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# PROPERTIES OF LINEAR CONTINUOUS DYNAMICAL SYSTEM TRANSFER FUNCTION PARAMETERS ESTIMATES OBTAINED BY THE METHOD OF AUXILIARY VARIABLES

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**Abstract.** The present paper considers a constant-parameter estimation algorithm for an analog transfer function by discrete observations of the object's input and output variables. The algorithm is based on supplementary variables and least-squares methods. It is assumed, that the order of the transfer function is known and the derivatives of the input and output variables are non-measurable. The supplementary variables and their derivatives are constructed from discrete observations of the input and output variables by applying a numerically realized analog filter. Investigation results for the estimate properties are presented. The results were obtained by the method of statistical simulation.

**Key words:** transfer function, identification, parameter estimation, method of supplementary variables, least-square method, method of numerical simulation.

**1. Introduction.** Solution of control tasks for genuine single-measured linear continuous dynamical objects requires a sufficiently exact model for each case. Object model is to be constructed in the form of analog transfer function $H(s)$, where $s$ is a complex variable of the Laplas transform. Coefficient estimates for the transfer function $H(s)$ are obtained from experimental data by applying different identification methods. However it is hard obtain adequate estimates for an analog transfer function, as the derivatives of the input and output variables cannot be measured directly, while their estimations obtained by means of numerical methods contain rather large errors. Eykhoff (1975), Middleton and Goodwin (1990) state that this trouble can be overcome by applying the method of auxiliary variables.

Johansson (1990) suggested an interesting method for the construction of auxiliary variables for the purpose of coefficient estimation either for a nonlinear differential equation or for a system of such equations. Full mathematical justification for this method is missing, neverthemore its efficiency can be illustrated by means of characteristic examples.

Until now, there have been no publications on the problem of parameter selection for the filter, applied for the construction of auxiliary variables and their derivatives, as well as publications on the properties of estimates, obtained by applying the method of auxiliary variables. This fact labours the implementation of the above mentioned method in practice.

The aim of this article is to investigate the properties of coefficient estimates, obtained by applying the method of auxiliary variables for an analog transfer function by discrete input and output observances. A certain analogous filter is applied for the construction of auxiliary variables and their derivatives. The filter consist of $m$ sequentially connected equal aperiodical units, where $m$ is the order of the continuous dynamical object that is being identified.

**2. Identification algorithm and the investigation task.** The object for the identification is a linear continuous dynamical system, defined by a differential equation

$$y^{(m)}(t) + a_1 y^{(m-1)}(t) + a_2 y^{(m-2)}(t) + \ldots + a_m y(t)$$
$$= b_1 u^{(m-1)}(t) + b_2 u^{(m-2)}(t) + \ldots + b_m u(t) + \xi(t), \quad (1)$$

where $u(t)$ is the input variable, $a_i$, $b_i$ $(i = \overline{1,m})$ are constant parameters, $\xi(t)$ is normally distributed white noise. Then the analog transfer function of the system being identified is

$$H(s) = \frac{y(s)}{u(s)} = \frac{B(s)}{A(s)}$$
$$= \frac{b_1 s^{m-1} + b_2 s^{m-2} + \ldots + b_m}{s^m + a_1 s^{m-1} + a_2 s^{m-2} + \ldots + a_m}, \quad (2)$$

where $u(s)$ and $y(s)$ are the Laplas transform result of the variables $u(t)$ and $y(t)$ correspondingly, $B(s)$ and $A(s)$ are polynomials in the complex Laplace transform variable $s$.

The method of auxiliary variables is applied for obtaining the estimates of vector parameters $a^T = (a_1, \ldots, a_m)$, $b^T = (b_1, \ldots, b_m)$. This method assumes the transform of the variables $u(t)$, $y(t)$, by a filter with a transfer function of the $m$-th order, e.g.

