INFORMATICA, 2014, Vol. 25, No. 1, 21–36

Decision Making with Unknown Data: Development of ELECTRE Method Based on Black Numbers

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Received: December 2011; accepted: March 2013

Abstract. In multi criteria Decision Making, the decision maker wants to find the best alternative among a set of alternatives in order to satisfy a set of criteria. Traditionally, decision making models are based on crisp data. The shortcoming of these data in capturing the reality and lack of information persuaded researchers to develop decision making methods with uncertain data. In this paper, the ELECTRE method is extended with black numbers, under ambiguous environment. The proposed method is applied in a supplier selection problem. It's an outstanding method that can be used in real world problems with ill-defined and incomplete data.

Key words: multiple criteria Decision Making, ELECTRE, unknown data; black number, supplier selection

1. Introduction

The history of operations research science in its structured and exhaustive form illustrates that this field is a response to the questions of managers, decision makers, and resource owners in having a criteria to judge their decisions. In fact, decision makers always seek a criterion to evaluate their decisions favorite. In this context, decision making methods arise when decision maker simultaneously envisage various criteria for evaluating his or her decisions favorite (Kuo *et al.*, 2008). Such a problem is the subject of multiple criteria decision making (MCDM) methods. This class is further divided into multi objective decision making and multi attribute decision making (MADM) (Climaco, 1997). The problem of MADM often arises when there is the issue of choice or comparison. Because there are often numerous and antithetic criterions in real decision making problems, the MCDM methods became a commonly used branches of operations research science, during last decades (Figueira *et al.*, 2004; Triantaphyllou, 2000; Zavadskas and Turskis, 2011; Antuševičiene *et al.*, 2011).

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Several categorizations have been developed for MADM methods. Hwang and Yoon (1995) categorized MADM methods into two compensatory and non compensatory models. Triantaphyllou (2000) extensively compares, both theoretically and empirically, real-life MCDM issues. Priority based, outranking, distance-based, and mixed methods are also applied to various problems (Vahdani *et al.*, 2010a).

A series of MCDM models use what is known as “outranking relations” to rank a set of alternatives. The elimination and choice translating reality (ELECTRE) method and its derivatives play a prominent role in this group. The ELECTRE approach was first introduced in 1966 (Benayoun et al.,1966). The origins of ELECTRE methods go back to 1965 at the European consultancy company SEMA. At that time, a research team from SEMA worked on a concrete, multiple criteria, and real-world problem regarding decisions dealing with the development of new activities in firms (Figueira *et al.*, 2004). The main idea of this method is based on outranking relations, concordance and discordance concepts (Roy and Vanderpooten, 1997). This method uses concordance and discordance indices to analyze the outranking relations (De Almeida, 2007). Soon after the introduction of the first version known as ELECTRE I (Gal and Hanne, 1999), this approach was evolved into a number of other variants. Today, the most widely used versions are ELECTRE II (Moore, 1979; Roy, 1968; Roy and Bertier, 1973), ELECTRE III (Roy and Bertier, 1971; Roy, 1978), ELECTRE IV (Roy and Hugonnard, 1982a, 1982b), ELECTRE IS (Roy and Skalka, 1984; Younes *et al.*, 2000) and ELECTRE TRI (Dias and Climaco, 2000; Mousseau and Slowinski, 1998; Yu, 1992).

