

Energy Cost Optimization by Adequate Transmission Rate Dividing in Wireless Communication System

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Received: March 2007

Abstract. Energy cost is the main constraint in modern wireless communication system. A powerful scheme due to optimal energy cost is provided for a single node server in this paper. In wireless communications, the total energy for transmitting packets can be reduced by proper regulating the service rate due to different packet sizes. In our study, a generic method applying the Lagrange Multiplier methods for optimizations is proposed. We show the energy cost is a convex function and it is easy to achieve the optimization. Our contribution focused on minimizing the total energy cost induced by the transmission energy in a single server with multi-queues. The methodology presented in this paper can effectively save the energy cost due to energy consumption in wireless communication systems.

Keywords: Lagrange multiplier, capacity ratio, reliable transmission, optimal energy cost.

1. Introduction

Packet transmission technology has been widespread used in wireless networks. Heterogeneous media such as voice, video and data can be efficiently transmitted using the resources of wireless channels. The key concern in wireless transmission is the energy consumption. The energy of a mobile station (MS) is limited from the fact that the battery has to regularly recharged, this induce a wide interest on developing low power system. In (Kelly and Weber, 2000), a minimization of energy used by a node in wireless data networks to transmit packet information within a dead line (delay time threshold) is considered. In (Prabhakar *et al.*, 2002), a lazy schedule that judiciously changes packet transmission time was proposed to minimize the energy to transmit packets over wireless links.

In this paper a new method for energy cost optimization is proposed for a single server with N multi-queues. For simplicity, the packet size in each queue is assumed to be fixed and different queue behaves different packet size. The energy as a function of transmission rate (transmission rate and service rate are used interchangeably in this paper) and packet size are optimized for minimum energy cost. In our multi-queue system, the delay

in each input queue is controlled in the tolerant range by assumption. The transmission rate is adequately allocated for each queue to achieve the requirement of minimum energy cost.

The problem to minimize the energy consumption for a single server with N input queues is performed by generically dividing the transmission rate according the packet size. In our study, we prove the energy cost is a convex function and the optimization is in existing based on the convex characteristics. The terminology proposed here can be widely used in wireless system for energy saving.

The rest of this paper is organized as follows: Section 2 is the system model descriptions, Section 3 presents the mathematical analysis, Section 4 includes numerical results and discussions and Section 5 is our conclusion.

2. System Model

The relation between the power consumption and the transmission rate (in bits per second) is well studied in information theory (Cover and Thomas, 1991). Let R be the transmission rate, then for an Additive White Gaussian Noise (AWGN) channel with bandwidth W is presented as follows:

$$R = W \log_2 \left(1 + \frac{P_{av}}{WN_0} \right) \quad \text{bits per second,} \quad (1)$$

where P_{av} the transmission power and N_0 is the noise power. Without loss of generality, the channel bandwidth W and the noise power N_0 can be assumed to be constant. From (1), evidently, the magnitude of the transmission rate R is proportional to the value of the transmission power P_{av} . I.e., R is the upper bound for a given P_{av} and equivalently P_{av} is the lower bound to achieve a given rate R .

In our study, a single server node with N channels is adopted for analysis as Fig. 1 shows. For simplicity we assume each channel capacity is large enough to storage the input traffics and no overflow occurs. In Fig. 1 an accumulator is used prior to each queue to accumulate the input traffics every t seconds, i.e., the packet enters to the queue buffer every t interval. Let λ_i denotes the arrival rate in queue i , then each packet size equal to $\lambda_i t$ in queue i . Therefore the N channels can be taken equivalently as N input queues, where each queue is modeled as an M/M/1 queueing model as Fig. 2 shown. Hence, channel and queue are taken to be equivalent in the sequel.

Obviously, in (1), high transmission rate corresponds to short transmission time and high transmission power. Conversely, low transmission rate corresponds to long transmission time and low transmission power. Obviously, long transmission time will benefit on power consumption since long transmission time corresponding to low transmission rate as depicted in (1). However, the transmission time of any one packet cannot be long arbitrarily as this would induce un-tolerable delay. E.g., voice transmission is delay-sensitive and over-delay is not permitted in real time environments.