$$F(s) = \frac{1}{(1+\tau s)^m} = \frac{1}{1 + \sum_{j=1}^{m} \alpha_j s^j}, \quad (3)$$

where $\tau$ is the time constant for the filter. The result of the above defined transforms is a pair of auxiliary variables

$$y_f(t) = F(s)y(t), \quad (4)$$
$$u_f(t) = F(s)u(t). \quad (5)$$

By inserting the definition of the transfer function (3) into the equations (4), (5) we get differential equations for the auxiliary variables:

$$y_f(t)+\alpha_1 y_f'(t)+\alpha_2 y_f''(t)+\ldots+\alpha_m y_f^{(m)}(t)=y(t), \quad (6)$$

$$u_f(t)+\alpha_1 u_f'(t)+\alpha_2 u_f''(t)+\ldots+\alpha_m u_f^{(m)}(t)=u(t). \quad (7)$$

Obtained differential equations can be rewritten as corresponding differential equation systems of the first order:

$$\left.\begin{array}{l} x_1'(t)=x_2(t),\\ x_2'(t)=x_3(t),\\ \ldots\ldots\ldots\ldots\ldots\ldots\\ \ldots\ldots\ldots\ldots\ldots\ldots\\ x_m'(t)=-\dfrac{\alpha_{m-1}}{\alpha_m}x_m(t)-\ldots-\dfrac{\alpha_1}{\alpha_m}x_2(t)\\ \qquad -\dfrac{1}{\alpha_m}x_1(t)+\dfrac{1}{\alpha_m}y(t), \end{array}\right\} \quad (8)$$

$$\left.\begin{array}{l} z_1'(t)=z_2(t),\\ z_2'(t)=z_3(t)\\ \ldots\ldots\ldots\ldots\ldots\ldots\\ \ldots\ldots\ldots\ldots\ldots\ldots\\ z_m'(t)=-\dfrac{\alpha_{m-1}}{\alpha_m}z_m(t)-\ldots-\dfrac{\alpha_1}{\alpha_m}z_2(t),\\ \qquad -\dfrac{1}{\alpha_m}z_1(t)+\dfrac{1}{\alpha_m}u(t), \end{array}\right\} \quad (9)$$

where $x_1(t)=y_f(t)$, $z_1(t)=u_f(t)$.

By solving the equation systems (8) and (9) we obtain corresponding values for all components of the state vectors $x^T(t)=(x_1(t),\ldots,x_m(t))$, $z^T(t)=(z_1(t),\ldots,z_m(t))$ and the derivatives $x_m'(t)=y_f^m(t)$, $z_m'(t)=u_f^m(t)$. It is obvious that $x_i(t)=y_f^{(i-1)}(t)$ and $z_i(t)=u_f^{(i-1)}(t)$.

The solution accuracy depends upon the efficiency of the solution method being applied. Corresponding values of the variables $y(t)$ and $u(t)$ are known only at discrete time moments $t_k = \Delta t \cdot k$ ($k = 0, 1, 2, \ldots$) so it is possible to apply the Runge-Kutta method for the solution in the systems (8), (9). The differential equation (1) can be rewritten as

$$A(s)y(t) = B(s)u(t) + \xi(t).$$

After multiplying both sides of that equation by $F(s)$ we get

$$A(s)F(s)y(t) = B(s)F(s)u(t) + F(s)\xi(t)$$

or

$$A(s)y_f(t) = B(s)u_f(t) + \xi_f(t), \tag{10}$$

where $\xi_f(t) = F(s)\xi(t)$.
Relation ship (10) can be rewritten in the expanded form

$$\begin{aligned} &y_f^m(t_k) + a_1 y_f^{(m-1)}(t_k) + a_2 y_f^{(m-2)}(t_k) + \ldots \\ &+ a_m y_f(t_k) = b_1 u_f^{(m-1)}(t_k) + b_2 u_f^{(m-2)}(t_k) + \ldots \\ &+ b_m u_f(t_k) + \xi_f(t_k). \end{aligned} \tag{11}$$

Equation (11) shows that by applying the method of linear regression analysis we can obtain estimates of the parameters $a_i$, $b_i$ $(i = \overline{1,m})$ of the differential equation (1) by the auxiliary variables $y_f(t_k)$, $u_f(t_k)$ and their derivatives. Thus, the equation (11) can be named as the identification equation.

