

A Dynamic Network Interdiction Problem

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Abstract. We present a novel dynamic network interdiction model that accounts for interactions between an interdictor deploying resources on arcs in a digraph and an evader traversing the network from a designated source to a known terminus, wherein the agents may modify strategies in selected subsequent periods according to respective decision and implementation cycles. For the resulting minimax model, we develop a reformulation that facilitates a direct solution procedure using commercial software or via a proposed alternating heuristic. We examine certain related stability and convergence issues, demonstrate special convergence cases, and provide insights into the computational performance of the solution procedures.

Keywords: network interdiction, resource allocation, minimax flow problems, dynamic formulation, alternating heuristic.

1. Introduction

The act of interdicting flow through a network is most often modeled in the form of a *static* two-player, two-stage, sequential game with perfect information (i.e., a Stackelberg game), in which an interdictor allocates resources, followed by the subsequent decisions made by an evader to direct flow through the network from a source to a terminus. As with most models, this is a simplification of reality. Accordingly, in this work we seek to enhance the foregoing modeling approach by considering the *dynamic* interaction between agents within the context of such a network interdiction problem, wherein opponent strategies are not static. A motivation for our model is to provide an application framework to examine, and possibly validate, the observe-orient-decide-act cycle (a.k.a., OODA loop), a theory developed by Boyd (1986), which serves as the foundation for military operational planning cycles as motivated by his following maxim for successful operations:

“Observe-orient-decide-act more inconspicuously, more quickly, and with more irregularity as basis to keep or gain initiative as well as shape and shift main effort: to repeatedly and unexpectedly penetrate vulnerabilities and weaknesses exposed by that effort or other effort(s) that tie-up, divert, or drain-away adversary attention (and strength) elsewhere.”

To this end, we extend over the temporal domain the problem of minimizing the maximum flow pertaining to an evader from a single source to a known terminus, considering the simultaneous allocation of multiple resource types to achieve (partial) arc interdictions, but with additional objective function penalties as well as an allowance for different durations for the interdicator and evader OODA loops. We then reformulate the model to facilitate a direct solution as a mixed-integer nonlinear program, and we examine stability issues under three conditions for *relative* loop lengths and for two categories of problem structure.

The network interdiction problem has been examined for several decades within the context of a variety of modeling approaches, optimization objectives, and solution techniques. Pertinent to our efforts is the previous research on network interdiction to minimize an adversary's maximum flow through a network. Wollmer (1964) examined this problem on a planar graph under the assumption of no (or uniform) interdiction costs, and developed a dynamic programming approach to optimally identify a prescribed number of arcs for removal (i.e., *discrete* or *binary* interdiction). Wollmer (1969), as well as McMasters and Mustin (1970), examined a variant of this work to consider an adversary seeking to minimize a prescribed flow cost through a network, wherein flow costs are linearly proportional to the arc capacities. Ghare *et al.* (1971) examined a similar problem, but without the assumption of a planar graph, and developed an exact branch-and-bound algorithm. This problem was also addressed by Phillips (1993), where it was referred to as the *Network Inhibition Problem*. Several variants were formulated that allow for partial interdiction of arcs, all of which were proven to be NP-Hard. In 1993, Wood published a seminal work for network interdiction modeling, incorporating existing graph theory techniques and introducing some new variations to expand the applicability of the models. Of particular note, Wood proposed different deterministic network interdiction formulations to account for partial arc interdiction, multiple sources and sinks, undirected networks, multiple resources, and multiple commodities, and designed effective solution techniques.

In contrast, the extension of network interdiction strategies and responses over multiple cycles in the temporal domain is found less frequently in the published literature. Exceptions include the exploration of network interdiction within trilevel optimization frameworks by Brown *et al.* (2006) and, more recently, by Lim and Smith (2008). Also, Lunday and Sherali (2009a) consider such a framework in which an interdicator deploys overt resources prior to an evader's selection of a path through a network to maximize the subsequent probability of evasion, followed, in turn, by the deployment of additional (covert) resources of which the evader is unaware. These recent trends account for an increasing number of interactions between opponents in the interdiction problem and reflect a greater need to shift towards dynamic model formulations involving multiple strategic and response cycles, which is the principal motivation for this paper. In addition, our dynamic network interdiction model also builds upon the general multi-objective approach of others (Royset and Wood, 2007), applying preemptive weights within a nonpreemptive formulation (Sherali and Soyster, 1983), and we examine related stability and convergence issues.

The remainder of this paper is organized as follows. In Section 2, we propose and formulate a dynamic network interdiction model that seeks to minimize the maximum value of a regret function that is comprised of a weighted combination of the interdictor's costs, the evader's maximum flow, and the evader's penalties due to interdiction. The resulting minimax problem is reformulated in Section 3 as a mixed-integer nonlinear program to facilitate a direct solution approach, either using a standard software or, particularly for larger-sized instances, via a proposed alternative fixing heuristic. Thereafter, in Section 4, we study certain stability, convergence, and computational issues related to the players' strategies. We demonstrate the efficacy of the alternative heuristic in Section 5, and we conclude the paper in Section 6 with a summary and recommendations for future research.

2. Dynamic Network Interdiction Problem

In this section, we outline the underlying assumptions for our model, the dynamic network interdiction problem, and then present the notation along with the model formulation as a mixed-integer minimax programming problem.

2.1. Modeling Assumptions

For our model, we assume that the topology and characteristics of the network are static, i.e., the set of nodes and arcs and the uninterdicted upper bounds on arc flows are constant, as are the interdiction costs for each arc and resource type combination. Lim and Smith (2008) have examined a problem involving network characteristic changes over time, but this aspect is outside the modeling scope of the dynamic process considered herein. In contrast, we consider the dynamic interaction between the strategies and responses of the interdictor and evader based on the evolving state of the network resulting from their periodic, sequential decisions.

Within a game theoretic framework, the static network interdiction problem is a two-player, two-stage, sequential game with perfect information (i.e., a Stackelberg game). By adding the temporal domain, we extend the game to a finite, or possibly infinite, number of moves. However, a simple temporal replication of the static model would merely create a series of identical two-stage subgames. To affect a dynamic model, we propose a set of behavioral assumptions to construct a finite series of two-player, *simultaneous* (i.e., strategic) and *sequential* games (Osborne, 2004), wherein a simultaneous game occurs during any period in which both players may change strategies, and a sequential game occurs otherwise.

2.1.1. Temporal Domain

We first define a *reaction time* as the time necessary for an actor (interdictor or evader) to implement an OODA loop, i.e., to (a) observe the current state of the network; (b) orient the current problem within the context of the objectives; (c) decide on changes to strategy; and (d) act to implement the changes. In order to facilitate modeling, we assume that the

strategic decisions are such that both the interdicator and evader reaction times, denoted γ^I and γ^E , respectively, can be reasonably taken to be constant integer units of time.

Considering this definition, we assume that the dynamic network interdiction occurs over a fixed horizon of time, indexed by $\gamma = \Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma$, wherein the discretization of time (Δ_γ) is based on the greatest common factor of interdicator and evader reaction times, and where Γ is an integer multiple of Δ_γ . This requires that interdicator and evader strategies may change only in certain specified time periods, which might be different, and we impose constraints accordingly.

Note that for an initial network state at $\gamma = 0$, we may assume that both actors have implemented an initial strategy as determined by an optimal solution to the underlying static model. This is an appropriate assumption if we are interested in the evolution of interdicator-evader strategies over time, rather than their growth from a null solution. Otherwise, we assume an initial solution with no interdiction resources applied to the network and no evader flow.

2.1.2. Strategic Objectives

To manifest a true dynamic behavior, we assume that the interdicator does not merely seek to minimize the maximum flow of an evader; such an objective would only consider network characteristics to determine the interdicator's strategy. Instead, the objective must consider other time-variant factors. Accordingly, we formulate a multiobjective program in which the interdicator seeks to minimize a maximum regret function that is comprised of a linearly weighted combination of three objectives summed over all time periods: (1) the maximum evader flow through the network; (2) the cost of interdicator actions; and (3) penalties incurred upon the evader by attempting to send flow in excess of the interdicted arc capacities. As such, our objective formulation is related to multi-criteria decision making (e.g., see Steuer (1986) and Zavadskas *et al.* (2009) on possible techniques to address such problems). Whereas the measure of maximum flow is no different from the static model, we will elaborate on the other measures after a formal presentation of the model in Section 2.2.

