INFORMATICA, 2008, Vol. 19, No. 2, 161–190

Interactive Aggregation/Disaggregation Dichotomic Sorting Procedure for Group Decision Analysis Based on the Threshold Model

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Received: March 2007

Abstract. In this paper, a new multi-criteria decision-making procedure is presented, which captures preferential information in the form of the threshold model. It is based on the ELECTRE-like sorting analysis restricted by the localization principle, which enables high adaptability of the decision model and reduces the cognitive load imposed on the decision-makers. It lays the foundation for the introduction of three concepts that have been previously insufficiently supported by outranking methods – semiautomatic derivation of criteria weights according to the selective effects of discordance and veto thresholds, convergent group consensus seeking, and autonomous multi-agent negotiation. The interdependent principles are justified, and the methodological solutions underlying their implementation are provided.

Keywords: decision-making, group decisions and negotiations, pseudo-criterion, outranking relation, preference aggregation/disaggregation, nonlinear optimization, fuzzy sets, agents.

1. Introduction

Multi-criteria decision analysis is a widely used methodology in science, engineering and management (Triantaphyllou and Baig, 2005). It refers to making preference decisions over the available alternatives that are characterized by multiple, usually conflicting criteria. Often, the decision-makers have to deal with insufficiently accurate and uncertain data (Huynh *et al.*, 2006; Peldschus and Zavadskas, 2005), or have to engage into group negotiations in order to reach a consensus or a compromise solution (Raiffa *et al.*, 2002).

There exist several approaches to multi-criteria decision analysis. Among the most popular are the utility theory (Keeney and Raiffa, 1976), the Analytic Hierarchy Process (Saaty, 1980; Triantaphyllou, 2000), the ideal solution based methods, such as TOPSIS (Zavadskas and Zakarevicius, 2006), qualitative or ordinal scale evaluations (Larichev, 2001; Moshkovich *et al.*, 2002; Petrovsky, 2001), and methods that capture preferential information in the form of threshold model (Roy, 1996). It has been proven that the concept of pseudo-criterion, which is founded on the indifference and preference thresholds, deals in an effective and practical way with imprecision, indetermination and uncertainty of available numerical data (Miettinen and Salminen, 1999). The treatment of pseudo-criteria results in the construction of outranking relations on pairs of alternatives, or between alternatives and reference profiles that delimit predefined categories/classes. In this way, four distinct situations – strong/weak preference, indifference and incomparability – are modelled obeying the axiom of limited comparability. Some evidence that it is unrealistic to assume comparability of options as a general case for a rational decision-maker has been provided by Rauschmayer (2001).

The principal outranking methods belong to the PROMETHEE (Brans and Vincke, 1985) and ELECTRE (Roy, 1991) families. Several extensions and applications of these methods have been presented in the literature (Azar *et al.*, 2001; Theil and Mroz, 2001). Yet, classical decision-making procedures, which elicit preferential information according to the nature of pseudo-criteria, have a disadvantage that they demand from the decision-makers to set a lot of parameters, putting a substantial cognitive load on them. Moreover, two additional drawbacks appear as a consequence of many required inputs: the quantitative model is insufficiently adaptable, and an inadequate insight into the derivation of results is given. To overcome these difficulties, several interactive methods have been introduced.

The aggregation/disaggregation approach (Jacquet–Lagrèze and Siskos, 2001) has been applied to the ELECTRE TRI method (Mousseau *et al.*, 2000) using outranking relations with the purpose of sorting. A mathematical programming problem is solved to infer preferential parameters from a set of assignment examples based on holistic judgments provided by the decision-maker (Mousseau and Slowinski, 1998). Because the simultaneous derivation of all parameter values requires solving a non-linear program with non-convex constraints, only a subset of parameters is inferred at a time, while maintaining remaining ones fixed (Ngo The and Mousseau, 2002). Dias *et al.* (2002) argue that inference programs should be considered as problems to be solved several times in an interactive learning process through which the decision-maker continuously revises his preferences as he obtains the results from the model. Their method integrates parameter inference with robustness and consistency analysis.

Miettinen and Salminen (1999) have proposed an ELECTRE III based search procedure in the criteria weight space. It finds the weight vector according to which a chosen alternative is ranked as the best one. Jaszkiewicz and Slowinski (1997) have considered choice problems with large alternative sets by describing three interactive exploration procedures. Interactive trichotomy segmentation (Jaszkiewicz and Ferhat, 1999) is founded on the localization principle. Preferences are modelled in the neighbourhood of a single reference profile in the nondominated set. The model accepts good alternatives, rejects uninteresting ones and defines alternatives that can neither be accepted nor rejected with regard to the available information.

The method, which is introduced in this paper, uses the localization principle in order to reduce the cognitive load imposed on the decision-makers, and to enable high adaptability of the decision model. Yet, contrary to the aforementioned pseudo-criterion based approaches, it does not focus solely on providing interaction, iterativeness and analytical capabilities, but strives to attain two further objectives:

- group consensus seeking that relies on the aggregation/disaggregation approach and can be easily applied to fully automated agent based negotiation, and
- semiautomatic derivation of criteria weights according to the selective strengths of veto and discordance thresholds.

The rest of the paper is organized as follows. In Section 2, the reasons for the introduction of three proposed principles – problem localization, weight derivation, and group consensus seeking – are discussed. The methodological solutions underlying the implementation of these interdependent concepts are explained in Sections 3 through 6, respectively. Finally, Section 7 concludes the paper by giving a resume and some directions for further work.

2. Justification of the Proposed Principles

2.1. *Dichotomic Sorting Based Problem Localization*

According to Jaszkiewicz and Ferhat (1999), specifying a single reference profile, which is a vector of desired values on criteria domains, and searching for alternatives better than this profile is one of the most natural ways to solve multi-criteria decision-making problems. This approach is considered as sorting, since it refers to the absolute assignment of a set of options into existing ordinal categories/classes (Zopounidis and Doumpos, 2002). While $m \cdot (m-1)/2$ pairwise comparisons between alternatives have to be made in the case of ranking analysis, sorting has the advantage that only m pieces of information about class memberships suffice. As the global problem of assigning alternatives to $p+1$ ordered categories is further reduced to the dichotomic partition of the solution set, several additional benefits appear:

- Providing a single referential value on each criterion domain and a single value of each threshold is cognitively much easier for the decision-maker than setting p-times more fixed inputs or defining functional thresholds, which are used by a global preference model.
- Because of mental and time constraints, the decision-maker is rarely capable of altering many reference vectors at once. It is therefore difficult for him to figure out how different profiles affect alternative evaluation. But when he concentrates on only one profile instead, it is relatively easy for him to modify referential values. By doing so, he can tighten or loosen demands and see what effect this has on selection. As a consequence, learning about the problem situation, the model, and advantages/weaknesses of alternatives is improved.
- As the dispersion of alternatives across classes is reduced, comparability of individuals' results is increased.
- The unification of opinions becomes an easy task, because fewer parameters are required.

2.2. *Group Consensus Seeking*

There exist many methods for group decision-making (Matsatsinis and Samaras, 2001), yet only a few capture preferences in the threshold form. Leyva–Lopez and Fernandez–Gonzalez (2003) give some criticism of the existing approaches. They argue that these techniques rest on a poor heuristic, which makes a decision about the consensus solution difficult to support. The PROMETHEE method (Brans *et al.*, 1997) cannot guarantee unanimity among group members because it uses a net flow weighted sum function and has a compensatory nature. It is, however, supplemented by the GAIA analysis, which can help in reaching a compromise. The idea of GAIA has been further developed by Espinasse *et al.* (1997). Their approach relies heavily on the guidance of a human moderator who has to interpret six types of GAIA planes. Although a set of interpretation rules is provided, this task is difficult and complex. Colson (2000) has introduced JUDGES, a system which compares rankings of team members. It uses four visual aids to expose disagreements. Jabeur *et al.* (2004) have defined a procedure to obtain a collective preorder from partial preorders of decision-makers, who can apply either the ELECTRE III or the PROMETHEE method. A version of ELECTRE TRI for groups exists as well (Dias and Clímaco, 2000). It focuses on finding the best and the worst class that an alternative may attain with regard to constraints on individually set imprecise parameters. Finally, several variations of Kendall's τ ensure a compromise by determining the correlation between rank-orders (Emond and Mason, 2002).

Most of these approaches have a disadvantage that a very complex discussion during group meetings is required. The problem solving process thus has to be very well structured, and a responsible, potentially overly demanding role has to be imposed on the human moderator, especially when the group is large or when there exist many contradicting scenarios. This weakness is additionally emphasized by the fact that the decision support system is incapable of advising participants on how to adjust inputs when considerable discrepancies between personal opinions emerge. Considering the abovementioned drawbacks, an active mechanism for iterative group consensus seeking is needed that would take into account the following demands: all preferential parameters are important in group decision-making, the level of consensus has to be known, the decision-making process has to be democratic, the activity of the human moderator should be reduced to the minimum, and the system should be able to tell the decision-maker how he can modify his input parameters so that they will correspond to the preferences of the whole group. It is crucial for a convergent negotiation procedure to adopt the aggregation/disaggregation paradigm (Bregar, 2005). According to evaluated alternatives that reflect the global preference structure of both the individual decision-maker as well as the complete group, it has to automatically adjust the parameters of the model so that it becomes capable of reproducing the agreed upon consensual solutions.

