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A Multi-Attribute Decision Making Model Based on Distance from Decision Maker's Preferences

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Abstract. This paper proposes a new multi-attribute ranking procedure based on distance from decision-maker preferences. This method has two phases. In the first phase, the decision maker is asked to define the preferred performance for each attribute. In the second phase, Weighted Sum method and new distance-based normalization procedure are used to determine the overall performance rating of alternatives.

Keywords: multi-attribute, decision making, preferred performance, weighted sum method, ideal point approach.

1. Introduction

Multi-attribute decision making (MADM) refers to screening, prioritizing, ranking, or selecting a set of alternatives usually under independent, incommensurate or conflicting attributes (Saremi *et al.*, 2009; Hwang and Yoon, 1981). A MADM problem can be concisely expressed in the matrix format as shown below:

$$
D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}
\begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\
\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{12} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{1n} & x_{2n} & \cdots & x_{mn} \end{bmatrix} \end{array},
$$
$$
W = [w_1, w_2, \ldots, w_n],
$$

where $A_1, A_2, \ldots, A_m$ are feasible alternatives, $C_1, C_2, \ldots, C_n$ are attributes (criteria), x_{ij} is the performance rating of ith alternative with respect to jth attribute, and w_j is a weight (significance) of jth attribute.

In a typical MADM evaluation, attributes can be classified into two main categories: cost attributes and benefit attributes. In the case of benefit attributes, the higher score is assigned to the alternative which performance rating is higher, i.e., preferable is a maximum of jth attribute. In contrast to the previous, in the case of cost attributes, higher score is assigned to the alternative which performance rating is lower, i.e., the minimum of jth attribute is preferable.

In addition to cases in which decision-makers prefer the higher or lower performance ratings, there are also cases where decision-makers express their preferences using the preferred performance ratings. For example, in the case of the computer selection IT specialists can provide some recommendations in relation to the characteristics of computers, i.e., to define some necessary or desirable characteristics of the computers. In these cases, an alternative, i.e., computer, whose performances are equal to desirable performances, compared to all attributes, is potentially the best alternative. However, in the real-world cases of evaluation, the performance ratings of real alternatives usually are different from the necessary or desirable performance ratings, at least in relation to one attribute.

The attributes that are used for evaluation of alternatives sometimes can be mutually dependent, to some extent. As a result, alternatives whose performance ratings in some way deviate from the preferred performance ratings may be more acceptable. For example, an alternative could be much more acceptable if any of its performance ratings, according to a benefit attribute, exceeded the preferred performance rating without a significant increase of performance ratings of some cost attributes or a slightly worse performance rating of a benefit attribute which significantly affect the decrease of performance ratings of cost attributes.

The best solution, i.e., the most acceptable alternative, is an alternative whose performance provides the best compromise between performance ratings of benefit and cost attributes.

The rest of the paper is organized as follows. In Section 2 the Weighted Sum method is presented; in Section 3, two characteristic normalization procedures are described in details; in Section 4 the normalization procedure, based on preferred performance ratings and our approach, is proposed. An illustrative example is provided and discussed in Section 5. Finally, Section 6 contains the conclusions of the paper.

2. The Weighted Sum Method

The Weighted Sum (WS) method, more often known as the Simple Additive Weighted (SAW) method (MacCrimmon, 1968), is probably the best known and most widely used MADM method (Hwang and Yoon, 1981; Chen and Hwang, 1992; Chou *et al.*, 2008).

The basic logic of the WS method is to obtain a weighted sum of performance ratings of each alternative over all attributes (MacCrimmon, 1968; Hwang and Yoon, 1981; Chen and Hwang, 1992; Yoon and Hwang, 1995). Therefore, the overall performance rating of each alternative is obtained by using the following formula:

$$S_i = \sum_{j=1}^{n} w_j \cdot r_{ij}, \tag{1}$$

where S_i is the overall performance rating of the ith alternative; w_j is the weight of jth attribute; and r_{ij} is a normalized performance rating of ith alternative with respect to jth attribute.

As it can be seen from the formula (1), we should have normalized performance ratings in order to eliminate computation problems that can be caused by using different units of measures in a decision-making matrix.