Furthermore, we assume that the interdicator and evader employ myopic strategies. That is, each reaction cycle is constrained such that during a time period (γ) in which an actor implements a strategy, that actor assesses the state of the network and the opponent's strategy, then makes a decision on the subsequent strategy to implement in period $(\gamma + \gamma^I)$ or $(\gamma + \gamma^E)$, as appropriate. We also assume that the evaluation of the network and strategies to support decision-making are based only on the situation in the particular action period γ , i.e., the decisions are not affected by the state of the network in time periods between an observation and a subsequent action.

As another dynamic element in our model, we consider strategic costs incurred by the interdicator, as measured in consistent monetary units. We assume that these costs include strategy implementation costs, strategy adjustment costs, and resource level adjustment costs. We also assume that these costs are subject to both an overall budget as well as a budget specific to each time period. The discretization of budget authorizations over fiscal periods often occurs in practice for interdicator agencies. To account for the cost of implementing interdicator strategies, we assume that these costs are linearly proportional to the

interdiction level of each arc and resource-type combination. In considering the costs for adjusting strategies by raising or lowering specific resource type allocations on a given arc between time periods, we also assume that these costs are linearly proportional to the amount of change. We maintain that the foregoing proportional factors for costs may vary based on the resource type and the arc on which the interdicator applies and/or alters a strategy, but is invariant over time. Furthermore, we allow for different marginal costs for *increasing* versus *decreasing* applications of a strategy. Finally, we incorporate available resource level adjustment costs, assuming that the interdicator may alter the availability of resources types within specified bounds between periods, and subject to an overall bound on the availability of each resource type. Each of these changes to resource availabilities incurs a cost specific to the period in which it occurs, and the marginal costs may also differ for increasing or decreasing resource type availabilities.

Our proposed model also considers a third component in the objective function: linear penalties on the evader for attempting to send flow on arcs in excess of their respective interdicted capacities. In application, these penalties result from a misperception between the state of the network when the current strategy was decided upon and the actual network state during implementation.

2.2. Dynamic Network Interdiction – Model Formulation

To formulate our model, we define the following sets, decision variables, and parameters based on the stated assumptions from Section 2.1:

Set Notation:

- $\gamma \in \{0, \Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$: the time domain, discretized as previously outlined with the following terminology:
 - γ^I, γ^E : interdicator and evader reaction times, respectively.
 - Δ_γ : the discretization of time, where Δ_γ is the greatest common factor of γ^I and γ^E .
 - Γ : time horizon (an integer multiple of Δ_γ).
 - $\Gamma^I \equiv n_I \gamma^I, \Gamma^E \equiv n_E \gamma^E$: the final periods in which the interdicator and evader can implement changes, where $n_I = \lfloor \frac{\Gamma}{\gamma^I} \rfloor$ and $n_E = \lfloor \frac{\Gamma}{\gamma^E} \rfloor$.
- $i \in N$: set of nodes in the network.
- $(i, j) \in A$: set of directed arcs in the network.
- $G[N, A]$: the underlying network.
- $k \in \mathcal{K}$: set of resource types for interdiction.

Primary Decision Variables:

- p_{ijk}^γ : the percentage of capacity reduction for arc (i, j) during time period γ as affected by the application of interdicator resource type k .
- R_k^γ : the total amount of units of resource type k procured for interdiction during time period γ .
- x_{ij}^γ : the maximal flow solution on arc (i, j) based on the residual capacity in time period γ .

- y_{ij}^γ : the amount of non-negative flow that the evader *attempts* to transport over arc (i, j) during time period γ . (For time period $\gamma = 0$, we assume $y_{ij}^0 = x_{ij}^0$, $\forall (i, j) \in A$.)

Secondary Decision Variables (influenced by (p, R, x, y)):

- $p_{ijk}^{\gamma+}, p_{ijk}^{\gamma-}$: the *increase* and *decrease*, respectively, in percentage of capacity reduction for arc (i, j) between time periods $(\gamma - \gamma^I)$ and γ , $\forall \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma\}$, as affected by the application of interdicator resource type k .
- δ_{ijk}^γ : a binary decision variable to enforce the logical constraint that only $p_{ijk}^{\gamma+}$ or $p_{ijk}^{\gamma-}$ may be non-zero for any combination of $\gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma\}$, $(i, j) \in A$, $k \in \mathcal{K}$.
- $R_k^{\gamma+}, R_k^{\gamma-}$: the *increase* and *decrease*, respectively, in total amount of units of resource k available for interdiction between time periods $(\gamma - \gamma^I)$ and γ , $\forall \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma\}$.
- ψ_k^γ : a binary decision variable to enforce the logical constraint that only $R_k^{\gamma+}$ or $R_k^{\gamma-}$ may be non-zero for any combination of $\gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma\}$, $k \in \mathcal{K}$.
- z^γ : the maximum flow capacity through the network from node s to node t in time period γ .
- $y_{ij}^{\gamma+}$: the amount of flow the evader attempts to transport over arc (i, j) during time period γ *in excess* (if at all) of the interdicted capacity during time period γ .

Auxiliary Decision Variables (defined for notational convenience):

- C^γ : the total cost (in monetary units) for the interdicator to change strategies between time periods $(\gamma - \Delta_\gamma)$ and γ .
- D^γ : the total cost (in monetary units) for the interdicator to implement a strategy during time period γ .
- E^γ : the total cost (in monetary units) for the interdicator to change levels of resource types between time periods $(\gamma - \Delta_\gamma)$ and γ .
- P^γ : the penalty incurred by the evader (not necessarily in monetary units) by attempting to send units of flow across the network in excess of interdicted arc capacities during time period γ .

Parameters:

- $w = (w_C, w_P, w_F)$: a vector of positive relative weights in the multi-criteria objective function, corresponding to the interdicator costs, evader penalty, and maximum flow, respectively. Given w_C and w_P , we determine w_F so as to weight the network flow component in the objective function preemptively higher in order to ensure the definitional role of z^γ (see Proposition 1 below).
- u_{ij} : uninterdicted flow capacity for arc (i, j) .
- B : the net budget (in monetary units) available for the interdicator over the duration of the time periods examined.
- B^γ : the budget (in monetary units) available for the interdicator for use between time periods $(\gamma - \Delta_\gamma)$ and γ .
- c_{ijk}, d_{ijk} : the cost (in *resource* and *monetary* units, respectively) to completely interdict arc (i, j) using resource k during any time period.

- $\alpha_{ijk}, \omega_{ijk}$: the cost (in monetary units) to *add* or *remove*, respectively, a unit of interdiction resource k on arc (i, j) between time periods $(\gamma - \Delta_\gamma)$ and γ . (Recall that we do not assume $\alpha_{ijk} = \omega_{ijk}$, as resource deployment and retraction costs may differ.)
- R_k : the maximum amount of units of resource k available for interdiction over all periods. For examination of strategy evolution, we recommend utilizing $R_k^0 < R_k$ to consider any resource types that are not fully procured at the outset of the problem. In order to model the *addition* of a resource type over time, we set $R_k^0 \equiv 0$ for the resource type of interest. One can further affix this value to zero for a subset of immediate subsequent time periods, if the model should account for a resource type that will only be available for procurement after a specified point in time.
- A_k, Ω_k : the cost (in monetary units) to *increment* or *decrement*, respectively, the availability of resource type k by a single unit between any time periods $(\gamma - \Delta_\gamma)$ and γ . We assume $A_k + \Omega_k \geq 0, \forall k \in \mathcal{K}$, to prevent a benefit to the interdictor by cyclically incrementing and decrementing resources without relevance to the network interdiction problem. (We do not assume $A_k = \Omega_k$, as procurement costs are assuredly greater than costs of withdrawing resource availability; moreover, Ω_k might be negative, reflecting a savings.)
- g_{ij} : the positive penalty cost (not necessarily in monetary units) per unit of flow that an evader attempts to send on arc (i, j) in excess of the interdicted capacity of the arc, for any given time period.
- $\hat{x}_{ij}^0, \hat{z}^0, \hat{y}_{ij}^0, \hat{p}_{ijk}^0, \hat{R}_k^0$: the respective values for $x_{ij}^\gamma, z^\gamma, y_{ij}^\gamma, p_{ijk}^\gamma, R_k^\gamma$, at time period $\gamma = 0$, which characterize the initial state of the network and the opponent strategies.