Recently, a powerful methodology that combines the principles of group decision-making, sorting, outranking and aggregation/disaggregation has been introduced (Demart *et al.*, 2006). It has been implemented with the IRIS system (Dias and Mousseau, 2003). It is efficient, but has a few limitations which are addressed by the interactive procedure proposed in this paper:

- it does not concentrate on other preference parameters than criteria weights;
- the determination of criteria weights is based on holistic assessments of alternatives, while the values of other preference parameters, such as veto thresholds, are not considered;
- a complicated and time consuming discussion of group members is required, which relies heavily on the human mediator and has to be structured as a Delphi process (Turoff and Hiltz, 1996) for which no outline has been provided;
- the mechanism aims primarily at directing the decision-makers, and hence does not enable fully automated agent based negotiation;
- the mechanism does not identify the decision-maker who has to conform to the other group members, leaving this judgement to the moderator;
- the robustness of the decision is not measured with regard to the allowed deviation/change of preferential parameters.

Majority of aggregation/disaggregation techniques for group decision-making are based on the multi-attribute utility theory. They apply the UTA/UTASTAR methods (Jacquet–Lagrèze and Siskos, 2001) to derive partial value functions of individual decision-makers. Afterwards, they aggregate utilities of alternatives with some averaging operator. Matsatsinis *et al.* (2005) have found that such representations of group preferences guarantee neither a consensus nor a good compromise because individual assessments may be considerably different. Therefore, they have incorporated several criteria to measure the decision-makers' satisfaction over the aggregated rank-order of alternatives. Since they have consequently increased the cognitive load, it is a necessity for any efficient new method not to average the individuals' evaluations.

2.3. *Semiautomatic Weight Derivation*

In the past, guidelines for specifying realistic values of the indifference, preference and veto thresholds have been introduced (Rogers and Bruen, 1998a). On the other hand, the assessment of criteria importance weights remains a hard and time consuming task. Because it is difficult to elicit weights, a couple of structured techniques have been defined, such as the Analytic Hierarchy Process (Saaty, 1980), Simos' procedure (Figueira and Roy, 2002) or the "resistance to change" grid (Rogers and Bruen, 1998b), that aim at reducing the cognitive load, and that help people make reasonable estimates. Most existing methods, however, have their origin in the multi-attribute utility theory. The direct rating method (von Winterfeldt and Edwards, 1986) requires the decision-maker to provide exact weights by considering relative preferences for swings from the worst to the best options on each attribute, or by comparing attributes to the least significant one. This method has been advanced by several approaches that allow for the specification of linear constraints on weight intervals in order to deal with imprecise judgments and incomplete information (Eum *et al.*, 2001; Mateos *et al.*, 2003). By applying the trade-off procedure (Keeney and Raiffa, 1976), the decision-maker determines probabilities for which he is indifferent between gambles and sure consequences. Salo and Hämäläinen (2001) have proposed the PRIME method combining the trade-off approach with AHP-like decomposed ratio judgments. Several techniques that translate criteria importance ranks into weight approximations have also been introduced (Roberts and Goodwin, 2002). In addition, optimization programs that derive weights on the ground of evaluated alternatives have been defined (Miettinen and Salminen, 1999).

These approaches are, however, incapable of automatic weight determination according to a given problem situation, which is reflected through values of other input parameters. The presented research bridges this gap. A mechanism is proposed that obtains criteria weights by considering the selective influence of veto and discordance thresholds.

Although a relationship between the veto threshold v_j and the criterion weight w_j has been discussed in the past (Rogers and Bruen, 1998a), it has neither been mathematically founded nor practically applied. The v_j threshold characterises the conditions under which a criterion can prevent an outranking relation. It conveys the idea that outranking of b by a is vetoed when a performs much worse than b for any criterion. If the difference in value between a and b exceeds the preference threshold p_j, or the discordance threshold u_j (Mousseau and Dias, 2003), respectively, the discordance rises above zero. Similarly, the magnitude of criterion valuation difference at which the outranking relation is with certainty opposed is represented by the v_j threshold. This has the effect of neutralising the mechanism of veto for the criteria of lesser importance while making it an essential factor for the most significant ones. The nearer v_j is to p_j, the lower the criterion valuation difference is at which the veto is imposed, and the more the v_j threshold affects the overall outranking of one option over another. In this way, the correlation between decreasing values of $v_j - p_j$ and increasing values of w_j is implied, and it can be argued that the veto threshold is connected to the importance rating of a criterion.

In the case when available alternatives are sorted into $p + 1 > 2$ categories, the intercriterion veto influence is locally limited to two adjacent categories. This means that various settings of the veto threshold cause the reassignment of an alternative to at most the adjoining category. Results of the alternative sorting analysis are therefore largely dependent on referential values of p profiles, and not so much on the v_j thresholds' magnitudes. For this reason, the introduced weight derivation mechanism is restricted by the localization principle.

The proposed procedure to specify the criteria weights relies upon the assumption that the importance of a criterion is closely related to its ability to exclude cases from the category of good alternatives. The discordance test is thus employed as the basis for the weight derivation procedure. It is essential, though, that the complementary concordance test should not have any influence on the importance of a criterion. This test is founded on the indifference and preference thresholds, which provide intracriterion preferential information. Their effect has to be compensated in order to deal with imprecision, indetermination and uncertainty of data. Because, by definition, the weights are introduced specifically to determine the compensation rates, it is their task to restrain the effect of the indifference and preference thresholds, and not the opposite.

3. Dichotomic Alternative Sorting

In order to implement the localized alternative sorting analysis, a slightly modified version of the ELECTRE TRI method is used. The set of alternatives is partitioned into two exclusive categories – C^+ and C^-. All acceptable choices belong to the positive category C^+, while unsatisfactory ones are members of the negative category C^-. The categories are delimited by the profile b, which is defined as a vector of n referential values on criteria domains. Let this vector be denoted as $(g_1(b), g_2(b), \ldots, g_n(b))$. Let similarly $g_j(a_i)$ denote the value of an alternative $a_i \in A$ that is measured with regard to a criterion $x_j \in X$. The assignment of each alternative to either the positive or the negative class then results from comparisons of values $g_j(a_i)$ with values $g_j(b)$, where $j = 1, \ldots, n$. Because numerical evaluations are subject to imprecision, indetermination and uncertainty, and because people are unable to perceive small differences in data, it is essential that an alternative does not have to outperform the profile on all criteria to be sorted into the positive class C^+. Weaknesses on some criteria are therefore admissible and can be compensated with advantages on other criteria. Two parameters allow for compensation – the indifference threshold q_j and the preference threshold p_j. On the basis of these thresholds, two valued outranking relations are built for each criterion expressing the degree to which the alternative a_i outperforms the profile b (b outperforms a_i, respectively):

$$s_j(a_i, b) = \max\Big(\min\big((g_j(a_i) - g_j(b) - q_j)/(p_j - q_j), 1\big), 0\Big),$$
$$s_j(b, a_i) = \max\Big(\min\big((g_j(b) - g_j(a_i) - q_j)/(p_j - q_j), 1\big), 0\Big).$$

In real-life problems, alternatives having poor values are not taken into consideration. This means that certain criterion weaknesses are not accepted to be compensated by good values on some other criteria. To model partial incompensation, the discordance concept is applied. It is based on the veto threshold v_j and the discordance threshold u_j, where $v_j \geqslant u_j \geqslant p_j \geqslant q_j$. The latter is introduced because it is not necessary that the point at which preference becomes obvious coincides with the lower boundary of intolerance, as is the case with other methods of the ELECTRE type. These methods are compelled to avoid weak veto effects by taking into account only a subset of discordance indices exceeding the overall level of concordance. The influence of veto is thereby implicitly dependent on the concordance concept, and cannot be directly and distinctly controlled by the decision-maker.

The threshold model may lead to the incomparability relation which occurs when there exist at least two conflicting criteria. In this case, neither the alternative a_i is treated to be at least as good as the profile b nor the opposite. Since the profile b represents the delimitation of the categories C^+ and C^-, it cannot be clearly stated whether the alternative should be assigned to C^+ or to C^-. Consequently, the membership of a_i is undetermined. A solution to this problem could be the introduction of the incomparability category. The approach gives an adequate insight into the characteristics of alternatives and thus enables high adaptiveness of preferences. However, an additional class hinders the comparison and the unification of group members' choices. Besides, the incomparability category has to be an empty subset at the end of the performed analysis as it is meant to show alternatives that are neither acceptable nor unsatisfactory at a certain point in time. The decision-maker still hesitates over the status of these alternatives, but they eventually have to be unambiguously sorted.

The localization principle thus calls for the prevention of the incomparability relation. So, the u_j and v_j thresholds are treated asymmetrically. This is justified by the noncompensatory nature of the veto concept and originates from the explicitly regarded primary viewpoint of the logical evaluation of the truthfulness of the alternative assignment to the positive category. This fixed point of view implicitly determines the complementary logical evaluation, which confirms or rejects the truthfulness of the assignment to the negative category. The positive semantics is mathematically denoted as: $a_i \in C^+ \Rightarrow a_i \notin C^-,\ a_i \notin C^+ \Rightarrow a_i \in C^-$.