It is natural to assume that the accuracy of the obtained estimates depends upon the standard deviation value $\sigma_\xi$ of the object noise $\xi(t_k)$, $t_k = k\Delta t$, $k = 0, 1, 2, \ldots$, on the time constant $\tau$ of the forming filter for the auxiliary variables and their derivatives, as well as on the fact which value is to be considered as the output of the regression identification model

(11). In accordance with the selection of the regression model output the following regression models can be obtained from the equation (11)

$$y_f^m(t_k) = -a_1 y_f^{(m-1)}(t_k) - a_2 y_f^{(m-2)}(t_k) - \dots$$
$$- a_m y_f(t_k) + b_1 u_f^{(m-1)}(t_k) + b_2 u_f^{(m-2)}(t_k) + \dots$$
$$+ b_m u_f(t_k) + \xi_f(t_k), \tag{12}$$

$$y_f^{(m-1)}(t_k) = \frac{1}{a_1}(-y_f^{(m)}(t_k) - a_2 y_f^{(m-2)}(t_k) - \dots$$
$$- a_m y_f(t_k) + b_1 u_f^{(m-1)}(t_k) + b_2 u_f^{(m-2)}(t_k) + \dots$$
$$+ b_m u_f(t_k) + \xi_f(t_k)), \tag{13}$$

$$\vdots$$

$$y_f(t_k) = \frac{1}{a_m}(-y_f^{(m)}(t_k) - a_1 y_f^{(m-1)}(t_k) - \dots$$
$$- a_{m-1} y_f'(t_k) + b_1 u_f^{(m-1)}(t_k) + b_2 u_f^{(m-2)}(t_k) + \dots$$
$$+ b_m u_f(t_k) + \xi_f(t_k)). \tag{14}$$

On the basis of the above presented equations the following investigation tasks can be formulated: to define the dependence of the standard deviation of the parameter estimates for the model (1) on the standard deviation $\sigma_\xi$ for the random noise $\xi(t_k)$, on the time constant $\tau$ for the filter and on the form of the identification model (12)–(14), by applying the method of statistical digital simulation.

**3. Technique and results of the digital simulation.** Digital simulation of the parameter estimation algorithms for the models (12) and (14) assumes identification of the transfer function of a linear continuous system

$$H(s) = \frac{K(T_2 s + 1)}{T_1^2 s^2 + 2\gamma T_1 s + 1}, \quad T_2 > T_1, \ \gamma = 0,5 \div 1 \tag{15}$$

It is necessary to estimate parameters $T_1$, $T_2$, $\gamma, K$ of the transfer function $H(s)$ by the observances of the transition processes. It is convenient for this purpose to rewrite the equation (15) in the form (2)

$$H(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2} = \frac{B(s)}{A(s)}, \tag{16}$$

where

$$a_1 = \frac{2\gamma}{T_1}, \ a_2 = \frac{1}{T_1^2}, \ b_1 = \frac{KT_2}{T_1^2}, \ b_2 = \frac{K}{T_1^2}. \tag{17}$$

For the calculation of auxiliary variables a filter of the second order is applied

$$F(s) = \frac{1}{(1+\tau s)^2} = \frac{1}{1 + 2\tau s + \tau^2 s^2}. \tag{18}$$

Equation for the filtering process are

$$y_f(t) + 2\tau y'_f(t) + \tau^2 y''_f(t) = y(t),$$
$$u_f(t) + 2\tau u'_f(t) + \tau^2 u''_f(t) = u(t),$$

and their transform in the form of Koshi equation systems are

$$\left.\begin{aligned} x'_1(t) &= x_2(t), \\ x'_2(t) &= -\frac{1}{\tau^2}x_1(t) - \frac{2}{\tau}x_2(t) + \frac{1}{\tau^2}y(t), \end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned} z'_1(t) &= z_2(t), \\ z'_2(t) &= -\frac{1}{\tau^2}z_1(t) - \frac{2}{\tau}z_2(t) + \frac{1}{\tau^2}u(t), \end{aligned}\right\} \tag{20}$$

where $x_1(t) = y_f(t)$, $z_1(t) = u_f(t)$. From (19), (20) using the observances $y(t_k)$, $u(t_k)$, in the transition processes and by applying the Runge-Kutta method, we obtain the auxiliary variables $y_f(t_k)$, $u_f(t_k)$ and their derivatives $y'_f(t_k)$, $y''_f(t_k)$, $u'_f(t_k), u''_f(t_k)$ $(t_k = k\cdot\Delta t,\ k = \overline{1,s})$.