Based on these decision variable definitions, we denote a period-specific solution to be

$$\nu^\gamma = \begin{cases} (p^\gamma, p^{\gamma+}, p^{\gamma-}, \delta^\gamma, R^\gamma, R^{\gamma+}, R^{\gamma-}, \psi^\gamma, x^\gamma, z^\gamma, y^\gamma, y^{\gamma+}, y^{\gamma-}, C^\gamma, D^\gamma, E^\gamma, P^\gamma), \\ \quad \forall \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma\}, \\ (p^\gamma, R^\gamma, x^\gamma, z^\gamma, y^\gamma, y^{\gamma+}, y^{\gamma-}, C^\gamma, D^\gamma, E^\gamma, P^\gamma), \quad \forall \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma\}, \end{cases}$$

wherein each component is a vector over the arc and/or resource-type indices as defined above. Hence, a feasible solution to the model is given by:

$$\nu = (\nu^\gamma, \gamma = 0, \Delta_\gamma, \dots, \Gamma).$$

Accordingly, the proposed *dynamic network interdiction problem* (DNIP) can be formulated as follows:

$$\begin{aligned} \text{DNIP: } & \min_{p,R} \max_{x,y} \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} [w_C(D^\gamma + C^\gamma + E^\gamma) - w_P P^\gamma + w_F z^\gamma], \quad (1) \\ & \text{subject to } \sum_{j:(i,j) \in A} x_{ij}^\gamma - \sum_{j:(j,i) \in A} x_{ji}^\gamma = \begin{cases} z^\gamma, & \text{if } i = s \\ 0, & \text{if } i \neq s, t \\ -z^\gamma, & \text{if } i = t \end{cases}, \quad \forall i \in N, \\ & \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \quad (2) \end{aligned}$$

$$\sum_{(i,j) \in A} c_{ijk} p_{ijk}^{\gamma} \leq R_k^{\gamma}, \quad \forall k \in \mathcal{K}, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (3)$$

$$x_{ij}^{\gamma} \leq u_{ij} \left(1 - \sum_{k \in \mathcal{K}} p_{ijk}^{\gamma}\right), \quad \forall (i, j) \in A, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (4)$$

$$\sum_{k \in \mathcal{K}} p_{ijk}^{\gamma} \leq 1, \quad \forall (i, j) \in A, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (5)$$

$$(x_{ij}^{\gamma}, p_{ijk}^{\gamma}) \geq 0, \quad \forall (i, j) \in A, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (6)$$

$$D^{\gamma} = \sum_{(i,j) \in A} \sum_{k \in \mathcal{K}} d_{ijk} p_{ijk}^{\gamma}, \quad \forall \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (7)$$

$$p_{ijk}^{\gamma} = p_{ijk}^{(\gamma - \Delta_{\gamma})}, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (8)$$

$$R_k^{\gamma} = R_k^{(\gamma - \Delta_{\gamma})}, \quad \forall k \in \mathcal{K}, \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (9)$$

$$x_{ij}^{\gamma} = x_{ij}^{(\gamma - \Delta_{\gamma})}, \quad \forall (i, j) \in A, \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (10)$$

$$z^{\gamma} = z^{(\gamma - \Delta_{\gamma})}, \quad \forall \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (11)$$

$$y_{ij}^{\gamma} = \begin{cases} x_{ij}^{(\gamma - \gamma^E)}, & \forall (i, j) \in A, \gamma \in \{\gamma^E, 2\gamma^E, \dots, \Gamma^E\}, \\ y_{ij}^{(\gamma - \Delta_{\gamma})}, & \forall (i, j) \in A, \gamma \notin \{\gamma^E, 2\gamma^E, \dots, \Gamma^E\}, \end{cases} \quad (12)$$

$$p_{ijk}^{\gamma} - p_{ijk}^{(\gamma - \gamma^I)} = p_{ijk}^{\gamma+} - p_{ijk}^{\gamma-}, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (13)$$

$$p_{ijk}^{\gamma+} \leq \delta_{ijk}^{\gamma}, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (14)$$

$$p_{ijk}^{\gamma-} \leq 1 - \delta_{ijk}^{\gamma}, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (15)$$

$$\delta_{ijk}^{\gamma} \in \{0, 1\}, \quad \forall (i, j) \in A, k \in \mathcal{K}, \forall \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (16)$$

$$(p_{ijk}^{\gamma+}, p_{ijk}^{\gamma-}) \geq 0, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (17)$$

$$C^{\gamma} = \begin{cases} \sum_{(i,j) \in A} \sum_{k \in \mathcal{K}} c_{ijk} (\alpha_{ijk} p_{ijk}^{\gamma+} + \omega_{ijk} p_{ijk}^{\gamma-}), & \forall \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \\ 0, & \forall \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \end{cases} \quad (18)$$

$$R_k^{\gamma} - R_k^{(\gamma - \gamma^I)} = R_k^{\gamma+} - R_k^{\gamma-}, \quad \forall k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (19)$$

$$R_k^{\gamma+} \leq R_k \psi_k^{\gamma}, \quad \forall k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (20)$$

$$R_k^{\gamma-} \leq R_k (1 - \psi_k^{\gamma}), \quad \forall k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (21)$$

$$\psi_k^{\gamma} \in \{0, 1\}, \quad \forall k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (22)$$

$$R_k^{\gamma} \leq R_k, \quad \forall k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (23)$$

$$(R_k^{\gamma}, R_k^{\gamma+}, R_k^{\gamma-}) \geq 0, \quad \forall k \in \mathcal{K}, \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \quad (24)$$

$$E^{\gamma} = \begin{cases} \sum_{k \in \mathcal{K}} (A_k R_k^{\gamma+} + \Omega_k R_k^{\gamma-}), & \forall \gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \\ 0, & \forall \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}, \end{cases} \quad (25)$$

$$C^{\gamma} + D^{\gamma} + E^{\gamma} \leq B^{\gamma}, \quad \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (26)$$

$$\sum_{\gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}} (C^{\gamma} + D^{\gamma} + E^{\gamma}) \leq B, \quad (27)$$

$$y_{ij}^{\gamma+} \geq y_{ij}^{\gamma} - u_{ij} \left(1 - \sum_{k \in \mathcal{K}} p_{ijk}^{\gamma}\right), \quad \forall (i, j) \in A, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (28)$$

$$y_{ij}^{\gamma+} \leq u_{ij}, \quad \forall (i, j) \in A, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (29)$$

$$y_{ij}^{\gamma+} \geq 0, \quad \forall (i, j) \in A, \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (30)$$

$$P^{\gamma} = \sum_{(i,j) \in A} g_{ij} y_{ij}^{\gamma+}, \quad \forall \gamma \in \{\Delta_{\gamma}, 2\Delta_{\gamma}, \dots, \Gamma\}, \quad (31)$$

$$x_{ij}^0 = \hat{x}_{ij}^0, \quad \forall (i, j) \in A, \quad (32)$$

$$z^0 = \hat{z}^0, \quad (33)$$

$$y_{ij}^0 = \hat{y}_{ij}^0, \quad \forall (i, j) \in A, \quad (34)$$

$$p_{ijk}^0 = \hat{p}_{ijk}^0, \quad \forall (i, j) \in A, k \in \mathcal{K}, \quad (35)$$