It is only important whether an alternative is good enough to be assigned to C^+ and not whether it is convenient for C^-. This is utterly reasonable, since alternatives belonging to C^+ are solely chosen for further analysis or for implementation. In practice, asymmetry means that an alternative a_i with very poor values on some criteria is excluded from C^+. It is not important though, if the profile b does not reach one or more veto thresholds when compared with a_i, because this information does not confirm that a_i is a member of the C^+ class nor does it prevent the assignment of a_i to the C^- class. Yet small weaknesses of an alternative should be compensated. Hence, the indifference and preference thresholds are treated symmetrically, thereby leading to the symmetrically-asymmetrical interpretation of preferential information.

To express the degree of concordance with the assertion "the alternative a_i belongs to the C^+ category", the indices $s_j(a_i, b)$ and $s_j(b, a_i)$ are aggregated:

$$c_j(a_i) = \frac{1}{2} \cdot \Big(s_j(a_i, b) + \big(1 - s_j(b, a_i)\big)\Big).$$

It is assured that $c_j(a_i)$ is a fuzzy averaging operator (Zimmermann, 1996) because all of its operands are defined on the [0, 1] interval and because the following inequality holds:

$$\min\big(s_j(a_i, b), 1 - s_j(b, a_i)\big) \leqslant c_j(a_i) \leqslant \max\big(s_j(a_i, b), 1 - s_j(b, a_i)\big).$$

For the sake of compensation of small weaknesses, the indices $c_j(a_i)$ are combined so that each is scaled by the weighting coefficient w_j which represents the voting power of the jth criterion and determines its contribution to the aggregation:

$$c(a_i) = \sum_{j=1..n} w_j \cdot c_j(a_i), \quad \text{where} \sum_{j=1..n} w_j = 1.$$

In contrast to the ELECTRE type methods and based on the proposed semantics, instead of two discordance indices, only one such index is defined for each criterion:

$$d_j(a_i) = \max\Big(\min\big((g_j(b) - g_j(a_i) - u_j)/(v_j - u_j), 1\big), 0\Big).$$

The indices $d_j(a_i)$ express the degree of discordance with the assertion "the alternative a_i belongs to the C^+ category". The overall non-discordance relation is grounded in two ways:

$$\tilde{d}'(a_i) = \prod_{j=1..n} \big(1 - d_j(a_i)\big),$$
$$\tilde{d}''(a_i) = 1 - d(a_i), \quad \text{where } d(a_i) = \max_{j=1..n} d_j(a_i).$$

The first non-discordance index considers the veto effects of all criteria, while the latter limits itself to a single discordance degree computed with the fuzzy union operator. The interpretation is that an alternative cannot be excluded from the positive class with greater certainty than it is excluded according to a criterion on which its performance is the poorest.

The higher the value of the jth discordance index $d_j(a_i)$ is, the weaker a_i is according to x_j, and the more its overall credibility decreases. In the case of nonadmissible deficiencies, the non-discordance index has to be low enough to exclude a_i from the C^+ category. The valued outranking relation is therefore obtained as a result of the multiplication:

$$\sigma(a_i) = c(a_i) \cdot \tilde{d}(a_i), \text{ so that } \tilde{d}(a_i) = \tilde{d}'(a_i) \text{ or } \tilde{d}(a_i) = \tilde{d}''(a_i).$$

As $\sigma(a_i) = 0.5$ denotes strict equality among the alternative and the profile, the classical λ-cut may be used to determine the "crisp" membership of the alternative:

$$a_i \in C^+ \Leftrightarrow \sigma(a_i) \geqslant \lambda, \text{ where } \lambda \in [0.5, 1].$$

Because of the introduced positive semantics and because the index $c_j(a_i)$ combines the indices $s_j(a_i, b)$ and $s_j(b, a_i)$, there is no need to explicitly verify whether the alternative is a member of the negative category. This prevents logical nonsense, which can – according to Bisdorff (2000) – occur when applying standard outranking methods. Fig. 1 gives the graphical interpretation of the criterion-wise concordance and discordance indices taking part in the logical evaluation of the assertion "the alternative a_i belongs to the C^+ category".

The outranking school has abandoned the classical framework of a preference structure based solely on the relations of preference and equality, in an attempt to be more realistic and closer to the bounded rationality of the decision-makers (Stewart and Losa, 2003). Since the nontransitive relation of incomparability no longer exists in conjunction with the presented method, appropriate metrics are defined that indicate conflicting alternatives, and help express sensible values of input parameters.

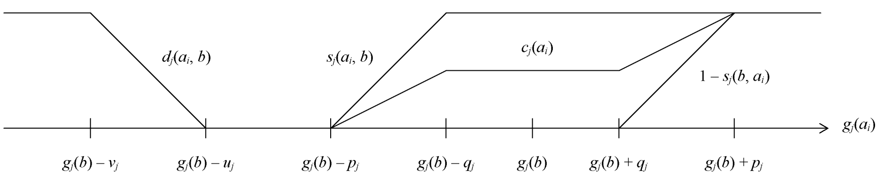

Fig. 1. The degrees of concordance and discordance with the assertion "a_i belongs to C^+" with respect to a maximized criterion.

4. Robustness Metrics

Sensitivity/robustness analysis is one of key concepts in the field of multi-criteria decision-aiding (Saltelli *et al.*, 1999). Many approaches to it have been introduced in the past (Saltenis, 1998; Vincke, 1999). However, since they are designed for specific existing decision-making methods, an original robustness analysis technique is required to enable fully automated pseudo-criterion based group consensus seeking and multi-agent negotiation.

Three types of distance metrics are defined. They reflect the minimum changes of weight, veto and preference vectors that cause the reassignment of an alternative to the other category. When, considering the alternative a_i, any of these measures is low, the membership of a_i is insufficiently robust, since only a slight modification of preferences may result in a different decision. Group members thus have to focus primarily on boundary alternatives and strive to clarify the reasons for their selection. Moreover, when the process is automated in an agent based setting, the interpretation of robustness degrees becomes a prerequisite to adjust input parameters in a correct manner, increase the consensus level and start a new iteration.

The most simple task is to find the smallest change of the weight vector $w = (w_1, \ldots, w_n)$ so that the reassignment of an alternative to the other class occurs, that is: $a_i \in C_k^+ \to a_i \in \tilde{C}_k^-$ or equivalently $a_i \in C_k^- \to a_i \in \tilde{C}_k^+$. The problem is solved with an optimization program:

$$\Delta_w(a_i) = \min\Big[\sum_{j=1..n} (\,|w_j - \tilde{w}_j|\,)^P / \Delta_w^{\max}\Big]^{1/P} \text{ by deriving } w_j,\ \forall j = 1, \ldots, n,$$

subject to

$$\sigma(a_i) = d(a_i) \cdot \Big(\sum_{j=1..n} w_j \cdot c_j(a_i)\Big) = \lambda,$$

$$\sum_{j=1..n} w_j = 1, \quad lw_j \leqslant w_j \leqslant uw_j,\ \forall j = 1, \ldots, n.$$

Here, $\tilde{w}_j$ are current and w_j newly defined weights, while lw_j and uw_j are lower and upper bounds of allowed weight intervals. The value of the parameter P, $1 \leqslant P \leqslant \infty$, determines which one of the L_P distance metrics is used. The obtained distance has to be normalized by division with $\Delta_w^{\max}$ which denotes the largest possible change of the weight vector. For the case when all criteria weights are allowed to take any value on the interval between 0 and 1 ($\forall j$: $dw_j = uw_j - lw_j = 1$), the vector changes maximally when exactly two of its components move from one extreme to the other: $w_i = 1, \forall k \neq i$: $w_k = 0 \to w_j = 1$, $i \neq j$, $\forall k \neq j$: $w_k = 0$. In this special situation, $\Delta_w^{\max}$ equals to 2. However, for arbitrary differences dw_j, such that $\neg\forall j$: $dw_j = 1$, the following mathematical program is solved:

$$\Delta_w^{\max} = \max\Big[\sum_{j=1..n} \big(|w_j^{\text{end}} - w_j^{\text{start}}|\big)^P\Big]^{1/P} \text{ by deriving } w_j^{\text{start}}, w_j^{\text{end}},\ \forall j = 1, \ldots, n,$$

subject to

$$\sum_{j=1..n} w_j^{\text{start}} = 1,$$
$$\sum_{j=1..n} w_j^{\text{end}} = 1,$$
$$lw_j \leqslant w_j^{\text{start}} \leqslant uw_j, \quad \forall j = 1, \ldots, n,$$
$$lw_j \leqslant w_j^{\text{end}} \leqslant uw_j, \quad \forall j = 1, \ldots, n.$$