Under many conditions, exact data are inadequate to model the real-life situations. These situations are called as uncertainty and many researchers developed some structures such as bounded data, ordinal data, fuzzy data, and grey numbers in response to such situations. In fact, most of the decisions aren’t made on the basis of well known calculations and there is a lot of ambiguity and uncertainty in decision making problems (Riabacke, 2006). In this context, Deng (1982) developed the Grey system theory and presented grey Decision-making systems (Deng, 1989). Many authors investigated grey system theory in decision making. Zhanga *et al.* (2005) by emphasis on attractiveness of qualitative inputs in multiple attribute problems and its uncertainty presented the method of grey related analysis to this problem using interval fuzzy numbers. Liu and Lin (2006) in their article explored a more effective method to study the information content of grey numbers and used an axiomatic approach for the measurement of information content of given grey number. The grey system has been applied in many fields. Satapathy et al. (2006) in their article dealt with the assessment of fiber contribution to the performance of friction materials based on various possible combinations of organic fibers and used grey relation analyzing in compliance with the existing set of incomplete data. Noorul Haq and Kannan (2007) develop an effective and efficient hybrid normalized multi criteria decision making model for evaluating and selecting the vendor using an Analytical Hierarchy Process (AHP) and Fuzzy Analytical Hierarchy Process (FAHP) and an integrated approach of Grey Relational Analysis (GRA) in a Supply Chain Model (SCM). Li *et al.* (2007) proposed a new grey-based approach to deal with the supplier selection problem. Their work procedure is as follows: firstly, the weights and ratings of attributes for all alternatives are described by linguistic variables that can be expressed in grey numbers. Secondly, using a grey possibility degree, the ranking order of all alternatives is determined. Lin *et al.* (2008) applied the TOPSIS method and grey numbers operations to deal with the problem of uncertain information. Zavadskas *et al.* (2008) used a multiple criteria method of complex proportional assessment of alternatives with grey relations (COPRAS-G) in multi criteria problem of matching of managers to construction projects. Kuo *et al.* (2008) proposed a grey rational analysis method for solving multi attribute decision making problem and compared the application of this method in two cases: facility layout and dispatching rules selection problem. Amiri *et al.* (2008) proposed a new method of ranking alternatives based on interval grey data by ELECTRE method. Zavadskas *et al.* (2009) developed COPRAS method by applying grey numbers and used it in Contractors' selection in construction problem. Vahdani *et al.* (2010a) also extended another approach for applying ELECTRE method by interval weights and data. In this article we extend a new method to rank alternatives by ELECTRE method, when our criteria's weight and decision matrix's data are black numbers. Stanujkic et al. (2012a, 2012b) proposed extended versions of the ration system part of MOORA method to determine the preferable alternative among all possible alternatives, when performance ratings are given as intervals or grey numbers. Hashemkhani Zolfani *et al.* (2012a) have applied Fuzzy AHP, SAW-G and TOPSIS Grey methods to evaluate the progress of some projects for establishing rural telephone centers in Iran. Zavadskas *et al.* (2010) have used SAW-G and TOPSIS Grey techniques for SELECTING contractors for construction works. Turskis and Zavadskas (2010) have presented ARAS method as a novel method applied ARAS-G for selecting potential suppliers. Rezaeiniya *et al.* (2012) applied ANP to find relative weights among criteria and COPRAS-G method to rank alternatives. Balezentis *et al.* (2011) used the multi-moora method for ranking EU member states' efforts in seeking strategy's Europe 2020 goals. Hashemkhani Zolfani *et al.* (2012b) have proposed a personal selection system based on AHP and complex proportional assessment of alternatives with grey relations (COPRAS-G) method. Ranjan Maity *et al.* (2012) in order to select cutting tool material for machine performance, have applied COPRAS-G method. Chatterjee and Chakraborty (2012) have focused on application of EXPROME2, COPRAS-G, ORESTE and OCRA to prioritize a set of the best and worst materials.

The rest of the paper is organized as follows: Section 2 briefly introduces the original ELECTRE method. Then, a short review on the concept and basic calculation (algebraic operations) of grey and black numbers is done in Section 3. In Section 4 after introducing MCDM problems with black weights and data, an algorithm is presented to extend ELECTRE method which deals with black weights and data. In Section 5, the proposed algorithmic method is illustrated by applying it to an example. Section 6 consists of conclusions and future work.

2. The ELECTRE

Suppose a decision making problem consists of m alternatives $\{A_1, A_2, \ldots, A_m\}$ which is evaluated based on n criterion $\{C_1, C_2, \ldots, C_n\}$ and x_{ij} is the value of ith alternative in jth criterion. The ELECTRE, as pointed in introduction, uses the concept of "outranking relationship". The outranking relationship of $A_k \to A_l$ says that even though two alternatives k and l don't dominate each other mathematically, the DM accepts the risk of regarding A_k as almost surely better than A_l. This method is consist of a pair-wise comparison of alternatives based on the degree to which evaluations of the alternatives and the preference weights confirm or contradict the pair wise dominance relationship between alternatives (Hwang and Yoon, 1995).

It starts from the data of the decision matrix and here assumes that the sum of the weights of all criteria (w_i, $i = 1, 2, \dots, n$) equals to 1. For an ordered pair of alternatives (A_j, A_k), the concordance index c_{jk} is the sum of the all weights for those criteria where the performance score of A_j is least as high as that of A_k, i.e.,

$$c_{jk} = \sum_{l:\, x_{jl} \geqslant x_{kl}} w_l, \quad j, k = 1, 2, \dots, n, \ j \neq k. \tag{1}$$

The computation of the discordance d_{jk} index is a bit more complicated: $d_{jk} = 0$ if $a_{jl} \succ a_{kl}$, $l = 1, 2, \dots, n$, i.e., the discordance index is zero if A_j performs better than A_k on all criteria. Otherwise,

$$d_{jk} = \frac{\max_{l \in D_{kl}} |V_{jl} - V_{kl}|}{\max_{l \in J} |V_{jl} - V_{kl}|}, \tag{2}$$

which D_{kl} is the set of criteria that alternative k is preferred to alternative l. A concordance threshold C^* and discordance threshold d^* are then defined. Then, A_j outranks A_k if the $c_{jk} \succ C^*$ and $d_{jk} \prec d^*$, i.e., the concordance index is above and the discordance index is below its threshold, respectively. This outranking defines a partial ranking on the set of alternatives. Consider the set of all alternatives that outrank at least one other alternative and are themselves not outranked. This set contains the promising alternatives for this decision problem. Interactively changing the level thresholds, we also can change the size of this set (Kuo *et al.*, 2008; Vahdani *et al.*, 2010b).

