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Multi-Criteria Inventory Classification by Using a Fuzzy Analytic Network Process (ANP) Approach

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Abstract. Inventory management is an important part of production planning process for enterprises. Decisions for strategies to determine when and how many to buy or make can be made by classifying the inventory items based on their sorts. In this evaluation, ABC inventory classification is one of the most commonly used approaches. In this study, a fuzzy analytic network process approach was proposed to determine the weights of the criteria and the scores of the inventory items were determined with simple additive weighting by using linguistic terms. Applying fuzzy ANP to a multi-criteria inventory classification problem is the novelty of this study in the related literature. In addition, the application area of the problem which is the management of the engineering vehicles' items in a construction firm is different from the other studies.
Keywords: multi-criteria decision making, fuzzy ANP, ABC classification, inventory control.

1. Introduction

Inventory management is an important issue in organizations to analyze and to track the levels of the inventory items. Quantity of the items can be elicited and the questions such as how many items and when are required can be answered. In addition, the most used items with their seasons can be determined and purchasing plans related to production plans can be prepared. But the control of the inventory becomes harder because of the large amount of data. In this case, inventory items should be classified based on their features. While some of them are more important and they should be taken care mostly by management, the others are less important and decision systems can be simpler. Therefore, inventory classification is required to evaluate inventory items and to give necessary attention to the related items. ABC analysis is the most commonly used technique in inventory control to classify the inventory items based on their features. ABC analysis is based on the Pareto principle. The Pareto principle was first applied to inventory systems by Dickie (1951) for General Electric and it was called ABC analysis. In this analysis, items are divided into three classes as A, B and C. While A items are the few important items, C items are the many unimportant items. B items are between A and C items (Sipper and Bulfin, 1997).

In traditional ABC classification, only the criterion of annual dollar usage is used, but in some conditions this criterion is not sufficient for evaluation. For example, an inexpensive item may be classified as A because of its importance to the operation of the firm. Flores and Whybark (1987) recommended a two dimensional classification where the first was the annual dollar usage and the second was criticality. Cohen and Ernst (1988) used a statistical technique called cluster analysis to group items across many dimensions, including criticality. Analytic hierarchy process (AHP) was applied to some multi-criteria inventory classification problems by Flores *et al.* (1992), Gajpal *et al.* (1994), Partovi and Hopton (1993) and Partovi and Burton (1993). Reynolds (1994) provided a classification scheme to help managers for focusing on important items.

Meta-heuristics were also used for inventory classification such as genetic algorithm and artificial neural network approach by Guvenir and Erel (1998) and Partovi and Anandarajan (2002). Ramanathan (2006) proposed a weighted linear optimization model for multi-criteria ABC inventory classification by using a DEA-like model. Zhou and Fan (2007) obtained the most favorable and the least favorable scores for each item according to Ramanathan's model. Ng (2007) proposed a weighted linear model for multi-criteria ABC inventory classification. The Ng model could be solved without a linear optimizer by using transformations, so it is simple to apply. Hadi-Vencheh (2010) proposed an extended version of Ng model. It was a simple nonlinear programming model and determined a common set of weights for all the items. Chen *et al.* (2008) used case-based multiple-criteria ABC analysis with additional criteria, such as lead time and criticality. In their study, decisions from cases as input preferences over alternatives were represented intuitively using weighted Euclidean distances. Rezaei and Dowlatshahi (2010) used a rule-based multi-criteria approach for inventory classification problem. Hadi-Vencheh and Mohamadghasemi (2011) proposed an integrated fuzzy analytic hierarchy process-data envelopment analysis (FAHP-DEA) for multiple criteria ABC inventory classification.

Inventory classification is a multi-criteria decision making (MCDM) problem that includes multiple criteria such as annual dollar usage, average unit cost, lead time, criticality, reparability, number of requests, scarcity, durability, perish ability, reparability, demand, stock ability and so on.

As seen in the literature, MCDM approaches can be used for these problems. In this study, the fuzzy Analytic Network Process (ANP) approach was used to evaluate the criteria that affect the inventory control system and to determine the scores of the inventory items. This study is different from the others in terms of the fuzzy ANP approach and the application area that is applied to a construction firm for obtaining an ABC classification system.

In the next section, the fuzzy ANP approach is defined. Then in Section 3, an application with a real-life problem is given. Proposed approach and traditional ABC analysis results are compared. Finally, the paper concludes with some remarks.

2. Fuzzy ANP Approach

The analytic network process (ANP) is a generalization of the analytic hierarchy process (AHP). The AHP was proposed by Saaty in 1980 as a method of solving socioeconomic decision making problems and has been used to solve a wide range of problems. The AHP is a framework of logic and problem-solving that spans the spectrum from instant awareness for fully integrated consciousness by organizing perceptions, feelings, judgments and memories into a hierarchy of forces which have influence on decision results. In the AHP approach, the system elements are assumed to be uncorrelated and uni-directionally influenced by a hierarchical relationship (Saaty, 2000). The AHP approach is one of the MCDM approaches with extensive applications in a wide variety of areas such as selection, evaluation, planning and development, decision making, forecasting, and so on (Hadi-Vencheh and Mohamadghasemi, 2011).