A harder problem is to measure the robustness of discordance and veto thresholds. An advanced metric is needed that allows for the aggregation of partial discordance indices with the product operator $\tilde{d}'(a_i)$, and indicates the minimal threshold changes that would cause the reassignment of the observed alternative:

$$\Delta_v(a_i) = \min \Big[\sum_{j=1..n} (\delta_j)^P / \sum_{j=1..n} \big(2 \cdot (g_j(b) - p_j - D_j^-)\big)^P \Big]^{1/P}$$
$$\text{by deriving } u_j \text{ and } v_j, \quad \forall j = 1, \ldots, n,$$

subject to

$$\sigma(a_i) = c(a_i) \cdot \prod_{j=1..n} \Big(1 - \max\big(\min((g_j(b) - g_j(a_i) - u_j)/(v_j - u_j), 1), 0\big)\Big) = \lambda,$$
$$\delta_j = |u_j - \tilde{u}_j| + |v_j - \tilde{v}_j| + \big|(v_j - u_j) - (\tilde{v}_j - \tilde{u}_j)\big|, \quad \forall j = 1, \ldots, n,$$
$$p_j \leqslant u_j \leqslant v_j \leqslant b_j, \quad \forall j = 1, \ldots, n.$$

The program minimizes the distances between new and previous values of discordance and veto thresholds. It also pays regard to the distances between different thresholds ($|v_j - u_j|$), to prevent anomalies that can occur if thresholds converge towards the same value. The obtained distance is, again, normalized according to the least favourable situation, which appears when one threshold changes its value from one possible extreme to the other (that is from the lower bound of the criterion domain to $g_j(b) - p_j$, or the opposite), while the second remains in the starting extreme position. The presented program demonstrates the problematic of finding the smallest change of u_j and v_j thresholds that causes the reassignment of an alternative. Yet, it deals with piecewise linear functions with unknown segments. For this reason, it is substituted with a different optimization program. For each criterion value $g_j(a_i)$ of the alternative a_i, an appropriate partial discordance degree is found so that the product of these degrees equals the required overall discordance $\tilde{d}(a_i) = \lambda / c(a_i)$ calculated by dividing the fixed cut level λ with the fixed concordance index. The k_j coefficient of a linear function is then derived for each criterion x_j according to $g_j(a_i)$ (x-axis) and $\tilde{d}_j(a_i)$ (y-axis). The induced function determines the u_j threshold (at $y = 0$) and the v_j threshold (at $y = 1$), and minimizes the distance metric:

$$\Delta_v(a_i) = \min \Big[\sum_{j \in F} (\delta_j)^P / \sum_{j \in E} \big(2 \cdot (b_j - p_j - D_j^-) \big)^P \Big]^{1/P}$$
$$\text{by deriving } \tilde{d}_j(a_i) \text{ and } k_j, \quad \forall j \in F,$$

subject to

$$E = \{1, \ldots, n\}, \quad F \subseteq E,$$
$$\prod_{j \in F} \big(1 - \tilde{d}_j(a_i)\big) \cdot \prod_{j \in E \setminus F} \big(1 - d_j(a_i)\big) = \tilde{d}(a_i), \quad 0 \leqslant \tilde{d}_j(a_i) \leqslant 1, \ \forall j \in F,$$
$$\delta_j = \delta_j^u + \delta_j^v + \delta_j^{uv}, \quad \forall j \in F,$$
$$\delta_j^u = u_j - g_j(a_i) + \tilde{d}_j(a_i)/k_j, \quad \forall j \in F,$$
$$\delta_j^v = v_j - g_j(a_i) - \big(1 - \tilde{d}_j(a_i)\big)/k_j, \quad \forall j \in F,$$
$$\delta_j^{uv} = v_j - u_j - 1/k_j, \quad \forall j \in F,$$
$$\big(1 - \tilde{d}_j(a_i)\big) / \big(D_j^+ - D_j^- - g_j(a_i)\big) \leqslant k_j \leqslant \tilde{d}_j(a_i) / \big(g_j(a_i) - p_j\big), \quad \forall j \in F.$$

To cope with its high complexity, the problem is divided into subproblems and solved with an algorithm:

```
if d̃(aᵢ) ⩽ 1
    E = {1, ..., n}
    for each k = 1, ..., n
        for each k-subset of E
            F ← current k-subset
            d̃ⱼ(aᵢ) = dⱼ(aᵢ), ∀j ∈ F
            minimize Δᵥ(aᵢ) by repeating
                find new d̃ⱼ(aᵢ), ∀j ∈ F, so that
                    ∏_{j∈F}(1 − d̃ⱼ(aᵢ)) · ∏_{j∈E\F}(1 − dⱼ(aᵢ)) = d̃(aᵢ)
                for each j ∈ F
                    find kⱼ that minimizes δⱼ according to gⱼ(aᵢ) and d̃ⱼ(aᵢ)
                Δᵥ(aᵢ) = ‖(δ_{F(1)}, ..., δ_{F(k)})‖_P
            if (solution for F is feasible) ∧(Δᵥ(aᵢ) decreases)
                F̄ = F
                d̄ⱼ(aᵢ) = d̃ⱼ(aᵢ), ∀j ∈ F
                k̄ⱼ = kⱼ, ∀j ∈ F
    uⱼ = gⱼ(aᵢ) − d̄ⱼ(aᵢ)/k̄ⱼ, ∀j ∈ F̄
    vⱼ = gⱼ(aᵢ) + (1 − d̄ⱼ(aᵢ))/k̄ⱼ, ∀j ∈ F̄
```

It should be noticed that is not always necessary to infer new u_j and v_j values for each criterion. Therefore, all possible combinations of criteria are generated and evaluated. Only the one that is feasible and yields the lowest distance is kept. If $\tilde{d}(a_i) > 1$, the algorithm does not start, since in this case a_i cannot be reassigned by adjusting the u_j and v_j thresholds in isolation.

The problem of finding the smallest changes of indifference and preference thresholds that cause the classification of an alternative into a different category is very similar to the one described above. The required degree of concordance is $\tilde{c}(a_i) = \lambda/d(a_i)$. The optimization is slightly more demanding because it has to deal with symmetry of partial concordance indices. This difficulty is overcome by multiplying each $\tilde{c}_j(a_i)$ index with a sign that is determined by comparing the $g_j(a_i)$ and $g_j(b)$ values:

$$\Delta_p(a_i)=\min\Big[\sum_{j\in F}(\delta_j)^P/\sum_{j\in E}(2\cdot u_j)^P\Big]^{1/P} \text{ by deriving } \tilde{c}_j(a_i) \text{ and } k_j,\ \forall j\in F$$

subject to

$$\begin{aligned}
&E=\{1,\dots,n\}, \quad F\subseteq E,\\
&\frac{1}{2}\cdot\sum_{j\in F} w_j\cdot\Big(1+\operatorname{sgn}\big(g_j(a_i)-g_j(b)\big)\cdot\tilde{c}_j(a_i)\Big)+\sum_{j\in E\setminus F} w_j\cdot c_j(a_i)=\tilde{c}(a_i),\\
&0\leqslant \tilde{c}_j(a_i)\leqslant 1, \quad \forall j\in F,\\
&\delta_j=\delta_j^q+\delta_j^p+\delta_j^{qp}, \quad \forall j\in F,\\
&\delta_j^q=q_j-g_j(a_i)+\tilde{c}_j(a_i)/k_j, \quad \forall j\in F,\\
&\delta_j^p=p_j-g_j(a_i)-\big(1-\tilde{c}_j(a_i)\big)/k_j, \quad \forall j\in F,\\
&\delta_j^{qp}=p_j-q_j-1/k_j, \quad \forall j\in F,\\
&\tilde{c}_j(a_i)/g_j(a_i)\leqslant k_j\leqslant \big(\tilde{c}_j(a_i)-1\big)/\big(g_j(a_i)-u_j\big), \quad \forall j\in F.
\end{aligned}$$

The $\Delta_w(a_i), \Delta_v(a_i)$ and $\Delta_p(a_i)$ degrees should be aggregated with a fuzzy weighted averaging operator (Ribeiro and Marques-Pereira, 2003), because the least robust that certain parameters are, the stronger impact they have on weakening the overall robustness:

$$r(a_i)=1-\big(\varpi_{(j)}\cdot\Delta_w(a_i)+\varpi_{(k)}\cdot\Delta_v(a_i)+\varpi_{(l)}\cdot\Delta_p(a_i)\big),$$

where $j\neq k\neq l$ and $\varpi_{(\phi)}=(4-\phi)/\sum_{\varphi=1}^{3}\varphi$.

5. Process of Group Decision-Making

5.1. *Compromise, Consensus and Agreement Degrees*

To direct the process of group decision-making, in the sense of preference unification, the consensus and agreement measures are defined. These measures are similar to the ones that have been proposed by Herrera–Viedma *et al.* (2002). Their method, however, relies on direct specification of fuzzy preferential relations and combines individuals' solutions by applying an ordered weighted geometric operator (Herrera *et al.*, 2001). The presented method, on the contrary, implements the pseudo-criterion concept to elicit input information and does not force an aggregated rank-order of alternatives on the decision-makers.