$$R_k^0 = \hat{R}_k^0, \quad \forall k \in \mathcal{K}. \quad (36)$$

The objective function (1) represents the interdictor seeking to minimize the maximum of a regret function, comprised of the sum over all periods of the weighted interdictor costs, evader penalty, and evader maximum flow. Note that the evader penalty component is subtracted, as we previously defined $w_P > 0$. The negative coefficient on the evader penalty in (1) in this minimax formulation reflects the interdictor's goal to maximize the minimum evader penalty, within the context of the relative weightings of components within the regret function. Constraints (2)–(6) impose the period-specific physical constraints on arc-wise and maximum flows based on constrained resource applications, with indexing over the temporal domain. Constraint (2) enforces conservation of flow in the network at each node, with z^{γ} as the maximum flow through the network between the start and terminus nodes. Constraint (3) restricts the application of each resource type subject to resource availability. Constraint (4) bounds the maximum flow on each arc by the modified upper bound due to the interdictive effects of the applied resources. Constraint (5) limits the level of interdiction for each arc to at most complete interdiction. Constraint (6) enforces non-negativity of the x_{ij}^{γ} - and p_{ijk}^{γ} -decision variables, and Constraint (7) computes the cost of interdictor strategy implementation. We model the restrictions on player decisions due to their respective reaction times in Constraints (8)–(12). Constraints (8)–(10) ensure that the interdictor cannot change elements of strategy in time periods other than those that are multiples of the reaction time, γ^I , which likewise restricts the maximum flow in (11). Constraint (12) determines that an evader will only change behavior in time periods that are multiples of the corresponding reaction time, γ^E , and will do so based on a myopic observation of the state of the network at the *beginning* of the OODA loop (to allow for time to revise strategies). The model determines the interdictor changes and related costs via Constraints (13)–(27). Constraint (13) computes the the absolute positive and negative changes to arc and resource type combinations for periods in which the interdictor may alter strategies, Constraints (14)–(16) ensure that only one of $p_{ijk}^{\gamma+}$ or $p_{ijk}^{\gamma-}$ may be non-zero for any time period, and Constraint (17) enforces these fluctuations to be non-negative. (Without Constraints (14)–(16) and (17), both fluctuations would be positive in the case where

α_{ijk} and ω_{ijk} are non-zero and of opposite sign, with $\alpha_{ijk} + \omega_{ijk} < 0$.) Constraint (18) calculates the cost due to period-specific fluctuations as determined via Constraints (13)–(17). Constraint (19) calculates the (absolute) changes in resource type availabilities for periods in which the interdicator may alter strategies, as bounded by Constraints (20)–(24). For this subset of periods, Constraints (20)–(22) ensure that only one of $R_k^{\gamma+}$ or $R_k^{\gamma-}$ may be non-zero for any period and resource type combination. Constraint (23) provides upper bounds on the absolute availability of all resource types in each period γ , $\gamma \in \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}$, and the non-negativity of these resource availabilities and inter-cycle fluctuations are enforced by Constraint (24). Upper and lower bounds are induced on R_k^γ , $\forall k \in \mathcal{K}$, $\gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}$ via Constraint (9). Constraint (25) determines period-specific costs due to (absolute) changes in resource type availabilities, as calculated by Constraints (19)–(24). Constraint (26) enforces the budget specific to each time period, while Constraint (27) enforces the overall budget. The model utilizes Constraints (28)–(31) to determine the evader penalties for the third component of the objective function, based on the evader possibly attempting to transport units of flow that exceed interdicted arc capacities. Constraint (28) enforces a lower bound on the amount of flow the evader attempts to transport over each arc and time period in excess (if at all) of the interdicted capacity, Constraints (29) provides upper bounds, and Constraint (30) ensures non-negativity of this measure. Constraint (31) determines the penalty cost for each time period, based on the calculation of $y_{ij}^{\gamma+}$ via Constraints (28)–(30). Given that $x_{ij}^0 \leq u_{ij}$ and $y_{ij}^0 \leq u_{ij}$, $\forall (i, j) \in A$, and noting (4), we have that $y_{ij}^\gamma \leq u_{ij}$, $\forall (i, j) \in A$, $\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$. Hence, with $\underline{p}^\gamma > 0$, the objective function (1) will enforce $y_{ij}^{\gamma+} = \max\{0, y_{ij}^\gamma - u_{ij}(1 - \sum_{k \in \mathcal{K}} p_{ijk}^\gamma)\}$, $\forall (i, j) \in A$, $\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$ via (28) and (30), whence (29) is essentially redundant. Nevertheless, we retain (29) for convenience in further analysis and algorithmic implementation. Finally, Constraints (32)–(36) impose the initial states of the process.

Observe that by (18)–(24), we can rewrite the objective function (1) as

$$\min_{p, R} \left\{ \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} w_C (D^\gamma + C^\gamma + E^\gamma) + \max_{x, y} \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} [-w_P P^\gamma + w_F z^\gamma] \right\}. \quad (37)$$

Hence, given w_P , in order to ensure the definitional maximum flow role of z^γ , we determine w_F so that the relative weighting in the inner maximization problem in (37) assures the preemptive relationship

$$\sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} z^\gamma \gg \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} [-w_P P^\gamma]. \quad (38)$$

This is accomplished via Proposition 1 as stated below.

PROPOSITION 1. Given a decision maker’s choice of w_P , suppose that we correspondingly select

$$w_F = 1 + \frac{(\Gamma/\Delta_\gamma)[w_P UB^P]}{\epsilon}, \tag{39}$$

where $UB^P = \sum_{(i,j) \in A} g_{ij} u_{ij}$, and where $\epsilon > 0$. Then, the preemptive relationship (38) is satisfied within a tolerance ϵ with respect to the deviation of $\sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} z^\gamma$ from its maximum attainable value.

Proof. Denote the components within the inner maximization objective function in (37) as

$$f_1 \equiv \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} z^\gamma, \quad \text{and} \quad f_2 \equiv \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} [-w_P P^\gamma],$$

and let $f_{1\max}$, $f_{2\max}$, and $f_{2\min}$ indicate the maximum or minimum attainable values for the corresponding functions over the feasible region for the inner maximization problem in Problem DNIP, given any feasible (p, R) to the outer problem. Following Sherali and Soyster (1983), to enforce that an optimal solution to this inner maximization problem does not deviate more than ϵ from $f_{1\max}$, we must have that:

$$w_F f_{1\max} + f_{2\min} > w_F (f_{1\max} - \epsilon) + f_{2\max}. \tag{40}$$

Solving Equation (40) for w_F , we obtain

$$w_F > \frac{f_{2\max} - f_{2\min}}{\epsilon}. \tag{41}$$

From Constraints (29), (30), and (31), we obtain that $0 \leq P^\gamma \leq UB^P$, where UB^P is as defined in the proposition. Hence, regardless of the outer problem solution (p, R) , we have that $f_{2\max} \leq 0$ and $f_{2\min} \geq (\Gamma/\Delta_\gamma)(-w_P UB^P)$, which yields

$$f_{2\max} - f_{2\min} \leq (\Gamma/\Delta_\gamma)[w_P UB^P]. \tag{42}$$

Hence, from (42), using w_F given by (39) satisfies (41), and so (39) yields a valid choice for w_F .

REMARK 1. Note that by enforcing the definitional role of z^γ via Proposition 1, the parameter w_F can be quite large, which might also induce a relatively higher priority to minimizing the maximum flow (though not necessarily preemptively) with respect to the overall objective function. However, this can be counter-balanced by subsequently adjusting the parameter w_C so that the ratio w_C/w_F reflects the interdictor’s relative priority

for minimizing costs (the first term in (37)) versus minimizing the maximum flow. Also, in regard to Proposition 1, note that whereas we can derive w_F to exactly satisfy (38) by using an equivalent integerized model for DNIP (assuming rational data) and applying a derivation similar to that in Bazaraa *et al.* (2005) to obtain a minimal possible decrement in $f_{1\max}$ that would suffice for use as ϵ in (41) (or (39)), this would involve computing maximal absolute determinants of bases, which is practically intractable. Besides, such a theoretical value of ϵ would be much too small, resulting in an inordinately large value of w_F in (39). Hence, in our computations, we shall rely on using (39) for a specified tolerance of $\epsilon \equiv 10^{-3}$.