The ANP approach is an extended version of the AHP approach that can be used to assess a dynamic multi-directional relationship among decision attributes (Hamalainen and Seppalainen, 1986; Saaty, 1988; Saaty and Takiawz, 1986). It has been defined as a non-linear, network relationship among various factors. It allows for the capability to model more complex and dynamic environments which are influenced by ever changing external forces (Meade and Sarkis, 1998). The ANP approach is proposed to overcome the problem of interdependence and feedback between criteria or alternatives. The main difference between AHP and ANP is the ANP's capability of handling interrelationships between decision levels and attributes by obtaining the composite weights through the development of a supermatrix (Saaty, 1996; Huang *et al.*, 2005; Shyur, 2006).

Most values of qualitative criteria are not clear, so it is not easy to make decisions with crisp numbers. Fuzzy numbers and linguistic variables support decision makers to express the subjective judgments. Therefore, the fuzzy ANP approach is thought to be a more suitable approach to obtain realistic results. Some researchers have applied the fuzzy ANP based approach to solve complex decision making problems in different areas.

Mikhailov and Singh (2003) applied the fuzzy preference programming method to the ANP for deriving priorities from different types of uncertain ratio scale judgments and developed a prototype decision support system. Büyüközkan *et al.* (2004) studied on a fuzzy ANP approach to improve the quality of the responsiveness to customer needs and technical design requirements with Quality Function Deployment. Mohanty *et al.* (2005) used fuzzy ANP along with fuzzy cost analysis in selecting Research and Development projects. Kahraman *et al.* (2006) proposed an integrated framework based on fuzzy-QFD and a fuzzy optimization model to determine the product technical requirements to be considered in designing a product. The coefficients of the objective function were obtained by a fuzzy analytic network process (ANP) approach. Dağdeviren *et al.* (2008) used fuzzy ANP to calculate the faulty behavior risk (FBR) in work system. Tuzkaya and Önüt (2008) studied on evaluating alternative modes of transport by using fuzzy ANP with a case study between Turkey and Germany. Önüt *et al.* (2009) proposed a supplier evaluation approach based on the analytic network process (ANP) and the technique for order performance by similarity to ideal solution (TOPSIS) methods to help a telecommunication company in the GSM sector in Turkey under the fuzzy environment. Guneri *et al.* (2009) applied fuzzy ANP to the selection of an appropriate location for a shipyard. Chen and Chen (2010) used a decision-making trial and evaluation laboratory (DEMATEL), a fuzzy analytical network process (FANP), and a technique for order preference by similarity to an ideal solution (TOPSIS) forming order to develop an innovation support system (ISS) that considers the interdependence and the relative weights of each measurement criterion. Dağdeviren and Yüksel (2010) measured the sectoral competition level (SCL) of an organization within the framework of Porter's five forces analysis by using fuzzy ANP. Yüksel and Dağdeviren (2010) studied on the integration of Balanced Scorecard and fuzzy ANP to determine the performance level of a business on the basis of its vision and strategies. Shen *et al.* (2010) integrated fuzzy set theory and ANP to propose an innovative model for distinguishing strong financial prospect stocks among high book-to-market stocks. Önüt *et al.* (2011) proposed fuzzy ANP in selecting container port. Liou *et al.* (2011) combined fuzzy preference programming and ANP for strategic alliance partner selection in the airline industry.

2.1. *Fuzzy Numbers and Linguistic Variables*

Zadeh (1965) pioneered the use of fuzzy set theory to address problems involving fuzzy structure. In a universe of discourse X, a fuzzy subset $\tilde{A}$ of X is defined with a membership function $\mu_{\tilde{A}}(x)$ that maps each element x in X to a real number in the interval $[0,1]$. The function value of $\mu_{\tilde{A}}(x)$ signifies the grade of membership of x in Kaufmann and Gupta's (1991) study.

Fuzzy numbers are a fuzzy subset of real numbers, representing the expansion of the idea of the confidence interval. According to the definition of Laarhoven and Pedrycz (1983), a triangular fuzzy number (TFN) should possess the following basic features:

A fuzzy number $\tilde{A}$ on R to be a TFN if its membership function $\mu_{\tilde{A}}(x): R \to [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-L)/(M-L), & L \leqslant x \leqslant M,\ L \neq M, \\ (U-x)/(U-M), & M \leqslant x \leqslant U,\ M \neq U, \\ 0, & \text{otherwise}. \end{cases} \tag{1}$$

where L and U stand for the lower and upper bounds of the fuzzy number $\tilde{A}$, respectively, and M is for the modal value. The TFN can be denoted by $\tilde{A} = (L, M, U)$ and the following is the operational laws of two TFNs $\tilde{A}_1 = (L_1, M_1, U_1)$ and $\tilde{A}_2 = (L_2, M_2, U_2)$, as shown in Chen and Hwang's (1993) study:

$$\text{Addition of a fuzzy number } \oplus: \quad \tilde{A}_1 \oplus \tilde{A}_2 = (L_1, M_1, U_1) \oplus (L_2, M_2, U_2) = (L_1 + L_2, M_1 + M_2, U_1 + U_2). \tag{2}$$

$$\text{Subtraction of a fuzzy number } \Theta: \quad \tilde{A}_1 \Theta \tilde{A}_2 = (L_1, M_1, U_1) \Theta (L_2, M_2, U_2) = (L_1 - U_2, M_1 - M_2, U_1 - L_2) \quad \text{for } L_i > 0,\ M_i > 0,\ L_i > 0. \tag{3}$$

