Design of Fixed and Adaptive Companding Quantizer with Variable-Length Codeword for Memoryless Gaussian Source

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Abstract. The problem we address in this paper is the design of a quantizer that in comparison to the classical fixed-rate scalar quantizers provides more sophisticated bit rate reduction while restricting the class of quantizers to be scalar. We propose a switched variable-length code (VLC) optimal companding quantizer composed of two optimal companding scalar quantizers, the inner and the outer one, both designed for the memoryless Gaussian source of unit variance. Quantizers composing the proposed quantizer have a different codebook sizes and a different compressor functions. Particularly, we assume a smaller size of the inner quantizer's codebook in order to provide assignment of the shorter codewords to the high probability low amplitude speech samples belonging to the support region of the inner quantizer. We study the influence of codebook size of the inner and the outer quantizer on the Signal to Quantization Noise Ratio ($SQNR$). In such a manner the conclusion of the proposed quantizer significance in speech compression is distinctly shown in the paper. For the proposed quantizer model and its forward adaptive version the $SQNR$ robustness analysis in a wide variance range is also presented in the paper. It is shown that our multi-resolution quantizer can satisfy G.712 Recommendation for high-quality quantization at the bit rate of 6.3 bit/sample achieving the compression of 1.7 bit/sample over the G.711 quantizer.

Keywords: variable-length code, companding technique, Gaussian source.

1. Introduction

Quantization deals with the digital or, more specifically, binary representation of signals. All the audio and video encoding algorithms typically include a quantization module. Quantization can be: (i) with memory or memoryless, depending upon whether the encoding rules rely or not on past samples; (ii) uniform or nonuniform based on the step-size or the quantization levels employed; (iii) scalar or vector depending upon each sample is quantized individually, or a block of samples is quantized jointly. An algorithm for design

of the optimal quantizer for a source with known distribution was developed by Lloyd and Max (1960). However, this algorithm is very time consuming for the large number of quantization levels. One solution which overcomes this problem is defined by the companding quantizer model (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Sayood, 2006; Hanzo *et al.*, 2007). Quantizers based on the companding model have simple realization structure and performance close to the optimal ones. The design of such quantizers is also more efficient than the one of the Lloyd-Max's quantizers since it does not require the iterative method. This difference is very notable for some commonly used sources including the Gaussian source, we consider in the paper. The reason of considering the Gaussian source is that the first approximation to the short-time-averaged probability density function (PDF) of speech amplitudes is provided by Gaussian PDF (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Sayood, 2006; Hanzo *et al.*, 2007). In case of a high-quality narrowband speech coding, widely used coders are 64 kbit/s nonuniform scalar quantizers based on segmental logarithmic companding characteristic. This solution is standardized and is known as G.711 quantizer (ITU-T, 1972). Its main disadvantage is that bit rate decrease brings the significant reduction in quality (6 dB per 1 bit). Nowadays, when new bandwidth limited communication systems are deployment, this can be unfavorable. Because of that, more sophisticated speech compression has become essential component in telecommunications. This has prompted extensive research in the area of speech coding during the last decade (Hiwasaki *et al.*, 2006, 2008a, 2008b; Perić and Nikolić, 2007; Nikolić and Perić, 2008; Perić *et al.*, 2009, 2010; Na, 2008, 2011). Namely, fixed-rate scalar quantizers for Gaussian source have already been the topic of our earlier research (Perić *et al.*, 2008; Perić *et al.*, 2010). In these two papers, the switched-adaptive quantization technique is utilized for a high-quality quantization of speech signals. The challenge to meet not only G.712 Recommendation for a high-quality speech coding (ITU-T, 2001), but also to provide the gain in the performance over the G.711 quantizer has driven much of the research in this area and several speech coding algorithms have been developed and eventually adopted in international standards (Hiwasaki *et al.*, 2006, 2008a, 2008b).

However, although a great number of a quantizers have been developed to provide an additional enhancement of the speech signal quality, especially for the VoIP applications, where G.711 quantizers are used, there is still a need to continue the research in this field. In this paper we propose switched VLC optimal companding quantizer, composed of two optimal companding quantizers having a different number of representation levels and a different compressor functions. The aim of presented research is to find a simple manner to realize a quantizer system having high-quality performance but maintaining robustness in a wide variance range of input signal. Additionally, our goal is to obtain a flexible compression algorithm, where the flexibility is related to the more sophisticated choice of bit rate while maintaining the necessary quality.