3. Solution Procedures

In this section, we reformulate Problem DNIP as an equivalent mixed-integer nonlinear program by taking the dual of the inner maximization problem. This consequently enables the application of available commercial software such as BARON (Ryoo and Sahinidis, 1996), which is designed for mixed-discrete nonconvex formulations, to derive a global optimal solution.

Toward this end, define the associated dual variables for the relevant constraints as shown in Table 1. Then, writing the dual to the inner maximization problem, we obtain the following equivalent representation, Problem **P**, where we have denoted $p_{ij}^{0\gamma}$ as the nonnegative slack in Constraint (5), and where $\Psi_{ij}^\gamma \equiv \theta_{ij}^\gamma + \tau_{ij}^\gamma$, $\forall(i, j) \in A$, $\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$:

Table 1
Dual variables for reformulating problem DNIP

Dual variable	Definition	Associated constraint
β	$\beta_i^\gamma, \forall i \in N, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$	(2)
θ	$\theta_{ij}^\gamma, \forall(i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$	(4)
μ	$\mu_{ij}^\gamma, \forall(i, j) \in A, \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}$	(10)
λ	$\lambda^\gamma, \forall \gamma \notin \{\gamma^I, 2\gamma^I, \dots, \Gamma^I\}$	(11)
σ	$\sigma_{ij}^\gamma, \forall(i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$	(12)
τ	$\tau_{ij}^\gamma, \forall(i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$	(28)
ϕ	$\phi_{ij}^\gamma, \forall(i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$	(29)
π	$\pi^\gamma, \forall \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$	(31)
ξ	$\xi_{ij}, \forall(i, j) \in A$	(32)
ζ	ζ	(33)
η	$\eta_{ij}, \forall(i, j) \in A$	(34)

$$\mathbf{P:} \quad \min \sum_{\gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}} \left[w_C(C^\gamma + D^\gamma + E^\gamma) + \sum_{(i,j) \in A} u_{ij}(p_{ij}^{0\gamma} \Psi_{ij}^\gamma + \phi_{ij}^\gamma) \right] \\ + \sum_{(i,j) \in A} (\hat{x}_{ij}^0 \xi_{ij} + \hat{y}_{ij}^0 \eta_{ij}) + \hat{z}^0 \zeta \tag{43}$$

$$\text{s.t. } \beta_t^\gamma - \beta_s^\gamma = w_F \\ + \begin{cases} -\lambda^\gamma + \lambda^{(\gamma+\Delta_\gamma)}, & \text{if } \gamma \pmod{\gamma_I} \neq 0 \text{ and } (\gamma + \Delta_\gamma) \pmod{\gamma_I} \neq 0, \\ -\lambda^\gamma, & \text{if } \gamma \pmod{\gamma_I} \neq 0 \text{ and } (\gamma + \Delta_\gamma) \pmod{\gamma_I} = 0, \\ \lambda^{(\gamma+\Delta_\gamma)}, & \text{if } \gamma \pmod{\gamma_I} = 0 \text{ and } (\gamma + \Delta_\gamma) \pmod{\gamma_I} \neq 0, \\ 0, & \text{if } \gamma \pmod{\gamma_I} = 0 \text{ and } (\gamma + \Delta_\gamma) \pmod{\gamma_I} = 0, \end{cases} \\ \forall \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \tag{44}$$

$$\beta_i^\gamma - \beta_j^\gamma + \theta_{ij}^\gamma \\ \geq \begin{cases} -\mu_{ij}^\gamma + \mu_{ij}^{(\gamma+\Delta_\gamma)} + \sigma_{ij}^{(\gamma+\gamma_E)}, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} \neq 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} \neq 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} = 0 \end{cases}, \\ -\mu_{ij}^\gamma + \mu_{ij}^{(\gamma+\Delta_\gamma)}, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} \neq 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} \neq 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} \neq 0 \end{cases}, \\ -\mu_{ij}^\gamma + \sigma_{ij}^{(\gamma+\gamma_E)}, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} \neq 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} = 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} = 0 \end{cases}, \\ -\mu_{ij}^\gamma, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} \neq 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} = 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} \neq 0 \end{cases}, \\ \mu_{ij}^{(\gamma+\Delta_\gamma)} + \sigma_{ij}^{(\gamma+\gamma_E)}, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} = 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} \neq 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} = 0 \end{cases}, \\ \mu_{ij}^{(\gamma+\Delta_\gamma)}, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} = 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} \neq 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} \neq 0 \end{cases}, \\ \sigma_{ij}^{(\gamma+\gamma_E)}, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} = 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} = 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} = 0 \end{cases}, \\ 0, & \text{if } \begin{cases} \gamma \pmod{\gamma_I} = 0 \\ (\gamma + \Delta_\gamma) \pmod{\gamma_I} = 0 \\ (\gamma + \gamma_E) \pmod{\gamma_E} \neq 0 \end{cases}, \end{cases} \\ \forall (i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \tag{45}$$

$$\sigma_{ij}^\gamma + \tau_{ij}^\gamma = \begin{cases} \sigma_{ij}^{(\gamma+\Delta_\gamma)}, & \text{if } (\gamma + \gamma_E) \pmod{\gamma_E} \neq 0, \\ 0, & \text{if } (\gamma + \gamma_E) \pmod{\gamma_E} = 0, \end{cases} \\ \forall (i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \tag{46}$$

$$\phi_{ij}^\gamma - \tau_{ij}^\gamma - g_{ij} \pi^\gamma \geq 0, \quad \forall (i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \tag{47}$$

$$\Psi_{ij}^\gamma = \theta_{ij}^\gamma + \tau_{ij}^\gamma, \quad \forall (i, j) \in A, \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \tag{48}$$

$$\pi^\gamma = -w_P, \quad \forall \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}, \tag{49}$$

$$\xi_{ij} = \sigma_{ij}^{\gamma E} + \mu_{ij}^{\Delta\gamma}, \quad \forall (i, j) \in A, \quad (50)$$

$$\zeta = \lambda^{\Delta\gamma}, \quad \forall (i, j) \in A, \quad (51)$$

$$\eta_{ij} = \sigma_{ij}^{\Delta\gamma}, \quad \forall (i, j) \in A, \quad (52)$$

$$(\theta_{ij}^{\gamma}, \phi_{ij}^{\gamma}, \tau_{ij}^{\gamma}, \Psi_{ij}^{\gamma}) \geq 0, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \in \{\Delta\gamma, 2\Delta\gamma, \dots, \Gamma\}, \quad (53)$$

constraints (3), (7)–(9), (13)–(27), (35)–(36),

$$\sum_{k \in \mathcal{K}} p_{ijk}^{\gamma} + p_{ij}^{0\gamma} = 1, \quad \forall (i, j) \in A, \gamma \in \{\Delta\gamma, 2\Delta\gamma, \dots, \Gamma\}, \quad (54)$$

$$p_{ij}^{0\gamma} \geq 0 \quad \text{and} \quad p_{ijk}^{\gamma} \geq 0, \quad \forall (i, j) \in A, k \in \mathcal{K}, \gamma \in \{\Delta\gamma, 2\Delta\gamma, \dots, \Gamma\}, \quad (55)$$

where the nonnegativity of the Ψ_{ij}^{γ} -variables are implied by the other restrictions in (48) and (53), but are retained in (53) for convenience in implementation.

Observe that Problem P is a nonlinear mixed-integer 0-1 program, where the only nonlinearity appears in the bilinear terms $p_{ij}^{0\gamma} \psi_{ij}^{\gamma}$ within the objective function (43). Because the direct application of the commercial solver BARON requires the scaling of the weights in (43) to overcome convergence difficulties when solving certain categories of DNIP instances, as will be discussed in the following section, we also propose the following alternating heuristic that addresses the bilinear $p_{ij}^{0\gamma} \Psi_{ij}^{\gamma}$ -terms in (43) via an iterative fixing technique, as in Cooper (1964).

