# A Fully Secure Revocable ID-Based Encryption in the Standard Model 

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#### Abstract

Revocation problem is a critical issue for key management of public key systems. Any certificate-based or identity (ID)-based public key systems must provide a revocation method to revoke misbehaving/compromised users from the public key systems. In the past, there was little work on studying the revocation problem of ID-based public key systems. Most recently, Tseng and Tsai presented a novel ID-based public key system with efficient revocation using a public channel, and proposed a practical revocable ID-based encryption (called RIBE). They proved that the proposed RIBE is semantically secure in the random oracle model. Although the ID-based encryption schemes based on the random oracle model can offer better performance, the resulting schemes could be insecure when random oracles are instantiated with concrete hash functions. In this paper, we employ Tseng and Tsai's revocable concept to propose a new RIBE without random oracles to provide full security. We demonstrate that the proposed RIBE is semantically secure against adaptive-ID attacks in the standard model.


Keywords: revocation, identity-based encryption, standard model, bilinear pairing.

## 1. Introduction

In the certificate-based public key systems, the certificates make publicly available the mapping between identities and public keys. In order to eliminate the required certificates in the certificate-based public key systems, Shamir (1984) presented a good idea that a user's identity such as social security number, e-mail address or telephone number may be viewed as the user's public key. Boneh and Franklin (2001) followed Shamir's idea to propose the first practical identity (ID)-based encryption (IBE), in which there are two roles: a trusted private key generator (PKG) and users. Users can authenticate themselves to the PKG and then the PKG generates the corresponding private keys to the users. Boneh and Franklin's IBE was built on the progress in elliptic curves with bilinear pairings such as Weil, Tate and Ate pairings. Subsequently, the study of ID-based cryptographic mechanisms using bilinear pairings has received a great attention from researchers. A large number of literatures have been presented such as Cha and Cheon
(2003), Choi et al. (2004), Bellare et al. (2004b), Waters (2005), Chen et al. (2007), Tseng et al. (2008), Boneh and Hamburg (2008), Choi et al. (2008), Tseng et al. (2009), Wu and Tseng (2010), Liu and Huang (2010), Ren et al. (2010), Wu et al. (2011), Chen et al. (2012).

In the public key systems, several situations require a revocation mechanism to remove a misbehaving/compromised user before its intended expiration date. For example, the user's company may request revocation if the user leaves the company and is no longer entitled to use the public key. In the certificate-based public key systems, certificate revocation list (CRL) (Housley et al., 2002) is generally used to revoke the users' public keys, and then users can know misbehaving/compromised users by querying the CRL. Using the CRL technique to remove misbehaving/compromised users is an efficient approach for the certificate-based public key systems. Actually, efficient revocation is a well-studied problem in the certificate-based public key systems (Aiello et al., 1998; Micali, 2002; Gentry, 2003; Elwailly et al., 2004; Goyal, 2007). However, the ID-based public key systems have eliminated the need of certificates, thus the CRL method and the related revocation solutions will not be well-suited for ID-based public key systems.

For the revocation problem in the ID-based public key systems, Boneh and Franklin (2001) suggested that the PKG generates all non-revoked users' new private keys for each time period and then the PKG uses a secure channel to transmit these periodic private keys to non-revoked users. As a result, the PKG and each non-revoked user must encrypt and decrypt these periodic private keys, respectively. And the total size of the periodic update keys grows linearly with the number of non-revoked users. For reducing the PKG and user's periodic workload, Boldyreva et al. (2008) used a binary tree structure to construct a revocable ID-based encryption that reduces the key update size to logarithmic in the number of users. However, each user must keep $3 \log n$ private keys while the PKG needs to maintain a binary tree data structure of $n$ leaf nodes, where $n$ denotes the total number of users. Boldyreva et al. (2008) proved that their RIBE is secure in the relaxed selective-ID model (Canetti et al., 2003), in which adversaries must choose the target identity before the system begins to be operated. Later on, Libert and Vergnaud (2009) improved Boldyreva et al.'s scheme to present an adaptive-ID secure RIBE scheme. However, the mentioned three schemes above need a secure channel to transmit the users' new private keys for each time period. For the PKG and non-revoked users, it raised enormous computation workload of encryption and decryption procedures, respectively.

Quite recently, Tseng and Tsai (2012) presented a new ID-based public key setting and its associated revocation mechanism with a public channel. They partitioned a user's private (decryption) key into two components including an initial secret key and a time update key. The initial secret key is fixed and unchanged, while the time update key is changed along with time period. The PKG periodically generates new time update keys for non-revoked users, the PKG then sends them to users using a public channel. Non-revoked users may update their own decryption keys while the PKG stops to issue the new time update keys of the revoked users. It can eliminate the requirement for the secure channel established between the PKG and each user. In such a case, no encryption or decryption is required the PKG and each non-revoked user. In the meantime, Tseng
and Tsai also proposed a practical revocable ID-based encryption (called RIBE) with a public channel. They proved that their RIBE scheme is secure in an adaptive-ID model, but their RIBE scheme only provided the security in the random oracle model (Bellare and Rogaway, 1993).

Although the ID-based encryption schemes based on the random oracle model can offer better performance, the resulting schemes could be insecure when random oracles are instantiated with concrete hash functions (Canetti et al., 1998; Bellare et al., 2004a; Boneh and Boyen, 2004a). Canetti et al. (2003) presented an IBE scheme without random oracles, in which the security is proven in the relaxed selective-ID model. Subsequently, Boneh and Boyen (2004a) provided more practical IBE schemes in the selective-ID model without random oracles. Afterwards, Boneh and Boyen (2004b) presented a fully secure IBE scheme in the adaptive-ID model without random oracles. For improving the efficiency of Boneh and Boyen's IBE scheme, Waters (2005) also proposed a fairly efficient IBE scheme without random oracles. Furthermore, Gentry (2006) proposed a new fully secure IBE scheme to reduce the required public parameters in Waters's scheme, but it relies on a stronger complexity assumption called the augmented bilinear Diffie-Hellman exponent (ABDHE) assumption (Gentry, 2006).

In this paper, we will employ the revocable concept and the framework presented by Tseng and Tsai (2012) to propose a new revocable ID-based encryption (RIBE) in the standard model (without random oracles) to provide full security. For security analysis, as the adversary model presented in Tseng and Tsai's RIBE scheme, the attackers consists of two kinds: an inside adversary (or a revoked user) and an outside adversary. We will give formal security analysis of the proposed RIBE schemes for the inside and the outside adversaries, respectively. Under the bilinear decision Diffie-Hellman problem (Boneh and Franklin, 2001), we demonstrate that the proposed RIBE scheme is semantically secure against adaptive-ID attacks in the standard model. Meanwhile, we discuss the transformation technique from a chosen-plaintext secure (CPA) RIBE scheme to a chosen-ciphertext secure (CCA) RIBE scheme.

The remainder of the paper is organized as follows. Preliminaries are given in Section 2. In Section 3, we present the definitions and security notions of revocable ID-based encryption (RIBE) with a public channel. Section 4 gives the concrete RIBE scheme. In Section 5, we analyze the security of the proposed RIBE scheme. Discussions and comparisons are presented in Section 6. Conclusions are given in Section 7.