Multiplication of a fuzzy number $\otimes$:
$$\tilde{A}_1 \otimes \tilde{A}_2 = (L_1, M_1, U_1) \otimes (L_2, M_2, U_2) = (L_1L_2, M_1M_2, U_1U_2) \quad \text{for } L_i > 0,\ M_i > 0,\ U_i > 0. \tag{4}$$

Division of a fuzzy number $\emptyset$:
$$\tilde{A}_1 \emptyset \tilde{A}_2 = (L_1, M_1, U_1) \emptyset (L_2, M_2, U_2) = (L_1/U_2, M_1/M_2, U_1/L_2) \quad \text{for } L_i > 0,\ M_i > 0,\ L_i > 0. \tag{5}$$

Reciprocal of a fuzzy number :
$$\tilde{A}_1^{-1} = (L_1, M_1, U_1)^{-1} = (1/U_1, 1/M_1, 1/L_1) \quad \text{for } L_i > 0,\ M_i > 0,\ U_i > 0. \tag{6}$$

In this paper, the computational technique is based on the following fuzzy scale defined by Kahraman *et al.* (2006) as seen in Table 1. Linguistic variables are primarily used to assess the linguistic ratings given by decision makers for pairwise comparisons of the importance of criteria in FANP.

The scores of the inventory items for each criterion are also determined by a way of using linguistic terms as "very high", "high", "medium", "low" and "very low". Linguistic variables proposed by Cheng *et al.* (1999) are used in the study as given in Table 2. The membership functions of these linguistic variables are also given in Fig. 1.

Table 1
Linguistic scales for importance

Linguistic scale for importance	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal	(1,1,1)	(1,1,1)
Equally important (EI)	(1/2,1,3/2)	(2/3,1,2)
Weakly more important (WMI)	(1,3/2,2)	(1/2,2/3,1)
Strongly more important (SMI)	(3/2,2,5/2)	(2/5,1/2,2/3)
Very strongly more important (VSMI)	(2,5/2,3)	(1/3,2/5,1/2)
Absolutely more important (AMI)	(5/2,3,7/2)	(2/7,1/3,2/5)

Table 2
Linguistic values and mean of fuzzy numbers

Linguistic values for cost sub-criteria	Linguistic values for benefit sub-criteria	The mean of fuzzy numbers
Very low	Very high	1
Low	High	0.75
Medium	Medium	0.5
High	Low	0.25
Very high	Very low	0

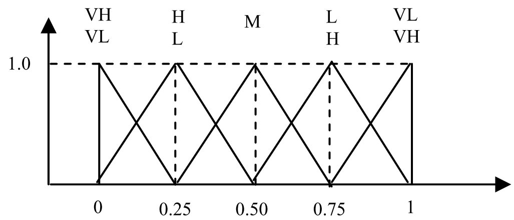

Fig. 1. Membership functions of linguistic values for sub-criteria rating.

2.2. *The Proposed Approach*

In the study, the fuzzy ANP approach was applied to relieve the uncertainness of ANP method. The fuzzy comparison ratios were used to determine the criteria and sub-criteria weights from subjective judgments. There are several procedures to attain the priorities in FAHP and FANP. The fuzzy least square method (Xu, 2000), geometric mean method (Buckley, 1985), the direct fuzzification of the method of Csutora and Buckley (2001), synthetic extend analysis (Chang, 1996), Mikhailov's fuzzy preference programming (Mikhailov, 2000, 2003) and two-stage logarithmic programming (Wang *et al.*, 2005) are some of these methods. Mikhailov's fuzzy preference programming (Mikhailov, 2000, 2003) is used in this study, thus it is able to derive consistency value and crisp priorities vector of pairwise comparison matrices. The model of the proposed approach is based on the maximum decision rule, known from the game theory. The maximum rule has also been applied by Bellman and Zadeh (1970) for solving decision-making problems in uncertain environment. Zimmermann (1990) used the same decision rule for fuzzy linear problems.

The nonlinear optimization model by Mikhailov (2000, 2003) is as follows:

$$\begin{aligned} &\text{Max} \quad \lambda, \\ &\text{s.t.} \quad (m_{ij} - l_{ij})\lambda w_j - w_i + l_{ij} w_j \leqslant 0, \\ &\qquad (u_{ij} - m_{ij})\lambda w_j + w_i - u_{ij} w_j \leqslant 0, \\ &\qquad \sum_{k=1}^{n} w_k = 1, \quad w_k > 0,\ k = 1, 2, \ldots, n, \\ &\qquad i = 1, 2, \ldots, n-1,\ j = 2, 3, \ldots, n,\ j > i, \end{aligned} \tag{7}$$

where n is the number of criteria, l_{ij}, u_{ij} and m_{ij} are the lower, upper bounds and most likely value of triangular fuzzy numbers in pairwise comparisons matrix, respectively, when experts compare ith criterion (alternative) with respect to jth criterion (alternative) and their values have been presented in Table 1, w_j is the weight of criterion j and λ is the consistency index.