A number of normalization methods, by different complexity, have been proposed. Some MADM methods require the use of a certain normalization methods. For example, the COPRAS method (Zavadskas *et al.*, 1994) is based on the use of linear transformations – the Sum method, while the authors of the MOORA method (Brauers and Zavadskas, 2006) argue that the square root of the sum of squares of each alternative per attribute, also known as Vector normalization method, is the only appropriate normalization method for this method.

Contrary to the above group of MADM methods, some other MADM methods have their recommended normalization methods, but they can also be used with other normalization methods. For example, the TOPSIS method (Hwang and Yoon, 1981) is based on the use of a vector normalization method, but it can also be used with some other normalization methods (Wang and Chang, 2007; Mahdavi *et al.*, 2008; Wu *et al.*, 2009), especially when the evaluation is performed in a fuzzy environment.

The WS method can be used with various normalization methods. However, the aggregative function, used in the WS method, does not make a difference between benefit and cost type attributes, that is why the normalization procedure used with WS method must at the same time transform performance ratings of cost type attributes into the adequate benefit performance ratings.

3. Normalization Procedures

In MADM methods, normalization procedures are used to eliminate different units of measure in which performance ratings of attributes are expressed. Many authors suggest the use of various normalization procedures. Two characteristic normalization procedures:

- Linear Scale Transformation, Max Method; and
- Linear Scale Transformation, MaxMin Method,

are presented below.

3.1. *Linear Scale Transformation, Max Method*

This method provides the simplest normalization procedure. In Linear Scale Transformation – Max (LST-Max) method, the performance rating of each alternative is divided by a maximum performance rating for that attribute.

For benefit attributes the normalized performance ratings are calculated (Zavadskas and Turskis, 2008; Van Delft and Nijkamp, 1977) using the following formula:

$$r_{ij} = \frac{x_{ij}}{x_j^+}, \tag{2}$$

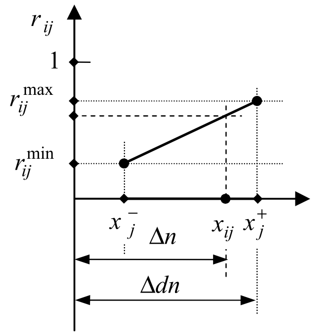

Fig. 1. The performance-based normalization, the LST-Max method.

where x_{ij} is a performance rating of ith alternative with respect to jth attribute; and x_j^+ is the largest performance rating of jth attribute.

This way of normalization, where the performance rating of alternative is used as the nominator of ratio $\Delta n/\Delta dn$, can be classified as the performance-based normalization procedure (see Fig. 1).

In order to transform cost to benefit type performances, the normalized performance ratings are calculated using the following formula:

$$r_{ij} = x_j^- / x_{ij}, \tag{3}$$

where x_j^- is the smallest performance rating of the considered attribute.

In addition to the procedure discussed above, the performance-based normalization procedures also include the well-known:

- Linear Scale Transformation – Sum method (LST-Sum); and
- Vector normalization (VN),

where performance ratings are also used as nominators.

Unlike the procedure discussed above, in Linear Scale Transformation – Sum method (4) the sum of all performance ratings, with respect to the considered attribute, is used as the denominator, while the Vector normalization (5) uses the square root of sum of squares of performance ratings as the nominator (Van Delft and Nijkamp, 1977).

$$r_{ij} = x_{ij} \Big/ \sum_{i=1}^{n} x_{ij}, \tag{4}$$

$$r_{ij} = x_{ij} \Big/ \left(\sum_{i=1}^{n} x_{ij}^2 \right)^{1/2}. \tag{5}$$

Nominators, used in (4) and (5), have an effect on values of normalized performance ratings, but do not change anything fundamentally in relation to the formula (2).

Using these normalization procedures, that belong to the performance-based normalization procedures, performance ratings are transformed into dimensionless values that are in the interval [0,1], or more precisely in the interval $[x_j^-/\Delta dn_j, x_j^+/\Delta dn_j]$, while the alternative with the best performance rating has the highest value of normalized performance rating.

As can be seen from (2), (4) and (5), the performance-based normalization procedures do not permit inclusion of the decision-makers preferences in the process of normalization.

3.2. *Linear Scale Transformation, MaxMin Method*

This normalization method considers both the maximum and minimum performance ratings of attributes during the calculation (Zavadskas and Turskis, 2008; Weitendorf, 1976). The normalized value r_{ij} is obtained by using the formula:

$$r_{ij} = \frac{x_{ij} - x_j^-}{x_j^+ - x_j^-}, \tag{6}$$

for benefit attributes and by using the following formula:

$$r_{ij} = \frac{x_j^+ - x_{ij}}{x_j^+ - x_j^-}, \tag{7}$$

for cost attributes.