## 2. Preliminaries

In this section, we briefly introduce the concept of bilinear pairings and the related mathematical assumptions. Bilinear pairings such as Weil, Tate and Ate pairings that have been used to establish efficient ID-based encryption (Boneh and Franklin, 2001; Baek and Zheng 2004; Sakai and Kasahara, 2003; Waters, 2005). For the details of the relationship between the security levels and speed of pairing computations, please refer to (Galbraith et al., 2008; Wu and Tseng, 2010) for full descriptions.

### 2.1. Bilinear Pairings

Let $G_{1}$ and $G_{2}$ be two multiplicative cyclic groups of large prime order $p$, and $g$ be a generator of $G_{1}$. We say that the map $\hat{e}$ : $G_{1} \times G_{1} \rightarrow G_{2}$ is an admissible bilinear map if it satisfies the following properties:
(1) Bilinearity: $\hat{e}\left(g^{a}, g^{b}\right)=\hat{e}(g, g)^{a b}$ for $g \in G_{1}$ and $a, b \in Z_{p}^{*}$.
(2) Non-degeneracy: There exist $g_{1}, g_{2} \in G_{1}$ such that $\hat{e}\left(g_{1}, g_{2}\right) \neq 1$.
(3) Computability: For any $g_{1}, g_{2} \in G_{1}$, there exists an efficient algorithm to compute $\hat{e}\left(g_{1}, g_{2}\right) \in G_{2}$.
It is obvious that since $\hat{e}\left(g^{a}, g^{b}\right)=\hat{e}(g, g)^{a b}$, it implies

$$
\hat{e}\left(g^{a} g^{b}, g^{c}\right)=\hat{e}\left(g^{a+b}, g^{c}\right)=\hat{e}(g, g)^{(a+b) c}=\hat{e}(g, g)^{a c+b c}=\hat{e}\left(g^{a}, g^{c}\right) \hat{e}\left(g^{b}, g^{c}\right)
$$

and

$$
\hat{e}\left(g^{a}, g^{b} g^{c}\right)=\hat{e}\left(g^{a}, g^{b+c}\right)=\hat{e}(g, g)^{a(b+c)}=\hat{e}(g, g)^{a b+a c}=\hat{e}\left(g^{a}, g^{b}\right) \hat{e}\left(g^{a}, g^{c}\right)
$$

Full descriptions of groups, maps and other parameters are discussed in Boneh and Franklin (2001), Waters (2005), Paterson and Schuldt (2006).

### 2.2. Related Mathematical Assumptions

Here, we present two mathematical problems and define a security assumption for bilinear pairings on which our scheme is based.

- Bilinear Diffie-Hellman (BDH) problem: Given $g, g^{a}, g^{b}, g^{c} \in G_{1}$ for unknown $a, b, c \in Z_{p}^{*}$, this problem is to compute $\hat{e}(g, g)^{a b c} \in G_{2}$.
- Bilinear Decision Diffie-Hellman (BDDH) problem: Given $g, g^{a}, g^{b}, g^{c} \in G_{1}$ for some $a, b, c \in Z_{p}^{*}$ and $K \in G_{2}$, this problem is to decide whether $K=\hat{e}(g, g)^{a b c}$.

DEFINITION 1 (BDDH assumption). Given $g, g^{a}, g^{b}, g^{c} \in G_{1}$ for some $a, b, c \in Z_{p}^{*}$ and $K \in G_{2}$, there exists no probabilistic polynomial-time adversary $\mathcal{A}$ with non-negligible probability who can decide whether $K=\hat{e}(g, g)^{a b c}$. The successful probability (advantage) of the adversary $\mathcal{A}$ is presented as

$$
\operatorname{Adv}_{A}=\operatorname{Pr}\left[\mathcal{A}\left(g, g^{a}, g^{b}, g^{c}, \hat{e}(g, g)^{a b c}\right)=1\right]-\operatorname{Pr}\left[\mathcal{A}\left(g, g^{a}, g^{b}, g^{c}, K\right)=1\right]
$$

where $K \in G_{2}$ is chosen uniformly at random and the probability is over the random choice consumed by the adversary $\mathcal{A}$.

### 2.3. Notations

We define the following notations that are used throughout this paper:

- $\hat{e}$ : an admissible bilinear map, $\hat{e}: G_{1} \times G_{1} \rightarrow G_{2}$.
- $g$ : a generator of the group $G_{1}$.
- id: the identity-related information of a user.
- $d_{i d}$ : the user's initial secret key.
- $t$ : a time period, where $1 \leqslant t \leqslant z$ and $z$ denotes the total number of time periods.
- $d_{i d, t}$ : a user $i d$ 's time update key for time period $t$.
- $D_{i d, t}$ : a user id's entire decryption key for time period $t$. Note that the user's entire decryption key $D_{i d, t}$ is obtained by the user's initial secret key $d_{i d}$ and time update key $d_{i d, t}$.


## 3. Framework and Security Notions of RIBE

Tseng and Tsai (2012) have defined the framework and security notions of revocable IDbased encryption (RIBE) with a public channel. Under their framework of RIBE, a user's decryption key is divided into two components including a fixed initial secret key and a changed time update key along with time periods. As a result, the framework will add one time key update algorithm except for four algorithms defined in ID-based encryption (IBE) proposed by Boneh and Franklin (2001). We follow the revocable concept, the framework and security notions presented by Tseng and Tsai (2012). Here, we present the framework and security notions of revocable ID-based encryption (RIBE) with a public channel.

### 3.1. Framework

In this subsection, the framework of revocable ID-based encryption with a public channel is formally defined as follows.

Definition 2. A revocable ID-based encryption (RIBE) with a public channel has 5-tuple of polynomial time algorithms $(\mathcal{G}, \mathcal{I K} \mathcal{E}, \mathcal{T} \mathcal{K} \mathcal{U}, \mathcal{E}, \mathcal{D})$ as follows:

- System setup algorithm $\mathcal{G}$ : Take a security parameter $l$ and the total number $z$ of all time periods as input, the algorithm returns the system secret key and the public parameters Parms. The public parameters Parms are made public and implicitly inputted to all the following algorithms.
- Initial key extract algorithm $\mathcal{I K E}$ : Take the system secret key and a user's identityrelated information id as input, the algorithm returns the user's initial secret key $d_{i d}$.
- Time key update algorithm $\mathcal{T} \mathcal{K} \mathcal{U}$ : For a time period $t$, take the system secret key and a user identity-related information $i d$ as input, the algorithm returns the user's time update key $d_{i d, t}$. Note that the non-revoked user can use the initial secret key $d_{i d}$ and the time update key $d_{i d, t}$ to obtain the entire decryption key $D_{i d, t}$.
- Encryption algorithm $\mathcal{E}$ : For a time period $t$, take an identity id and a message $M$ as input, the algorithm generates a ciphertext $C$.
- Decryption algorithm $\mathcal{D}$ : Take a ciphertext $C$ and the user's entire decryption key $D_{i d, t}$ as input, the algorithm returns a plaintext $M$.