This paper is organized as follows: Section 2 provides a detailed description of the novel switched VLC optimal companding quantizer. In Section 3, we discuss which codebook sizes of two companding quantizers, composing the proposed quantizer, should be chosen in order to satisfy G.712 Recommendation. Section 4 presents the adaptive vari-

ant of our quantizer model which is tested on the speech coding. Section 5 concludes the paper about the possibilities of the proposed algorithm application in speech processing.

2. Design of Switched Variable-Length Code Optimal Companding Quantizer

The set of real numbers $t_0, t_1, \ldots, t_N$, called decision thresholds satisfying

$$-\infty = t_0 < t_1 < \cdots < t_{N-1} < t_N = +\infty, \tag{1}$$

and the set $y_1, y_2, \ldots, y_N$, called representation levels satisfying

$$y_j \in \alpha_j = (t_{j-1}, t_j], \quad j = 1, \ldots, N, \tag{2}$$

characterize the N-level fixed rate scalar quantizer (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Sayood, 2006; Hanzo *et al.*, 2007). Every N-level fixed rate scalar quantizer is associated with the partition of the set of real numbers R into N cells α_j. There are several models of scalar quantizers that are based on different quantization techniques. The stucture of nonuniform quantizer, consisting of a compressor, a uniform quantizer and an expandor in cascade, is called compandor (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Sayood, 2006; Hanzo *et al.*, 2007). Nonuniform quantization can be achieved by compandor in the following way: compressing the input signal x using a nonuniform compressor function $c(x)$, then by quantizing the compressed signal $c(x)$ employing a uniform quantizer $Q_u(c(x))$, and finally by expanding the quantized version of the compressed signal using a nonuniform inverse compressor function $c^{-1}(Q_u(c(x)))$. In the case of the optimal compressor function $c \colon R \to [-1, 1]$ defined by Judell and Scharf (1986):

$$c(x) = \begin{cases} -1 + 2\dfrac{\int\limits_{-t}^{x} p^{1/3}(x)\,dx}{\int\limits_{-t}^{0} p^{1/3}(x)\,dx + \int\limits_{0}^{t} p^{1/3}(x)\,dx}, & -t < x < 0, \\[4mm] -1 + 2\dfrac{\int\limits_{-t}^{0} p^{1/3}(x)\,dx + \int\limits_{0}^{x} p^{1/3}(x)\,dx}{\int\limits_{-t}^{0} p^{1/3}(x)\,dx + \int\limits_{0}^{t} p^{1/3}(x)\,dx}, & 0 < x < t, \end{cases} \tag{3}$$

the decision thresholds t_i and the representation levels y_i of the companding quantizer are determined by the solutions of the following equations:

$$c(t_i) = t_{u,i}, \quad i = 0, 1, \ldots, N, \qquad c(y_i) = y_{u,i}, \quad i = 1, \ldots, N, \tag{4}$$

where $t_{u,i}$ are decision thresholds, and $y_{u,i}$ are representation levels of the corresponding uniform quantizer, and t denotes the support region threshold between granular and overload region of an optimal companding quantizer.

In this paper we consider the Gaussian PDF of the input source, which is one of the most commonly used distributions (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Hanzo *et al.*, 2007). The probability density function for a random variable with a Gaussian distribution, mean μ and variance σ^2 is given by:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \tag{5}$$

In the rest of the paper, without loss of generality, we assume that information source is Gaussian source with memoryless property and zero mean value. The PDF of this source is given by:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{-\frac{x^2}{2\sigma^2}}. \tag{6}$$

Assuming further the unit variance, $\sigma^2 = 1$, by direct evaluation we obtain:

$$\int\limits_0^t p^{1/3}(x)\,dx = \sqrt{3}\left(\frac{\pi}{4}\right)^{1/3} \mathrm{erf}\left(\frac{t}{\sqrt{6}}\right), \tag{7}$$

where $\mathrm{erf}\,(x)$ is the error function:

$$\mathrm{erf}\,(x) = \frac{2}{\sqrt{\pi}} \int\limits_0^x e^{-u^2}\,du. \tag{8}$$

In the rest of the paper we assume symmetry about zero in the quantizer characteristic. This symmetry is an intuitionally expected result when the input has a PDF that is symmetrical about zero. The Gaussian PDF, we consider here, is indeed symmetrical about zero.