The procedure for determining the evaluation criteria weights by FANP can be summarized as follows:

Step 1. Define criteria and sub-criteria and construct the pairwise comparison matrices among all the criteria/sub-criteria in the system. Assign linguistic terms to the pairwise comparison matrices by asking which criteria should be emphasized more and how much. Local weights are calculated by using 's Mikhailov (2000, 2003) model.

Step 2. Construct the network which shows the interdependencies of the criteria and construct the inner dependence matrices for each criterion. In the comparison, the questions such as which criterion will influence criterion 1 more; criterion 2 or 3 and how much are answered. Interdependent weights of the criteria are computed by multiplying the inner dependence matrix and the local weights of the criteria.

Step 3. Calculate the global weights of the sub-criteria by multiplying the local weights of the sub-criteria and the interdependent weights of the related criteria.

The procedure of calculating scores of the inventory items can be summarized as follows:

Step 1. Define the inventory items and assign linguistic variables to the items for each sub-criterion as proposed by Cheng *et al.* (1999).

Step 2. Transform the linguistic variables into the mean of the fuzzy numbers for each item.

Step 3. Use SAW method by using the weights of sub-criteria and the values of the inventory items as equation:

$$S_i = \sum_{c=1}^{n} w_c v_{ic}, \quad i = 1, 2, \ldots, m,\ c = 1, 2, \ldots, n. \tag{8}$$

The simple additive weighting method is also called the weighted sum method (Fishburn, 1997) and is the simplest and the widest used MCDM method. The sum of the weights must be 1. Each alternative is evaluated according to each criterion. S_i is the score of the inventory item based on the criteria. w_c is the weight of the cth criterion and v_{ic} is the value of ith items according to cth criteria.

Flow chart of the proposed approach is given in Fig. 2.

3. An Application of the Proposed Approach in a Construction Firm

The proposed approach is applied to a construction firm including two dozers, two excavators, a grader, a loader, a road roller, a crane, an oil machine on a truck, a generator, a compressor, a forklift, a battery charger, a welding machine, a small truck, 10 dumper trucks, a tow truck, a digger, a box truck and a crane on a truck. Also fifty workers are employed in the firm. In this study, the materials of these vehicles and tools are analyzed to manage the inventory. The materials used for construction buildings are important to be analyzed and similar applications have been found in the literature. But on the other hand, another important issue is the inventory management of the engineering vehicles for the construction firms. These vehicles have lots of items that should be tracked; otherwise scheduled jobs cannot be continued. Also all of the items cannot be tracked continuously, therefore the classification is required to see the most important items. In this application, some of these items were tried to be analyzed by traditional ABC and the fuzzy ANP approach, and then a comparison was made.

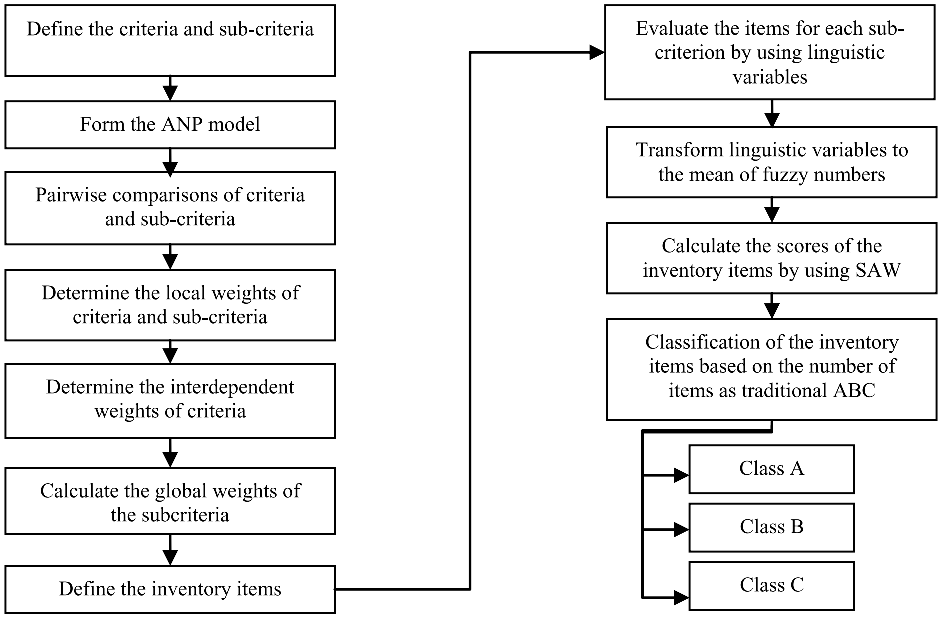

Fig. 2. The flow chart of the proposed approach.

In traditional ABC analysis, class A items constitute the most important class of inventories so far as the proportion in the total value of inventory. The class A items consist of approximately 15% of the total items, accounting for 80% of the economic value of total material usage. These items merit a tightly controlled inventory system with constant attention to the purchase management. A larger effort per item on only a few items will cost only moderately, but the effort can result in larger savings. Class B items constitute an intermediate position and constitute approximately 35% of the total items, accounting for approximately 15% of the total material consumption. These items merit a formalized inventory system and periodic attention by the purchase management. Class C items are quite insignificant. It consists of the remaining 50% items, accounting only 5% of the economic value of total material usage. Quite relaxed inventory procedures are used. The unit of a criterion must not change from item to item. For that reason, the annual dollar usage, the measuring unit of which is dollars, is used in traditional ABC analysis. According to the traditional ABC analysis 15 items are classified as class A, 18 items are classified as class B and 34 items are classified as class C.