### 3.2. Security Model

By the standard security model of ID-based encryption (IBE) in Boneh and Franklin (2001), Boneh and Boyen (2004a), Waters (2005), the indistinguishability (IND) of encryption is under selective/adaptive-ID and chosen-plaintext/chosen-ciphertext attacks. In which, the selective-ID model means that before the system begins to be operated, the adversary has to decide which identities it would like to attack. The selective-ID version proposed by Canetti et al. (2003) is a weaker security than the adaptive-ID version. Tseng and Tsai (2012) extended Boneh and Franklin's notions to define the security model of revocable ID-based encryption that includes the indistinguishability of encryption under adaptive-ID, chosen-plaintext attacks (IND-RID-CPA), as well as the indistinguishability of encryption under adaptive-ID, chosen-ciphertext attacks (IND-RID-CCA). Here, we first define a security game, and then present the definitions of both IND-RID-CPA and IND-RID-CCA attacks.

Security game. The semantic security under adaptive-ID attacks for revocable IDbased encryption is defined using the following game between a challenger and an adversary:

- Phase 1 . The challenger $\mathcal{B}$ runs the system setup algorithm $\mathcal{G}$ of RIBE to generate a system secret key and produce the public parameters Parms. Then the challenger $\mathcal{B}$ gives the adversary $\mathcal{A}$ the Parms and keeps the system secret key to itself.
- Phase 2. The adversary $\mathcal{A}$ may issue a number of different queries to $\mathcal{B}$ as follows:
- Initial key extract query (id). Upon receiving this query with identity-related information id, the challenger $\mathcal{B}$ runs the initial key extract algorithm $\mathcal{I K E}$ to return the user's initial secret key $d_{i d}$ to $\mathcal{A}$.
- Time key update query (id, t). Upon receiving this query with $(i d, t)$, the challenger $\mathcal{B}$ runs the time key update algorithm $\mathcal{T} \mathcal{K} \mathcal{U}$ to return the user's time update key $d_{i d, t}$ to $\mathcal{A}$.
- Decryption query (id, $t, C$ ). Upon receiving the query, the challenger $\mathcal{B}$ accesses the entire decryption key $D_{i d, t}$. The entire decryption key $D_{i d, t}$ is implicitly obtained by issuing the initial key extract query (id) and the time key update query $(i d, t)$. The challenger $\mathcal{B}$ runs the decryption algorithm $D$ to decrypt the ciphertext $C$. Then it returns $D\left(D_{i d, t}, C\right)$ to $\mathcal{A}$.
- Phase 3. The adversary $\mathcal{A}$ gives a target identity $i d^{*}$, a plaintext pair $\left(M_{0}^{*}, M_{1}^{*}\right)$ and a time period $t^{*}$ to B. $\mathcal{B}$ chooses a random $\gamma \in\{0,1\}$ and computes $C^{*}$ by running the encryption algorithm $E\left(\right.$ Parms, $\left.i^{*}, t^{*}, M_{\gamma}^{*}\right)$. Then B sends $C^{*}$ to $\mathcal{A}$.
- Phase 4. The adversary $\mathcal{A}$ may issue more queries as in Phase 2. A restriction here is that either $i d^{*}$ or $\left(i d^{*}, t^{*}\right)$ is disallowed to be queried in the initial key extract query or the time key update query, respectively. The other restriction is that (id, $t, C) \neq\left(i d^{*}, t^{*}, C^{*}\right)$.
- Phase 5. The adversary $\mathcal{A}$ outputs $\gamma^{\prime} \in\{0,1\}$ and wins this game if $\gamma^{\prime}=\gamma$.

We define the adversary $\mathcal{A}$ 's advantage in attacking a RIBE scheme in the security game as

$$
\operatorname{Adv}_{\mathcal{A}}(l)=\left|\operatorname{Pr}\left[\gamma=\gamma^{\prime}\right]-1 / 2\right|
$$

DEFINITION 3 (IND-RID-CCA). We say that a RIBE scheme is $\left(\tau, q_{E}, q_{U}, q_{D}, \varepsilon\right)$ -IND-RID-CCA secure against adaptive-ID, chosen ciphertext attacks if no probabilistic polynomial-time adversary $\mathcal{A}$ that has a non-negligible $\varepsilon$ against the RIBE scheme within a running time $\tau$ and making at most $q_{E}$ initial key extract queries, $q_{U}$ time key update queries, and $q_{D}$ decryption queries.

For the security of adaptive-ID, chosen plaintext attacks (IND-RID-CPA), the adversary $\mathcal{A}$ cannot issue the decryption queries of Phases 2 and 4 in the security game above.

DEFINITION 4 (IND-RID-CPA). We say that a RIBE scheme is ( $\tau, q_{E}, q_{U}, \varepsilon$ )-IND-RIDCPA secure against adaptive-ID, chosen plaintext attacks if no probabilistic polynomialtime adversary $\mathcal{A}$ that has a non-negligible $\varepsilon$ against the RIBE scheme within a running time $\tau$ and making at most $q_{E}$ initial key extract queries and $q_{U}$ time key update queries.

## 4. The Proposed RIBE Scheme

The RIBE scheme without random oracles consists of five algorithms: the system setup, the initial key extract, the time key update, the encryption and the decryption algorithms.

- System setup: As in Waters (2005), Paterson and Schuldt (2006), a trusted private key generation (PKG) takes a security parameter $l$ and the total number $z$ of all time periods as input. Two groups $G_{1}, G_{2}$ of prime order $p>2^{l}$, an admissible bilinear map $\hat{e}: G_{1} \times G_{1} \rightarrow G_{2}$ and a generator $g$ of $G_{1}$ are generated by the PKG. The PKG sets two collision-resistant hash functions $H_{n}:\{0,1\}^{*} \rightarrow\{0,1\}^{n}$ and $H_{m}:\{0,1\}^{*} \rightarrow\{0,1\}^{m}$, where $n$ and $m$ are fixed lengths. Then the PKG randomly chooses two secret values $\alpha, \beta \in Z_{p}^{*}$ and computes $g_{1}=g^{\alpha+\beta} \in G_{1}$. In addition, the PKG also chooses a random value $g_{2} \in G_{1}$, two random values $u^{\prime}, t^{\prime} \in G_{1}$, as well as two vectors $U=\left(u_{i}\right)$ of length $n$ and $T=\left(t_{j}\right)$ of length $m$, where $u_{i}, t_{j} \in G_{1}$ for $i=1,2, \ldots, n$ and $j=1,2, \ldots, m$. Finally, the PKG returns the system secret key $=\left(g_{2}^{\alpha}, g_{2}^{\beta}\right)$ and the public parameters Parms $=$ $\left(G_{1}, G_{2}, \hat{e}, g, g_{1}, g_{2}, H_{m}, H_{n}, u^{\prime}, U, t^{\prime}, T\right)$.
- Initial key extract: Given a user's identity-related information $i d \in\{0,1\}^{*}$, the PKG computes $v=H_{n}(i d)$. Here, $v$ is a bit string of length $n$ representing an identity $i d$, and $v_{i}$ denotes the $i$ th bit of $v$. Let $\mathrm{U} \subset\{1,2, \ldots, n\}$ be the set of index $i$ such that $v_{i}=1$, for $i=1,2, \ldots, n$. The PKG chooses a random $r_{v} \in Z_{p}^{*}$ and uses $u_{i}$ to compute the initial key $d_{i d}=\left(d_{i d 1}, d_{i d 2}\right)=\left(g_{2}^{\alpha}\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}}, g^{r_{v}}\right)$ or $\left(g_{2}^{\alpha}\left(u^{\prime} \prod u_{i}^{v_{i}}\right)^{r_{v}}, g^{r_{v}}\right)$ for $i=1,2, \ldots, n$. Finally, the PKG transmits $d_{i d}$ to the user via a secure channel.
- Time key update: Let $v t=H_{m}(i d, t)$ be a bit string of length $m$ representing the identity-related information $i d$ and the time period $t$, and $v t_{j}$ denotes the $j$ th bit of $v t$. Let $\mathrm{T} \subset\{1,2, \ldots, m\}$ be the set of index $j$ such that $v t_{j}=1$, for $j=1,2, \ldots, m$. The PKG chooses a random $r_{t} \in Z_{p}^{*}$ and uses $t_{j}$ to compute the time update key $d_{i d, t}=\left(d_{i d, t 1}, d_{i d, t 2}\right)=\left(g_{2}^{\beta}\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{t}}, g^{r_{t}}\right)$ or
$\left(g_{2}^{\beta}\left(t^{\prime} \prod t_{j}^{v t_{j}}\right)^{r_{t}}, g^{r_{t}}\right)$ for $j=1,2, \ldots, m$. The PKG sends $d_{i d, t}$ to the user via a public channel. Thus, the non-revoked user can use $d_{i d}$ and $d_{i d, t}$ to compute his/her entire decryption key for the time period $t$ as