Switched quantizer we propose in this paper is composed of two companding quantizers, the inner and the outer one. These two quantizers are designed according to a different optimal compressor functions. Particularly, the inner companding quantizer Q_1, having N_1 representation levels, is defined on the inner interval $I = [-t_1, t_1]$ while the outer companding quantizer Q_2, having N_2 representation levels, is defined on the outer interval $O = (-\infty, -t_1) \cup (t_1, +\infty)$. Value t_1 is called threshold value. For the assumed Gaussian PDF of unit variance the optimal compressor function $c_1: I \rightarrow [-1, 1]$ of the inner companding quantizer is:

$$c_1(x) = \frac{\mathrm{erf}\left(\frac{x}{\sqrt{6}}\right)}{\mathrm{erf}\left(\frac{t_1}{\sqrt{6}}\right)} \mathrm{sgn}(x), \quad |x| < t_1. \tag{9}$$

Further, for the inner companding quantizer the decision thresholds $t_{I,i}$ and the representation levels $y_{I,i}$ are defined by:

$$t_{I,i,-} = -t_{I,i,+} = c_1^{-1}\left(\frac{2i - N_1}{N_1}\right), \quad i = 0, 1, \ldots, N_1/2, \tag{10}$$

$$y_{I,i,-} = -y_{I,i,+} = c_1^{-1}\left(\frac{2i - 1 - N_1}{N_1}\right), \quad i = 1, \ldots, N_1/2. \tag{11}$$

Using (9), by direct computation we obtain the following expressions:

$$t_{I,i,-} = \sqrt{6}\,\mathrm{erf}^{-1}\left(\frac{2i - N_1}{N_1}\mathrm{erf}\left(\frac{t_1}{\sqrt{6}}\right)\right), \quad i = 0, 1, \ldots, N_1/2, \tag{12}$$

$$y_{I,i,-} = \sqrt{6}\,\mathrm{erf}^{-1}\left(\frac{2i - 1 - N_1}{N_1}\mathrm{erf}\left(\frac{t_1}{\sqrt{6}}\right)\right), \quad i = 1, \ldots, N_1/2. \tag{13}$$

The decision thresholds of the outer companding quantizer Q_2 in each of the two symmetric intervals composing the outer region O are ordered such that we have $-\infty = t_{O,0,-} < t_{O,1,-} < \ldots < t_{O,N_2/2,-} = -t_1$ and $t_1 = t_{O,N2/2,+} < t_{O,N2/2-1,+} < \ldots < t_{O,0,+} = +\infty$. Similarly, one can define the order of the corresponding positive and negative representation levels $y_{O,i,+}$ and $y_{O,i,-}$, $i = 1, \ldots, N_2/2$. The optimal compressor function $c_2(x)\colon O \longrightarrow [-1, 1]$ defining the outer companding quantizer Q_2 can be calculated from (3), where $0 \to t_1$ and $t = +\infty$:

$$c_2(x) = \begin{cases} -\frac{\mathrm{erf}(x/\sqrt{6}) - \mathrm{erf}(t_1/\sqrt{6})}{1 - \mathrm{erf}(t_1/\sqrt{6})}, & -\infty < x < -t_1, \\ \frac{\mathrm{erf}(x/\sqrt{6}) - \mathrm{erf}(t_1/\sqrt{6})}{1 - \mathrm{erf}(t_1/\sqrt{6})}, & t_1 < x < \infty. \end{cases} \tag{14}$$

The decision thresholds and the representation levels of the companding outer quantizer Q_2 are computed from the following relations:

$$t_{O,i,-} = -t_{O,i,+} = c_2^{-1}\left(\frac{2i - N_2}{N_2}\right), \quad i = 0, 1, \ldots, N_2/2, \tag{15}$$

$$y_{O,i,-} = -y_{O,i,+} = c_2^{-1}\left(\frac{2i - 1 - N_2}{N_2}\right), \quad i = 1, \ldots, N_2/2. \tag{16}$$

By an explicit computation using the relation (14) we obtain the following expressions:

$$t_{O,i,-} = \sqrt{6}\,\mathrm{erf}^{-1}\left(\frac{2i - N_2}{N_2}\left(1 - \mathrm{erf}\frac{t_1}{\sqrt{6}}\right) - \mathrm{erf}\frac{t_1}{\sqrt{6}}\right),$$
$$i = 0, 1, \ldots N_2/2, \tag{17}$$

$$y_{O,i,-} = \sqrt{6}\,\mathrm{erf}^{-1}\left(\frac{2i - 1 - N_2}{N_2}\left(1 - \mathrm{erf}\frac{t_1}{\sqrt{6}}\right) - \mathrm{erf}\frac{t_1}{\sqrt{6}}\right),$$
$$i = 1, \ldots N_2/2. \tag{18}$$