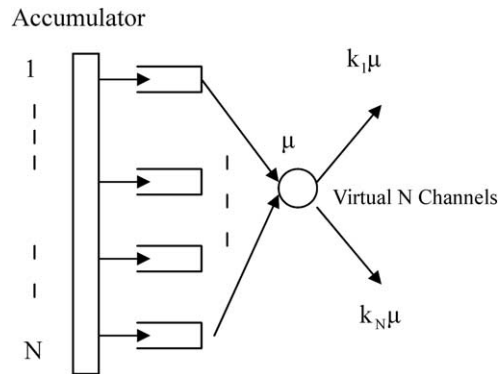


Fig. 1. Service rate dividing for N channels.

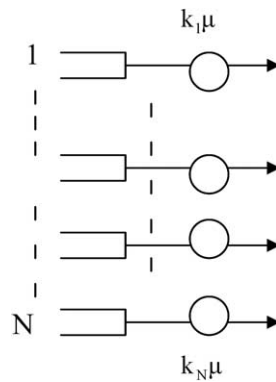


Fig. 2. Equivalent virtual channels of Fig. 1.

In this paper the system is assumed operating in steady state, then the constraint $\sum_{i=1}^N \lambda_i \leq \mu$ is satisfied for stationary system, where μ is the total system transmission rate and λ_i is the arrival rate of the queue i . The optimal scheme for energy saving is performed under the assumptions that the packet delay in each queue is under the threshold value.

In Fig. 2, the input stream arrive to the queue is treated as individual M/M/1 queue served with FIFO policy. Since we consider a conservative system, the single server (e.g., the routing node in ad-hoc networks) will not be idle when there are packets waiting for transmissions. From (1), the consumed energy is a function of the transmission rate. To achieve the minimum energy consumption, the transmission rate should be divided appropriately for each queue for packet transmissions.

Let b_i be the packet size in bits corresponding to queue i , then b_i equals $\lambda_i t$ since packet enters to each queue every t seconds. Hence, high arrival rate corresponds to large magnitude of packet size. Let μ be the total service rate (its value is taken to be one unit for convenience) and k_i be the positive fraction of service rate allocated for packets in the queue i , where $0 \leq k_i \leq 1$ with the constraints $\sum_{i=1}^N k_i = 1$.

In steady state, we divide the service rate into N parts for the N input queues. Unlike divide the service rate proportional to the packet size, the optimization on energy cost is performed approximately in reverse order of the packet size, i.e., larger packet is allocated with lower value of transmission rate. Where queue i is allocated with the service rate equal to $k_i\mu$. Therefore, different packet size is allocated with different transmission rates to achieve minimum total energy cost.

3. Mathematic Analysis

Consider a single node with N channels in wireless networks. Where each channel has an average signal power constraint P_{av} , an Additive White Gaussian Noise (AWGN) power N_0 and fixed channel bandwidth W in frequency response. Therefore for channel i , the Shannon channel capacity (Cover and Thomas, 1991; Kleinrock, 1974) is given by

$$C_i = \frac{1}{2} \log_2 \left(1 + \frac{P_{av}}{WN_0} \right) \quad (2)$$

bits per dimension. From (2) we have

$$P_{av} = WN_0(2^{2C_i} - 1) = \epsilon C_i \varepsilon_i, \quad (3)$$

where ε_i is the energy consumption per bit and ϵ is the reliable transmission probability. From (3) we obtain

$$\varepsilon_i = \frac{1}{\epsilon C_i} WN_0(2^{2C_i} - 1). \quad (4)$$

Hence, for queue i , the value C_i in (2) is related to the transmission rate by (Cover and Thomas, 1991)

$$k_i\mu = C_i 2W. \quad (5)$$

Let T denotes the total energy cost to transmit N packets with one packet from each queue, i.e., packet 1 corresponds to queue 1, packet i corresponds to queue i and packet N corresponds to queue N etc. Also let b_i denotes the packet size (bits per packet) and d_i denotes the cost per one unit of energy of queue i , then from Eqs. 2–5 we have

$$\begin{aligned} T &= \sum_{i=1}^N b_i d_i \varepsilon_i = \sum_{i=1}^N \frac{b_i d_i}{\epsilon C_i} WN_0(2^{2C_i} - 1) \\ &= \sum_{i=1}^N \frac{2W b_i d_i}{\epsilon k_i \mu} WN_0 \left(2^{\frac{k_i \mu}{W}} - 1 \right). \end{aligned} \quad (6)$$