Alternating Heuristic (AH)

Step 1. Given a heuristic relative improvement tolerance $\epsilon_2 > 0$ on the objective function value, compute a lower bound on $p_{ij}^{0\gamma}$ via (3), (9), (23), (54), and (55) as $\bar{p}_{ij}^{0\gamma} \equiv \max\{0, 1 - \sum_{k \in \mathcal{K}} \frac{R_{kk}}{c_{ijk}}\}$, $\forall (i, j) \in A$, and solve Problem P by replacing $p_{ij}^{0\gamma} \Psi_{ij}^{\gamma}$ with $\bar{p}_{ij}^{0\gamma} \Psi_{ij}^{\gamma}$, $\forall (i, j) \in A$, in (43) to attain a lower bounding solution. Let $(p^0, \Psi) = (\hat{p}^0, \hat{\Psi})$ in this resulting solution.

Step 2. Fix $\Psi \equiv \hat{\Psi}$ in P and solve the resulting problem to obtain an optimal revised solution vector \hat{p}^0 for p^0 , and let \hat{V}_1 be the corresponding optimal objective function value of (43).

Step 3. Fix $p^0 \equiv \hat{p}^0$ in P and solve the resulting problem to obtain an optimal revised solution vector $\hat{\Psi}$ for Ψ , and let \hat{V}_2 be the corresponding optimal objective function value of (43). If $|(\hat{V}_1 - \hat{V}_2)/\hat{V}_1| < \epsilon_2$, terminate the procedure and prescribe the resulting solution. Otherwise, proceed to Step 4.

Step 4. Continue alternating between the solutions of the restricted problems in Steps 2 and 3 until any solution fails to improve the objective function value in (43) by at least $100\epsilon_2\%$, whence terminate the procedure with the final solution obtained.

4. Illustration of Stability and Convergence Behavior

In this section, we consider two types of problem structures as embodied by two simple illustrative examples and examine the convergence of period-specific optimal strategies for three combinations of interdictor and evader reaction times or decision cycle lengths, $(\gamma^I, \gamma^E) \in \{(2, 2), (2, 3), (3, 2)\}$, and three time horizons, $\Gamma \in \{6, 12, 18\}$. We examine Problem P for two instances with network topology and parameters displayed in Figs. 1a and 1b, which have the following structural difference. Denote Z to be the number of distinct optimal solutions for deployment of the resources $R_k, \forall k \in \mathcal{K}$, in the problem of minimizing the maximum flow through the network for a static formulation. The instance in Fig. 1a has $Z = 1$, and for such a case, we expect that, given sufficient budgetary resources and time periods, and a relatively small ratio w_C/w_F , the distinct strategy that minimizes the maximum evader flow will be manifested as the stable equilibrium (i.e., under such an outcome, the interdictor would not choose to incur costs related to the redeployment of resources *for the purposes of inflicting penalties* upon the evader). On the other hand, for the instance displayed in Fig. 1b, the interdiction costs have been modified to ensure that alternative minimax flow solutions exist, thereby yielding $Z > 1$ and allowing for the possibility of the interdictor altering strategies between periods in order to inflict evader penalties, depending on the respective weights (w_C, w_P) and the related cost parameters.

Note that the value $\epsilon = 10^{-3}$ as specified in Fig. 1 is the parameter used for determining w_F via Proposition 1, whereas we employ a tighter relative optimality tolerance of 10^{-9} (i.e., $10^{-7}\%$) for solving Problem P via the commercial solver BARON Version 8.1.5 (Ryoo and Sahinidis, 1996) using the GAMS modeling language on a computer having an Intel Model T7100 Core 2 Duo Processor (dual core with a 1.8 GHz speed) and 2.0 GB of RAM. For our analysis, we assume that the interdictor begins with no available resources, and that the evader begins with the maximum uninterdicted flow through the network $(\hat{z}^0, \hat{x}_{ij}^0, \forall (i, j) \in A)$, along with a consistent perception thereof by way of setting $\hat{y}_{ij}^0 = \hat{x}_{ij}^0, \forall (i, j) \in A$. We employed BARON with a time limit of 360, 1800, and 3600 CPU seconds for the respective time horizons of $\Gamma \in \{6, 12, 18\}$.

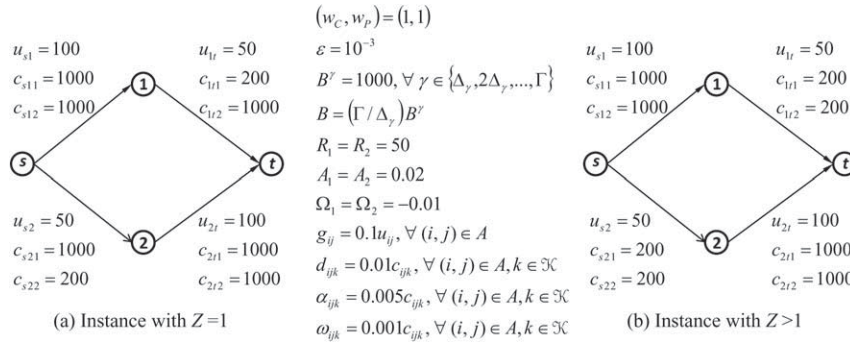


Fig. 1. Two structurally different DNIP instances.

The derived value for w_F varies for each instance in accordance with Proposition 1. For all three cases of $(\gamma^I, \gamma^E) \in \{(2, 2), (2, 3), (3, 2)\}$, we have $\Delta_\gamma = 1$, and the computation of w_F by Proposition 1 for both instances in Fig. 1, given their identical uninterdicted upper bounds on arc flows, $u_{ij}, \forall (i, j) \in A$, results in $w_F = \{1 + (1.5 \times 10^7), 1 + (3.0 \times 10^7), 1 + (4.5 \times 10^7)\}$ for the respective time horizons $\Gamma \in \{6, 12, 18\}$.

REMARK 2. In order to ensure the fidelity of the solver BARON, we scaled the objective function in (1) to yield $w_F = 10^6$ for each instance examined, with the parameters (w_C, w_P) linearly scaled accordingly. Without such scaling, the solver returned *null solutions*, wherein the interdictor did not procure or deploy any resources. Null solutions do represent a global optimal solution for *cost-prohibitive instances* of DNIP, wherein the procurement and/or deployment costs for resources exceed their interdiction benefit in (1); e.g., via empirical analysis, this occurs when $w_C > 10^8$ for the instances presented in Fig. 1 with $\Gamma = 6$. However, the parameters for the instances in Fig. 1 are not cost-prohibitive, and we further ensured the implausibility of null solutions (without scaling of the objective function) by setting $w_C \equiv 0$ and running both instance structures for the different $(\Gamma, \gamma^I, \gamma^E)$ -combinations. However, BARON still reported null solutions. Therefore, our proposed scaling of the objective function is necessary from the viewpoint of assuring BARON's fidelity, and we apply this scaling technique for all the runs reported in the remainder of this section. We further note that we did not apply a scaling method as for example that utilized by Kurilovas and Dagiene (2009), wherein the weights are linearly scaled such that their sum equals one, in order to prevent further fidelity issues that would result from $(w_P, w_C) \rightarrow (0, 0)$, as effected by our application of preemptive weighting within a nonpreemptive formulation.

Note that for $w_P \equiv 0$, Proposition 1 yields $w_F = 1$ as a legitimate choice for any $\epsilon > 0$. Accordingly, we tested the different $(\Gamma, \gamma^I, \gamma^E)$ -combinations with $(w_C, w_P, w_F) = (1, 0, 1)$ for both instances represented in Fig. 1, and verified that, in the absence of penalties inflicted upon the evader, the interdictor strategy indeed converges to a minimax net flow solution and does not exhibit oscillations, while minimizing costs associated with the redeployment of resources. This validates that the model reflects the intended interdictor and evader behaviors in the absence of evader penalties.