In Linear Scale Transformation – MaxMin (LST-MaxMin) method, instead of using performance ratings as nominators, the distance between performance ratings of alternatives and appropriate reference points is used; therefore, this type of normalization can be classified as the distance-based normalization procedure (see Fig. 2).

When the LST-MaxMin method is used a for benefit attribute, the distance is calculated as the difference between the performance rating of the considered alternative and the worst performance rating of all alternatives according to the considered attribute ($\Delta n = x_{ij} - x_j^-$); or for cost attributes, as the difference between the best performance rating of all alternatives and performance rating of the considered alternative

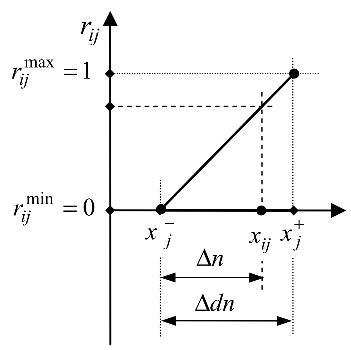

Fig. 2. The distance-based normalization, the LST-MaxMin method.

$(\Delta n = x_j^+ - x_{ij})$. Using the described procedure, the obtained distances are greater than or equal to zero $\Delta n \geqslant 0$.

The denominator used in (6) and (7), $\Delta dn_j = x_j^+ - x_j^-$, transform the obtained distances to dimensionless values that belong to the interval [0,1], whereby the normalized performance of the alternative with the best performance ratings to the considered attribute has the value 1, and worst has the value 0.

In addition to the discussed normalization procedure, the LST-MaxMin method, in the distance-based normalization procedures may also be included, less frequently used, Juttler (1966) and Juttler–Korth (Korth, 1969) normalization procedures.

For normalization Juttler (Zavadskas and Turskis, 2008; Brauers *et al.*, 2008; Juttler, 1966) proposed the following formula:

$$r_{ij} = \frac{x_j^+ - x_{ij}}{x_j^+}, \tag{8}$$

for benefit attributs. A similar formula was proposed by Korth (1969):

$$r_{ij} = 1 - \left| \frac{x_j^+ - x_{ij}}{x_j^+} \right|. \tag{9}$$

Some disadvantages of Juttler and Juttler–Korth normalization procedures are being discussed in Zavadskas and Turskis (2008) and Brauers *et al.* (2008).

4. The Proposed Model for Ranking Alternatives Based on Preferred Performance Ratings

To highlight the advantage that can be achieved by using the preferred performance ratings of attributes during the evaluation of alternatives, we propose the use of the Weighted Sum method, which includes the following steps:

- define the preferred performance rating for each attribute;
- normalize performance ratings;
- apply the aggregation procedure to determine the overall ranking index for each alternative; and
- select the most acceptable alternative or rank alternatives.

Step 1. Define the preferred performance ratings for each attribute

After creating the initial decision-making matrix, the first step in our methodology is formation of a virtual alternative $A^* = \{x_1^*, x_2^*, \ldots, x_n^*\}$, whose elements x_j^* are the preferred performance ratings of attributes.

Preferred performance ratings are assigned by the decision-maker according to his/her preferences. If the preferred performance rating of any attribute is not assigned, it can be determined by using the following formula (Zavadskas and Turskis, 2010):

$$x_j^* = \left\{ \left(\max_i x_{ij} \middle| j \in J^{\max} \right), \left(\min_i x_{ij} \middle| j \in J^{\min} \right) \right\}, \tag{10}$$

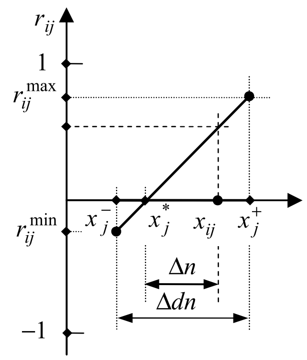

Fig. 3. The distance-based normalization, with respect to preferred performance ratings.

where x_j^* is the preferred performance rating of jth attribute; $J^{\max}$ is associated with benefit attributes; and $J^{\min}$ is associated with cost attributes.