$$
\begin{aligned}
D_{i d, t} & =\left(D_{1}, D_{2}, D_{3}\right) \\
& =\left(d_{i d 1} \cdot d_{i d, t 1}, d_{i d 2}, d_{i d, t 2}\right) \\
& =\left(g_{2}^{\alpha+\beta} \cdot\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}} \cdot\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{t}}, g^{r_{v}}, g^{r_{t}}\right) .
\end{aligned}
$$

- Encryption: For a time period $t$, given a message $M$ and a receiver $i d$, a sender also computes two bit strings $v=H_{n}(i d)$ and $v t=H_{m}(i d, t)$ to obtain two sets $U$ and T . Thus, the sender computes $u^{\prime} \prod_{i \in U} u_{i}$ and $t^{\prime} \prod_{j \in T} t_{j}$. Then the sender chooses a random number $r \in Z_{p}^{*}$ and computes the ciphertext as

$$
\begin{aligned}
C & =\left(C_{1}, C_{2}, C_{3}, C_{4}\right) \\
& =\left(\hat{e}\left(g_{1}, g_{2}\right)^{r} \cdot M, g^{r},\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r},\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r}\right) .
\end{aligned}
$$

- Decryption: Given a ciphertext $C=\left(C_{1}, C_{2}, C_{3}, C_{4}\right)$, the receiver can use his/her entire decryption key $D_{i d, t}=\left(D_{1}, D_{2}, D_{3}\right)$ to decrypt $C$ as follows:

$$
\begin{aligned}
C_{1} \cdot & \frac{\hat{e}\left(D_{2}, C_{3}\right) \cdot \hat{e}\left(D_{3}, C_{4}\right)}{\hat{e}\left(D_{1}, C_{2}\right)} \\
= & \left(\hat{e}\left(g_{1}, g_{2}\right)^{r} \cdot M\right) \cdot \frac{\hat{e}\left(g^{r_{v}},\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r}\right) \cdot \hat{e}\left(g^{r_{t}},\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r}\right)}{\hat{e}\left(g_{2}^{\alpha+\beta}\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}} \cdot\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{t}}, g^{r}\right)} \\
= & \left(\hat{e}\left(g_{1}, g_{2}\right)^{r} \cdot M\right) \\
& \times \frac{\hat{e}\left(g^{r},\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}} \cdot\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{t}}\right)}{\hat{e}\left(g_{2}^{\alpha+\beta}, g^{r}\right) \cdot \hat{e}\left(\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}} \cdot\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{t}}, g^{r}\right)} \\
= & M .
\end{aligned}
$$

## 5. Security Analysis

In the proposed RIBE scheme, the entire decryption key is divided into two parts, the initial secret key and the time update key. As the mentioned IND-RID-CPA attack in Definition 4, the adversary is allowed to obtain either the initial secret key or the time update key in the security game. Hence, we consider two types of adversaries to simplify the security proof. One is the outside adversary and the other is the inside adversary (or a revoked user). The outside adversary is allowed to issue all queries in the security game except for the initial key extract query on $i d^{*}$. The inside adversary is allowed to issue all queries in the security game except for the time key update query on $\left(i d^{*}, t^{*}\right)$. In the
following theorems, we use the similar technique in Waters (2005), Paterson and Schuldt (2006) to prove that the proposed RIBE scheme is semantically secure for the outside and the inside adversaries, respectively.

Theorem 1. In the standard model, the proposed RIBE scheme is a semantically outsider-secure RIBE scheme (IND-O-RID-CPA) under the BDDH assumption. Concretely, assume that there is an outside adversary $A$ that has an advantage $\varepsilon$ against the proposed RIBE scheme within a running time $\tau$ and $A$ can make at most $q_{E}>0$ initial key extract queries and $q_{U}>0$ time key update queries. Then the proposed RIBE scheme is $\left(\tau, q_{E}, q_{U}, \varepsilon\right)$-IND-O-RID-CPA secure assuming that the BDDH problem is $\left(\tau^{\prime}, \varepsilon^{\prime}\right)$-intractable, where $\tau^{\prime}=\tau+O\left(\left(n \cdot q_{E}+m \cdot q_{U}\right) \cdot \tau_{1}+\left(q_{E}+q_{U}\right) \cdot \tau_{2}\right)$ and $\varepsilon^{\prime}=\frac{\varepsilon}{4 q_{E}(n+1)}$, in which $\tau_{1}$ and $\tau_{2}$ denote the executing time of a multiplication in $G_{1}$ and an exponentiation in $G_{1}$, respectively.

Proof. Assume that an adversary $\mathcal{A}$ can break the proposed RIBE scheme. Using the adversary A , we can construct a challenger $\mathcal{B}$ in the security game to solve the BDDH problem. We assume that the challenger $\mathcal{B}$ is given $\left\langle G_{1}, G_{2}, \hat{e}, g, g^{a}, g^{b}, g^{c}, K\right\rangle$ as an instance of the BDDH problem, where $a, b, c \in Z_{p}^{*}$ and $K \in G_{2}$. $\mathcal{B}$ would like to decide whether $K=\hat{e}(g, g)^{a b c} . \mathcal{B}$ simulates the challenger in the security game for A as follows.