Examining the instance represented in Fig. 1a for the case where neither the overall nor period-specific budgets represent active constraints in an optimal solution, the results were consistent across all $(\Gamma, \gamma^I, \gamma^E)$ -combinations considered. In an optimal solution, the interdictor utilizes the maximum available amount of each resource type at the earliest opportunity (i.e., $R_k^{\gamma^+} = R_k$, for $\gamma = \gamma^I, \forall k \in \mathcal{K}$, and $R_k^{\gamma^+} = 0$ otherwise), and deploys them in a manner to minimize the maximum flow for the remaining periods, resulting in $p_{1t1}^\gamma = p_{s22}^\gamma = 0.25$ for $\gamma \in \{\gamma^I, \gamma^I + \Delta_\gamma, \gamma^I + 2\Delta_\gamma, \dots, \Gamma\}$, and $p_{ijk}^\gamma = 0$ otherwise. For several other instances of Problem DNIP having a distinct optimal solution to minimize the maximum flow, and without budget limitations, we observed a similar attainment of a stable strategic equilibrium in a single decision cycle.

Table 2
 Optimal p_{ijk}^γ -values for the instance of Fig. 1a with $(\Gamma, \gamma^I, \gamma^E) = (6, 3, 2)$ and $B^\gamma = 100, \forall \gamma \in \{\Delta_\gamma, \dots, \Gamma\}$

γ	p_{s11}^γ	p_{s21}^γ	p_{1t1}^γ	p_{2t1}^γ	p_{s12}^γ	p_{s22}^γ	p_{1t2}^γ	p_{2t2}^γ
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0.2357	0	0	0.2497	0	0
4	0	0	0.2357	0	0	0.2497	0	0
5	0	0	0.2357	0	0	0.2497	0	0
6	0	0	0.25	0	0	0.25	0	0

In contrast to the foregoing single-cycle convergence to an equilibrium solution, the outcome was quite different for the case of active budget constraints at optimality in the instance represented in Fig. 1a (enforced by setting $B^\gamma = 100, \forall \gamma \in \{\Delta_\gamma, \dots, \Gamma\}$, where a single-period budget equals the total cost to procure all available resources). For a particular representative instance having $(\Gamma, \gamma^I, \gamma^E) = (6, 3, 2)$, Table 2 presents the optimal p_{ijk}^γ -values, with dashed lines demarcating the interdictor’s decision cycles (with implied initial values of $p_{ijk}^0 = 0, \forall (i, j) \in A, k \in \mathcal{K}$). These results exhibit two noteworthy characteristics. First, convergence to the same equilibrium strategies as before is still attained, albeit at a later time period ($\gamma = 6$). Second, the interdictor procures less than the maximum available resources in period $\gamma^I = 3$, and then given these resources, allocates them in that same period to minimize the maximum evader flow. Within the context of the regret function (1) or (43), this is preferable to procuring all the resources $R_k, \forall k \in \mathcal{K}$, during period $\gamma^I = 3$, but then not having any remaining capital to deploy any of these available resources until the next decision cycle, i.e., when $\gamma = 2\gamma^I = 6$.

Next, consider the instance in Fig. 1b. Using a relative optimality tolerance of $10^{-7}\%$ as before, the solver BARON attained a global optimal solution for six of the nine $(\Gamma, \gamma^I, \gamma^E)$ -combinations within the specified time limit. However, for the instances with $(\Gamma, \gamma^I, \gamma^E) \in \{(6, 3, 2), (12, 2, 3), (12, 3, 2)\}$, the solver terminated due to the imposed computational time limit, but it did attain relative optimality gaps of $\{0.00017\%, 0.64\%, 0.47\%\}$, respectively. Upon extending the computational time limit to 18,000 CPU seconds (i.e., 5 CPU hours) for these instances, an optimal solution was only attained for $(\Gamma, \gamma^I, \gamma^E) = (12, 2, 3)$ after 9552 CPU seconds, but no improvement resulted in the objective function value for the other two instances. (We mention here that when we replaced $p_{ij}^{0\gamma}$ in (43) with $(1 - \sum_{k \in \mathcal{K}} p_{ijk}^\gamma), \forall (i, j) \in A$, before utilizing the solver BARON to optimize Problem P, the resulting formulation prevented BARON from converging to an optimal solution for any of the $(\Gamma, \gamma^I, \gamma^E)$ -combination instances within 18,000 CPU seconds of computational effort.)

Table 3
 Reported optimal (expected) non-zero p_{ijk}^γ -values for the instance of Fig. 1b with
 $(\gamma^I, \gamma^E) \in \{(2, 2), (2, 3), (3, 2)\}$, given $\Gamma = 12$, $(w_C, w_P, w_F) = (0.\bar{3}, 0.\bar{3}, 10^6)$,
 and $\epsilon = 0.001$, terminated at 18,000 CPU seconds (i.e., 5 CPU hours) of effort

γ	$(\gamma^I, \gamma^E) = (2, 2)$				$(\gamma^I, \gamma^E) = (2, 3)$				$(\gamma^I, \gamma^E) = (3, 2)$			
	p_{s21}^γ	p_{1t1}^γ	p_{s22}^γ	p_{1t2}^γ	p_{s21}^γ	p_{1t1}^γ	p_{s22}^γ	p_{1t2}^γ	p_{s21}^γ	p_{1t1}^γ	p_{s22}^γ	p_{1t2}^γ
1	0	0	0	0.25	0	0	0	0	0	0	0	0
2	0.25	0	0	0.25	0.25	0	0.2306	0.0194	0	0	0	0
3	0.25	0	0	0.25	0.25	0	0.2306	0.0194	0.2475	0.0025	0	0.25
4	0.25	0	0	0.25	0.0841	0.1659	0	0.25	0.2475	0.0025	0	0.25
5	0.25	0	0	0.25	0.0841	0.1659	0	0.25	0.2475	0.0025	0	0.25
6	0.25	0	0	0.25	0.0841	0.1659	0.1402	0.1098	0.2052	0.0448	0	0.25
7	0.25	0	0	0.25	0.0841	0.1659	0.1402	0.1098	0.2052	0.0448	0	0.25
8	0.25	0	0	0.25	0.1146	0.1354	0.25	0	0.2052	0.0448	0	0.25
9	0.25	0	0	0.25	0.1146	0.1354	0.25	0	0.2360	0.0140	0	0.25
10	0.25	0	0	0.25	0.1061	0.1439	0.0256	0.2244	0.2360	0.0140	0	0.25
11	0.25	0	0	0.25	0.1061	0.1439	0.0256	0.2244	0.2360	0.0140	0	0.25
12	0.25	0	0	0.25	0.1061	0.1439	0.25	0	0.2360	0.0140	0.0621	0.1879

Table 3 presents the non-zero (and expected non-zero) p_{ijk}^γ -values in the reported optimal solutions for $(\gamma^I, \gamma^E) = \{(2, 2), (3, 2)\}$ as well as the incumbent solution at termination for $(\gamma^I, \gamma^E) = (3, 2)$, for the particular case of $\Gamma = 12$. As before, the dashed lines demarcate the interdicator's decision cycles (with implied initial values of $p_{ijk}^0 = 0, \forall (i, j) \in A, k \in \mathcal{K}$).

The optimal and near-optimal solutions produced in Table 3 exhibit that, as in the case of the instance of Fig. 1a, the interdicator utilizes the maximum available amount of each resource type at the earliest opportunity and deploys them in a manner to minimize the maximum flow for the remaining periods. Furthermore, as evident from Table 3, we observed oscillations in strategies for some $(\Gamma, \gamma^I, \gamma^E)$ -combinations, although not necessarily at a period within $\{2\gamma^I, 3\gamma^I, \dots, \Gamma\}$. Note that, given the prescribed interdicator strategy, the evader minimizes penalties among the resulting alternative maximal flow solutions, as preemptively enforced by Proposition 1. The same interdicator pattern of resource procurement and deployment, as well as similar patterns of oscillatory behavior, were obtained for $\Gamma \in \{6, 18\}$ (not shown here). Also, similar to the differing nature of the oscillations among the (γ^I, γ^E) -combinations in Table 3, we observed no identifiable trend over the (γ^I, γ^E) -combinations in the solutions obtained for the time horizons of $\Gamma \in \{6, 18\}$.