3. Grey Numbers and Their Extension to Black

The solution of each multi criteria problem begins with constructing the decision-making matrix X. In this matrix, the values of the criteria x_{ij} may be real numbers, intervals, probability distributions, possibility distributions, qualitative labels or grey numbers.

Before developing the ELECTRE method based on Grey numbers, some definitions are presented to introduce these numbers.

Definition 1. A grey system is defined as a system involves non deterministic information. If we show clear information of system in white color and consider unknown information in black, so information related to most of natural systems aren't white (completely known) or black (completely unknown), of curse are combined; it means grey (Li *et al.*, 2007). The meaning of being "grey" can be as is shown in Table 1.

Table 1
Meaning of information.

	White	Grey	Black
Information	Known	Incomplete	Unknown
Appearance	Bright	Grey	Dark
Process	Old	Replace old with new	New
Property	Order	Complexity	Chaos
Methodology	Positive	Transaction	Negative
Attitude	Seriousness	Tolerance	Indulgence
Conclusion	Unique solution	Multiple solution	Ni results

DEFINITION 2. A grey number (Lin *et al.*, 2004) is a number whose exact value is unknown, but a range within which the value lies is known. There are the several types of grey numbers.

- Grey numbers with only lower limits: $\otimes G \in [x, \infty)$ or $\otimes G(x)$, where a fixed real value x represents the lower limit of the grey number $\otimes G$.
- Grey numbers with only upper limits: $\otimes G \in [-\infty, x)$ or $\otimes G(x)$, where x is a fixed real number or an upper limit of the grey number $\otimes G$.
- Interval grey number is the number with both lower limit and upper limit: $\otimes G \in [\underline{x}, \overline{x}]$.
- Continuous grey numbers and discrete grey numbers: The grey numbers taking a finite number of values or a countable number of values in an interval are called discrete. The continuously taking values, which cover an interval, are continuous.
- Black and white numbers: When $\otimes G \in (-\infty, \infty)$ or $\otimes G \in (\otimes G_1, \otimes G_2)$, i.e., when $\otimes G$ has not upper neither and lower limits, or the upper and the lower limits are all grey numbers, $\otimes G$ is called a black number. When $\otimes G \in [\underline{x}, \overline{x}]$ and $\underline{x} = \overline{x}$, $\otimes G$ is called a white number (Zavadskas *et al.*, 2009).

DEFINITION 3. Two main operations on grey numbers $\otimes G_1 = [\underline{G}_1, \overline{G}_1]$ and $\otimes G_2 = [\underline{G}_2, \overline{G}_2]$ are as follow (Li *et al.*, 2007):

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \overline{G}_1 + \overline{G}_2], \tag{3}$$

$$\otimes G_1 - \otimes G_2 = [\underline{G}_1 - \underline{G}_2, \overline{G}_1 - \overline{G}_2], \tag{4}$$

$$\otimes G_1 \times \otimes G_2 = \big[\min(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2), \max(\underline{G}_1\underline{G}_2, \underline{G}_1\overline{G}_2, \overline{G}_1\underline{G}_2, \overline{G}_1\overline{G}_2)\big], \tag{5}$$

$$\otimes G_1 \div \otimes G_2 = [\underline{G}_1, \overline{G}_1] \times \left[\frac{1}{\overline{G}_2}, \frac{1}{\underline{G}_2}\right]. \tag{6}$$

DEFINITION 4. length of grey number is calculated as (Li *et al.*, 2007):

$$L(\otimes G) = [\overline{G} - \underline{G}]. \tag{7}$$

Definition 5. $\otimes G_1 \leqslant \otimes G_2$ for two grey numbers $\otimes G_1 = [\underline{G}_1, \overline{G}_1]$ and $\otimes G_2 = [\underline{G}_2, \overline{G}_2]$ is defined as (Li *et al.*, 2007):

$$P\{\otimes G_1 \leqslant \otimes G_2\} = \frac{\max(0, L^* - \max(0, \overline{G}_1 - \underline{G}_2))}{L^*}, \tag{8}$$

which:

$$L^* = L(\otimes G_1) + L(\otimes G_2). \tag{9}$$

Definition 6. Four relationships are assumed between two grey numbers $\otimes G_1$ and $\otimes G_2$:

- If $\underline{G}_1 = \underline{G}_2$ and $\overline{G}_1 = \overline{G}_2$ are two equal grey numbers. So $\otimes G_1 = \otimes G_2$ and also: $P\{\otimes G_1 \leqslant \otimes G_2\} = 0.5$;
- If $(\underline{G}_2 \succ \overline{G}_1)$ grey number $\otimes G_2$ is greater than $\otimes G_1$, so $\otimes G_2 \succ \otimes G_1$ and also: $P\{\otimes G_1 \leqslant \otimes G_2\} = 1$;
- If $(\overline{G}_2 \prec \underline{G}_1)$ grey number $\otimes G_2$ is smaller than $\otimes G_1$, so $\otimes G_2 \prec \otimes G_1$ and also: $P\{\otimes G_1 \leqslant \otimes G_2\} = 0$;
- When there is a common part in both grey numbers, if $P\{\otimes G_1 \leqslant \otimes G_2\} \prec 0.5$ so $\otimes G_2 \prec \otimes G_1$ and if $P\{\otimes G_1 \leqslant \otimes G_2\} \succ 0.5$ so $\otimes G_2 \succ \otimes G_1$.