With the help of the transfer function (16) we get the following differential equations

$$A(s)y_f(t_k) = B(s)u_f(t_k), \tag{21}$$

$$y''_f(t_k) + a_1 y'_f(t_k) + a_2 y_f(t_k) = b_1 u'_f(t_k) + b_2 u_f(t_k). \tag{22}$$

In accordance with the selection of the output variable for the regression model, we can consider three different regression models

$$y''_f(t_k) = -a_1 y'_f(t_k) - a_2 y_f(t_k) + b_1 y'_f(t_k) + b_2 u_f(t_k), \tag{23}$$

$$\begin{aligned} y'_f(t_k) = {} & \frac{1}{a_1}(-a_1 y''_f(t_k) - a_2 y_f(t_k) + b_1 u'_f(t_k) \\ & + b_2 u_f(t_k), \end{aligned} \tag{24}$$

$$\begin{aligned} y_f(t_k) &= \frac{1}{a_2}(-y''_f(t_k) - a_1 y'_f(t_k) + b_1 u'_f(t_k) + b_2 u_f(t_k)) \\ &= -\alpha_1 y''_f(t_k) - \alpha_2 y'_f(t_k) + \beta_1 u'_f(t_k) \\ &\quad + \beta_2 u_f(t_k), \end{aligned} \tag{25}$$

where $\alpha_1 = \frac{1}{a_2}$, $\alpha_2 = \frac{a_1}{a_2}$, $\beta_1 = \frac{b_1}{a_2}$, $\beta_2 = \frac{b_2}{a_2}$.

In case model (23) is used for identification purposes the estimates $\widehat{a}_1$, $\widehat{a}_2$, $\widehat{b}_1$, $\widehat{b}_2$ of the parameters $a_1$, $a_2$, $b_1$, $b_2$ are obtained in accordance with the least squares method by the observances $y_f(t_k)$, $y'_f(t_k)$, $y''_f(t_k)$, $u_f(t_k)$, $u'_f(t_k)$ $(k = \overline{1,s})$.

Parameter estimates for the analogous transfer function (15) can be obtained from the relationship (17), using the estimates $\widehat{a}_1$, $\widehat{a}_2$, $\widehat{b}_1$, $\widehat{b}_2$ :

$$\widehat{T}_1 = \frac{1}{\sqrt{\widehat{a}_2}}, \tag{27}$$

$$\widehat{T}_2 = \frac{\widehat{b}_1}{\widehat{b}_2}, \tag{28}$$

$$\widehat{K} = \frac{\widehat{b}_2}{\widehat{a}_2}, \tag{29}$$

$$\widehat{\gamma} = \frac{\widehat{a}_1}{2\sqrt{\widehat{a}_2}}. \tag{30}$$

Variances of the estimates $\widehat{T}_1$, $\widehat{T}_2$, $\widehat{K}$ and $\widehat{\gamma}$ can be obtained by the variance estimates for random values $\widehat{a}_1$, $\widehat{a}_2$, $\widehat{b}_1$, and $\widehat{b}_2$. For this purpose the functions (27)–(30) are expanded into the Taylor's series. Only the first two members in these series are used for defining the mathematical expectation and variance values for the estimates $\widehat{T}_1$, $\widehat{T}_2$, $\widehat{K}$ and $\widehat{\gamma}$. Also it is assumed that the estimates $\widehat{a}_1$, $\widehat{a}_2$, $\widehat{b}_1$, and $\widehat{b}_2$ are normally distributed. This leads us to the following relationships:

$$\widehat{\sigma}^2_{\widehat{T}_1} = \frac{\widehat{\sigma}^2_{\widehat{a}_2}}{4\widehat{a}_2^3}, \tag{31}$$

$$\widehat{\sigma}^2_{\widehat{T}_2} = \frac{\widehat{\sigma}^2_{\widehat{b}_1}}{\widehat{b}_2^2} + \frac{\widehat{b}_1^2\widehat{\sigma}^2_{\widehat{b}_2}}{\widehat{b}_2^4}, \tag{32}$$