- Phase 1: The challenger $\mathcal{B}$ sets $l_{v}=2 q_{E}$, and randomly chooses an integer $k_{v}$ with $0 \leqslant k_{v} \leqslant n$. We assume that $l_{v}(n+1)<p$ for the given values of $q_{E}$ and $n$. The challenger $\mathcal{B}$ randomly chooses an integer $x^{\prime} \in Z_{l_{v}}$ and a vector $X=\left(x_{i}\right)$ of length $n$, where $x_{i} \in Z_{l_{v}}$ for $i=1,2, \ldots, n$. The challenger $\mathcal{B}$ randomly chooses an integer $y^{\prime} \in Z_{p}$ and a vector $Y=\left(y_{i}\right)$ of length $n$, where $y_{i} \in Z_{p}$ for $i=$ $1,2, \ldots, n$. Finally, the challenger $\mathcal{B}$ randomly chooses an integer $z^{\prime} \in Z_{p}$ and a vector $Z=\left(z_{j}\right)$ of length $m$, where $z_{j} \in Z_{p}$ for $j=1,2, \ldots, m$. We define three functions for $v=H_{n}(i d)$ and $v t=H_{m}(i d, t)$ as follows:

$$
\begin{aligned}
& F(v)=x^{\prime}+\sum_{i \in U} x_{i}-l_{v} k_{v} \\
& J(v)=y^{\prime}+\sum_{i \in U} y_{i} \\
& L(v t)=z^{\prime}+\sum_{j \in T} z_{j}
\end{aligned}
$$

The challenger $\mathcal{B}$ chooses a value $\beta \in Z_{p}$ as the secret value of the time update key, then assigns $g_{1}=g^{a} g^{\beta}, g_{2}=g^{b}, u^{\prime}=g_{2}^{-l_{v} k_{v}+x^{\prime}} g^{y^{\prime}}, u_{i}=g_{2}^{x_{i}} g^{y_{i}}, t^{\prime}=g^{z^{\prime}}$, and $t_{j}=g^{z_{j}}$, for $1 \leqslant i \leqslant n$ and $1 \leqslant j \leqslant m$.

- Phase $2: \mathcal{B}$ respectively responds the initial key extract query with identity id and the time key update query with ( $i d, t$ ) as follows.
- Initial key extract query (id): Upon receiving this query with identity $i d$, the challenger $\mathcal{B}$ computes $v=H_{n}(i d)$ and then computes $F(v)$ and $J(v)$. If
$F(v)=0 \bmod p$, the challenger $\mathcal{B}$ reports failure and terminates. If $F(v) \neq 0$ $\bmod p$, the challenger $\mathcal{B}$ chooses a random $r_{v} \in Z_{p}$ and computes the initial secret key $d_{i d}$ as follows.

$$
\begin{aligned}
d_{i d} & =\left(d_{i d 1}, d_{i d 2}\right) \\
& =\left(\left(\frac{g_{1}}{g^{\beta}}\right)^{-J(v) / F(v)} \cdot\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}},\left(\frac{g_{1}}{g^{\beta}}\right)^{-1 / F(v)} g^{r_{v}}\right) .
\end{aligned}
$$

Now, we show that $d_{i d}=\left(d_{i d 1}, d_{i d 2}\right)$ is a valid initial secret key as follows.

$$
\begin{aligned}
d_{i d 1} & =\left(\frac{g_{1}}{g^{\beta}}\right)^{-J(v) / F(v)} \cdot\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}} \\
& =\left(\frac{g_{1}}{g^{\beta}}\right)^{-J(v / F(v)} \cdot\left(g_{2}^{-l_{v} k_{v}+x^{\prime}} g^{y^{\prime}} \cdot \prod_{i \in U} g_{2}^{x_{i}} g^{y_{i}}\right)^{r_{v}} \\
& =\left(\frac{g_{1}}{g^{\beta}}\right)^{-J(v) / F(v)} \cdot\left(g_{2}^{-l_{v} k_{v}+x^{\prime}} g^{y^{\prime}} \cdot g_{2}^{\sum_{i \in U} x_{i}} g^{\sum_{i \in U} y_{i}}\right)^{r_{v}} \\
& =\left(\frac{g_{1}}{g^{\beta}}\right)^{-J(v) / F(v)} \cdot\left(g_{2}^{F(v)} g^{J(v)}\right)^{r_{v}} \\
& =g_{2}^{a}\left(g_{2}^{F(v)} g^{J(v)}\right)^{-a / F(v)} \cdot\left(g_{2}^{F(v)} g^{J(v)}\right)^{r_{v}} \\
& =g_{2}^{a}\left(g_{2}^{F(v)} g^{J(v)}\right)^{r_{v}-a / F(v)} \\
& =g_{2}^{a}\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{v}^{\prime}}
\end{aligned}
$$

and

$$
\begin{aligned}
d_{i d 2} & =\left(\frac{g_{1}}{g^{\beta}}\right)^{-1 / F(v)} g^{r_{v}} \\
& =g^{r_{v}-a / F(v)} \\
& =g^{r_{v}^{\prime}}
\end{aligned}
$$

where $r_{v}^{\prime}=r_{v}-a / F(v)$.

- Time key update query (id, $t$ ): Upon receiving the time key update query with (id, $t$ ), the challenger $\mathcal{B}$ chooses a random $r_{t} \in Z_{p}$ and uses the secret value $\beta \in Z_{p}$ to compute the time update key as follows.

$$
d_{i d, t}=\left(d_{i d, t 1}, d_{i d, t 2}\right)=\left(g_{2}^{\beta} \cdot\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{t}}, g^{r_{t}}\right)
$$

- Phase 3: The adversary $\mathcal{A}$ gives a target identity $i d^{*}$, a plaintext pair ( $M_{0}^{*}, M_{1}^{*}$ ) and a time period $t^{*}$ to $\mathbf{B}$. The challenger $\mathcal{B}$ first chooses a random $\gamma \in\{0,1\}$.

Then the challenger $\mathcal{B}$ computes $v^{*}=H_{n}\left(i d^{*}\right)$ and $v t^{*}=H_{m}\left(i d^{*}, t^{*}\right)$. The challenger $\mathcal{B}$ uses $v^{*}$ and $v t^{*}$ to compute $F\left(v^{*}\right), J\left(v^{*}\right)$ and $L\left(v t^{*}\right)$. If $F\left(v^{*}\right) \neq 0$ $\bmod p$, the challenger $\mathcal{B}$ reports failure and terminates. If $F\left(v^{*}\right)=0 \bmod p$, the challenger $\mathcal{B}$ constructs a ciphertext $C^{*}$ as follows.

$$
C^{*}=\left(K \cdot \hat{e}\left(g^{c}, g^{b}\right)^{\beta} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v^{*}\right)}, g^{c L\left(v t^{*}\right)}\right) .
$$

Now, we show that verify $C^{*}$ is a valid ciphertext as follows.

$$
\begin{aligned}
C^{*} & =\left(K \cdot \hat{e}\left(g^{c}, g^{b}\right)^{\beta} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v^{*}\right)}, g^{c L\left(v t^{*}\right)}\right) \\
& =\left(\hat{e}(g, g)^{a b c} \cdot \hat{e}\left(g^{\beta}, g^{b}\right)^{c} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v^{*}\right)}, g^{c L\left(v t^{*}\right)}\right) \\
& =\left(\hat{e}\left(g^{a}, g^{b}\right)^{c} \cdot \hat{e}\left(g^{\beta}, g^{b}\right)^{c} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v^{*}\right)}, g^{c L\left(v t^{*}\right)}\right) \\
& =\left(\hat{e}\left(g_{1}, g_{2}\right)^{c} \cdot M_{\gamma}^{*}, g^{c},\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{c},\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{c}\right) .
\end{aligned}
$$