Finally we can write decision thresholds and representation levels of the joint switched optimal companding quantizer Q as follows:

$$t = (-\infty, \ldots t_{O,N2/2-1,-}, -t_1, t_{I,1}, \ldots, t_{I,N1/2-1}, t_1, t_{O,N2/2-1,+},$$
$$\ldots + \infty), \tag{19}$$

$$y = (y_{O,0,-}, \ldots y_{O,N2/2-1,-}, y_{I,0}, y_{I,1}, \ldots, y_{I,N1/2-1}, y_{O,N2/2-1,+},$$
$$\ldots y_{O,0,+}). \tag{20}$$

For the proposed quantizer, the first bit of the codeword corresponding to the source sample x is equal 1 if the sample belongs to the inner region ($x \in I = [-t_1, t_1]$), otherwise it is equal 0. Other $\log_2 N_1$ (or $\log_2 N_2$) bits of the codeword are formed by an index of the quantization cell x belongs to. Since the length of the codeword is $k_1 + 1$ or $k_2 + 1$, ($N_i = 2^{k_i}$), depending on whether $x \in I$ or not, our quantizer is a variable-length quantizer (VLC – variable-length coder). In general, with variable length quantizers compression is achieved by assigning shorter codewords to the more frequent symbols and longer codewords to the less frequent ones (Gersho and Gray, 1992; Hanzo *et al.*, 2007). The average bit rate of the considered quantizer can be computed by:

$$R(Q) = P_1 \left(\log_2 N_1 + 1 \right) + P_2 \left(\log_2 N_2 + 1 \right), \tag{21}$$

where P_i, $i = 1, 2$, are probabilities that one signal sample belongs to the inner and to the outer region, respectively. The additional bits, in previous formula, carries necessary side information for decoding, i.e., which region, and which number of quantization levels is used in encoding process. Since:

$$P_1 = \int_{-t_1}^{t_1} p(x) \, dx, \tag{22}$$

$$P_2 = 1 - P_1, \tag{23}$$

for the average bit rate we obtain:

$$R(Q) = \operatorname{erf}\left(\frac{t_1}{\sqrt{2}}\right)(\log_2 N_1 + 1) + \left(1 - \operatorname{erf}\left(\frac{t_1}{\sqrt{2}}\right)\right)(\log_2 N_2 + 1). \tag{24}$$

Further, we can express threshold t_1 as a function of R, N_1 and N_2, in the following way:

$$t_1 = \sqrt{2} \operatorname{erf}^{-1}\left(\frac{R - 1 - \log_2 N_2}{\log_2 N_1 - \log_2 N_2}\right). \tag{25}$$

It is obvious that the choice of the threshold t_1 has an effect on the probabilities P_1 and P_2, and thus on $R(Q)$ which is therefore much more sophisticated R and can be easily changed in relation to the case where R is obtained as $R = \log_2(N_1 + N_2)$. For instance,

one can determine the threshold t_1 that provides a high-quality quantization specified by the G.712 Recommendation (ITU-T, 2001).

The quality of the N-level scalar quantizer Q, for a source that is characterized as a continuous random variable with PDF $p(x)$, is commonly measured by the distortion defined by the expected mean square error between original and quantized signal (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Sayood, 2006; Hanzo *et al.*, 2007):

$$D(Q) = E\big(X - Q(X)\big)^2 = \sum_{i=1}^{N} \int_{t_{i-1}}^{t_i} (x - y_i)^2 p(x)\,dx. \tag{26}$$

The N-level quantizer Q^* is optimal for the source X, if there is no other N-level quantizer Q, such that $D(Q) < D(Q^*)$. Denote by D_1 and D_2 the distortions of the inner and the outer quantizer. Also denote by N_1 and N_2 the number of representation levels of the inner and the outer quantizer, respectively. In this model, distortions D_1 and D_2 can be approximated using Bennett's integral as follows (Bennett, 1948):

$$D_1 = D(Q_1) = \frac{2}{3N_1{}^2}\left(\int_0^{t_1} p^{1/3}(x)\,dx\right)^3, \tag{27}$$

$$D_2 = D(Q_2) = \frac{2}{3N_2{}^2}\left(\int_{t_1}^{\infty} p^{1/3}(x)\,dx\right)^3. \tag{28}$$

Finally, total distortion $D = D_1 + D_2$ for the assumed Gaussian source of unit variance can be represented, using (6), (27) and (28), as the function of t_1, N_1 and N_2 as follows:

$$D(Q) = \frac{\sqrt{3}\pi}{2N_1{}^2}\operatorname{erf}\big(t_1/\sqrt{6}\big)^3 + \frac{\sqrt{3}\pi}{2N_2{}^2}\big(1 - \operatorname{erf}\big(t_1/\sqrt{6}\big)\big)^3. \tag{29}$$

It is easy to prove that there exists one global minimum, which is the solution of the equation $D'(t_1) = 0$, and we apply the Newton iterative method to find the numerical solution.

