR versus missing data: Results for comparative study between the proposed algorithm ( $\circ$ ) and the Burg algorithm ( $*$ ). SNR = 10 dB. Model order  $p = 18$ .

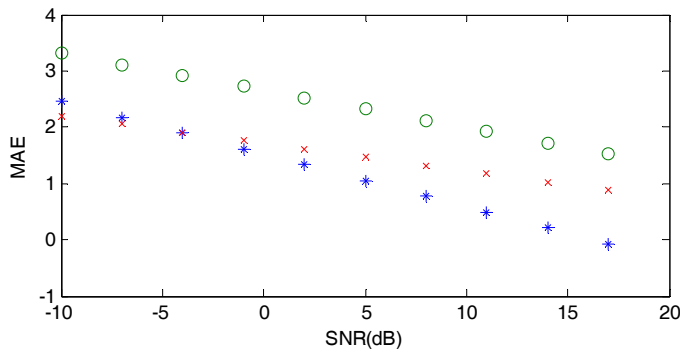


Fig. 6. MAE versus SNR: ( $\circ$ ) missing samples are replaced with samples from given earlier samples; ( $\times$ ) missing samples are replaced with zeros; ( $*$ ) missing samples are restored according to the proposed algorithm. Missing samples are in the interval (70, 120). Model order  $p = 18$ .

Figure 6 shows the dependence of the restoration error MAE on SNR. Model order  $p = 18$ . Missing samples are in the interval (70, 120). We analyze three cases: missing samples are restored using the proposed algorithm ( $*$ ), missing samples are replaced with zeros ( $x$ ), and missing samples are replaced with samples from the given earlier samples ( $\circ$ ).

## 5. Conclusions

We have presented the missing data restoration algorithm which uses the forward–backward AR model parameter estimates of neighbouring segments and the extrapolation technique. As neighbouring segments are modeled separately, the proposed algorithm may be applied for the restoration of missing samples of the non-stationary signals. The restoration error is minimized using the forward–backward signal model and the information from the neighbouring segments. Accurate restoration of the missing segment is possible if the signal is predictable, i.e., if the missing samples carry no more informa-

tion than that included in the given segments. For deterministic signals without noise, restoration error is zero, but for deterministic signals in noise, the accurate missing data restoration is impossible. The restoration error depends on the model order, signal to noise ratio, number of missing samples, and on the accuracy of the estimated parameters from the incomplete data. Restoration error increases with increasing length of the missing segment. The restoration error is largest in the middle of the missing segment. The simulation results also showed that in many cases the proposed algorithm reduces the missing data restoration error as compared with the Burg algorithm.

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**K. Kazlauskas** received a PhD degree from Kaunas Polytechnic Institute and a doctor habilius degree from Institute of Mathematics and Informatics and Vytautas Magnus University. He is senior researcher of the Recognition Processes Department at the Vilnius University Institute of Mathematics and Informatics, and a professor at the Informatics Department of Lithuanian University of Educational Sciences. His interests include signal processing, parameter estimation, and digital system design.

**R. Pupeikis** received PhD degree from the Kaunas Polytechnic Institute, Kaunas, Lithuania, 1979. He is a senior researcher at the Process Recognition Department of the Institute of Mathematics and Informatics of Vilnius University and Professor (Associate) at the Department of Electronic Systems of Vilnius Gediminas Technical University. His research interest include the classical and robust approaches of dynamic system identification as well technological process control.

## **Prarastųjų duomenų atkūrimo algoritmas**

Kazys KAZLAUSKAS, Rimantas PUPEIKIS

Straipsnyje pasiūlytas prarastųjų duomenų atkūrimo algoritmas esant Gauso triukšmams panaudojant autoregresijos parametų tiesioginio bei atgalinio įvertinimo metodą ir ekstrapoliavimą. Pirmiausia, tiesioginio bei atgalinio parametų įvertinimo metodu įvertinami gretimų segmentų parametrai. Po to, panaudojus ekstrapoliavimo metodą gretimiems segmentams, atkuriami prarastieji duomenys. Eksperimento rezultatai parodė, kad šis metodas yra pranašesnis už Burgo metodą.