Definition 7. When $\otimes G \in [\underline{x}, \overline{x}]$ is a grey number, its *absolute value* is the maximum of the absolute value of its endpoints: $|\otimes G| = \max(|\underline{G}|, |\overline{G}|)$ (Moore *et al.*, 2009).

Definition 8. If $\{\otimes G_1, \otimes G_2, \ldots, \otimes G_n\}$ is a set of grey numbers, their mean is calculated as (Moore *et al.*, 2009):

$$\otimes\mu = \frac{\otimes G_1 + \otimes G_2 + \ldots + \otimes G_n}{n} = [\underline{\mu}, \overline{\mu}].$$

Note 1. Now suppose that we have two black numbers $\otimes G_1 \in (\otimes\underline{G}_1, \otimes\overline{G}_1)$ and $\otimes G_2 \in (\otimes\underline{G}_2, \otimes\overline{G}_2)$. The calculation will be like as the above definition, but $\underline{G}_1$, $\underline{G}_2$, $\overline{G}_1$ and $\overline{G}_2$ replaced with $\otimes\underline{G}_1$, $\otimes\underline{G}_2$, $\otimes\overline{G}_1$ and $\otimes\overline{G}_2$.

Definition 9. When we have two black numbers such as $\otimes G_1 = \{[\underline{G}_{11}, \overline{G}_{11}], [\underline{G}_{12}, \overline{G}_{12}]\}$ and $\otimes G_2 = \{[\underline{G}_{21}, \overline{G}_{21}], [\underline{G}_{22}, \overline{G}_{22}]\}$, so their intersection is as:

$$\otimes G_1 \cap \otimes G_2 = \big\{\big([\underline{G}_{11}, \overline{G}_{11}] \cap [\underline{G}_{21}, \overline{G}_{21}]\big), \big([\underline{G}_{12}, \overline{G}_{12}] \cap [\underline{G}_{22}, \overline{G}_{22}]\big)\big\}.$$

According to Deng (1989), the GRA has some advantages:

- It involves simple calculations and requires a smaller number of samples; a typical distribution of samples is not needed.
- The quantified outcomes from the grey relational grade do not result in contradictory conclusions to qualitative analysis.
- The grey relational grade model is a transfer functional model that is effective in dealing with discrete data (Zavadskas *et al.*, 2009).

Table 2
MADM problem with black data.

Alternatives	Criterions			
	C_1 $(\otimes\underline{w}_1, \otimes\overline{w}_1)$	C_2 $(\otimes\underline{w}_2, \otimes\overline{w}_2)$	… …	C_n $(\otimes\underline{w}_n, \otimes\overline{w}_n)$
A_1	$(\otimes\underline{w}_{11}, \otimes\overline{w}_{11})$	$(\otimes\underline{w}_{12}, \otimes\overline{w}_{12})$	…	$(\otimes\underline{w}_{1n}, \otimes\overline{w}_{1n})$
A_2	$(\otimes\underline{w}_{21}, \otimes\overline{w}_{21})$	$(\otimes\underline{w}_{22}, \otimes\overline{w}_{22})$	…	$(\otimes\underline{w}_{2n}, \otimes\overline{w}_{2n})$
⋮	⋮	⋮	⋮	⋮
A_m	$(\otimes\underline{w}_{m1}, \otimes\overline{w}_{m1})$	$(\otimes\underline{w}_{m2}, \otimes\overline{w}_{m2})$	…	$(\otimes\underline{w}_{mn}, \otimes\overline{w}_{mn})$

4. ELECTRE Method Based on Black Numbers

Suppose a decision making problem as defined in Section 3 which contains m alternatives and n criteria to evaluate them. In this problem x_{ij} is the value of alternative A_i with respect to criterion C_j and it is not exactly known and only we know $x_{ij} \in (\otimes\underline{x}_{ij}, \otimes\overline{x}_{ij})$ which is a black number; besides the weights of criteria cannot be calculated exactly and we can just consider a black interval $w_j \in (\otimes\underline{w}_j, \otimes\overline{w}_j) = [(\underline{w}_{1j}, \overline{w}_{1j}), (\underline{w}_{2j}, \overline{w}_{2j})]$, $j = 1, 2, \dots, n$ for them, such that: $\sum_{j=1}^{n}(\underline{w}_{1j} + \overline{w}_{1j} + \underline{w}_{2j} + \overline{w}_{2j})/4 = 1$. In this situation, an MCDM problem with Black weights and data can be concisely expressed in form of a decision matrix as Table 2.