A compromise is ensured in a very simple way. An acceptable alternative is assigned to the positive category. It thereby receives one vote. As all group members operate on the same alternative set, votes are plainly added. Let o be the number of decision-makers and C_k^+ the subset of alternatives that are approved by the kth individual. Then the sum of votes for a_i is

$$\upsilon_i = \mathrm{card}(a_i \in C_k^+, \quad k = 1, \ldots, o).$$

Alternatives can now be ranked from the most preferable ones, for which $\upsilon = \max_{i=1..m} \upsilon_i$ holds true, to those that receive the least votes. It is thus clear how many participants in the decision-making process agree upon a given choice and it can never happen that a decision is made, which is not in accordance with the opinion of the majority of people involved.

Since a high level of comparability is attained as a consequence of the applied localization principle, it is an uncomplicated task to define the consensus measure. Let z_i be the consensus degree reached for the alternative a_i. The equality $z_i = 0$ holds true, if half of individuals in the group assign a_i to the C^+ category and the other half to the C^- category. The greatest possible separateness between decision-makers occurs in this case, so it cannot be determined whether a_i is an appropriate choice. On the contrary, z_i equals to 1 when all participants classify a_i into the same class, thereby making the group totally uniform. Let $\nu_i^+ = \upsilon_i$ and $\nu_i^- = o - \upsilon_i$ denote how many participants assign a_i to the C^+ class and to the C^- class, respectively. Then

$$z_i = \frac{\nu_i - \rho}{o - \rho}, \quad \text{where } \nu_i = \max(\nu_i^+, \nu_i^-).$$

Another measure is important for the sake of active preference unification. It is called the agreement degree. If the kth decision-maker assigns a_i to the same class as all the other group members, then he agrees with the majority opinion. Thus, $\zeta_i^k = 1$. On the contrary, $\zeta_i^k = 0$, if according to his preferential parameters, a_i belongs to the category that is in opposition to the collective choice. So, the more people that assign a_i to the same category as an individual does, the higher the level of agreement that is reached from the perspective of this person:

$$\zeta_i^k = \begin{cases} (\nu_i^+ - 1)/(o - 1), & a_i \in C_k^+; \\ (\nu_i^- - 1)/(o - 1), & a_i \in C_k^-. \end{cases}$$

An operator which aggregates the partial consensus indices (and similarly, the partial agreement indices), should not only ensure compensation but has to consider the weakest alternative as well. For this reason, Werners' fuzzy "and" (Zimmermann, 1996) is chosen:

$$Z = \gamma \cdot \min_{i=1..m} z_i + (1 - \gamma) \cdot \frac{\sum_{i=1..m} z_i}{m}, \quad \gamma \in [0, 1].$$

Many researchers and practitioners – like for instance Jabeur *et al.* (2004), Leyva-López and Fernández–González (2003), or Zhang and Lu (2003) – state that in real-world decision groups, it is often the case that certain actors may be considered as playing a "leading" role, because they hold the responsibility and the possibility for realizing the chosen solution, have recognized abilities and interests for specific problems, or occupy privileged positions. To enable a hierarchical decision-making situation, group members are given importance weights ω_k, $k = 1, \dots, o$, and the degrees of compromise, consensus and agreement are anewly defined:

$$\begin{aligned}
&\upsilon_i = \frac{\sum_{k \in E} \omega_k}{\sum_{k=1..o} \omega_k}, \text{ where } E = \{\forall k = 1, \dots, o\colon\ a_i \in C_k^+\},\\
&z_i = 2 \cdot (\nu_i - 1/2), \text{ where } \nu_i = \max(\upsilon_i, 1 - \upsilon_i),\\
&\zeta_i^k = \frac{\omega_k + \sum_{l \in F} \omega_l}{\sum_{l=1..o} \omega_l},\\
&\qquad \text{where } F = \left\{ \begin{array}{l} \forall l = 1, \dots, o,\ \ l \neq k\colon \\ (a_i \in C_k^+ \wedge a_i \in C_l^+) \vee (a_i \in C_k^- \wedge a_i \in C_l^-) \end{array} \right\}.
\end{aligned}$$

The only difference that should be noticed in comparison with the previous "democratic" definitions regards to the interpretation of the agreement degree. It is always higher than 0, because it is presupposed that a decision-maker agrees with himself. This is relevant when an autocratic individual is present in the group and when he is in contradiction with other team members. The decision support system must not demand from such a decision-maker to be the first to conform to the opinions of colleagues. It is obvious, though, that if the weight of this person would not be considered, exactly this kind of a situation would occur.

5.2. *Mechanism of Consensus Seeking*

The active mechanism of directing group members toward unified opinions is founded on the progressive increasing of the consensus degree Z toward the specified threshold ξ. At first sight, it seems that z_i has to be primarily raised for those alternatives which have reached a low degree of consensus ($z_i \approx 0$). Their status is namely absolutely undetermined. Neither can they be with certainty assigned to the positive nor to the negative category since there does not exist a prevailing majority of decision-makers that would have enough strength to approve or disprove the suitability of their selection. But such indetermination disables the decision support system to effectively advise a person about the adjustment of preferential parameters. In the case when exactly half of decision-makers assign a_i to C^+ and another half to C^-, it is not evident in which direction the category change should be carried out. Hence, the problem of reaching a consensus is approached from the other side. It is presumed that it is the most credible task to increase the degree of consensus for alternatives that already have a high z_i value. If it is close to 1 ($z_i \approx 1$, but compulsorily $z_i < 1$), two facts may be taken into account:

- It is clear toward which category the group opinion leans when evaluating an alternative with a high level of consensus. Thus, it is righteous to demand the category change from the individuals that oppose this opinion as they are in the absolute, uninfluential minority.

- At first, the group concentrates on alternatives that are rather uniformly judged by its members. A full agreement about very good alternatives can therefore quickly be found. As these alternatives generally suffice for the right decision, other alternatives need not be dealt with at all or can be left for later consideration.

The outline of the consensus seeking procedure is presented on Fig. 2. The decison-maker with the lowest degree of agreement is selected in each iteration. Since this individual is in the strongest opposition to the collective choice, his preferential attitude is the principal reason why the value of Z is not high enough. He has to adjust the values of input parameters to such an extent that someone else becomes the most contradictive group member. Since it is always the turn of the participant with the lowest computed agreement level, two gains arise:

- the values of ζ^k incessantly increase ensuring the convergence of Z toward the threshold ξ;
- equality among decision-makers can be guaranteed, as the only measure of the required conformation to the opinions of colleagues is the deviation from the collective choice, which is independent of the person's rank, except when hierarchical metrics are applied.

It is reasonable that the decision-maker reassigns only alternatives with a low value of the agreement index and with a low robustness level. Otherwise, either a satisfactory agreement degree is reached from this person's perspective, or his opinion is so firmly stated that the conformation to the group is not sensible in spite of a considerable contradiction with it. It is thus the obligation of the decision support system to show for each alternative its partial agreement index as well as data on its sensitivity. The obtained information enables manual selection of alternatives which are subject to the reassignment. This is essential because the decision-maker must be able to reject the proposed category changes. When he is convinced that his judgment is right, he may insist on his own choice. Other participants are thereby stimulated to rethink about the decision, en-

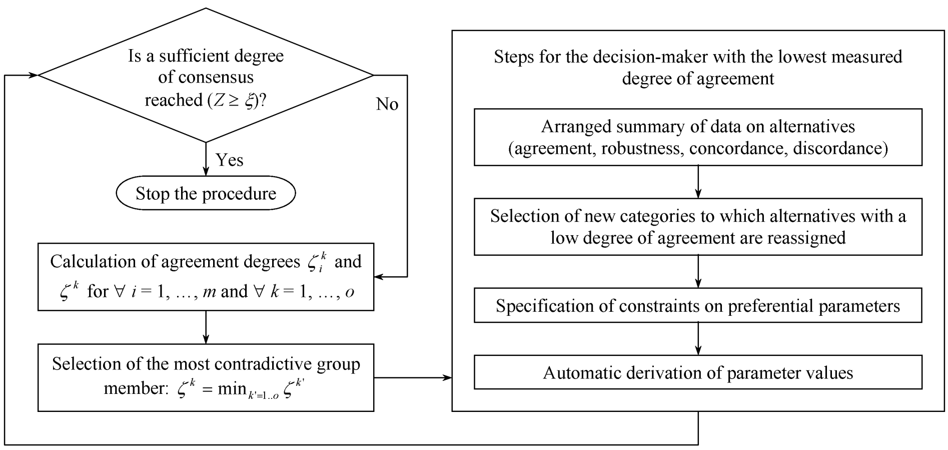

Fig. 2. The group consensus seeking procedure.

lighten their understanding of the problem from complementary viewpoints, and consider important facts that they have perhaps overlooked.