3. Numerical Results

Quality of a quantized signal is often specified in terms of Signal to Quantization Noise Ratio ($SQNR$), which is directly obtained from the distortion D using the following relation (Jayant and Noll, 1984; Gersho and Gray, 1992; Kondoz, 2004; Sayood, 2006; Hanzo *et al.*, 2007):

$$SQNR = 10\log\big(\sigma^2/D\big). \tag{30}$$

Table 1

Values of t_1, R, and $SQNR$ for the presented model and a different value of N_1 and N_2

N_1	N_2	t_1	R [bit/sample]	$SQNR$ [dB]
32	128	0.438810	7.32160	39.7356
16	128	0.241985	7.42637	38.8204
32	64	0.746042	6.45564	35.2986
16	64	0.438810	6.32160	33.7150

Table 2

Values of t_1^{34} and R^{34} for the presented model such that $SQNR = 34$ dB

N_1	N_2	t_1^{34}	R^{34} [bit/sample]
32	128	1.231717	6.43611
16	128	0.712567	6.42834
32	64	1.136107	6.25591
16	64	0.438810	6.32160

For $R \in (\min(\log_2 N_1, \log_2 N_2) + 1, \ \max(\log_2 N_1, \log_2 N_2) + 1)$, using equations (25), (29) and (30), we can get the $SQNR$ value as a function of R for our model. Computed values of the threshold t_1 and $SQNR$ for the presented model, designed for the Gaussian source of unit variance and a different values of N_1 and N_2, are given in the Table 1. As it can be noticed from the Table 1 a smaller size of the inner companding quantizer codebook is assumed, i.e., $N_1 < N_2$. This assumption is introduced in order to assign shorter codewords to the high probability low amplitude samples belonging to the inner interval. In such a manner the basic principe of VLC quantization is fulfilled. What also can be noted from the first and the second raw in the Table 1 is that the increase of the bit rate do not result in $SQNR$ increase. The explanation for this behavior can be found in the expression for the bit rate (24), where in addition to the influence of codebook sizes and side information we have a threshold t_1 effect manifested through the probability of coding with corresponding companding quantizer. Recall that the listed values of threshold t_1 are calculated by applying the Newton iterative method to solve the equation $D'(t_1) = 0$. Define t_1^{34} as a value of threshold such that $SQNR$ of the proposed quantizer, designed for the Gaussian source of unit variance, is 34 dB. Denote by R^{34} corresponding average bit rate. Table 2 presents values of t_1^{34} and R^{34} we have calculated for a different values of N_1 and N_2. From the Table 2 one can perceive that our quantizer designed for the Gaussian source of unit variance, can provide a high-quality quantization with about 6.25 bit/sample. Moreover, it can be noticed that in comparison to the classical fixed-rate companding quantizer, our quantizer enables more sophisticated bit rate reduction. Figure 1 shows $SQNR$ of our quantizer as a function of the signal variance in the case of a different choice of the threshold t_1^{34}. It is interesting to observe from Fig. 1 that for a different threshold t_1^{34} the maximum $SQNR$ is achieved at a different variances σ_i^2. Particularly, as the threshold value is higher, the maximum of $SQNR$ characteristic
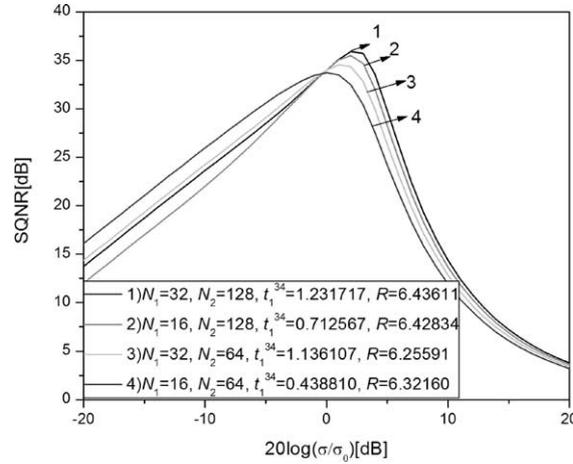
Fig. 1. $SQNR$ as a function of the signal variance for a different values of N_1, N_2 and t_1^{34}.