Now we propose a step by step approach to apply ELECTRE method for this kind of problems.

Step 1. Calculate the black normalized decision matrix. Transform the various scales into comparable scales by using Eq. (10).

$$n_{ij} = \frac{(\otimes\underline{x}_{ij}, \otimes\overline{x}_{ij})}{\sqrt{\sum_{i=1}^{m}[(\otimes\underline{x}_{ij}, \otimes\overline{x}_{ij})^2]}} = (\otimes\underline{n}_{ij}, \otimes\overline{n}_{ij}). \tag{10}$$

Which the multiplication, summation and division operations are performed as Definition 3 and Note 1. Then decision matrix is transformed into a normalized matrix as $\otimes N = [\otimes n_{ij}]_{m\times n}$ which its elements are black numbers.

Step 2. Calculate the weighted normalized decision matrix by using Eq. (11).

$$\otimes V = \otimes N. \otimes W. \tag{11}$$

The ijth element's of V matrix is $\otimes v_{ij} = \otimes n_{ij}. \otimes w_j = (\otimes\underline{n}_{ij}, \otimes\overline{n}_{ij})(\otimes\underline{w}_j, \otimes\overline{w}_j)$.

Step 3. Determine the black concordance and discordance set. The set of decision criteria $J = \{j \mid j = 1, 2, \dots, n\}$ is divided into two decision subsets. The concordance set C_{kl} of A_k and A_l is composed of all criteria which A_k is preferred to A_l.

$$C_{kl} = \{j \mid x_{kj} \geqslant x_{lj}\}; \quad k, l = 1, 2, \dots, m;\ k \neq l.$$

The complementary subset is called the discordance set, which is $D_{kl} = \{j \mid x_{kj} \prec x_{lj}\} = J - C_{kl}$.

To construct these two subsets, we need to compare pairs of black numbers. Assume we have two black numbers $\otimes G_1 = \{[\underline{G}_{11}, \overline{G}_{11}], [\underline{G}_{12}, \overline{G}_{12}]\}$ and $\otimes G_2 = \{[\underline{G}_{21}, \overline{G}_{21}], [\underline{G}_{22}, \overline{G}_{22}]\}$. First we examine whether their intersection is empty or not. If it's empty:

$$1.\ \text{If } \underline{G}_{11} \geqslant \overline{G}_{22} \text{ so } \otimes G_1 \succ \otimes G_2 \quad \text{and} \quad 2.\ \text{If } \overline{G}_{12} \leqslant \underline{G}_{22} \text{ so } \otimes G_2 \succ \otimes G_1.$$

If their intersection wouldn't be empty, first we calculate:

$$L_B(\otimes G_1) = [\underline{G}_{12}, \overline{G}_{12}] - [\underline{G}_{11}, \overline{G}_{11}] = [\underline{L}_1, \overline{L}_1],$$
$$L_B(\otimes G_1) = [\underline{G}_{22}, \overline{G}_{22}] - [\underline{G}_{21}, \overline{G}_{21}] = [\underline{L}_1, \overline{L}_1].$$

Then

$$L[L_B \otimes X] = \overline{L}_X - \underline{L}_X \quad \text{and} \quad L[L_B \otimes Y] = \overline{L}_Y - \underline{L}_Y,$$
$$L^* = L[L_B \otimes X] + L[L_B \otimes Y],$$
$$P(\otimes X \leqslant \otimes Y) = \frac{L^* - (\overline{X}_2 - \underline{Y}_1)}{L^*}.$$

Now consider these notices:

(1) $P(\otimes X \leqslant \otimes Y) \leqslant 0.5$ so $\otimes X \succ \otimes Y$;
(2) $P(\otimes X \leqslant \otimes Y) = 0.5$ so $\otimes X = \otimes Y$;
(3) $P(\otimes X \leqslant \otimes Y) \succ 0.5$ so $\otimes X \prec \otimes Y$.

Step 4. Calculate the black concordance matrix. In the process of ELECTRE, we must construct a concordance matrix which its klth element equals to concordance index of A_k and A_l. This index is equivalent to the sum of black weights of those criteria that form the C_{kl}. Thus, black concordance index $(\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl})$ is equal to:

$$(\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl}) = \sum_{j \in C_{kl}} (\otimes \underline{w}_j, \otimes \overline{w}_j). \tag{12}$$

The concordance index reflects the relative importance of A_k with respect to A_l. Obviously $0 \leqslant (\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl})/4 \leqslant 1$. A higher value of this index indicates the higher preference of A_k to A_l. The values of $(\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl})$ for all k and l form the black concordance matrix $[\otimes I]$.