- Phase 4: The challenger $\mathcal{B}$ responds to the initial key extract query or the time key update query as in Phase 2. A restriction here is that $i d^{*}$ is disallowed to be queried in the initial key extract query.
- Phase 5: The adversary $\mathcal{A}$ outputs its guess $\gamma^{\prime} \in\{0,1\}$, and wins this game if $\gamma^{\prime}=\gamma$.
In Phases 2 and 3, it is obvious that the challenger $\mathcal{B}$ perfectly simulates the initial secret key extraction, the time key update queries and the ciphertext $C^{*}$. We analyze the probability of the challenger $\mathcal{B}$ not aborting. In Phase 2, if $F(v)=0 \bmod p$, the challenger $\mathcal{B}$ reports failure and terminates. To make the analysis of the simulation easier, we force the challenger $\mathcal{B}$ to abort whenever $F(v)=0 \bmod l_{v}$. By the mentioned assumption $l_{v}(n+1)<p$, we can imply $0 \leqslant l_{v} k_{v} \leqslant p$ and $0 \leqslant x^{\prime}+\sum_{i \in U} x_{i} \leqslant p$. It is easy to see that $F(v)=0 \bmod p$ implies $F(v)=0 \bmod l_{v}$. On the other hand, in Phase 3, if $F\left(v^{*}\right) \neq 0 \bmod p$, the challenger $\mathcal{B}$ reports failure and terminates.

Let $q_{E}$ and $q_{U}$ be the total queries in the initial key extract query and the time key update query, respectively. To simplify the analysis, we define the events $A_{i}: F\left(v_{i}\right) \neq 0$ $\bmod l_{v}$ and $A^{*}: F\left(v^{*}\right)=0 \bmod p$. From the above analysis, the probability of the challenger $\mathcal{B}$ not aborting is

By the assumption $l_{v}(n+1)<p$, it leads to that $F(v)=0 \bmod p$ implies $F(v)=0$ $\bmod l_{v}$. Furthermore, this assumption gives that if $F(v)=0 \bmod l_{v}$, there will be an unique choice of $k_{v}$ with $0 \leqslant k_{v} \leqslant n$ such that $F(v)=0 \bmod p$. Since $k_{v}, x^{\prime}$ and $X$ are chosen randomly, we have the probability of the event $A^{*}$ as follows.

$$
\begin{aligned}
& \operatorname{Pr}\left[A^{*}\right]=\operatorname{Pr}\left[F\left(v^{*}\right)=0 \bmod p \wedge F\left(v^{*}\right)=0 \bmod l_{v}\right], \\
& \operatorname{Pr}\left[A^{*}\right]=\operatorname{Pr}\left[F\left(v^{*}\right)=0 \bmod l_{v}\right] \cdot \operatorname{Pr}\left[F\left(v^{*}\right)=0 \bmod p \mid F\left(v^{*}\right)=0 \bmod l_{v}\right]
\end{aligned}
$$

$$
\operatorname{Pr}\left[A^{*}\right]=\frac{1}{l_{v}} \cdot \frac{1}{n+1}
$$

We also have that

$$
\operatorname{Pr}\left[{\left.\underset{i=1}{q_{E}} A_{i} \mid A^{*}\right]=1-\operatorname{Pr}\left[\underset{i=1}{q_{E}} \neg A_{i} \mid A^{*}\right] \geqslant 1-\sum_{i=1}^{q_{E}} \operatorname{Pr}\left[\neg A_{i} \mid A^{*}\right]=1-\frac{q_{E}}{l_{v}} . . . ~ . ~}_{\text {ind }}\right.
$$

Hence, we can obtain that

We set $l_{v}=2 q_{E}$, then get the resulting probability of the challenger $\mathcal{B}$ not aborting is

$$
\operatorname{Pr}[\neg \text { abort }] \geqslant \operatorname{Pr}\left[\wedge_{i=1}^{q_{E}} A_{i} \wedge A^{*}\right] \geqslant \frac{1}{4 q_{E}} \cdot \frac{1}{n+1}
$$

According to the above descriptions for the challenger $\mathcal{B}$, it is obvious that it requires $O(n)$ multiplications and $O(1)$ exponentiations in the initial key extract query. It is also obvious that it requires $O(m)$ multiplications and $O(1)$ exponentiations in the time key update query. So we have $\tau^{\prime}=\tau+O\left(q_{E} \cdot n \cdot \tau_{1}+q_{U} \cdot m \cdot \tau_{1}+\left(q_{E}+q_{U}\right) \cdot \tau_{2}\right)$, where $\tau_{1}$ and $\tau_{2}$ denote the executing time of a multiplication in $G_{1}$ and an exponentiation in $G_{1}$, respectively.

Theorem 2. In the standard model, the proposed RIBE scheme is a semantically insidersecure RIBE scheme (IND-I-RID-CPA) under the BDDH assumption. Concretely, assume that there is an inside adversary $A$ that has an advantage $\varepsilon$ against the proposed RIBE scheme within a running time $\tau$ and $A$ can make at most $q_{E}>0$ initial key extract queries and $q_{U}>0$ time key update queries. Then the proposed RIBE scheme is $\left(\tau, q_{E}, q_{U}, \varepsilon\right)$ -IND-I-RID-CPA secure assuming that the BDDH problem is $\left(\tau^{\prime}, \varepsilon^{\prime}\right)$-intractable, where $\tau^{\prime}=\tau+O\left(\left(n \cdot q_{E}+m \cdot q_{U}\right) \cdot \tau_{1}+\left(q_{E}+q_{U}\right) \cdot \tau_{2}\right)$ and $\varepsilon^{\prime}=\frac{\varepsilon}{4 q_{U}(m+1)}$, in which $\tau_{1}$ and $\tau_{2}$ denote the executing time of a multiplication in $G_{1}$ and an exponentiation in $G_{1}$, respectively.

Proof. Assume that an adversary $\mathcal{A}$ can break the proposed RIBE scheme. Using the adversary A , we can construct a challenger $\mathcal{B}$ to solve the BDDH problem. We assume that the challenger $\mathcal{B}$ is given $\left\langle G_{1}, G_{2}, \hat{e}, g, g^{a}, g^{b}, g^{c}, K\right\rangle$ as an instance of the BDDH problem, where $a, b, c \in Z_{p}^{*}$ and $K \in G_{2} . \mathcal{B}$ would like to decide whether $K=\hat{e}(g, g)^{a b c} . \mathcal{B}$ simulates the challenger in the security game for $\mathcal{A}$ as follows.