Suppose the decision-maker states which conflicting alternatives he is prepared to reassign: $a_i \in C_k^+ \rightarrow a_i \in \tilde{C}_k^-$ or $a_i \in C_k^- \rightarrow a_i \in \tilde{C}_k^+$. New parameter values can then be automatically derived so that the required changes are attained for the chosen alternatives and so that other memberships are preserved. Parameters are inferred by solving an optimization program:

$$\text{maximize } \min\{\tau_i^+, \tau_i^-\}_{i=1..m}$$

subject to

$$\begin{aligned}
&\sigma(a_i) - \tau_i^+ = \lambda, \quad \forall a_i \in \tilde{C}_k^+,\\
&\sigma(a_i) + \tau_i^- = \lambda, \quad \forall a_i \in \tilde{C}_k^-,\\
&0 \leqslant q_j \leqslant p_j \leqslant u_j \leqslant v_j \leqslant b_j - D_j^-, \quad \forall j = 1, \ldots, n,\\
&lw_j \leqslant w_j \leqslant uw_j, \quad \forall j = 1, \ldots, n,\\
&\sum_{j=1..n} w_j = 1,\\
&\lambda \in [0.5, 1].
\end{aligned}$$

The category membership changes at the value of $\sigma(a_i) = \lambda$. The variables τ_i^+ and τ_i^- must be positive to ensure the assignment of an alternative to the proper category. The lowest of them is maximized by the optimization program for the purpose of achieving robustness. To acquire credible results, it is sensible that the decision-maker specifies additional constraints, for example intervals of suitable criteria weights. In this way, the modification of parameters that are in accordance with the individual's preferences is prevented.

5.3. *Automated Cooperative Agent Based Negotiation*

It is essential for an efficient group decision-making procedure that the cognitive burden of each individual decision-maker as well as the human moderator is reduced to the minimum. Since in certain problem solving situations the latter is potentially biased in favour of its own opinion or against the judgements of some group members, it is advisable that its activity is eliminated. The implemented system must hence be able to autonomously guide the problem solving process. Moreover, people are not always willing to directly engage into synchronous or asynchronous communication. There exist many types of problems and e-services in the context of which they express initial preferences and expect the information system to find the optimal available solution by representing their interests (Bichler *et al.*, 2003; Lomuscio *et al.*, 2003). It is thus crucial to enable decision-making and negotiation in an agent based setting.

The centralized agent negotiation architecture is modelled on Fig. 3. The central role is given to the analytical and mediation agent. It is responsible for preference aggregation and alternative sorting on the individual level; calculation of compromise, consensus,

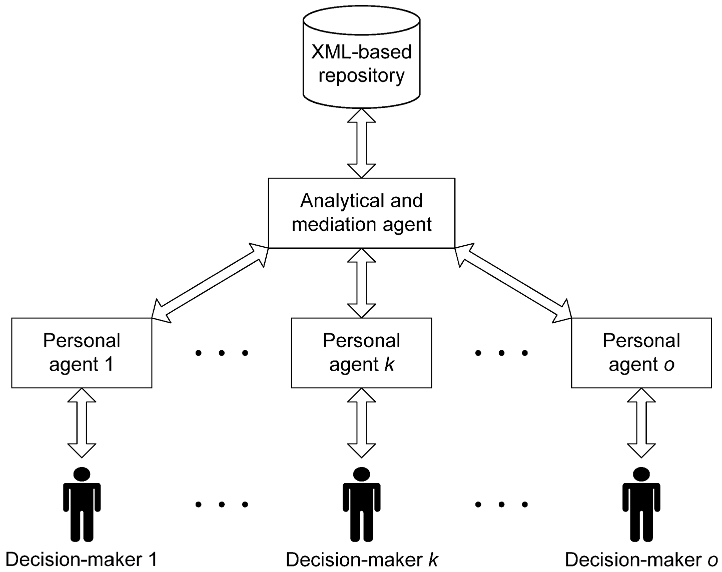

Fig. 3. Agent negotiation architecture.

agreement and robustness degrees; automatic adjustment of preferential parameters; negotiation process directing; and accessing the XML based repository. It communicates with personal agents that are not mutually connected. Each of them corresponds to a single decision-maker from whom it receives requests and preferential information, and in the name of whom it negotiates.

The defined centralized architecture has several advantages. It simplifies the collaboration between the agents because it eliminates the need for each one of them to inform all the others about its activities. The number of required interactions is therefore considerably decreased. In addition, only the mediation/analytical agent has to implement the decision-making logic with optimization facilities, and must provide support for the XML based repository. Personal agents are consequently regarded as thin clients. They merely offer the user interface to the decision-maker and ensure that his expectations are met.

In some cases, time constraints on synchronous preference aggregation/disaggregation, and in particular optimization algorithms, may be an issue (Dzemyda and Petkus, 2001). Then, the computational load is distributed through the network by transmitting it to all agents. Each personal agent has the responsibility to sort alternatives, compute the robustness degrees and infer parameters for the decision-maker that it represents. However, such agents cannot be implemented as thin clients, but rather as .NET or Java web services capable of executing complex optimizations.

As the agents strive to reach consensus, their behaviour tends to be cooperative, providing benefits to all involved parties and assuring an agreement on a commonly accepted solution. The protocol of agent based negotiation extends the consensus seeking mechanism for group decision-making. Its definition is given by means of an algorithm in which DM_k denotes the kth decision-maker, PA_k the kth personal agent and MAA the mediation/analytical agent.

for each $k = 1, \dots, o$
  DM_k specifies initial values of preferential parameters and their permissible deviations
  PA_k transmits initial parameter values to MAA
repeat
  MAA sorts alternatives for each PA_k into the categories C_k^+ and C_k^-
  MAA calculates the agreement, consensus and robustness degrees ζ_i^k, ζ^k, z_i, Z and r_i^k
  if $Z < \xi$
    MAA sorts PA_k in the ascending order so that $\mathrm{PA}_{(l)}$ exhibits the lth lowest ζ^k degree
    conformation ← false
    while $l \leqslant o$ and *conformation* = false
      reassignment← false
      for each $i = 1, \dots, m$
        if $\zeta_i^l < \frac{1}{2}$ and $r_i^l < \psi$
          MAA reassigns a_i so that $a_i \in C_l^+ \to a_i \in \widetilde{C}_l^-$ or $a_i \in C_l^- \to a_i \in \widetilde{C}_l^+$
          reassignment← true
      if *reassignment* = true
        MAA inferres robust values of preferential parameters according to $\widetilde{C}_l^+$ and $\widetilde{C}_l^-$
        MAA transmits inferred parameter values and initial/new assignments to $\mathrm{PA}_{(l)}$
        if permissible deviations of $\mathrm{PA}_{(l)}$ are not violated
          conformation ← true
        else // optional
          $\mathrm{DM}_{(l)}$ reconsiders parameter values and allowed deviations
          if $\mathrm{DM}_{(l)}$ accepts required adjustments
            conformation ← true
            $\mathrm{PA}_{(l)}$ transmits new parameter values to MAA
      if *conformation* = false
        $l \leftarrow l + 1$
    if $l > o$ and $\exists l' = \min l$: *reassignment* = true
      $\mathrm{PA}_{(l')}$ is forced to conformation by adjusting parameter values and/or deviations
until $Z \geqslant \xi$ or $\forall k = 1, \dots, o$: *reassignment* = false
if $Z < \xi$
  alternatives are ranked according to the degrees of compromise υ_i and robustness r_i

The mediation/analytical agent eliminates the need for a human moderator. Moreover, the cognitive load and the activity of each decision-maker are minimized. His only mandatory task is to specify initial values of preferential parameters and their permissible deviations that may not be exceeded by the mediation/analytical agent in the process of parameter inferrence. It is possible to provide this information in several forms:

- by specifying approximate linguistic modifiers of predetermined threshold magnitudes;
- by setting the exact initial values of parameters b_j, q_j, p_j, u_j, v_j and w_j, and the exact lower respectively upper bounds b_j^-, b_j^+, q_j^-, q_j^+, p_j^-, p_j^+,u_j^-, u_j^+, v_j^-, v_j^+, w_j^- and w_j^+ of parameters;
- by determining only the exact lower and upper bounds of allowed parameter intervals, in which case the central points of intervals are taken by the kth personal agent as the initial values b_j, q_j, p_j, u_j, v_j and w_j.

In the first case, the magnitudes of q_j, p_j, u_j and v_j thresholds are expressed with decibels (Rogers and Bruen, 1998a). Since the decibel is a dimensionless quantity without physical units, it is appropriate for criteria modelling within a wide range of decision problems. It can prevent subjectiveness in judgement, and thereby also potentialy irrational comprehension of imprecision and uncertainty. It determines the relationship between a variable S and a known reference S_0 by calculating the logarithm of their ratio:

$$\mathrm{dB} = 10 \cdot \log_{10}(S/S_0).$$

By extending the propositions of Rogers and Bruen, the magnitudes are predetermined in the following manner:

$$q_j = \Omega \cdot 2\mathrm{dB}, \quad p_j = \Omega \cdot 5\mathrm{dB}, \quad u_j = \Omega \cdot 10\mathrm{dB} \text{ and } v_j = \Omega \cdot 15\mathrm{dB},$$

where $\Omega \in \{0.2, 0.4, 0.6, 0.8, 1\}$ is a numerical modifier obtained with a direct transformation of a linguistic influence level $\varsigma \in \{$very weak, weak, moderate, strong, very strong$\}$. Different degrees of influence may be chosen for various thresholds by the decision-maker. He has to specify the initial modifiers Ω_j as well as the lower and upper bounds Ω_j^- and Ω_j^+. Based on these values, the initial, minimal and maximal threshold magnitudes are computed by the kth personal agent relative to the reference profiles b_j. The jth indifference threshold is set to

$$q_j = b_j \cdot (\mathrm{e}^Q - 1), \quad \text{where } Q = \frac{2\Omega_j}{10 \log_{10} \mathrm{e}}.$$

Analogously, the magnitudes of q_j^-, q_j^+, p_j, p_j^-, p_j^+, u_j, u_j^-, u_j^+, v_j, v_j^- and v_j^+ are calculated. It is the role of the kth personal agent to ensure that the kth decision-maker's requirements are fulfiled. The latter is given the opportunity to revise his preferences in each iteration of the negotiation process. However, he is not unconditionally obliged to that, since the negotiation mechanism is enabled to proceed without any human interaction. Once the decision-makers' initial preferences are set, the agents can autonomously collaborate, and automatically reach a common decision.