$$\widehat{\sigma}^2_{\widehat{K}} = \frac{\widehat{\sigma}^2_{\widehat{b}_2}}{\widehat{a}_2^2} + \frac{\widehat{b}_2\widehat{\sigma}^2_{\widehat{a}_2}}{\widehat{a}_2^4}, \tag{33}$$

$$\widehat{\sigma}^2_{\widehat{\gamma}} = \frac{\widehat{\sigma}^2_{\widehat{a}_1}}{4\widehat{a}_2} + \frac{\widehat{a}_1^2\widehat{\sigma}^2_{\widehat{a}_2}}{16\widehat{a}_2^3}. \tag{34}$$

In case of applying model (25), the following relationships are obtained

$$\widehat{T}_1 = \sqrt{\widehat{\alpha}_1}, \quad \widehat{\sigma}^2_{\widehat{T}_1} = \frac{\widehat{\sigma}^2_{\widehat{\alpha}_1}}{4\widehat{\alpha}_1}, \tag{35}$$

$$\widehat{T}_2 = \frac{\widehat{\beta}_1}{\widehat{\beta}_2}, \quad \widehat{\sigma}^2_{\widehat{T}_2} = \frac{\widehat{\sigma}^2_{\widehat{\beta}_1}}{\widehat{\beta}_2^2} + \frac{\widehat{\beta}_1^2\widehat{\sigma}^2_{\widehat{\beta}_2}}{\widehat{\beta}_2^4}, \tag{36}$$

$$\widehat{K} = \widehat{\beta}_2, \quad \widehat{\sigma}^2_{\widehat{K}} = \widehat{\sigma}^2_{\widehat{\beta}_2} \tag{37}$$

$$\widehat{\gamma} = \frac{\widehat{\alpha}_2}{2\sqrt{\widehat{\alpha}_1}}, \quad \widehat{\sigma}^2_{\widehat{\gamma}} = \frac{\widehat{\sigma}^2_{\widehat{\alpha}_2}}{4\widehat{\alpha}_1} + \frac{\widehat{\alpha}_2^2\widehat{\sigma}^2_{\widehat{\alpha}_1}}{16\widehat{\alpha}_1^3}. \tag{38}$$

Digital simulation was performed for two transfer functions in the form (15)

$$H_1(s) = \frac{1,6s + 1}{s^2 + 2s + 1}, \tag{39}$$

where $T_1 = 1;\ T_2 = 1,6;\ K = 1;\ \gamma = 1$,

$$H_2(s) = \frac{17s + 10}{s^2 + s + 1}, \tag{40}$$

where $T_1 = 1;\ T_2 = 1,7;\ K = 10;\ \gamma = 0,5$.

Identification results are presented in the Tables 1–6. Table 1 presents the estimates $\widehat{T}_1$, $\widehat{T}_2$, $\widehat{K}$, $\widehat{\gamma}$ for the transfer function $H_1(s)$ in case the regression model (23) in applied, and Table 2 correspondingly presents these estimates in case the regression model (25) is used. Parameter estimates are obtained for different standard deviation values of the disturbance $\xi$ (observances error) in the system output. Time constant for the filter is $\tau = 1$. Similar results for the transfer function $H_2(s)$ are presented in the Table 3 and 4. The results illustrate the fact that the most precise estimates are obtained in the case when the second-order derivative $y''_f(t_k)$ is considered as the output of the regression model, i.e. when the regression model (23) is used. The obtained results also show, that estimate quality for the model (25) decreases with the standard deviation of the disturbances at the object's output increasing, while the corresponding estimates for the model (23) change insufficiently with increasing, and provides rather good results even for high disturbance level. Similar results were obtained with different values of the time constant $\tau$ for the filter $F(s)$.