The application is performed by the proposed approach as explained in the following steps:

Step 1. Criteria and sub-criteria are defined as given in Table 3. The criteria and sub-criteria are determined according to the literature and the application area.

The ANP model is constructed as in Fig. 3. In the first stage the objective is defined. Criteria are defined in the second stage. The arrow in this stage shows the interdependence among the criteria. The sub-criteria lie in the third stage.

Table 3
Criteria and sub-criteria used in the proposed approach

Criteria	Sub-criteria
Price (C1)	Ordering cost of the material (C11)
	Holding cost of the material (C12)
	Unit cost of the material (C13)
Criticality (C2)	Number of requests (C21)
	Level of significance (C22)
	Availability (C23)
	Substitutability (C24)
Storage ability (C3)	Space requirement of the material (C31)
	Durability (C32)
	Tendency of obsolescence (C33)
Procurement process (C4)	Lead time of the material (C41)
	Accuracy of the orders (C42)
	Lot size of the material (C43)
Maintenance (C5)	Failure rate of the vehicle related to the material (C51)
	Maintenance frequency of the vehicle related to the material (C52)
	Risk of vehicle accident in the area (C53)

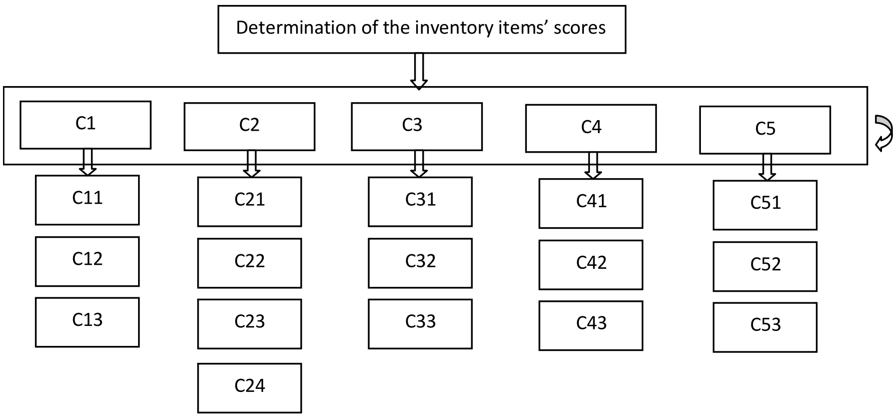

Fig. 3. The ANP model for the inventory items' scores.

Step 2. The pairwise comparisons of criteria and sub-criteria are made and local weights are calculated as given in Tables 4–8 and 9 respectively by using Table 1. Local weights are calculated by solving Mikhailov's (2000, 2003) model with Lingo 11.0 software. The consistency values (λ) are also measured by the model and therefore the comparison results can be more appropriate. If $\lambda = 1$, then the fuzzy pairwise comparison matrix is said to be consistent and if λ is negative, then the matrix is said to be strongly

Table 4
Local weights and pairwise comparison matrix of criteria

Criteria	C1	C2	C3	C4	C5	Weights
Price (C1)	(1,1,1)	(1/2,1,3/2)	(3/2,2,5/2)	(3/2,2,5/2)	(1,3/2,2)	0.27
Criticality (C2)	(2/3,1,2)	(1,1,1)	(3/2,2,5/2)	(3/2,2,5/2)	(1,3/2,2)	0.27
Storage ability (C3)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,1,1)	(2/3,1,2)	(1/2,2/3,1)	0.14
Procurement process (C4)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1/2,1,3/2)	(1,1,1)	(1/2,2/3,1)	0.14
Maintenance (C5)	(1/2,2/3,1)	(1/2,2/3,1)	(1,3/2,2)	(1,3/2,2)	(1,1,1)	0.18

$\lambda = 0.89$

Table 5
Local weights and pairwise comparison matrix of C1 sub-criteria

Sub-criteria	C11	C12	C13	Weights
C11	(1,1,1)	(2/3,1,2)	(2/5,1/2,2/3)	0.26
C12	(1/2,1,3/2)	(1,1,1)	(1/2,2/3,1)	0.28
C13	(3/2,2,5/2)	(1,3/2,2)	(1,1,1)	0.46

$\lambda = 0.69$

Table 6
Local weights and pairwise comparison matrix of C2 sub-criteria

Sub-criteria	C21	C22	C23	C24	Weights
C21	(1,1,1)	(2/5,1/2,2/3)	(2/5,1/2,2/3)	(1,3/2,2)	0.19
C22	(3/2,2,5/2)	(1,1,1)	(2/3,1,2)	(2,5/2,3)	0.34
C23	(3/2,2,5/2)	(1/2,1,3/2)	(1,1,1)	(2,5/2,3)	0.34
C24	(1/2,2/3,1)	(1/3,2/5,2)	(1/3,2/5,1/2)	(1,1,1)	0.13