Step 2. Normalize performance ratings

The second step in our methodology is the normalization of the decision-making matrix elements, and also the normalization of the preferred performance ratings of attributes.

Taking into account the preferred performance ratings of attributes, in order to calculate the normalized performance ratings, we combine the Weitendorf (1976) and Juttler (1966) approaches. Substituting the nominator in formula (6) by the nominator which represents the distance between the performance rating of ith alternative and the preferred performance rating of jth attribute (see Fig. 3), we have the following formula:

$$r_{ij} = \frac{x_{ij} - x_j^*}{x_j^+ - x_j^-}, \tag{11}$$

that can be used in case of benefit attributes. In the previous formula x_j^* represents the preferred performance rating of jth attribute.

In a similar way, by replacing the nominator in formula (7), we can get the formula for the normalization of cost attributes, in the following form:

$$r_{ij} = \frac{x_j^* - x_{ij}}{x_j^+ - x_j^-}. \tag{12}$$

In addition to the normalization, formulae (11) and (12) also perform the following transformations:

- In the case of benefit attributes, i.e., the optimization direction is maximization, normalized performance ratings are:
  - positive, $r_{ij} > 0$, if the performance rating x_{ij} of ith alternative is higher than the preferred performance rating of jth attribute;

- negative, $r_{ij} < 0$, if the performance rating of ith alternative is lower than the preferred.

- In the case of cost attributes, i.e., the optimization direction is minimization, normalized performance ratings r_{ij} are:
  - positive, $r_{ij} > 0$, if the performance rating x_{ij} of ith alternative is lower than the preferred performance rating of jth attribute;
  - negative, $r_{ij} < 0$, if performance rating of ith alternative is higher than the preferred.
- And finally, regardless of the attribute type, normalized performance ratings have the value zero, $r_{ij} = 0$, when the performance rating is equal to preferred performance rating, because the distance between the performance and the preferred performance is equal to zero.

After normalization, the next step in our methodology is determination of the overall performance ratings for considered alternatives.

Step 3. Aggregation phase

The overall performance ratings for considered alternatives are calculated using the formula (1), which is shown again below in order to make this presentation clearer:

$$S_i = \sum_{j=1}^{n} w_j \cdot r_{ij}. \tag{13}$$

Using the formula (13) we get values that belong to the interval $S_i \in [-1, 1]$. The overall performance ratings of aternatives which have a performance ratings equal to the preferred performance ratings are equal to zero, $S_i = 0$.

In a typical real-world MADM problems some performance ratings of alternatives usually deviate from the preferred performances. The proposed normalization procedure allows a compensation between better and worse performance ratings of alternatives.

Alternatives, whose some significantly better performance ratings successfully compensate the impact of worse performance ratings achieved with respect to the remaining attributes, $\sum_{r_{ij}>0} w_j r_{ij} - \sum_{r_{ij}<0} w_j r_{ij} > 0$, have the overall performance rating greater than zero, $S_i > 0$.

Contrary to the above mentioned case, the alternatives, whose better performance achieved on the basis of some attributes cannot compensate the impact of worse performance ratings achieved with respect to the remaining attributes, $\sum_{r_{ij}>0} w_j r_{ij} - \sum_{r_{ij}<0} w_j r_{ij} < 0$, have the overall performance rating less than zero, $S_i < 0$.

And finally, the alternatives whose better performances ratings are completely canceled by worse performance ratings,$\sum_{r_{ij}>0} w_j r_{ij} - \sum_{r_{ij}<0} w_j r_{ij} = 0$, have the overall performance rating equal to zero, $S_i = 0$.

By using the Weighted Sum method and the proposed distance-based normalization procedure, i.e., by applying (13), (11) and (12), we get the procedure for calculating overall performance ratings of alternatives which is similar to the procedure used in the newly proposed MOORA method (Brauers and Zavadskas, 2006), where in order to calculate overall performance ratings of alternatives (Brauers and Zavadskas, 2009) the following formula is used:

$$\ddot{y}_j^* = \sum_{i=1}^{i=g} s_i x_{ij}^* - \sum_{j=g+1}^{i=n} s_i x_{ij}^*, \tag{14}$$

where $\ddot{y}_j$ is the overall performance rating of jth alternative; s_i is the significance coefficient of ith attribute, x_{ij}^* is the normalized performance of jth alternative with respect to ith attribute; $i = 1, 2, \dots, g$ represent the benefit and $i = g + 1, g + 2, \dots, n$ represent the cost attributes.