is moved toward right, i.e., toward higher values of variances. $SQNR$ characteristic is distinctly asymmetrical and rapidly decreases from its maximum, such that $SQNR$ varies a lot in the observed variance range. Generally, any of non-adaptive (fixed) quantizers designed for the particular variance is not suitable for quantization of non-stationary signals that, as well as speech signals, have changing variance characteristic (Jayant and Noll, 1984; Gersho and Gray, 1992). In such situations, as it can be noticed from Fig. 1 and as it has been observed in Na (2008, 2011) the variance mismatch between the input signal's variance and the variance for which the quantizer is designed may occur. However, in short intervals the properties of the speech signal remain roughly constant, hence, it can be viewed as a local-stationary signal (Jayant and Noll, 1984; Gersho and Gray, 1992; Hanzo *et al.*, 2007). Accordingly, in order to process speech efficiently it is necessary to use same kind of adaptation or system that works on frame by frame basis. For that reason, in the next section, we consider forward adaptive version of the proposed quantizer model and its application in speech coding.

4. Application in Speech Coding

We have tested our coding scheme on the speech coding. The sample signal is taken from the base which is derived from the TIMIT corpus (Garofolo *et al.*, 1993). The TIMIT corpus of speech has been designed to provide speech data for the acquisition of acoustic-phonetic knowledge and for the development and evaluation of automatic speech recognition systems. For the purpose of testing, we consider the adaptive version of our model. We divide the input signal into the frames and for each frame we estimate the signal variance and normalize all samples before coding. Consider the n samples of the input signal $x_1, x_2, \ldots, x_n$ and assume that signal samples are divided in F frames each consisted of M samples. Furthermore denote by $x_{i,j}$ the jth sample of the ith frame

($i = 0, \ldots, F-1$ and $j = 0, \ldots, M-1$), i.e., $x_{i,j} = x_{iM+j}$. In the ith frame signal variance is estimated using $\sigma_i^2 = \frac{1}{M} \sum_{j=0}^{M-1} x_{i,j}^2$. The source samples are then normalized and sent to the quantizer. When received, the signal has to be denormalized by the same value of the squared root of the quantized variance that is used for normalization. For that purpose, we also need to transmit the information about the signal variance σ_i^2. It is quantized using N_g level log-uniform scalar quantizer. We consider the log-uniform scalar quantizer for variance quantization rather than the uniform scalar quantizer, since we have demonstrated that it could provide higher value of $SQNR$ (Nikolić and Perić, 2008). Signal samples are normalized to $\bar{x}_{i,j} = x_{i,j}/\tilde{\sigma}_i$, where $\tilde{\sigma}_i$ is squared root of the quantized signal variance, and then sent to the quantizer.

The representation levels and the decision thresholds of log-uniform quantizer $Q_{lu}(\sigma)$ are defined as:

$$\log(y_{lu,i}) = \log(\sigma_{\min}) + \frac{2i-1}{2N_g} \log\left(\frac{\sigma_{\max}}{\sigma_{\min}}\right), \quad i = 1, 2, \ldots, N_g, \tag{31}$$

$$\log(t_{lu,i}) = \log(\sigma_{\min}) + \frac{i}{N_g} \log\left(\frac{\sigma_{\max}}{\sigma_{\min}}\right), \quad i = 0, 1, 2, \ldots, N_g, \tag{32}$$

where $\sigma_{\min}^2$ and $\sigma_{\max}^2$ are respectively maximum and minimum possible values of the signal variance. In other words, log-uniform quantizer is the uniform quantizer in decibels scale. We consider the dynamic range of the variance ($20 \log(\sigma_{\max}/\sigma_{\min})$ [dB]) of 40 dB, $N_g = 16$ and $N_g = 32$ levels of log-uniform quantizer. The bit rate R corresponding to the proposed adaptive version is $R = R_{fix} + \log_2 N_g/M$, where R_{fix} refers to the number of bits per sample required for the fixed quantizer (24), while $\log_2 N_g/M$ define the number of bits per frame having length M that is required for transmission of the side information for adaptive version. Figure 2 shows theoretical dependencies of the $SQNR$
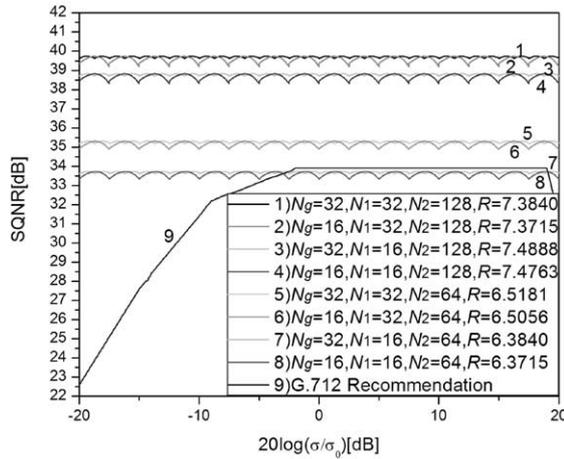
Fig. 2. Theoretical dependence of the $SQNR$ as a function of signal variance σ^2 (in decibels) for the presented adaptive model ($N_g = 16, 32$).