**Table 1.** Parameter estimates of the transfer function $H_1(s)$ for different disturbance levels in case regression model (23) is applied (time constant for the filter is $\tau = 1$)

| $\sigma_\xi(\%)$ | $\hat{T}_1$ | $\sigma_{\hat{T}_1}$ | $\hat{T}_2$ | $\sigma_{\hat{T}_2}$ | $\hat{K}$ | $\sigma_{\hat{K}}$ | $\hat{\gamma}$ | $\sigma_{\hat{\gamma}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.9993 | $0.4787\cdot10^{-2}$ | 1.5971 | $0.1558\cdot10^{-1}$ | 1.0003 | $0.1354\cdot10^{-1}$ | 0.9994 | $0.5175\cdot10^{-2}$ |
| 0.5 | 0.9818 | $0.2771\cdot10^{-1}$ | 1.5417 | $0.8835\cdot10^{-1}$ | 1.0040 | $0.7996\cdot10^{-1}$ | 0.9899 | $0.3029\cdot10^{-1}$ |
| 1 | 0.9680 | $0.5811\cdot10^{-1}$ | 1.4977 | 0.1821 | 1.0075 | 0.1704 | 0.9818 | $0.6403\cdot10^{-1}$ |
| 2 | 0.9625 | 0.1124 | 1.4792 | 0.3482 | 1.0124 | 0.3324 | 0.9764 | 0.1238 |

**Table 2.** Parameter estimates of the transfer function $H_1(s)$ for different disturbance levels in case regression model (25) is applied (time constant for the filter is $\tau = 1$)

| $\sigma_\xi(\%)$ | $\hat{T}_1$ | $\sigma_{\hat{T}_1}$ | $\hat{T}_2$ | $\sigma_{\hat{T}_2}$ | $\hat{K}$ | $\sigma_{\hat{K}}$ | $\hat{\gamma}$ | $\sigma_{\hat{\gamma}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.9994 | $0.1061\cdot10^{-1}$ | 1.5974 | $0.3758\cdot10^{-1}$ | 1.0002 | $0.1341\cdot10^{-2}$ | 0.9994 | $0.2058\cdot10^{-1}$ |
| 0.5 | 0.8581 | $0.2701\cdot10^{-1}$ | 1.1451 | $0.8280\cdot10^{-1}$ | 1.0024 | $0.3406\cdot10^{-2}$ | 0.9135 | $0.5348\cdot10^{-1}$ |
| 1 | 0.6302 | $0.3757\cdot10^{-1}$ | 0.5530 | $0.8621\cdot10^{-1}$ | 1.0019 | $0.4695\cdot10^{-2}$ | 0.7995 | $0.7928\cdot10^{-1}$ |
| 2 | 0.3870 | $0.4543\cdot10^{-1}$ | 0.1151 | $0.6717\cdot10^{-1}$ | 1.0007 | $0.5510\cdot10^{-2}$ | 0.7709 | 0.1199 |

**Table 3.** Parameter estimates of the transfer function $H_2(s)$ for different disturbance levels in case regression model (23) is applied (time constant for the filter is $\tau = 1$)

| $\sigma_\xi$ | $\hat{T}_1$ | $\sigma_{\hat{T}_1}$ | $\hat{T}_2$ | $\sigma_{\hat{T}_2}$ | $\hat{K}$ | $\sigma_{\hat{K}}$ | $\hat{\gamma}$ | $\sigma_{\hat{\gamma}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.0261 | $0.1579\cdot10^{-3}$ | 1.6987 | $0.1209\cdot10^{-2}$ | 10.003 | $0.5413\cdot10^{-2}$ | 0.4877 | $0.2179\cdot10^{-3}$ |
| 1 | 1.0227 | $0.4644\cdot10^{-2}$ | 1.6913 | $0.3530\cdot10^{-1}$ | 10.074 | 0.1601 | 0.4867 | $0.6416\cdot10^{-2}$ |
| 5 | 1.0094 | $0.2227\cdot10^{-1}$ | 1.6356 | 0.1643 | 10.306 | 0.7810 | 0.4736 | $0.3082\cdot10^{-1}$ |
| 10 | 0.9921 | $0.4233\cdot10^{-1}$ | 1.5180 | 0.3007 | 10.489 | 1.5264 | 0.4397 | $0.5836\cdot10^{-1}$ |