- Phase 1: The challenger $\mathcal{B}$ sets $l_{v t}=2 q_{U}$, and randomly chooses an integer $k_{v t}$ with $0 \leqslant k_{v t} \leqslant m$. We assume that $l_{v t}(m+1)<p$ for the given values of $q_{U}$ and $m$. The challenger $\mathcal{B}$ randomly chooses an integer $x^{\prime} \in Z_{l_{v t}}$ and a vector $X=\left(x_{i}\right)$ of length $m$, where $x_{i} \in Z_{l_{v t}}$ for $i=1,2, \ldots, m$. The challenger $\mathcal{B}$ also chooses
an integer $y^{\prime} \in Z_{p}$ randomly and a vector $Y=\left(y_{i}\right)$ of length $m$, where $y_{i} \in Z_{p}$ for $i=1,2, \ldots, m$. Finally, the challenger $\mathcal{B}$ randomly chooses an integer $z^{\prime} \in Z_{p}$ and a vector $Z=\left(z_{j}\right)$ of length $n$, where $z_{j} \in Z_{p}$ for $j=1,2, \ldots, n$. We define three functions for $v=H_{n}(i d)$ and $v t=H_{m}(i d, t)$ as follows.

$$
\begin{aligned}
& F(v t)=x^{\prime}+\sum_{i \in U} x_{i}-l_{v t} k_{v t}, \\
& J(v t)=y^{\prime}+\sum_{i \in U} y_{i} \\
& L(v)=z^{\prime}+\sum_{j \in T} z_{j} .
\end{aligned}
$$

The challenger $\mathcal{B}$ chooses a value $\alpha \in Z_{p}$ as the secret value of the initial secret key, then assigns $g_{1}=g^{\alpha} g^{a}, g_{2}=g^{b}, u^{\prime}=g_{2}^{-l_{v t} k_{v t}+x^{\prime}} g^{y^{\prime}}, u_{i}=g_{2}^{x_{i}} g^{y_{i}}, t^{\prime}=g^{z^{\prime}}$, and $t_{j}=g^{z_{j}}$ for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$.

- Phase 2: Upon receiving the initial key extract query with identity id and the time key update query with (id, $t$ ), $\mathcal{B}$ respectively responds to the following queries.
- Initial key extract query (id): Upon receiving the initial key extract query with identity $i d$, the challenger $\mathcal{B}$ chooses random $r_{v} \in Z_{p}$ and uses the secret value $\alpha$ to compute the initial secret key as follows.

$$
d_{i d}=\left(d_{i d 1}, d_{i d 2}\right)=\left(g_{2}^{\alpha} \cdot\left(t^{\prime} \prod_{j \in T} t_{j}\right)^{r_{v}}, g^{r_{v}}\right)
$$

- Time key update query (id, $t$ ): Upon receiving the time key update query with (id, $t$ ), the challenger $\mathcal{B}$ computes $v t=H_{m}(i d, t)$ and then computes $F(v t)$ and $J(v t)$. If $F(v t)=0 \bmod p$, the challenger $\mathcal{B}$ reports failure and terminates. If $F(v t) \neq 0 \bmod p$, the challenger $\mathcal{B}$ constructs a time update key $d_{i d, t}$. It chooses a random $r_{t} \in Z_{p}$ and computes the initial secret key as follows.

$$
\begin{aligned}
d_{i d, t} & =\left(d_{i d, t 1}, d_{i d, t 2}\right) \\
& =\left(\left(\frac{g_{1}}{g^{\alpha}}\right)^{-J(v t) / F(v t)} \cdot\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{t}},\left(\frac{g_{1}}{g^{\alpha}}\right)^{-1 / F(v t)} g^{r_{t}}\right) .
\end{aligned}
$$

Now, we show that $d_{i d, t}=\left(d_{i d, t 1}, d_{i d, t 2}\right)$ is a valid initial secret key as follows.

$$
\begin{aligned}
d_{i d, t 1} & =\left(\frac{g_{1}}{g^{\alpha}}\right)^{-J(v t) / F(v t)} \cdot\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{t}} \\
& =\left(\frac{g_{1}}{g^{\alpha}}\right)^{-J(v t) / F(v t)} \cdot\left(g_{2}^{-l_{v t} k_{v t}+x^{\prime}} g^{y^{\prime}} \cdot \prod_{i \in U} g_{2}^{x_{i}} g^{y_{i}}\right)^{r_{t}}
\end{aligned}
$$

$$
\begin{aligned}
& =\left(\frac{g_{1}}{g^{\alpha}}\right)^{-J(v t) / F(v t)} \cdot\left(g_{2}^{-l_{v t} k_{v t}+x^{\prime}} g^{y^{\prime}} \cdot g_{2}^{\sum_{i \in U} x_{i}} g^{\sum_{i \in U} y_{i}}\right)^{r_{t}} \\
& =\left(\frac{g_{1}}{g^{\alpha}}\right)^{-J(v t) / F(v t)} \cdot\left(g_{2}^{F(v t)} g^{J(v t)}\right)^{r_{t}} \\
& =g_{2}^{a}\left(g_{2}^{F(v t)} g^{J(v t)}\right)^{-a / F(v t)} \cdot\left(g_{2}^{F(v t)} g^{J(v t)}\right)^{r_{t}} \\
& =g_{2}^{a}\left(g_{2}^{F(v t)} g^{J(v t)}\right)^{r_{t}-a / F(v t)} \\
& =g_{2}^{a}\left(u^{\prime} \prod_{i \in U} u_{i}\right)^{r_{t}^{\prime}}
\end{aligned}
$$

and

$$
\begin{aligned}
d_{i d, t 2} & =\left(\frac{g_{1}}{g^{\alpha}}\right)^{-1 / F(v t)} g^{r_{t}} \\
& =g^{r_{t}-a / F(v t)} \\
& =g^{r_{t}^{\prime}}
\end{aligned}
$$

where $r_{t}^{\prime}=r_{t}-a / F(v t)$.

- Phase 3: The adversary $\mathcal{A}$ gives a target identity $i d^{*}$, a plaintext pair $\left(M_{0}^{*}, M_{1}^{*}\right)$ and a time period $t^{*}$ to $\mathcal{B}$. The challenger $\mathcal{B}$ first chooses a random $\gamma \in\{0,1\}$. Then the challenger $\mathcal{B}$ computes $v^{*}=H_{n}\left(i d^{*}\right)$ and $v t^{*}=H_{m}\left(i d^{*}, t^{*}\right)$. The challenger uses $v^{*}$ and $v t^{*}$ to compute $F\left(v t^{*}\right), J\left(v t^{*}\right)$ and $L\left(v^{*}\right)$. If $F\left(v t^{*}\right) \neq 0$ $\bmod p$, the challenger B reports failure and terminates. If $F\left(v t^{*}\right)=0 \bmod p$, the challenger $\mathcal{B}$ constructs a ciphertext $C^{*}$ as follows.

$$
C^{*}=\left(K \cdot \hat{e}\left(g^{c}, g^{b}\right)^{\alpha} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v t^{*}\right)}, g^{c L\left(v^{*}\right)}\right)
$$

Now, we show that $C^{*}$ is a valid ciphertext as follows.