In the following sections, the parameters W , b_i and d_i are taken to be constant for each queue for convenience. For packet transmissions, each packet (fixed size) in the

queue is allocated with a fraction of the total service rate (we assume service rate is one unit without loss of generality). With the aim of energy cost optimization, we have the following lemma:

Lemma 1. *For a single node server, let N be the total number of input queues. Let the packet size be fixed in each queue and packet size is different between queues, then if the delay is controlled in the tolerant range, the total energy cost is optimized under the following constraints.*

$$\frac{b_i}{k_i^2} \left[\frac{\mu}{W} k_i (\log 2) 2^{\frac{k_i \mu}{W}} - 2^{\frac{k_i \mu}{W}} + 1 \right] = \frac{b_j}{k_j^2} \left[\frac{\mu}{W} k_j (\log 2) 2^{\frac{k_j \mu}{W}} - 2^{\frac{k_j \mu}{W}} + 1 \right],$$

where $1 \leq i, j \leq N$.

Proof. By applying Lagrange multiplier method to (6), we complete the proof (detail proof is presented in the Appendix).

Lemma 2. *The function T in (6) is a convex monotone increasing function of the service rate fraction k_i for each i .*

Proof. Take the first derivative of (6) due to queue i , it is easy to verify that

$$\frac{\partial T}{\partial k_i} > 0.$$

This proves the characteristic of the monotone increasing function.

Similarly, taking double derivative of (6) it is easy to show that

$$\frac{\partial^2 T}{\partial k_i^2} > 0.$$

This proves the characteristic of convex function (detail proof is presented in the Appendix).

4. Numerical Results

The units in energy cost and packet size are normalized in the numerical calculations for simplicity. Fig. 3 shows the total energy cost as a function of the reliable transmission probability for four channels (packet size are 1, 2, 5, 7 units for channel 1 to channel 4 individually after normalizations) with three policies, namely, proportional, average and reverse policies. Proportional policy allocates the transmission rate for each channel according to the packet size, i.e., large size packet is allocated with larger service rate. On the other hand, reverse policy allocates transmission rate in reverse order as compared to the proportional policy, and the average policy allocates equal value of transmission rate to each channel.

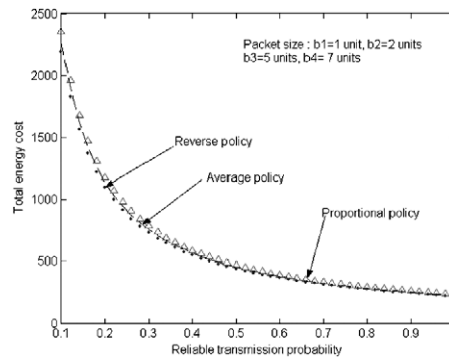


Fig. 3. Non-optimal energy cost.

Clearly, the total energy cost is a monotone decreasing function of the reliable transmission probability, i.e., the low reliable transmission probability corresponds to higher energy cost and high reliable transmission probability corresponds to lower energy cost. Hence in Fig. 3, the total energy cost by applying the reverse policy is better than the other two policies. This is desired from the fact that reverse policy has the effect to prolong the transmission time of long packets. The three policies shown in Fig. 3 are not optimal, in which reverse policy is the best (lowest energy cost) and the proportional policy is the worst (highest energy cost).

For convenience, we define the ratio of the total transmission rate μ to the value of channel capacity W as the capacity ratio. Fig. 4(a) shows the total energy comparisons for different policies of two channels with capacity ratio equal to 1. Similarly, Fig. 4(b)–(g) correspond to capacity ratio equal to 2, 5, 6.7, 7.7, 8.3, 10 respectively. Energy cost comparisons for channels greater than two can be treated in the same manner. The proposed policy in this paper having the optimal energy cost as compared with the other three policies, namely, proportional, average and reverse policies. Evidently, the proportional policy has the worst energy cost and our policy has the advantage of best saving on energy cost.

It is noted that for fixed value of the reliable transmission rate, the total energy cost is a decreasing function of the capacity ratio as Fig. 4(a)–(c) shows. On the other hand the total energy cost is an increasing function of the capacity ratio as Fig. 4(d)–(g) shown, where large value of the capacity ratio corresponds to large energy cost when the magnitude of the capacity ratio is large enough. This is reasonable from (6) and is verified by (8) in Appendix.