$\lambda = 0.79$

inconsistent. On the other hand, if λ is close to 1 then the fuzzy pairwise comparison matrix is said to be almost consistent. The consistency values of the models are also given in Tables 4–8 and 9 respectively. The model for criteria based on Table 4 is given as follows:

$$
\begin{aligned}
&\mathrm{Max} = \lambda; \\
&1/2 * \lambda * W2 - W1 + 1/2 * W2 <= 0; \\
&1/2 * \lambda * W2 + W1 - 3/2 * W2 <= 0; \\
&1/2 * \lambda * W3 - W1 + 3/2 * W3 <= 0; \\
&1/2 * \lambda * W3 + W1 - 5/2 * W3 <= 0; \\
&1/2 * \lambda * W4 - W1 + 3/2 * W4 <= 0;
\end{aligned}
$$

$1/2 * \lambda * W4 + W1 - 5/2 * W4 <= 0;$
$1/2 * \lambda * W5 - W1 + W5 <= 0;$
$1/2 * \lambda * W5 + W1 - 2 * W5 <= 0;$
$1/2 * \lambda * W3 - W2 + 3/2 * W3 <= 0;$
$1/2 * \lambda * W3 + W2 - 5/2 * W3 <= 0;$
$1/2 * \lambda * W4 - W2 + 3/2 * W4 <= 0;$
$1/2 * \lambda * W4 + W2 - 5/2 * W4 <= 0;$
$1/2 * \lambda * W5 - W2 + W5 <= 0;$
$1/2 * \lambda * W5 + W2 - 2 * W5 <= 0;$
$1/3 * \lambda * W4 - W3 + 2/3 * W4 <= 0;$
$\lambda * W4 + W3 - 2 * W4 <= 0;$
$1/6 * \lambda * W5 - W3 + 1/2 * W5 <= 0;$
$1/3 * \lambda * W5 + W3 - W5 <= 0;$
$1/6 * \lambda * W5 - W4 + 1/2 * W5 <= 0;$
$1/3 * \lambda * W5 + W4 - W5 <= 0;$
$W1 + W2 + W3 + W4 + W5 = 1;$
End

Step 3. The network of the interdependencies for the criteria is constructed as given in Fig. 4. According to the Fig. 4, while price is affected by all the other criteria, storage ability is affected only by procurement procedure. The other relations can be seen from the figure.

The inner dependence matrices for criterion C1 and C2 are constructed as in Tables 10–11 respectively.

The dependence matrix of the criteria is formed by using the computed relative importance weights as given in Table 12.

Table 7
Local weights and pairwise comparison matrix of C3 sub-criteria

Sub-criteria	C31	C32	C33	Weights
C31	(1,1,1)	(2/5,1/2,2/3)	(1/2,2/3,1)	0.22
C32	(3/2,2,5/2)	(1,1,1)	(1,3/2,2)	0.46
C33	(1,3/2,2)	(1/2,2/3,1)	(1,1,1)	0.32

$\lambda = 0.89$

Table 8

Local weights and pairwise comparison matrix of C4 sub-criteria

Sub-criteria	C41	C42	C43	Weights
C41	(1,1,1)	(2/3,1,2)	(1,3/2,2)	0.37
C42	(1/2,1,3/2)	(1,1,1)	(3/2,2,5/2)	0.41
C43	(1/2,2/3,1)	(2/5,1/2,2/3)	(1,1,1)	0.22

$\lambda = 0.69$

Table 9

Local weights and pairwise comparison matrix of C5 sub-criteria

Sub-criteria	C51	C52	C53	Weights
C51	(1,1,1)	(2/3,1,2)	(3/2,2,5/2)	0.39
C52	(1/2,1,3/2)	(1,1,1)	(2,5/2,3)	0.43
C53	(2/5,1/2,2/3)	(1/3,2/5,1/2)	(1,1,1)	0.18

$\lambda = 0.72$

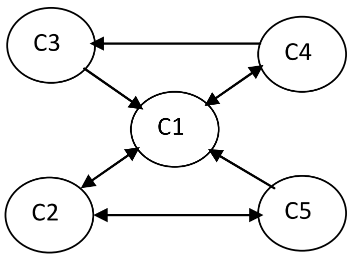

Fig. 4. The dependency of criteria.

Interdependent weights of the criteria are computed by multiplying the dependence matrix of the criteria and the local weights of the criteria as follows:

$$W_{\text{criteria}} = \begin{bmatrix} \text{C1} \\ \text{C2} \\ \text{C3} \\ \text{C4} \\ \text{C5} \end{bmatrix} = \begin{bmatrix} 1.00 & 0.40 & 0 & 1.00 & 0 \\ 0.34 & 1.00 & 0 & 0 & 1.00 \\ 0.25 & 0 & 1.00 & 0 & 0 \\ 0.22 & 0 & 1.00 & 1.00 & 0 \\ 0.19 & 0.60 & 0 & 0 & 1.00 \end{bmatrix} \times \begin{bmatrix} 0.27 \\ 0.27 \\ 0.14 \\ 0.14 \\ 0.18 \end{bmatrix} = \begin{bmatrix} 0.26 \\ 0.27 \\ 0.10 \\ 0.17 \\ 0.20 \end{bmatrix}.$$

Step 4. The global weights of the sub-criteria are calculated by multiplying the local weights of the sub-criteria and the interdependent weights of the related criteria. The global weights are shown in Table 13.