The proposed procedure is also similar to the procedure used by Kalibatas and Turskis (2008).

Step 4. Selection of the most acceptable alternative or ranking alternatives

In accordance with the previous consideration, in the case of ranking alternatives based on the preferred performance ratings of attributes, the most acceptable alternative is the alternative with the highest value of S_i, and the best alternative, A^*, is determined as

$$A^* = \max_i S_i = \max_i \sum_{j=1}^{n} w_j \cdot r_{ij}. \tag{15}$$

5. A Numerical Example

In this section, we consider a numerical example in order to explain accurately the proposed methodology. After that, as a comparative analysis, we compare the results from the proposed methodology and results obtained by using well-known MADM methods: SAW and TOPSIS.

5.1. *An Illustrative Example*

Suppose that the management of a company plans to replace the computers of employees in a department. Suppose also that, based on the characteristics of business activities that are performed in the department, IT specialist recommends the purchase of a model HP Compaq 6730b Notebook PC with the following preferred characteristics:

- CPU (speed) approximately 2.4 GHz;
- RAM (capacity) approximately 4 GB;
- HDD (capacity) approximately 250 GB.

In addition to the above-mentioned characteristics, i.e., attributes, the prices and warranty periods are also considered, so that the complete list of attributes on which the selection will be made contains the following attributes: Price, CPU (speed), RAM (capacity), HDD (capacity) and Warranty.

Table 1
Importance and weights of attributes

Attribute	Price	CPU	RAM	HDD	Warranty
Importance (1–10)	7	7	5	3	1
Weight	0.304	0.304	0.217	0.130	0.043

Table 2
Initial decision-making matrix – Characteristics of HP Compaq 6730b Notebook PC models

Attributes		C_1	C_2	C_3	C_4	C_5
		Price	CPU	RAM	HDD	Warranty
			Speed	Capacity	Capacity	Time
Unit of measures		$	GHz	GB	GB	Year(s)
Available				1; 2; 4	120–500	1; 3
Optimization type		min	max	max	max	max
Preferred ratings		1200.00	2.40	4	250	3
Weights (w_j)		0.304	0.304	0.217	0.130	0.043
No.	Model					
$A1$	FH008AW	1,549.00	2.53	2	160	3
$A2$	NQ281AW	1,499.00	2.53	2	250	3
$A3$	FH005AW	1,379.00	2.40	2	160	3
$A4$	KS183UT	1,259.00	2.40	4	*500*	3
$A5$	KS181UT	1,129.00	2.40	2	320	3
$A6$	KS179UT	1,049.00	2.40	4	320	1
$A7$	KS180UT	989.00	2.40	2	250	3

Suppose also that IT specialists, using the values on a scale of 1–10, assign the importance to these attributes. The assigned values and appropriate attribute weights are shown in Table 1.

Some of the most important characteristics of certain HP Compaq 6730b Notebook PC models (attributes), attributes weights, the optimization type and the preferred performance ratings of attributes are shown in Table 2.[1]

After creating the initial decision-making matrix, its normalization was carried out. Effects of normalization achieved by applying (11) and (12), are shown in Table 3.

In the next step, the weighted normalized decision-making matrix is formed by multiplying elements in columns of normalized decision-making matrix by the corresponding weights. The weighted normalized decision-making matrix is shown in Table 4.

[1]*Source*: `http://h10010.www1.hp.com/wwpc/us/en/en/WF25a/321957-321957-64295-321838-3955547-3687777.html` (02.12.2009).

Table 3
The effects of normalization

No.	Models	Attributes				
		C_1	C_2	C_3	C_4	C_5
$A1$	FH008AW	-0.623	1.000	-1.000	-0.265	0.000
$A2$	NQ281AW	-0.534	1.000	-1.000	0.000	0.000
$A3$	FH005AW	-0.304	0.000	-1.000	-0.265	0.000
$A4$	KS183UT	-0.105	0.000	0.000	0.735	0.000
$A5$	KS181UT	0.127	0.000	-1.000	0.206	0.000
$A6$	KS179UT	0.270	0.000	0.000	0.206	-1.000
$A7$	KS180UT	0.377	0.000	-1.000	0.000	0.000