Thus, we find that, given moderate time horizons, when the objective component weights of DNIP are scaled as per Remark 2, optimal solutions are obtained for instances having unique static minimax net flow interdicator strategies, and the results turn out to be either optimal or near-optimal for instances having alternative optimal static minimax net flow interdicator strategies.

5. Computational Performance with Larger-Sized Networks

In this section, we consider larger-sized networks for Problem P, and we compare the performance of direct optimization via the commercial solver BARON (Version 8.1.5) with the alternating heuristic AH prescribed in Section 3. Both procedures were coded using the GAMS modeling language while invoking the solver CPLEX (Version 11.1) to optimize the corresponding mixed-integer programs.

As a benchmark, we first tested the performance of Heuristic AH on the two network topologies displayed in Fig. 1, with $\Gamma = 12$ and over each of the relative decision cycle lengths, $(\gamma^I, \gamma^E) \in \{(2, 2), (2, 3), (3, 2)\}$. For each of these six instances, Heuristic AH terminated with an objective function value within 0.00% of that attained by the solver BARON, and with a reduction in the average required computational effort from 605.32 CPU seconds to 0.33 CPU seconds or, considering only the four instances for which BARON did not terminate due to the time limit of 1800 CPU seconds, a reduction in the average required computational effort from 7.98 CPU seconds to 0.32 CPU seconds.

We next tested both solution procedures on a set of directed grid networks (as described in Israeli and Wood, 2002) of different sizes with randomly generated arc attributes, where, given a source node, s , and a terminus node, t , there exist $m \times n$ transshipment nodes arranged in a grid of m rows and n columns. A non-interdictable arc exists from s to each transshipment node in the first column, and from each transshipment node in the last column to t . Furthermore, an interdictable arc exists from each node in row r and column c , i.e., in grid position (r, c) , to the nodes in positions $(r + 1, c)$, $(r - 1, c)$, $(r, c + 1)$, $(r + 1, c + 1)$, and $(r - 1, c + 1)$, provided that a node exists in the particular position, with the exception that there are no vertical arcs in the first or last columns.

Given such a transshipment network, the test instances were generated as follows, without specific regard to whether alternative minimax flow solutions exist. The interdictable arc capacities u_{ij} , $\forall (i, j) \in A$, were independently generated via a discrete uniform distribution on the interval $[30, 50]$. We again considered instances involving two-resources ($K = 2$), and we set $c_{ijk} = Q_{ijk}u_{ij}$, $\forall (i, j) \in A$, $k \in \mathcal{K}$, with Q_{ijk} being randomly generated via a discrete uniform distribution on the interval $[4, 20]$. We set $R_k = \lfloor 2\Phi_{\max} \rfloor$, $\forall k \in \mathcal{K}$, where Φ_{\max} is the maximal uninterdicted flow between s and t . We ensured that budgetary constraints would not be tight for an optimal solution by setting $B = B^\gamma = 3000R_1(\Gamma/\Delta_\gamma)$, $\forall \gamma \in \{\Delta_\gamma, 2\Delta_\gamma, \dots, \Gamma\}$. Furthermore, we generated the following parameters in the same manner as indicated for the instances in Fig. 1: (w_C, w_P) , ϵ , A_k , and Ω_k , $\forall k \in \mathcal{K}$, g_{ij} , $\forall (i, j) \in A$, and d_{ijk} , α_{ijk} , and ω_{ijk} , $\forall (i, j) \in A$, $k \in \mathcal{K}$.

Restricting our attention to $\Gamma = 12$, we tested the instances represented in Table 4 for $(\gamma^I, \gamma^E) \in \{(2, 2), (2, 3), (3, 2)\}$. We conducted tests on a computer having an Intel Model T7100 Core 2 Duo Processor and 2.0 GB of RAM, utilizing a relative optimality tolerance of 10^{-3} for BARON, a relative heuristic improvement tolerance of $\epsilon_2 = 10^{-3}$, and a time limit of 1800 CPU seconds (checked at the completion of any Step in Heuristic AH). For each of the larger-sized networks examined in Table 4, the solver BARON terminated after 1800 CPU seconds of preprocessing effort with a feasible solution but

Table 4
Performance of heuristic AH on transshipment networks with an $m \times n$ grid of nodes and with $\Gamma = 12$: relative improvement(%) over BARON and CPU time

(m, n)	$(\gamma^I, \gamma^E) = (2, 2)$		$(\gamma^I, \gamma^E) = (2, 3)$		$(\gamma^I, \gamma^E) = (3, 2)$	
	Imp.	Time	Imp.	Time	Imp.	Time
(5, 5)	69.52	1.434	69.46	1.246	67.98	0.998
(5, 10)	69.68	1.761	71.15	1.810	69.52	1.779
(10, 5)	71.33	1.543	71.72	1.672	66.77	1.214
(10, 10)	72.48	8.010	65.91	8.521	62.10	5.553

with a lower bound of 0. For Heuristic AH, Table 4 reports the relative % improvement of the incumbent objective function value from that attained by BARON, as well as the computational effort (in CPU seconds) utilized by the heuristic.

This experimentation demonstrates the superior efficacy of the heuristic AH over BARON for large-sized instances of Problem P, for which it achieved a solution having an objective function value that was, on average, 68.97% lower than the value reported by BARON upon its premature termination at 1800 CPU seconds, and while requiring an average of 2.962 CPU seconds of computational effort.

6. Conclusions and Recommendations

In this paper, we have proposed and formulated a novel multi-objective dynamic network interdiction problem, and have developed a solution procedure for which the commercial software BARON (Version 8.1.5) attains optimal or, for moderate time horizons, near-optimal solutions, depending on the presence of alternative optimal static strategies that minimize the maximum evader flow. We have investigated certain stability and oscillatory issues predicated on the latter structural property using two representative problem instances, and we have also proposed an alternating heuristic procedure and demonstrated its efficacy relative to BARON on larger-sized instances.

For future study, we propose that our model be modified to account for interdicator costs using goal programming. For government bureaucratic agencies that act as an interdicator, budgets are often soft constraints. Agencies do underspend and overspend their budgets, both in fiscal quarters and fiscal years. They are penalized for overspending (hopefully), but there is also an associated penalty for underspending a budget, as funds may be diverted to other competing budget-deficit agencies. Conceptually, a combination of these factors implies the possibility that overspending may incur a net *gain* up to a certain threshold, and a *penalty* beyond it. Therefore, an extended model might consider a fourth component in the weighted objective function: a penalty cost for variations in expenditures from authorized budget levels. Typically, this would be weighted much lower than the terms pertaining to the interdiction being performed, but this would depend on the severity of the interdiction scenario and the overall fiscal state of the interdicator's

budgetary resources. Furthermore, we suggest the expansion of the model formulation to account for arc-wise superadditive synergy between resources, whether in linear, convex, or general nonlinear forms, as examined by Lunday and Sherali (2009b). Finally, we propose the development of customized algorithms to solve the dynamic network interdiction problem, wherein a suitable relaxation is designed and embedded within a branch-and-bound framework.

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Dinaminis tinklų saugos modelis

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Pateikiamas naujas dinaminis tinklų saugos modelis aprašantis tarpusavio sąveiką tarp tinklų saugos agento paskirstančio savo resursus grafo lankams ir pažeidėjo agento bandančio kirsti tinklą nuo pradinio iki galinio taško. Agentai gali periodiškai keisti savo strategijas priklausomai nuo sprendimų priimtų ir panaudotų ankstesniuose cikluose. Kuriant galutinę minimakso tipo modelį buvo panaudotas naujas formulavimas palengvinantis tiesioginio sprendimo procedūrą naudojant komercinę programinę įrangą arba siūlomą alternatyvią euristiką. Nagrinėjami stabilumo ir konvergavimo klausimai bei specialūs pavyzdžiai tikslu parodyti skaičiavimo galimybes.