Step 5. Inventory items are defined for the construction firm and linguistic variables are assigned to the items for each sub-criterion by using Table 2.

Table 10

The inner dependence matrix of the criteria based on C1

C1	C2	C3	C4	C5	Relative importance weights
C2	1	(1,3/2,2)	(1,3/2,2)	(3/2,2,5/2)	0.34
C3	(1/2,2/3,1)	1	(1/2,1,3/2)	(1,3/2,2)	0.25
C4	(1/2,2/3,1)	(2/3,1,2)	1	(1/2,1,3/2)	0.22
C5	(2/5,1/2,2/3)	(1/2,2/3,1)	(2/3,1,2)	1	0.19

$\lambda = 0.68$

Table 11

The inner dependence matrix of the criteria based on C2

C2	C1	C5	Relative importance weights
C1	1	(1/2,2/3,1)	0.40
C5	(1,3/2,2)	1	0.60

$\lambda = 0.99$

Table 12

The dependence matrix of the criteria

Criteria	C1	C2	C3	C4	C5
Price (C1)	1.00	0.40	0	1.00	0
Criticality (C2)	0.34	1.00	0	0	1.00
Storage ability (C3)	0.25	0	1.00	0	0
Procurement process (C4)	0.22	0	1.00	1.00	0
Maintenance (C5)	0.19	0.60	0	0	1.00

Step 6. The linguistic variables are transformed into the mean of the fuzzy numbers for each item.

Step 7. Scores of the inventory items are calculated by using SAW method.

Centre knife for dozer is evaluated as given in Table 14 and the score of the items is calculated as 0.695. The other inventory items are also evaluated in the similar way.

Step 8. The items are ranked based on their score values in descending order and classified to the classes such as A, B and C.

The number of the items in the classes is determined according to the same number of the items in the classes of traditional ABC analysis. Then, the first 15 items are classified as A class, the next 18 items are classified as class B and the rest are classified as class C as given in Table 15. Also a comparison of the proposed approach and the traditional ABC analysis can be seen from Table 15. While claw of digger is classified as class A by the proposed approach, it is classified as class C by the traditional ABC analysis. This is because the only criterion taken into account is the annual dollar usage in the traditional

Table 13
The global weights of the criteria

Criteria	Sub-criteria	Local weights	Global weights
Price (C1)	C11	0.26	0.07
(0.26)	C12	0.28	0.07
	C13	0.46	0.12
Criticality (C2)	C21	0.19	0.05
(0.27)	C22	0.34	0.09
	C23	0.34	0.09
	C24	0.13	0.04
Storage ability (C3)	C31	0.22	0.02
(0.10)	C32	0.46	0.05
	C33	0.32	0.03
Procurement process (C4)	C41	0.37	0.06
(0.17)	C42	0.41	0.07
	C43	0.22	0.04
Maintenance (C5)	C51	0.39	0.08
(0.20)	C52	0.43	0.08
	C53	0.18	0.04

Table 14
The evaluation table of centre knife for dozer

Sub-criteria	Global weights	Linguistic variable	Scale value
C11	0.07	H	0.75
C12	0.08	H	0.75
C13	0.12	VH	1.00
C21	0.05	H	0.75
C22	0.09	M	0.50
C23	0.09	L	0.75
C24	0.03	M	0.50
C31	0.02	H	0.75
C32	0.05	L	0.75
C33	0.04	L	0.75
C41	0.07	M	0.50
C42	0.07	L	0.75
C43	0.04	L	0.75
C51	0.07	M	0.50
C52	0.08	M	0.50
C53	0.03	H	0.75
		Score	0.695