$$[\otimes I] = \begin{bmatrix} - & (\otimes \underline{I}_{12}, \otimes \overline{I}_{12}) & \dots & (\otimes \underline{I}_{1n}, \otimes \overline{I}_{1n}) \\ (\otimes \underline{I}_{21}, \otimes \overline{I}_{21}) & - & \dots & (\otimes \underline{I}_{2n}, \otimes \overline{I}_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (\otimes \underline{I}_{n1}, \otimes \overline{I}_{n1}) & (\otimes \underline{I}_{n2}, \otimes \overline{I}_{n2}) & \dots & - \end{bmatrix}.$$

Step 5. Calculate the discordance matrix. A second index, called the discordance index reflects the degree to which the evaluations of a certain alternative A_k are worse than the evaluations of competing A_l. The discordance index d_{kl} is as Eq. (13):

$$d_{kl} = \frac{\max_{j \in D_{kl}} |\otimes v_{kj} - \otimes v_{lj}|}{\max_{j \in J} |\otimes v_{kj} - \otimes v_{lj}|}. \tag{13}$$

The values of d_{kl} for all k and l form the concordance matrix D.

$$D = \begin{bmatrix} - & d_{12} & \dots & d_{1m} \\ d_{21} & - & \dots & d_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ d_{m1} & d_{m2} & \dots & - \end{bmatrix}.$$

Step 6. Specify the effective concordance matrix. The elements of black concordance matrix have to be compared against a *veto threshold* which expresses the power attributed to a given criterion against the assertion "a outrank b", when the difference between two alternatives value in each criterion is greater than threshold. This veto threshold for concordance matrix is defined as average of its elements. So:

$$\otimes I = (\otimes \underline{I}, \otimes \overline{I}) = \sum_{k=1}^{m} \sum_{l=1}^{m} \frac{(\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl})}{m(m-1)}. \tag{14}$$

Based on this threshold we construct a Boolean matrix F (effective concordance matrix) as:

$$f_{kl} = \begin{cases} 1 & \text{if } (\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl}) \geqslant (\otimes \underline{I}, \otimes \overline{I}), \\ 0 & \text{if } (\otimes \underline{I}_{kl}, \otimes \overline{I}_{kl}) \geqslant (\otimes \underline{I}, \otimes \overline{I}). \end{cases} \tag{15}$$

In this matrix, $f_{kl} = 1$ indicates that alternative A_k is dominant and preferred to A_l.

Step 7. Specify the effective discordance matrix. Such as Step 6, elements of black concordance matrix have to be compared against a *veto threshold.* This threshold is defined as follow:

$$\overline{d} = \sum_{k=1}^{m} \sum_{l=1}^{m} \frac{d_{kl}}{m(m-1)}. \tag{16}$$

Then we construct a Boolean matrix G (effective discordance matrix) as:

$$g_{kl} = \begin{cases} 1 & \text{if } d_{kl} \leqslant \overline{d}, \\ 0 & \text{if } d_{kl} \succ \overline{d}. \end{cases} \tag{17}$$

These veto thresholds are suggested values, and we can increase or decrease them if the dominant conditions of step9 wouldn't be satisfied.

Step 8. Determine the aggregate dominance matrix. Now we calculate the intersection of the matrix F and$G$. The elements of this matrix are defined by:

$$e_{kl} = f_{kl} \cdot g_{kl}. \tag{18}$$

Step 9. Eliminate the less favorable alternatives. The aggregate matrix E's elements show the outranking relations between alternatives. If $e_{kl} = 1$, this means that A_k is preferred to A_l in both concordance and discordance criteria. But A_k still would be dominated by the other alternatives. So the condition under which A_k is an attractive alternative will be as:

$$\begin{aligned} &e_{kl} = 1, \quad \text{for at least one } l;\ l = 1, 2, \dots, m, \\ &e_{lk} = 0, \quad \text{for all } l;\ l = 1, 2, \dots, m,\ l \neq k. \end{aligned} \tag{19}$$

Otherwise, we can determine the dominated alternative from E matrix. If any column of matrix has one element of 1, then this column's related alternative is dominated by the corresponding row. So we easily eliminate such columns.

5. Numerical Example

In this section, a numerical example is presented to illustrate the application of ELECTRE method using black numbers. In this case we compared four suppliers based on five criteria, chosen from Dickson's criteria as quality (C_1), technical capability (C_2), performance history (C_3), packaging ability (C_4), management and organization (C_5) (Dickson, 1966). In this study we have used linguistic variables to show the decision maker's preferences in order to deploy the range of ambiguous responses. Table 3 illustrates a guideline to transform linguistic variables into black numbers. Also decision maker may express that ith alternative is preferred 6 times to jth alternative. We can transform this crisp number into a black number as [(5, 6), (6, 7)].

Table 3
Definition of linguistic variables for the ratings.

Alternative k may be … than I	Equivalent black number
Very poor (VP)	[(0, 0), (1, 1.5)]
Poor (P)	[(0, 0.5), (2.5, 3.5)]
Moderately poor (MP)	[(0, 1.5), (4.5, 5.5)]
Fair (F)	[(2.3, 3.5), (6.5, 7.5)]
Moderately good (MG)	[(4.5, 5.5), (8, 9.5)]
Good (G)	[(5.5, 7.5), (9.5, 10)]
Very good (VG)	[(8.5, 9.5), (10, 10)]

Table 4
The decision matrix with black data.

