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SEQUENTIAL DETECTION OF MANY CHANGES IN SEVERAL UNKNOWN STATES OF DYNAMIC SYSTEMS

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Abstract. An essentially new method for sequential detection of many abrupt or slow changes in several unknown states of dynamic systems is presented. This method is based on the sequential nonlinear mapping into two-dimensional vectors of many-dimensional vectors which describe the present system states. The expressions for sequential nonlinear mapping are obtained. The mapping preserves the inner structure of distances between the vectors. Examples are given.

Key words: states of a dynamic system, sequential detection of changes, sequential nonlinear mapping.

1. Introduction. Often there is a necessity to watch the states of various dynamic objects or technological processes, their changes for an unlimited time. Those changes may be abrupt or slow. Dynamic objects or technological processes may be described by various parametric models. When the state of an object changes the parameters of the model change as well. If the object is described by a random process generated by this object, then the object state is described by the data characterizing random process. Thus, in all cases we can decide about the object state or its abrupt or slow change according to the same data or their changes. The object can have several states and we need watch the states and detect their changes sequentially and independently of the history. We shall still add that it is convenient to watch the object state and its changes marking it by some mark on the PC screen. According to the mark position we can make a decision of the object state and its change if the mark position changes.

For decision of these problems it is necessary to have a method of sequential detection of many changes in several unknown properties of random processes. There are many methods of detection of changes in the properties of random processes in the scientific publications (Kligienė and Telksnys, 1984; Basseville and Benveniste, 1986; Nikiforov, 1983), but there are no methods to solve the above mentioned problems.

In this paper we present an essentially new method for sequential detection of many changes in several unknown states of dynamic systems. This method is based on the sequential nonlinear mapping on the plane of vectors of the parameters, given by dynamic systems.

2. Statement of the problem. Let a dynamic system (DS) be in any state s_i of the set of possible states: $s_i \in S$. We can watch L parameters at the output of DS. Those parameters can be of any physical nature (then we must introduce the scale coefficient for each parameter). We can watch a random process too. That process may be described by a proper mathematical model, e.g., autoregressive (AR) sequence:

$$z_t - m = -\sum_{i=1}^{p} a_i(z_{t-i} - m) + bv_t, \qquad (1)$$

where we have $L = p + 2$ parameters: m, b, a_i $(i = 1, \dots, p)$, which will be estimated from the local-stationary segment of the watched process.

It is necessary to map the L-dimensional vectors sequentially nonlinearly into two-dimensional vectors in order to represent the present state by some mark on the screen of PC and, having in mind the existence of particular states, to identify the current state, a deviation from it or a transfer to another state when the mark changes its position.

3. Solution of the problem. At the very beginning we have to carry out the nonlinear mapping of the M vectors ($M \geqslant 2$) simultaneously. We shall use for that the expressions in (Sammon, 1969). It is very convenient when the M vectors include all the possible states S of DS. Then the screen is "fixed" from the very beginning because of the automatic scale. After that we have to realize the sequential nonlinear mapping of the arriving vectors and, in such a way, we can watch the present state, its changes and deviations from it for a practically unlimited time. In order to formalize the method we shall mark by N this practically unlimited number of the arriving vectors.

Thus, let us have $M+N$ vectors in the L-hyperspace. We shall mark them X_i, $i = 1, \dots, M$; X_j, $j = M+1, \dots, M+N$. M vectors are already simultaneously mapped into two-dimensional vectors Y_i, $i = 1, \dots, M$. Now we need to sequentially map the L-dimensional vectors X_j into two-dimensional vectors Y_j, $j = M+1, \dots, M+N$. Here the mapping expressions will change into sequential mapping expressions, respectively. First, before performing iterations it is expedient to put the two-dimensional vectors being mapped in the same initial conditions, i.e., $y_{jk} = c_k$, $j = M+1, \dots, M+N$; $k = 1, 2$. Note, that in the case of simultaneous mapping of the first M vectors, the initial conditions must be chosen in a random way. Let the distance between the vectors X_i and X_j in the L-hyperspace be defined by d^x_{ij} and on the plane – by d^y_{ij}, respectively. This algorithm uses the Euclidean distance measure, because if we have no a priori knowledge concerning the data, we would have no reason to prefer any metric over the Euclidean metric (Sammon, 1969).