Table 3

$SQNR_{avg}$ and R of adaptive quantization for the presented model

N_1	N_2	N_g	R [bit/sample]	$SQNR_{avg}$ [dB]
32	128	32	7.3840	39.6952
32	128	16	7.3715	39.5526
16	128	32	7.4888	38.7815
16	128	16	7.4763	38.6418
32	64	32	6.5181	35.2669
32	64	16	6.5056	35.1393
16	64	32	6.3840	33.6873
16	64	16	6.3715	33.5656

Table 4

Values of t_1^{34}, and R^{34}, for the adaptive version of the presented model such that $SQNR_{\min} > 34$ dB, i.e., satisfying G.712 Recommendation

N_1	N_2	N_g	t_1^{34}	R^{34} [bit/sample]
32	128	32	1.22506	6.50361
32	128	16	1.25132	6.47164
16	128	32	0.72497	6.46791
16	128	16	0.73560	6.43592
32	64	32	1.14438	6.31497
32	64	16	1.13841	6.30495

values as a function of signal variance σ^2 (in decibels) for presented adaptive quantizer model ($M = 80$, $N_g = 16, 32$). From the Fig. 2 one can notice that our multi-resolution quantizer provides a different level of signal quality, i.e., of the $SQNR$. Moreover, one can perceive that by increasing the level number of log-uniform quantizer, the bit rate is slightly increased, but the $SQNR$ characteristic is significantly flattened. Particularly, one can observe that in the considered variance range $SQNR$ is almost constant for $N_g = 32$. Accordingly, $N_g = 32$ is a good choice for the number of log-uniform variance quantizer levels. Using the basic definition for the average $SQNR$:

$$ SQNR_{avg} = \frac{1}{K} \sum_{i=1}^{K} 10 \log \left(\sigma_i^2 / D(\sigma_i) \right), \tag{33} $$

where K defines the number of the particular variances that are calculated in the assumed variances range of 40 dB, we have calculated values presented in the Table 3 for adaptive version of our quantizer. It has already been mentioned and explained how the codebook sizes, amount of the side information and the threshold choice influence on $SQNR$ and R of the corresponding non-adaptive quantizer. The same conclusion can be derived for the adaptive version of our quantizer model. In Table 4 are given values of t_1^{34} and R^{34}
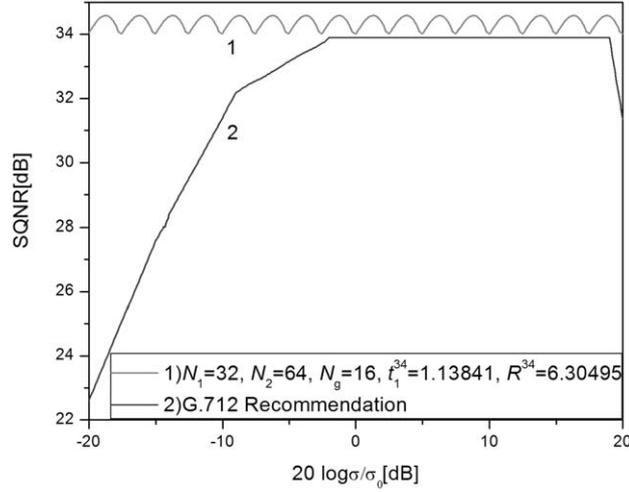
Fig. 3. Adaptive version of the presented model with values of t_1^{34} and R^{34} such that $SQNR_{min} > 34$ dB, i.e., satisfying G.712 Recommendation.