	C_1	C_2	C_3	C_4	C_5
w_j	$\begin{bmatrix}(0,05,0.15),\\(0.2,0.4)\end{bmatrix}$	$\begin{bmatrix}(0.01,0.11),\\(0.2,0.48)\end{bmatrix}$	$\begin{bmatrix}(0.12,0.19),\\(0.22,0.27)\end{bmatrix}$	$\begin{bmatrix}(0.14,0.17),\\(0.2,0.29)\end{bmatrix}$	$\begin{bmatrix}(0.05,0.15),\\(0.26,0.34)\end{bmatrix}$
A_1	$\begin{bmatrix}(6.5,7.5),\\(8.5,9)\end{bmatrix}$	$\begin{bmatrix}(1,1.3),\\(2.75,3.95)\end{bmatrix}$	$\begin{bmatrix}(6.5,7.25),\\(8,8.45)\end{bmatrix}$	$\begin{bmatrix}(0.5,1.5),\\(2,2.5)\end{bmatrix}$	$\begin{bmatrix}(6.5,7.3),\\(7.45,8.55)\end{bmatrix}$
A_2	$\begin{bmatrix}(3,4.2),\\(6.8,8)\end{bmatrix}$	$\begin{bmatrix}(2.4,3.5),\\(6.5,7.5)\end{bmatrix}$	$\begin{bmatrix}(4.5,5.5),\\(8,9.5)\end{bmatrix}$	$\begin{bmatrix}(5,5.95),\\(6.5,7)\end{bmatrix}$	$\begin{bmatrix}(2.5,3),\\(4.15,5.25)\end{bmatrix}$
A_3	$\begin{bmatrix}(5.5,7.5),\\(9.5,10)\end{bmatrix}$	$\begin{bmatrix}(1.5,3.5),\\(4,5.5)\end{bmatrix}$	$\begin{bmatrix}(1.5,2),\\(2.5,3.25)\end{bmatrix}$	$\begin{bmatrix}(8,9),\\(9.5,10)\end{bmatrix}$	$\begin{bmatrix}(5.5,6.5),\\(7,8.5)\end{bmatrix}$

Table 5
The weighted normalized decision matrix with black data.

	C_1	C_2	C_3	C_4	C_5
A_1	$\begin{bmatrix}(0.021,0.078),\\(0.149,0.399)\end{bmatrix}$	$\begin{bmatrix}(0.001,0.018),\\(0.107,0.632)\end{bmatrix}$	$\begin{bmatrix}(0.059,0.119),\\(0.189,0.284)\end{bmatrix}$	$\begin{bmatrix}(0.006,0.022),\\(0.037,0.077)\end{bmatrix}$	$\begin{bmatrix}(0.025,0.099),\\(0.189,0.328)\end{bmatrix}$
A_2	$\begin{bmatrix}(0.010,0.044),\\(0.119,0.354)\end{bmatrix}$	$\begin{bmatrix}(0.002,0.047),\\(0.254,1.199)\end{bmatrix}$	$\begin{bmatrix}(0.041,0.090),\\(0.189,0.319)\end{bmatrix}$	$\begin{bmatrix}(0.056,0.087),\\(0.119,0.215)\end{bmatrix}$	$\begin{bmatrix}(0.010,0.041),\\(0.106,0.201)\end{bmatrix}$
A_3	$\begin{bmatrix}(0.018,0.078),\\(0.167,0.443)\end{bmatrix}$	$\begin{bmatrix}(0.001,0.047),\\(0.156,0.880)\end{bmatrix}$	$\begin{bmatrix}(0.014,0.033),\\(0.059,0.109)\end{bmatrix}$	$\begin{bmatrix}(0.090,0.131),\\(0.174,0.307)\end{bmatrix}$	$\begin{bmatrix}(0.021,0.088),\\(0.178,0.326)\end{bmatrix}$

Now according to Table 4, the decision matrix with black data is as follow:

In this step we established the weighted normalized decision matrix (Table 5) by Eqs. (10) and (11).

Now we determine the black concordance and discordance set based on Step 3, comparing pairs of alternatives in terms of per attribute. These sets are determined as follow:

$$\begin{aligned}
&D_{12}=\{1,5\}, && C_{13}=\{3\}, && C_{21}=\{2,3,4\}, && C_{23}=\{2,3\},\\
&D_{12}=\{2,3,4\}, && D_{13}=\{1,2,4,5\}, && D_{21}=\{1,5\}, && D_{23}=\{1,4,5\},\\
&C_{31}=\{1,2,4,5\}, && C_{32}=\{1,4,5\}, && && \\
&D_{31}=\{3\}, && D_{32}=\{2,3\}. && &&
\end{aligned}$$