Table 4
Weighted normalized decision-making matrix

No.	Models	Attributes				
		C_1	C_2	C_3	C_4	C_5
		min	max	max	max	max
$A1$	FH008AW	-0.190	0.304	-0.217	-0.035	0.000
$A2$	NQ281AW	-0.163	0.304	-0.217	0.000	0.000
$A3$	FH005AW	-0.092	0.000	-0.217	-0.035	0.000
$A4$	KS183UT	-0.032	0.000	0.000	0.096	0.000
$A5$	KS181UT	0.039	0.000	-0.217	0.027	0.000
$A6$	KS179UT	0.082	0.000	0.000	0.027	-0.043
$A7$	KS180UT	0.115	0.000	-0.217	0.000	0.000

After that, the overall performance rating for any of the considered alternative can be finally determined.

Ranking results, as well as the sums of weighted normalized performance ratings of alternatives obtained on the base of attributes with significantly better or worse performance ratings compared to the preferred performance ratings, are shown in Table 5.

Based on the results shown in Table 5, it can be easily seen that the best ranked is the alternative A6 (Model: KS179UT), with the overall performance rating of 0.065. The runner alternative is the alternative A4 (Model: KS183UT), with a slightly lower overall performance rating, which is 0.064.

5.2. *A Comparative Study*

In order to verify the proposed methodology, below is shown a comparison of results obtained by its application and as well as the results obtained by applying two significant MADM methods, and also known normalization procedures.

Table 5

Results of ranking alternatives

No.	Models	Attributes			
		$\Sigma w_j r_{ij}$		S	Rank
		$r_{ij} > 0$	$r_{ij} < 0$		
$A1$	FH008AW	0.304	−0.442	−0.137	5
$A2$	NQ281AW	0.304	−0.380	−0.076	3
$A3$	FH005AW	0.000	−0.344	−0.344	7
$A4$	KS183UT	0.096	−0.032	0.064	2
$A5$	KS181UT	0.065	−0.217	−0.152	6
A6	**KS179UT**	0.109	−0.043	**0.065**	**1**
$A7$	KS180UT	0.115	−0.217	−0.103	4

Table 6

The overall performance and ranking order of the alternatives

Models	Method									
	SAW$^{(\text{LST-Max})}$		SAW $^{(\text{LST-MaxMin})}$		TOPSIS		TOPSIS$^{(\text{M})}$		WS	
	I		II		III		IV		V	
	S	Rank	S	Rank	S	Rank	S	Rank	S	Rank
FH008AW	0.693	7	0.348	5	0.121	7	0.063	7	−0.137	5
NQ281AW	0.723	5	0.410	3	0.191	5	0.094	6	−0.076	3
FH005AW	0.702	6	0.141	7	0.182	6	0.099	5	−0.344	7
KS183UT	**0.919**	**1**	0.549	2	**0.777**	**1**	0.286	2	0.064	2
KS181UT	0.791	4	0.333	6	0.409	4	0.216	4	−0.152	6
KS179UT	0.891	2	**0.551**	**1**	0.708	2	**0.287**	**1**	**0.065**	**1**
KS180UT	0.810	3	0.382	4	0.423	3	0.249	3	−0.103	4

Results, obtained by using the SAW method with the LST-Max method, are shown in column I of Table 6, and the results obtained by using the same method with the LST-Max-Min method in column II. The column III of Table 6 contains the results obtained by applying the TOPSIS method.

The ranking order of alternatives, obtained by using different MADM methods, are summarized in Table 7.

As it can be seen from Tables 6 and 7, using the SAW method with the LST-Max normalization procedure as well as the TOPSIS method, the alternative A_4 was selected as the best alternative, while the alternative A_6 takes the second position. However, using the SAW method with the LST-MaxMin normalization procedure we have a reverse order of the two best placed alternatives.