Next, we compute the normalized error of distances H.

$$H_j = \frac{1}{\sum_{i=1}^{M} d^x_{ij}} \sum_{i=1}^{M} \frac{(d^x_{ij} - d^y_{ij})^2}{d^x_{ij}}, \quad j = M+1, \dots, M+N. \qquad (2)$$

For correct mapping we have to change the positions of vectors Y_j, $j = M+1, \dots, M+N$ on the plane in such a way that the error H_j would be minimal. This is achieved by using the steepest descent procedure. After the r-th iteration the error of distances will be

$$H_j(r) = \frac{1}{\sum_{i=1}^{M} d_{ij}^x} \sum_{i=1}^{M} [d_{ij}^x - d_{ij}^y(r)]/d_{ij}^x, \tag{3}$$

$$j = M+1, \dots, M+N;$$

here

$$d_{ij}^y(r) = \sqrt{\sum_{k=1}^{2} [y_{ik} - y_{jk}(r)]^2}, \tag{4}$$

$$i = 1, \dots, M; \qquad j = M+1, \dots, M+N.$$

During the $r+1$-iteration the coordinates of the mapped vectors Y_j will be

$$y_{jk}(r+1) = y_{jk}(r) - F \cdot \Delta_{jk}(r) \tag{5}$$

$$j = M+1, \dots, M+N; \qquad k = 1, 2;$$

where

$$\Delta_{jk}(r) = \frac{\partial H_j(r)}{\partial y_{jk}(r)} \Big/ \left| \frac{\partial^2 H_j(r)}{\partial y_{jk}^2(r)} \right|, \tag{6}$$

F is the coefficient for the correction of coordinates, and it is defined empirically to be $F = 0.35$;

$$\frac{\partial H_j}{\partial y_{jk}} = E \sum_{i=1}^{M} \frac{D \cdot B}{d_{ij}^x \cdot d_{ij}^y}, \tag{7}$$

$$\frac{\partial^2 H_j}{\partial y_{jk}^2} = E \sum_{i=1}^{M} \frac{1}{d_{ij}^x \cdot d_{ij}^y} \left[D - \frac{B^2}{d_{ij}^y} \left(1 + \frac{D}{d_{ij}^y} \right) \right], \tag{8}$$

where

$$E = -\frac{2}{\sum_{i=1}^{M} d_{ij}^x}, \qquad D = d_{ij}^x - d_{ij}^y, \qquad B = y_{jk} - y_{ik}.$$

When $H_j < \varepsilon$, where ε is chosen under concrete conditions, the iteration process is over and the result is shown on the PC screen. In fact it is enough to be $\varepsilon = 0.01$.

4. Simulation results. Let us take a DS which has $S = 4$ stationary states. The system is described by three-order autoregressive equation (1) with the parameters (see Table 1):

Table 1. The AR parameters of the states $1 \div 4$ of DS

STATE	a_1	a_2	a_3	b
1	0.3	0.1	0.1	1.0
2	0.2	0.65	0.4	1.0
3	-0.5	0.3	0.4	1.0
4	-0.3	-0.1	0.15	1.0

We detect the states of DS at $M + N = 16$ time moments. At first we shall take such a case when the number of initial simultaneous mapping of state vectors is equal to the number of stationary states of the DS: $M = S = 4$, and during the time moments $M = 1 \div 4$ the DS passes through all its states S. After that we shall detect the states of DS at the time moments $N = 5 \div 16$ sequentially. A priori the states of DS are known at the time moments (see Table 2).

Table 2. The states of DS at the time moments $M + N = 4 + 12 = 16$

MAPPING	SIMULTANT (i)				SEQUENTIAL (j)											
MARK	×				+											
TIME MOMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
STATE	1	2	3	4	1	4	2	4	1	3	3	2	1	3	4	2

At each time moment the AR parameters are estimated from locally-stationary segments of 256 length using the Yule–Walker equations (Box and Jenkins, 1970).

In Fig. 1 the results of mapping are presented, where at the first 4 time moments state vectors mapped simultaneously are denoted by mark × with the index which means the time moment number, and the state vectors mapped sequentially are denoted by mark + with the respective index.

Next, let us take the same DS, but the number of initial simultaneously mapped state vectors will be $M = 2$ and does not involve all the possible states of DS. A priori the changes of DS states are known and are presented in Table 3.