**Table 4.** Parameter estimates of the transfer function $H_2(s)$ for different disturbance levels in case regression model (25) is applied (time constant for the filter is $\tau = 1$)

| $\sigma_\xi$ | $\hat{T}_1$ | $\sigma_{\hat{T}_1}$ | $\hat{T}_2$ | $\sigma_{\hat{T}_2}$ | $\hat{K}$ | $\sigma_{\hat{K}}$ | $\hat{\gamma}$ | $\sigma_{\hat{\gamma}}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.0261 | $0.1658\cdot10^{-3}$ | 1.6987 | $0.1151\cdot10^{-2}$ | 10.003 | $0.2530\cdot10^{-2}$ | 0.4877 | $0.2713\cdot10^{-3}$ |
| 1 | 1.0188 | $0.4626\cdot10^{-2}$ | 1.6783 | $0.3180\cdot10^{-1}$ | 10.092 | $0.7060\cdot10^{-1}$ | 0.4851 | $0.7595\cdot10^{-2}$ |
| 5 | 0.9273 | $0.2046\cdot10^{-1}$ | 1.3947 | 0.1295 | 10.635 | 0.3120 | 0.4411 | $0.3533\cdot10^{-1}$ |
| 10 | 0.76280 | $0.3255\cdot10^{-1}$ | 0.9693 | 0.1849 | 11.283 | 0.4981 | 0.3567 | $0.6310\cdot10^{-1}$ |

**Table 5.** Parameter estimates of the transfer function $H_1(s)$ for different time constant $\tau$ values for the filter $F(s)$. $\sigma_\xi = 0$. Model used is (23)

| $\tau$ | $\widehat{T}_1$ | $\widehat{T}_2$ | $\widehat{K}$ | $\widehat{\gamma}$ |
|---|---|---|---|---|
| 0.25 | 2.0685 | 5.9882 | 1.2084 | 1.7417 |
| 0.5 | 1.0470 | 1.5756 | 1.0029 | 1.0224 |
| 1 | 0.9993 | 1.5973 | 1.0002 | 0.9994 |
| 2 | 0.9733 | 1.5937 | 1.0001 | 1.0054 |
| 5 | 0.9628 | 1.6104 | 0.9994 | 1.0189 |
| 10 | 0.9622 | 1.6268 | 1.0001 | 1.0275 |

**Table 6.** Parameter estimates of the transfer function $H_2(s)$ for different time constant $\tau$ values for the filter $F(s)$. $\sigma_\xi = 0$. Model used is (23)

| $\tau$ | $\widehat{T}_1$ | $\widehat{T}_2$ | $\widehat{K}$ | $\widehat{\gamma}$ |
|---|---|---|---|---|
| 0.25 | 1.2144 | 1.6127 | 10.3616 | 0.8695 |
| 0.5 | 1.0811 | 1.6938 | 10.0372 | 0.5420 |
| 1 | 1.0261 | 1.6987 | 10.0027 | 0.4877 |
| 2 | 1.0003 | 1.6989 | 10.0002 | 0.4811 |
| 5 | 0.9854 | 1.6991 | 10.0000 | 0.4833 |
| 10 | 0.9805 | 1.6999 | 10.0000 | 0.4852 |

Table 5 presents the parameter estimates for the transfer function $H_1(s)$ in case of different time constant $\tau$ values. In all these experiments model (23) was used. Corresponding results for the transfer function $H_2(s)$ are presented in the Table 6. The results show that the best estimates are obtained when the time constant $\tau$ for the filter $F(s)$ is close to the time constant $T_2$ of the transfer function $H(s)$.

## 4. Conclusions

1. Digital algorithm for the construction of auxiliary variables and their derivatives by discrete input/output observances was designed on the basis of a $m$-th order analog filter and by applying a numerical method for the solution of a system of differential equations. It was assumed that the object under identification was known.

2. It was proved by digital simulation that in the process of identification of a second-order dynamical object by discrete observances in the transition process, the best parameter estimates for the transfer function are obtained for the regression model (23). The forming-filter time constant value in this case must be as close, as possible to the time constant value for the object under identification. In this case good identification results are obtained also for rather high measurement error levels for the output variable.

3. The designed algorithm can be applied for the parameter estimation of an analog transfer function of a continuous dynamical object not only by the observances in the transition process, but also by the input/output signal observances of a different kind, if the corresponding identifiability conditions are satistid.

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