[bit/sample] for which the minimum value of $SQNR$ for adaptive version of our model does not fall below 34 dB. These numerical results obtained to maintain a high-quality quantization (ITU-T, 2001) the best reflect gains in compression that ranges up to 1.7 bit/sample over the G.711 quantizer. Fig. 3 shows $SQNR$ characteristic of the adaptive version of presented model designed for $t_1^{34} = 1.13841$ and the bit rate of $R^{34} = 6.30495$ such that $SQNR_{\min} > 34$ dB, i.e., such that satisfies G.712 Recommendation. This adaptive version of our quantizer model in the considered wide variance range provides a high-quality quantization along with the compression of about 1.7 bit/sample over the G.711 quantizer (ITU-T, 1972). Moreover, our quantizer model achieves the compression of $6.7 - 6.3 = 0.4$ bit/sample over the quantizer proposed in Perić *et al.* (2010). Particularly, in Perić *et al.* (2010) additional gain in the compression is achieved applying the lossless compression technique where the resulting gain over the G.711 quantizer amount to $8 - 6.37 = 1.63$ bit/sample. This compression gain is smaller than the one we achieved in this paper without applying some of the lossless compression techniques. Fig. 4 shows the experimental $SQNR$ values as a function of σ^2 for adaptive variant of our model, and different values of N_1, N_2, and N_g. For the purpose of the experiment, we assumed the frame size of $M = 80$ and total $F = 800$ frames. We determined the experimental values $SQNRex_i$ for each frame $i = 0, 1, \ldots, F - 1$. In Fig. 4 we show the input signal (the first upper graph) and $SQNRex_i$ values (the second lower graph). Note that the x scale, on the lower graphs, still represents the index of the sample (not the frame). The average $SQNR$ value of all frames is equal to $SQNRex = \frac{1}{F} \sum_{i=0}^{F-1} SQNRex_i$. Comparing the theoretical and experimental values of the average $SQNR$ for the several different values of the average bit rate R, one can calculated that there is a good agreement between the theory and experiment in all of the considered cases.
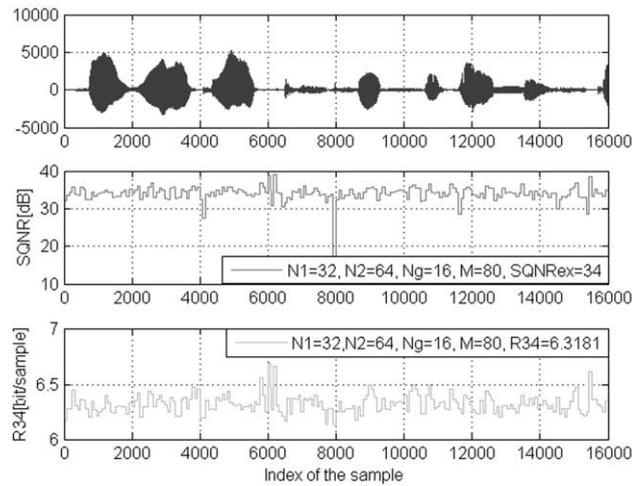
Fig. 4. An input speech signal (the first graph), experimental values of $SQNRex_i$ (the second graph) and R_i^{34} (the third graph) for the adaptive version of the presented model.

5. Conclusions

In this paper we have presented and analyzed performances of the new model of switched VLC optimal companding quantizer. The model is designed for a high-quality scalar quantization of Gaussian source in a wide variance range of input signal. According the analysis of the codebook size influence and the influence of the threshold t_1 choice on $SQNR$ it has been concluded that adaptive version of our quantizer model satisfies G.712 Recommendation, providing the compression of about 1.7 bit/sample over the G.711 quantizer (ITU-T, 1972), so it can be applied for a high-quality coding of speech signals. It has also been demonstrated that, in comparison to the fixed-rate scalar companding quantizers, the proposed multi-resolution quantizer provides much more sophisticated choice of the bit rate depending on the desired level of speech signal quality. Finally, good agreement between the theoretical and experimental values of the average $SQNR$ has been ascertained. Our quantizer gives a very simple realization structure and high-quality performances, hance it can be very useful in particular high-quality quantization of signals having Gaussian PDF.

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Fiksuotas ir adaptyvus kintamo kodinio žodžio ilgio kvantatorius Gauso šaltiniams be atminties

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Straipsnyje nagrinėjamas kintamo kodo ilgio skaliarinio kvantavimo uždavinys. Autoriai siūlo kintamo kodo ilgio kvantatorių sudaryti iš dviejų nuosekliai sujungtų skaliarinių kvantatorių, kiekvienas kurių yra optimizuotas darbui su Gauso šaltiniais be atminties. Šie kvantatoriai naudoja skirtingas kodines knygas bei skirtingas suspaudimo funkcijas. Straipsnyje autoriai įvertino naudojamų kodinių knygų dydžio įtaką kvantavimo kokybei ir parodė pasiūlytojo kvantatoriaus pranašumą koduojant kalbos signalus prieš fiksuoto kodo ilgio kvantatorius.