An agent is asked to conform to the others with regard to the alternative a_i only if a_i is not robustly sorted and if its evaluation contradicts judgments of more than half agents. In order to determine the sensitivity of assignments, the ψ threshold is introduced. The alternative a_i is hence robustly sorted into the C^+/C^- category when its $r(a_i)$ degree exceeds the ψ threshold.

It is reasonable that the most contradictive personal agent, exhibiting the lowest ζ^k degree, is subjected to conformation. However, if all of its assignments are robust or if the adjusted parameter values violate the kth decision-maker's constraints, it may be skipped, and the next most discordant personal agent may be chosen to negotiate. It is possible that the analytical and mediation agent must address several personal agents to find the one which is willing to accept the proposed changes. In the worst case, unsuccessful iteration over all personal agents causes the negotiation process to terminate without reaching a consensus. A compromise is then made by ranking alternatives in the descending order according to their υ_i levels. When a tie occurs, which means that $\upsilon_i = \ldots = \upsilon_j$ for two or more alternatives, it is resolved based on the multi-agent robustness degrees Γ_i. The higher the Γ_i degree is, the better a_i is. So:

$$a_i \succ a_j \Leftrightarrow (\upsilon_i > \upsilon_j) \vee \big((\upsilon_i = \upsilon_j) \wedge (\Gamma_i > \Gamma_j)\big),$$
$$a_i \approx a_j \Leftrightarrow (\upsilon_i = \upsilon_j) \wedge (\Gamma_i > \Gamma_j),$$

where $\succ$ and $\approx$ denote the relations of preference and indifference, respectively. Γ_i is obtained as the difference between the positive and negative robustness degrees:

$$\Gamma_i = \left|\Gamma_i^+ - \Gamma_i^-\right|,$$

so that

$$\Gamma_i^+ = \frac{\sum_{k \in E} r_i^k}{o}, \quad \text{where } E = \{\forall k = 1, \ldots, o\colon\ a_i \in C_k^+\},$$

and

$$\Gamma_i^- = \frac{\sum_{k \in F} r_i^k}{o}, \quad \text{where } F = \{\forall k = 1, \ldots, o\colon\ a_i \in C_k^-\}.$$

Each personal agent communicates with the mediation/analytical agent via XML based messages, which is a standard approach to invoke web services, and to interchange decision models (Kim, 2001). Because of the compactness of presentation, the data structures of these messages are defined in the DTD notation (W3C, 2006). Personal agents transmit preferential parameters of decision-makers in the PA_parameters data structure to the mediation/analytical agent, while the latter transmits inferred parameter values and original/new sorting results in the opposite direction by using the MAA_parameters_assignments root element.

```
<!ENTITY % common_parameters ”
   <!ELEMENT criteria (criterion+)>
   <!ELEMENT criterion (b, q, p, u, v, w)>
   <!ELEMENT b (#PCDATA)>
   <!ELEMENT q (#PCDATA)>
   <!ELEMENT p (#PCDATA)>
   <!ELEMENT u (#PCDATA)>
   <!ELEMENT v (#PCDATA)>
   <!ELEMENT w (#PCDATA)>
   <!ELEMENT lambda (#PCDATA)>
   <!ELEMENT alternatives (alternative+)>
   <!ATTLIST criterion c_index ID #REQUIRED>
   <!ATTLIST alternative a_index ID #REQUIRED>
”>
```

```
<!ELEMENT PA_parameters (criteria, alternatives, lambda)>
<!ELEMENT alternative (g+)>
<!ELEMENT g (#PCDATA)>
<!ATTLIST g c_reference IDREF #REQUIRED>
%common_parameters;
<!ELEMENT MAA_parameters_assignments (criteria, alternatives, lambda)>
<!ELEMENT alternative (original_category, new_category?)>
<!ELEMENT original_category (#PCDATA)>
<!ELEMENT new_category (#PCDATA)>
%common_parameters;
```

The mediation/analytical agent can access the XML repository in which several aspects of decision models are stored: general data on personal agents and/or decision-makers; criteria-wise values of alternatives and preferential parameters for each personal agent and for each iteration; classifications of alternatives for each personal agent and for each iteration, together with corresponding measures of compromise, consensus, agreement and robustness; and the final choice with its perceived real-life effectiveness evaluated after its implementation.

Historical data about problem solving processes can be used to extract deep organizational knowledge in all subsequent group decision-making or negotiation situations. Each current problem setting can be compared with existing ones by utilizing fuzzy measures of similarity. In the case when similarity reaches the specified cut-level, the consensus seeking mechanism is able to derive reliable conclusions and suggest the optimal feasible decision without going through the process of preference unification at all, or by performing just a small subset of otherwise required iterations. This mechanism represents an independent research topic.

6. Derivation of Criteria Weights

The weight derivation procedure consists of the following steps:

1. A fuzzy veto relation $V = \{((x_j, a_i), \mu_V(x_j, a_i)) | (x_j, a_i) \in \mathrm{X}\times\mathrm{A}\}$ is constructed by organizing partial discordance indices so that $\mu_V(x_j, a_i) = d_j(a_i)$, where $i = 1, \ldots, m, j = 1, \ldots, n$.
2. Partial selective strengths of criteria are calculated for each α-cut of the fuzzy relation V.
3. Partial selective strengths are consecutively joint by the use of an algorithm, which considers veto certainty and similarities between intermediate results.
4. Differences between complete selective strengths are transformed so that ratios of criteria weights are reflected through a pairwise comparison matrix.
5. The decision-maker modifies ratios in the comparison matrix according to his preferences.
6. Numerical values of weights are computed from the adjusted matrix.

All α-cuts of the fuzzy veto relation V are taken. The fuzzy weight derivation problem is thereby transformed into many corresponding subproblems formulated in the classical "crisp" sense. Obeying a heuristic rule, selective strengths of criteria are calculated for each crisp relation. The total selective strength of the jth criterion is computed according to the number of alternatives that are excluded from the positive category C^+ because of the discordance effect of the discordance and veto thresholds u_j and v_j, and simultaneously according to the number of other criteria from the set $X\backslash\{x_j\}$ that oppose a veto on the assignment of the same alternatives to the C^+ class. The partial selective strength of the criterion x_j considers only the ith alternative and the single cut-level α_k. It equals to zero when u_j and v_j do not contradict the assertion $a_i \in C^+$ or when all criteria oppose a veto on this assertion. It indicates to which degree a criterion outperforms the weakest criterion:

$$\varphi_{ji}^k = \begin{cases} \operatorname{card}\bigl(x_l \in X\backslash\{x_j\}\colon\ d_l(a_i) < \alpha_k\bigr), & d_j(a_i) \geqslant \alpha_k, \\ 0, & d_j(a_i) < \alpha_k. \end{cases}$$

It can be stated that the partial selective strength of the jth criterion, which excludes the ith alternative from the positive category, equals the number of criteria, which do not exclude the same alternative. The algorithm that joins the φ_{ji}^k indices rests on the following principles:

- The criterion x_j gains the highest possible selective strength, measured according to the alternative a_i, at the first cut for which the discordance degree $d_j(a_i)$ exceeds the α_k threshold. At levels $\alpha_{k'} > \alpha_k$, the indices equal to zero and need not be dealt with, while at $\alpha_{k'} < \alpha_k$, additional criteria might oppose a veto and thus $\varphi_{ji}^{k'} \leqslant \varphi_{ji}^k$ holds true.
- When $\varphi_{ji}^{k_1} = \ldots = \varphi_{ji}^{k_h}$ for adjacent $\alpha_{k_1} > \ldots > \alpha_{k_h}$, only the cut with the highest level is considered. This originates from the logical maximum concept: the highest certainty with which the partial strength φ_{ji}^k $(k = k_1, \ldots, k_h)$ affects the total strength Φ_j equals α_{k_1}, so the lower certainties $\alpha \in [\alpha_{k_h}, \alpha_{k_1})$ cannot intensify the influence of φ_{ji}^k.
- When the difference $\delta = \varphi_{ji}^k - \varphi_{ji}^{k'}$ exceeds 0 for $\alpha_{k'} < \alpha_k$, the total strength of x_j according to a_i falls by $\alpha_{k'} \cdot \delta$. The difference δ has to be lessened by the certainty factor of results. The decrease is a consequence of a weaker veto effect of one or more additional critera.
- Cut-levels $\alpha_k, k = 1, \ldots, l$, are treated as weights. The higher the cut-level is, the more certain is the crisp veto relation. Selective strengths of criteria, which are bound to cuts with levels $\alpha_k >> 0$, are consequently more substantial for a given problem situation than those, which correspond to low levels $\alpha_k \approx 0$. They are entitled to contribute to a greater extent to the total strengths.