Table 15

Comparison of the proposed approach and the traditional ABC analysis

No	Item	Score	Proposed approach	Traditional ABC	No	Item	Score	Proposed approach	Traditional ABC
3	centre knife-dozer	0.6950	A	A	24	storage battery-digger	0.3625	C	B
4	side knife-dozer	0.6525	A	A	30	brake chamber diaphragm (back)	0.3600	C	C
20	tyre-truck	0.6100	A	A	42	wheel nut and bolt	0.3550	C	B
1	adapter-excavator	0.5925	A	A	47	motor oil	0.3500	C	C
6	adapter-loader	0.5925	A	A	18	wiper motors	0.3475	C	B
5	claw-loader	0.5750	A	A	29	brake chamber diaphragm (front)	0.3475	C	C
32	clutch pressure	0.5175	A	A	2	Pawl-pin-segment set-excavator	0.3450	C	B
33	clutch disc	0.5075	A	A	27	brake chamber(front)	0.3450	C	C
8	starter motor	0.5050	A	A	23	storage battery-truck	0.3425	C	B
7	Claw- digger	0.4875	A	C	50	transmission oil	0.3300	C	C
21	tyre-small truck	0.4750	A	A	15	starter collector	0.3275	C	B
53	oil- filter	0.4450	A	C	12	strater bearing-truck	0.3275	C	C
54	fuel filter	0.4375	A	C	14	strater bearing-engineering vehicle	0.3275	C	C
59	V-belt-engineering vehicle	0.4300	A	B	43	spring	0.3225	C	C
55	air filter	0.4150	A	B	51	differential oil	0.3175	C	C
60	V-belt-truck	0.4000	B	B	48	hydraulic oil	0.3175	C	C
56	diesel filter	0.3950	B	C	13	strater bearing-small truck	0.3150	C	C
57	motor oil filter	0.3950	B	C	46	spring bearing	0.3100	C	C
36	axle oil box seal (back)	0.3950	B	A	49	grease oil	0.3075	C	C
38	drag link	0.3925	B	B	11	starter coal-engineering vehicle	0.2975	C	C
37	joint rod	0.3925	B	B	9	starter coal-truck	0.2975	C	C
58	hydraulic oil filter	0.3900	B	B	44	spring flange	0.2925	C	C
40	tyre ball (front)	0.3850	B	A	17	headlight lane	0.2925	C	C
28	brake chamber(back)	0.3825	B	B	10	starter coal-small truck	0.2750	C	C
35	axle oil box seal (front)	0.3825	B	B	52	antifreeze	0.2750	C	C
34	pressure ball	0.3800	B	B	19	wiper	0.2650	C	C
22	storage battery-dozer/exc.	0.3750	B	A	67	working clothes	0.2475	C	C
26	storage battery-roller	0.3750	B	A	62	helmet	0.2475	C	C
25	storage battery-loader	0.3750	B	B	65	worker shoes	0.2400	C	C
41	tyre ball (back)	0.3730	B	A	45	spring clutch hub pin	0.2375	C	C
31	brake shoe	0.3700	B	B	63	welding mask	0.2350	C	C
39	brake drum	0.3675	B	B	66	boot	0.2275	C	C
16	solenoid	0.3650	B	C	64	welding glasses	0.2250	C	C
					61	work gloves	0.2025	C	C

ABC analysis. The claw of the digger is often broken in the digging process; therefore it is clear that the criterion of maintenance (C5) with its sub-criteria is especially effective for the item. Also the other criteria have some influence on the related item. Similar examples can also be seen in the results.

Compared with the traditional ABC analysis, 48 inventory items of the proposed approach remain in the same classes. On the other hand, by applying the proposed approach, while 10 out of 15 items of class A in the traditional ABC analysis remain in the same class, the other 5 items are reclassified as class B. Out of 18 items in class B, while

10 items remain in the same class B, 2 items are transferred to class A and 6 items are transferred to class C. In addition, out of 34 items of class C in the traditional ABC analysis, while 28 items remain in the same class C, 3 items are reclassified to class A and 3 items to class B.

4. Conclusion

Large amounts of data have to be processed and analyzed in inventory management by enterprises. Therefore, classifying the inventory items based on their importance is a useful tool to control the inventory more efficiently. In this study, the fuzzy Analytic Network Process approach is used to analyze and to solve a multi-criteria inventory classification problem. The fuzzy ANP approach has been applied to different kinds of MCDM problems in the literature, but not to multi-criteria inventory classification problems. The determination of the criteria weights and evaluation of the inventory items are not easy tasks. Therefore the concept of fuzziness supports decision makers to make more flexible decisions in vagueness environments.

The proposed approach takes into account the price, the criticality, the storage ability, the procurement process and the maintenance with their sub-criteria. The interdependencies between criteria are also considered within the proposed approach. The weights of the criteria and sub-criteria are determined by using Mikhailov's (2000, 2003) model. In addition the consistency values are derived from the model, so the results may be more reliable. After determining the weights of the criteria and evaluating the inventory items based on the related criteria, the SAW method is applied to calculate the scores of the items. Finally, the classification is implemented.

A real example was investigated in a construction firm for the management of the engineering vehicles' items. Inventory management was required to prevent the disruption of the scheduled jobs. If the items of the engineering vehicles were absent or defective, the construction processes could not be continued. Therefore, the inventory items had to be tracked in a proper way. It was shown that the proposed approach is useful and effective to classify the inventory items of the construction firms. In addition, decision making with a large number of the inventory items can be easier and faster by using a decision support system. In this case, a proper decision support system can be designed to determine the weights, to evaluate and to classify the inventory items.

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Daugiakriterinis atsargų klasifikavimas taikant analitinio tinklinio proceso (ANP) metodą neraiškioje aplinkoje

Şafak KIRIŞ

Atsargų valdymas yra svarbi įmonės gamybos planavimo proceso dalis. Strateginiai sprendimai, siekiant nustatyti, kada ir kiek pirkti ar gaminti, gali būti priimami suklasifikavus atsargas pagal rūšis. Vienas iš dažniausiai šiam tikslui naudojamų būdų yra tradicinė ABC klasifikacija. Šiame tyrime siūloma taikyti neraiškųjį analitinį tinklinį procesą (*fuzzy ANP*) vertinimo kriterijų santykiniam reikšmingumui nustatyti. Atsargų grupės vertinamos taikant lingvistinę išraišką bei paprastųjų svorių sudėjimo (*SAW*) metodą. Šio tyrimo naujumas grindžiamas neraiškiojo analitinio tinklinio proceso naudojimu. Be to, taikymo sritis, t.y. mechanizmų valdymas statybos įmonėje, skiriasi nuo kitų tyrimų.