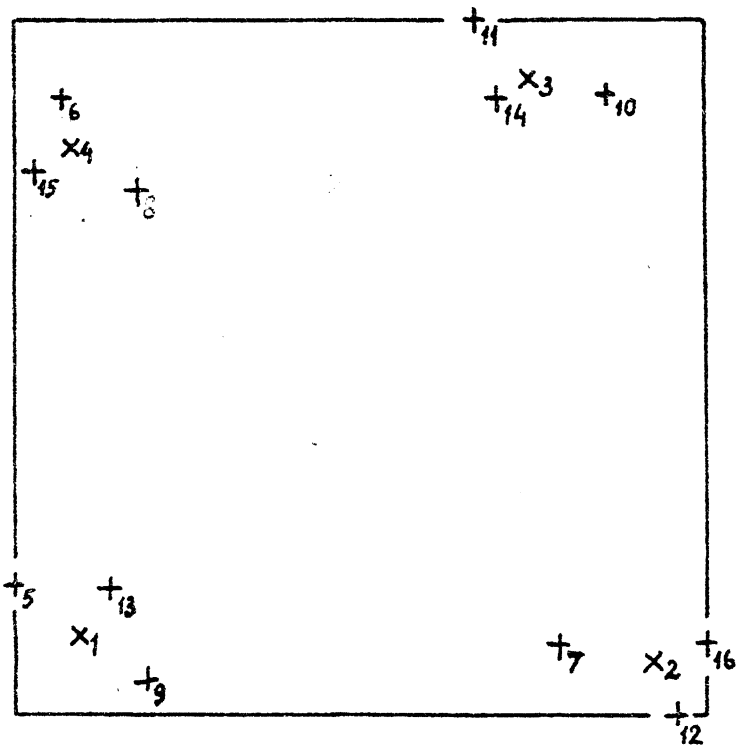

Fig. 1. The view on the PC screen of the mapped vectors of DS states for the first situation.

Table 3. The states of DS at the time moments $M + N = 2 + 14 = 16$

MAPPING	SIMULTANT (i)		SEQUENTIAL (j)													
MARK	×		+													
TIME MOMENT	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
STATE	1	2	3	4	1	4	2	4	1	3	3	2	1	3	4	2

The mapping results are presented in Fig. 2.

In Fig. 3 the situation similar to that of Fig. 2 is presented. The difference is at the time moment number 11, where a slow

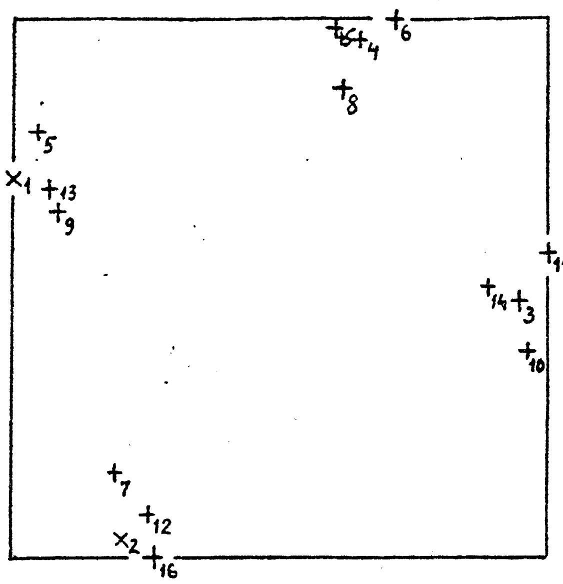

Fig. 2. The view on the PC screen of the mapped vectors of DS states for the second situation.

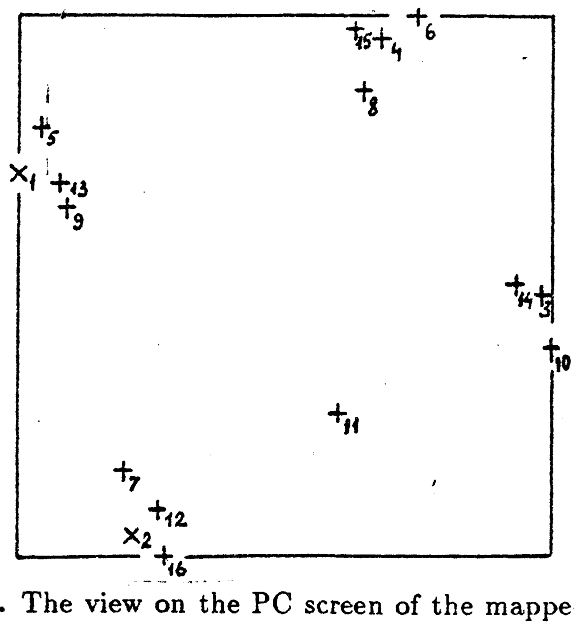

Fig. 3. The view on the PC screen of the mapped vectors of DS states for the third situation.

change of the state of DS took place, and it was between the states number 3 and number 2. In Fig. 3 this situation is clear.

5. Conclusions. The described method enables us to sequentially watch the dynamic system states, their jumpwise and slow changes on the screen of PC.

At the very beginning, before sequential watching of the states, it suffices to map simultaneously only $M = 2$ state vectors.

This method allows us to watch the dynamic system states, to detect their changes in fact without time limitations.

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