$$
\begin{aligned}
C^{*} & =\left(K \cdot \hat{e}\left(g^{c}, g^{b}\right)^{\alpha} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v t^{*}\right)}, g^{c L\left(v^{*}\right)}\right) \\
& =\left(\hat{e}(g, g)^{a b c} \cdot \hat{e}\left(g^{\alpha}, g^{b}\right)^{c} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v t^{*}\right)}, g^{c L\left(v^{*}\right)}\right) \\
& =\left(\hat{e}\left(g^{a}, g^{b}\right)^{c} \cdot \hat{e}\left(g^{\alpha}, g^{b}\right)^{c} \cdot M_{\gamma}^{*}, g^{c}, g^{c J\left(v t^{*}\right)}, g^{c L\left(v^{*}\right)}\right) \\
& =\left(\hat{e}\left(g_{1}, g_{2}\right)^{c} \cdot M_{\gamma}^{*}, g^{c},\left(u^{\prime} \prod_{j \in T} u_{i}\right)^{c},\left(t^{\prime} \prod_{j} t_{j}\right)^{c}\right) .
\end{aligned}
$$

- Phase 4: The challenger $\mathcal{B}$ responds to the initial key extract query or the time key update query as in Phase 2. A restriction here is that $\left(i d^{*}, t^{*}\right)$ is disallowed to be queried in the time key update query.
- Phase 5: The adversary $\mathcal{A}$ outputs its guess $\gamma^{\prime} \in\{0,1\}$, and wins this game if $\gamma^{\prime}=\gamma$.

The analysis is similar to Theorem 1. The probability of the challenger $\mathcal{B}$ not aborting is $\operatorname{Pr}[\neg$ abort $] \geqslant \frac{1}{4 q_{U}} \cdot \frac{1}{m+1}$. Then the successful probability (advantage) of the challenger $\mathcal{B}$ who can solve the BDDH problem is at least $\frac{\varepsilon}{4 q_{U}(m+1)}$. The executing time is $\tau+O\left(q_{E}\right.$. $\left.n \cdot \tau_{1}+q_{U} \cdot m \cdot \tau_{1}+\left(q_{E}+q_{U}\right) \cdot \tau_{2}\right)$, where $\tau_{1}$ and $\tau_{2}$ denote the executing time of a multiplication in $G_{1}$ and an exponentiation in $G_{1}$, respectively.

## 6. CCA Transformation and Comparisons

Here, we discuss the transformation technique from the proposed CPA-secure RIBE scheme to a CCA-secure RIBE scheme. We also make the comparisons between our proposed RIBE scheme and several IBE or RIBE schemes (Waters, 2005; Gentry, 2006; Tseng and Tsai, 2012).

Canetti et al. (2004) showed a generic conversion from a 2-level CPA-secure hierarchical ID-based encryption (HIBE) (Horwitz and Lynn, 2002; Gentry and Silverberg, 2002) to a CCA-secure IBE scheme by appending a one-time signature to the ciphertext, which is encrypted to an identity equal to the verification key. Boneh and Katz (2005) also improved the efficiency of Canetti et al.'s construction by using the MAC code instead of the one-time signature. Afterwards, Boyen et al.'s (2005) proposed a direct conversion approach avoiding the MAC codes and one-time signatures. In 2005, Waters adopted Canetti et al.'s generic conversion to obtain to a CCA-secure IBE scheme from a 2 level CPA-secure hierarchical ID-based encryption (HIBE). Following the idea in Waters (2005), we use our proposed scheme at the first level and employ the Boneh and Boyen's (2004a) CPA-secure IBE scheme at the second level to build a hybrid 2-level hierarchical revocable ID-based encryption (HRIBE) scheme. By the conversion techniques in (Canetti et al., 2004; Boneh and Katz, 2005), we can obtain an adaptive-ID, CCA-secure RIBE without random oracles. For the conversion approach in the random oracle model, Fujisaki and Okamoto (1999) presented a simple conversion from a weak public-key encryption scheme (IND-CPA) to a strong public-key encryption scheme (IND-CCA) in the random oracle model. Kitagawa et al. (2006) proposed an improvement on Fujisaki and Okamoto's (1999) conversion for ID-based encryption schemes. They can transform a weak ID-based encryption scheme (IND-ID-CPA) to a strong ID-based encryption scheme (IND-ID-CCA). Tseng and Tsai (2012) employed Kitagawa et al.'s conversion technique (2006) to obtain a CCA-secure RIBE scheme from their CPA-secure RIBE scheme in the random oracle model.

Table 1 lists the comparisons between our proposed RIBE scheme and some famous IBE or RIBE schemes (Waters, 2005; Gentry, 2006; Tseng and Tsai, 2012) in terms of underlying security assumption, security model, revocable functionality and computational cost. Note that we only compare the CPA-secure IBE or CPA-secure RIBE schemes in Table 1. Gentry's IBE scheme used a stronger security assumption called the augmented bilinear Diffie-Hellman exponent (ABDHE) assumption than the BDDH or BDH security assumptions which are used by the other three proposed schemes. Although Tseng and Tsai's RIBE scheme has the best performance, it could be insecure when random

Table 1
Comparisons between our proposed RIBE scheme and the previously proposed schemes

|  | Waters's IBE <br> $(2005)$ | Gentry's IBE <br> $(2006)$ | Tseng and Tsai's <br> RIBE (2012) | Our proposed <br> RIBE |
| :--- | :--- | :--- | :--- | :--- |
| Security <br> assumption | BDDH | ABDHE | BDH | BDDH |
| Security model Standard <br> model  | Standard <br> model | Random oracle <br> model | Standard <br> model |  |
| Revocable property | No | No | Yes | Yes |
| Pairing operation <br> for encryption | 1 | 2 | 1 | 1 |
| Pairing operation <br> for decryption | 2 | 1 | 1 | 3 |

oracles are instantiated with concrete hash functions (Canetti et al., 1998; Bellare et al., 2004a; Boneh and Boyen, 2004a). Our proposed RIBE scheme increase pairing operations as compared to Waters's and Gentry's IBE schemes for decryption, but the point is that our proposed RIBE scheme provides a flexible revocation mechanism with a public channel.

## 7. Conclusions

In this paper, we have proposed a fully secure RIBE scheme in the standard model (without random oracles) to provide robust security. We employed the revocable concept presented by Tseng and Tsai to provide an efficient and flexible revocation mechanism. For security analysis, we have demonstrated that the proposed RIBE scheme is semantically secure against adaptive-ID attacks in the standard model under the bilinear decision Diffie-Hellman assumption. For enhancing the practicality of ID-based public key systems, an efficient revocation mechanism must be involved in the design of various ID-based cryptographic schemes and protocols in the future.

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## Visiškai saugus susikompromitavusio vartotojo pašalinimas identifikatoriais gristame standartiniame šifravimo modelyje

Bet kuri sertifikatais arba identifikatoriais grịsta viešojo rakto šifravimo sistema privalo turéti galimybę pašalinti susikompromitavusius viešojo rato sistemos vartotojus. Neseniai Tseng ir Tsai pasiūlė naują identifikatoriais grịstą viešojo rakto sistemą (RIBE), kuri, naudodama neapsaugotą ryšio kanalą, efektyviai pašalina susikompromitavusius vartotojus. Straipsnyje pasiūlytas modifikuotas Tseng ir Tsai metodas, kuris užtikrina visišką sistemos saugumą ir nenaudoja atsitiktinio juodosios dèžès modelio.