According to Eq. (12), the black concordance matrix is developed based on concordance sets as:

$$[\otimes I]=\begin{array}{c}\\ A_1\\ A_2\\ A_3\end{array}\begin{array}{c}\begin{array}{ccc}A_1 & A_2 & A_3\end{array}\\ \begin{bmatrix} - & [(0.1,0.3),(0.46,0.74)] & [(0.12,0,19),(0.22,0.27)]\\ [(0.27,0.47),(0.62,1.04)] & - & [(0.13,0.3),(0.42,0.75)]\\ [(0.25,0.58),(0.86,1.51)] & [(0.24,0.47),(0.66,1.03)] & - \end{bmatrix}\end{array}.$$

Similarly based on Eq. (13) and discordance sets, the black discordance matrix is:

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} A_1 & A_2 & A_3 \end{array} \\ \begin{bmatrix} - & 1 & 1 \\ 0.3248 & - & 0.3619 \\ 0.3071 & 1 & - \end{bmatrix} \end{array}.$$

For example the d_{23} element of this matrix is calculated as follow:

$$\begin{aligned} d_{23} &= \frac{\max_{j \in D_{23}} |\otimes v_{2j} - \otimes v_{3j}|}{\max_{j \in J} |\otimes v_{kj} - \otimes v_{lj}|} \\ &= \frac{\max\{|\otimes v_{21} - \otimes v_{31}|, |\otimes v_{24} - \otimes v_{34}|, |\otimes v_{25} - \otimes v_{35}|\}}{\max\{|\otimes v_{21} - \otimes v_{31}|, |\otimes v_{22} - \otimes v_{32}|, \ldots, |\otimes v_{25} - \otimes v_{35}|\}}. \end{aligned}$$

Which its absolute value is derived from Table 5 and Definition 7 and Note 1. Based on Eqs. (14) and (16) we have:

$$\begin{aligned} \overline{\otimes I} &= \frac{[(0.1, 0.3), (0.46, 0.74)] + \ldots + [(0.24, 0.47), (0.66, 1.03)]}{3(2)} \\ &= [(0.185, 0.385), (0.540, 0.890)], \\ \overline{d} &= \frac{1 + 1 + 0.3071 + \cdots + 1}{3(2)} = 0.6656. \end{aligned}$$

The effective concordance and discordance matrixes based on Eqs. (15) and (17) are:

$$F = \begin{bmatrix} - & 0 & 0 \\ 1 & - & 0 \\ 1 & 1 & - \end{bmatrix}, \qquad G = \begin{bmatrix} - & 0 & 0 \\ 1 & - & 1 \\ 1 & 0 & - \end{bmatrix}.$$

If we multiply both F and G matrixes, based on Eq. (18), the aggregate matrix H will be calculated as follow:

$$H = \begin{bmatrix} - & 0 & 0 \\ 1 & - & 0 \\ 1 & 0 & - \end{bmatrix}.$$

The aggregate matrix H illustrates that A_1 is outperformed by both A_2 and A_3. But we cannot decide about preference relation between A_2 and A_3. This decision may need to reconsider the veto threshold in Steps 6 and 7, and take a more rigorous threshold to construct F and G matrices.

6. Conclusion

Decision making by deterministic data seems so restrictive, and it can't explain all features of a real world problem. In this paper, a method is developed considering the decision maker's ambiguity in determining the alternatives preferences. Sometimes, due to the lack of information about alternatives, decision maker doesn't exactly know how to determine his/her preferences and therefore expresses his/her judgments but statements like "I think it can be better than that one" which seems ambiguous. In order to cover an extended range of such statements, application of black numbers is proposed. For this purpose, here unknown numbers are considered for weights and evaluated alternatives value against criteria. The comparison was based on ELECTRE method and black data. Then a case is solved in order to compare three suppliers against five criteria and its results persuade us that it can make the survey more precise and applicable. For future research it's suggested to use black data in other ELECTRE methods and other MADM techniques. Also there is an opportunity to develop the black fuzzy numbers, beyond the fuzzy valued interval sets, in which all numbers of a fuzzy set are grey numbers.

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Sprendimų priėmimas su neapirėžtais duomenimis: ELECTRE metodo išvystymas taikant juoduosius skaičius

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Daugiakriterinių sprendimų priėmimas, tai toks priėmimas, kuomet sprendimų priėmėjas siekia rasti racionaliausią alternatyvą iš tarpusavyje susietų alternatyvų, siekiant patenkinti atitinkamų kriterijų skaičių. Tradiciškai, sprendimų priėmimo modelio pagrindas – tikslūs duomenys. Kada trūksta tikslios informacijos, sprendimų priėmimui tikslinga taikyti metodus su neapibrėžtais duomenimis. Šiame straipsnyje pateiktas ELECTRE metodas su juodaisiais skaičiais neapibrėžtoje aplinkoje. Pasiūlytas metodas taikomas tiekėjo parinkimo uždaviniui spręsti. Išsiskiriantis metodas, kuris gali būti taikomas realių problemų sprendimui su neapibrėžtais ir neužbaigtais duomenimis.