In the TOPSIS method, the positive-ideal solution is formed ot the base of the most desirable weighted normalized performance ratings in the case of benefit attributes, and

Table 7

Ranking order comparison

Method	Ranking order
SAW$^{(\text{LST-Max})}$	$A_4 \succ A_6 \succ A_7 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
SAW $^{(\text{LST-MaxMin})}$	$A_6 \succ A_4 \succ A_2 \succ A_7 \succ A_1 \succ A_5 \succ A_3$
TOPSIS	$A_4 \succ A_6 \succ A_7 \succ A_5 \succ A_2 \succ A_3 \succ A_1$
TOPSIS $^{(\text{M})}$	$A_6 \succ A_4 \succ A_7 \succ A_5 \succ A_3 \succ A_2 \succ A_1$
WS	$A_6 \succ A_4 \succ A_2 \succ A_7 \succ A_1 \succ A_5 \succ A_3$

the most undesirable in the case of cost attributes, as it is shown by the following formula:

$$A^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \left\{\left(\max_i v_{ij} \middle| j \in J^{\max}\right), \left(\min_i v_{ij} \middle| j \in J^{\min}\right)\right\}, \quad (16)$$

where v_{ij} is the weighted normalized performance rating of ith alternative according to jth attribute; $J^{\max}$ is associated with the benefit attributes; and $J^{\min}$ is associated the with cost attributes.

In order to perform a more realistic comparison, we also carried out the ranking of alternatives using the TOPSIS methods, whereby the positive-ideal solution is formed out of preferred performance ratings, as shown by the following formula:

$$A^+ = \{v_1^+, v_2^+, \dots, v_n^+\} = \{v_j^* | j \in 1, \dots, n\}, \quad (17)$$

where v_j^* is the weighted normalized preferred performance ratings of jth attribute.

The results obtained by applying the TOPSIS method, where the positive-ideal solution is formed by using the formula (17), are shown in column IV of Table 6. The same ranking order of the first two best-ranked alternatives as in the case of using our ranking procedure is achieved. This confirms the correctness of our methodology and also indicates that the ranking of alternatives based on preferred performance ratings with a higher degree satisfying decision-makers preferences.

Table 8 shows performances of two best-ranked models of computers, as well as the preferred performance rating of attributes used for its evaluation.

The alternative A_4 has performance ratings equal to the preferred performance ratings for attributes C_2, C_3 and C_5, and its performance ratings exceed the preferred for attributes C_1 and C_4. However, the attribute C_1 a is cost attribute whose performance is greater than the preferred and it has a negative effect on the overall performance.

The alternative A_6 has performance ratings equal the preferred for attributes C_2 and C_3. Its performance rating exceeds the desired performance for attribute C_4, and does not reach the preferred performance for attributes C_1 and C_5.

By using the proposed methodology the alternative A_6 its slightly worse performance rating obtained in relation to the attribute C_5 successfully compensates by a more preferable performance rating obtained in relation to the attribute C_1. Therefore, the alternative A_6 more closely meets the decision-makers preferences and it is selected as the most acceptable, i.e., the most appropriate from the set of offered alternatives.

Table 8

Comparative characteristic of the two best-ranked alternatives

Attributes		C_1	C_2	C_3	C_4	C_5
		Price	CPU	RAM	HDD	Warranty
Opt. direction		min	max	max	max	max
Weights		0.304	0.304	0.217	0.130	0.043
Preference		1200.00	2.40	4	250	3
No.	Model					
$A4$	KS183UT	1,259.00	2.40	4	*500*	3
$A6$	KS179UT	1,049.00	2.40	4	320	1

6. Conclusion

Ranking a set of alternatives based on their distance from an ideal point is a very actual multi-attribute decision making approach, and it is implemented in the frequently used TOPSIS method.

In the TOPSIS method, the ideal point concept has also been extended by distance from the negative-ideal point, so this method uses two ideal points, the positive-ideal point, which contains the most desirable performance ratings of attributes, and the negative-ideal point, which contains the most undesirable performance ratings.

This paper presents the methodology which allows ranking of alternatives on the basis of their distance from the ideal point, where the ideal point is formed according to the decision-makers preferences. The proposed methodology is much simpler compared to the methodology used by TOPSIS, and it also provides acceptable results.

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Daugiatikslis sprendimų priėmimo modelis, pagrįstas artumo sprendimų priėmėjo pageidaujamiems rezultatams nustatymu

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Šiame straipsnyje siūloma nauja daugiatikslė rangavimo procedūra, pagrįsta artumo sprendimų priėmėjo pageidaujamiems rezultatams nustatymu. Šį metodą sudaro du etapai. Pirmajame etape sprendimų priėmėjas yra prašomas apibrėžti pageidaujamą kiekvieno kriterijaus rezultatą. Antrajame etape pasvertos sumos metodas ir nauja atstumo nustatymu pagrįsta normalizavimo procedūra naudojami alternatyvų bendrajam efektyvumo reitingui nustatyti.