The simple algorithm that computes total selective strengths Φ_j is written in pseudocode. It holds: $\forall i,\ j = 1,\ \ldots, l$: $i < j \Rightarrow \alpha_i > \alpha_j$.

$\phi_{ji} = 0$, $\varphi_{ji}^0 = 0$ for $\forall i = 1, \ldots, m, \forall j = 1, \ldots, n$
$\forall k = 1, \ldots, l$
    $\forall i = 1, \ldots, m, \forall j = 1, \ldots, n$
        $\delta = \varphi_{ji}^{k-1} - \varphi_{ji}^k$

if $\delta \neq 0$, then $\phi_{ji} = \phi_{ji} - \alpha_k \cdot \delta$

$\Phi_j = \sum_{i=1..m} \phi_{ji}$

The Φ_j strengths are of great help to the decision-maker when specifying criteria weights, as they reflect the relationships between criteria importances. In addition, when the value of Φ_j is too high or too low, the domination or the discrimination of the jth criterion is exposed. This is a clear sign for the decision-maker to modify input parameters if he wishes that all criteria are adequately participating in the evaluation process.

Selective strengths must, however, not be directly interpreted as weights. The meaning of the "zero strength" has to be considered. If $\Phi_j = 0$, it does not imply that the weight w_j also equals to 0, since the inability of the criterion x_j to veto the assignment of any alternative to the positive class should not prevent its participation in the concordance test. Similarly, the maximal possible strength $\Phi_{\max} = m \cdot (n-1)$, which appears when a single criterion eliminates all alternatives from the C^+ class, has to be dealt with. And finally, computed strengths should be modified by the individual in order to properly match with his personal beliefs.

Zeleny (1982) has written about criteria in the sense of information sources. Each criterion weight is linked with inner information, which is generated by the set of alternatives, and with the decision-maker's subjective assessment of importance that is bound to his experience and knowledge. The concept of importance that is dependent on the internal information flows is hence represented by the selective strength, as it is sensitive to the chosen values of the u_j and v_j thresholds as well as to the changes in the set of alternatives. Since this implicit information must be enriched with explicitly provided input data, two interrelated problems arise:

1. the conversion of indices Φ_j to weights w_j, for $j = 1, \ldots, n$,
2. the representation of indices Φ_j in such a way that criteria importances are correctly and intelligibly expressed, and that the decision-maker is able to easily adjust them.

To solve both problems, the computed selective strengths may be transformed so that the relations between criteria are expressed in the notation of one of the available structured weight derivation techniques. Figueira and Roy (2002) believe that the Simos' procedure is the only one appropriate to be applied in conjunction with the pseudo-criterion based decision models, while Macharis *et al.* (2003) suggest the use of AHP matrices for the purpose of enhancing PROMETHEE through an additional weight determination mechanism.

The differences Δ_{ij} of each two selective strengths are thus transformed to be included in a $n \times n$ pairwise comparison matrix. This matrix contains ratios of criteria weights and is consistent with the concepts of AHP (Saaty, 1980) – it is reflexive, reciprocal and limited to the scale of 1 to 9. Because of the above illustrated "$\Phi_{\max}$ problem" and because the Φ_i/Φ_j ratio can be computed only if $\Phi_j \neq 0$, an approach is used that combines ratios with intervals.

Let the weight ratio be denoted as $r_{ij} \approx w_i/w_j$. The fundamental presumption is that r_{ij} increases linearly according to the difference $\Delta_{ij} = \Phi_i - \Phi_j$ between the selective strengths of two criteria. The ratio r_{ij} remains the same for all feasible values of Φ_i and Φ_j, if only the difference Δ_{ij} is constant. The interpretation is that the additional strength $d\Phi_j$, which is gained by the criterion x_j, influences the weight increase with equal intensity regardless of the initial value Φ_j. Since $\Phi_{\max}$ has the maximum possible priority over $\Phi_{\min} = 0$, it is evident that $r_{\max} = 9$ is assigned to $\Delta_{\max} = \Phi_{\max} - \Phi_{\min}$. Thus

$$r_{ij} = \frac{8}{\Delta_{\max}} \cdot \Delta_{ij} + 1.$$

Only non-negative differences are considered; if $\Delta_{ij} < 0$, a reciprocal value $r_{ij} = 1/r_{ji}$ is taken. The constant $b = 1$ ensures that $\Delta_{ij} = 0$ is transformed to the $r_{ij} = 1$ ratio which indicates total equality of criteria. The linear function does not guarantee matrix consistency, so the exponential function is also defined:

$$r_{ij} = (r_{\max})^E, \text{ where } E = \frac{\Delta_{ij}}{\Delta_{\max}},$$

such that $r_{ij} = 1$, if $\Delta_{ij} = 0$; $r_{ij} = r_{\max} = 9$, if $\Delta_{ij} = \Delta_{\max}$; and $r_{ik} = r_{ij} \cdot r_{jk}$ for $\Delta_{ik} = \Delta_{ij} + \Delta_{jk}$.

Experiments show that the inconsistency rates of pairwise matrices, which are constructed with the linear function, are low. They do not rise above the worst acceptable level of 0.1. The average value obtained for various numbers of alternatives and criteria is 0.01.

7. Conclusion

In this paper, the two-categorical alternative sorting analysis based on the pseudo-criterion concept was introduced. To enable this kind of analysis, the asymmetrical treatment of veto and discordance thresholds was grounded and applied. As a consequence of the localization principle, high adaptability of the quantitative decision model and high comparability of the individuals' results were achieved. Thereby, the foundation was laid to implement an active iterative mechanism for group consensus seeking, which is capable of automatic unification of decision-makers' opinions. A mathematical optimization program was used with the purpose of reaching robust conclusions, and the consensus and agreement measures were defined in order to ensure convergence of the proposed procedure. Several robustness metrics were also defined to allow for identification of alternatives suitable for automatic/manual reassignment.

In addition, the influence of the noncompensatory veto and discordance thresholds on the criteria weights was discussed. An approach for automatic weight derivation was proposed. It computes the selective strengths of criteria and transforms them into a pairwise comparison matrix, which can be adjusted by the decision-maker to correctly reflect his personal beliefs.

The weight derivation approach has been recently evaluated according to a comprehensive experimental model. Simulation has been performed with regard to four independent variables – applied method, number of alternatives, probability of veto, and distribution – as well as six dependent variables – sensitivity to values of input parameters, richness/extremness of weight discriminating information, proportion of rank perturbations in weak criteria orders between different applied methods, sensitivity to small changes of discordance indices, and sensitivity to adding either new alternatives or copies of existing alternatives. The results of the study have indicated the ability of selective strengths to provide good approximations of criteria weights. It has also been proven that the linear transformation of selective strengths induces near consistent pairwise comparison matrices, and that slightly inconsistent matrices generate more accurate and richer information on criteria weights than those which are obtained with the exponential function. Details on this empirical study will be presented in a separate paper.

The evaluation of the consensus seeking procedure is currently work in progress. A general framework has been defined (Bregar, 2005), which consists of twelve interrelated variables: guidance by the system; conflict resolution; convergence of opinions; cognitive load during analysis; initial cognitive load; level of imprecision, indetermination and uncertainty; ability of learning; ability of asynchronous interaction; time taken; thoroughness of problem domain analysis; robustness; and accuracy. Because of its high complexity, the framework requires combined application of two different research techniques – action research and statistical experiments in a controlled laboratory environment.

The introduced methodological solutions for consensus seeking, agent based negotiation and automatic weight derivation may be used either integratively in the context of a coherent interactive procedure, or in isolation. Due to substantial extensiveness, their applications will be the subject of a follow up paper. Although the localization principle leads to significant benefits, the weight derivation procedure and the group consensus seeking mechanism will be adapted to the other more general kinds of decision analyses as well. Particularly the case of ranking alternatives from the best to the worst ones will be considered within the scope of further research work.

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Interaktyvi kaupimo/padalinimo rikiavimo procedūra grupinei sprendimo analizei, pagrįsta slenksčio modeliu

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Šiame straipsnyje pristatyta nauja daugelio kriterijų sprendimų priėmimo procedūra, kuri surenka pirmenybės informaciją slenksčio modelyje. Ji yra pagrįsta rikiavimo analize, apribota lokalizavimo principo, leidžiančio sprendimo modelio prisitaikymą ir sumažinančio sprendikų pažinimo perkrovimą. Tai įgalina tris koncepcijas, kurios anksčiau buvo nepakankamai palaikomos kitų metodų: pusiau automatinis kriterijų svorių išvedimas priklausomai nuo atrankos pasekmių bei veto slenksčių, konverguojanti grupinio sutarimo paieška ir autonominės daugelio agentų derybos. Tarpusavyje priklausantys principai yra patikrinti ir realizacijos sprendimai pateikti.