If the channel capacity W is fixed for each channel, small value of the transmission rate will benefit on energy cost as depicted in (6). However, low transmission rate will induce large transmission delay, which is not permitted in delay-sensitive networks. E.g., in M/M/1 model, the delay equal to $1/(k_i\mu - \lambda_i)$, where $k_i\mu$ is the transmission rate and λ_i is the arrival rate for queue i , therefore small value of $k_i\mu$ correspond to large value of delay. Hence, large value of the transmission rate will benefit on delay, nevertheless too large of energy cost is not economic in practical real networks.

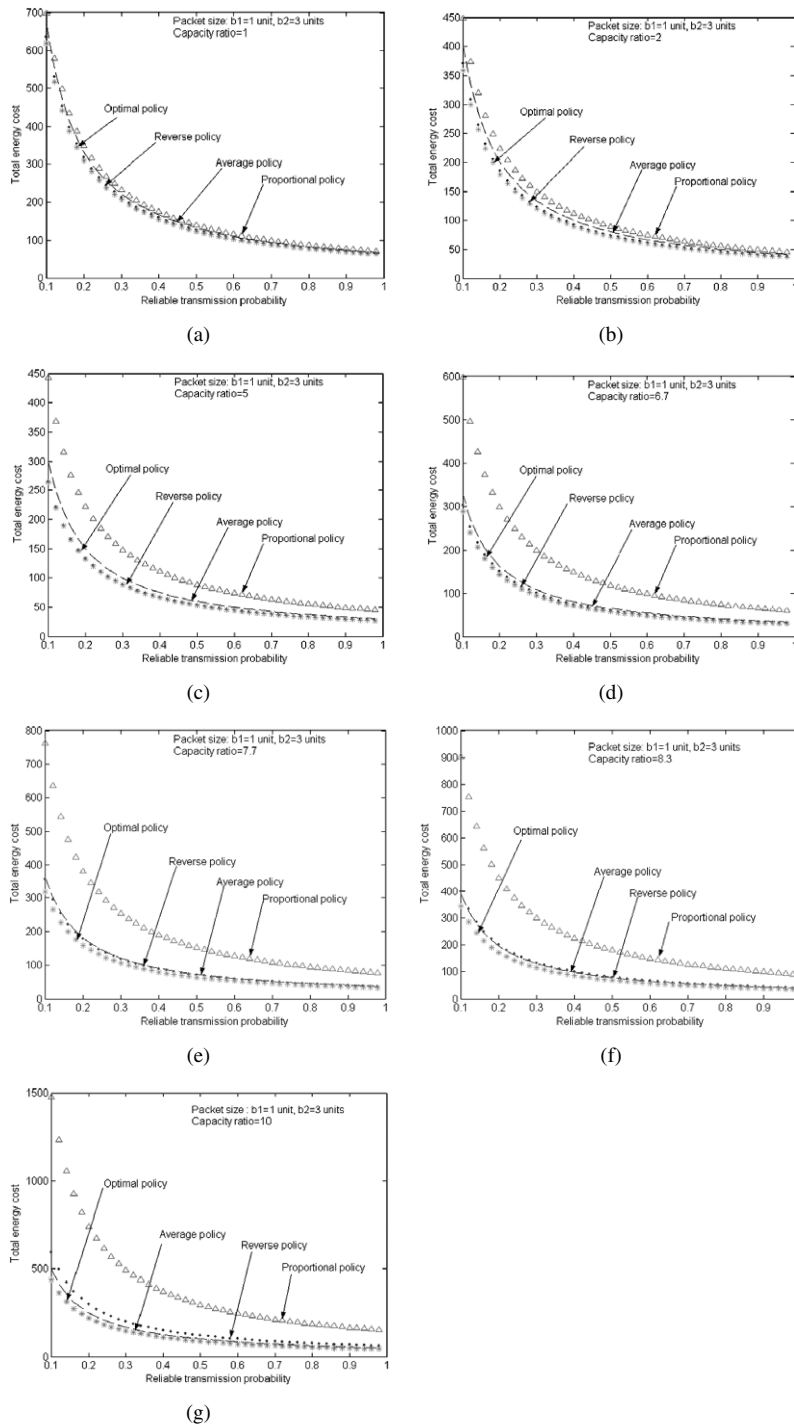


Fig. 4. Optimal energy cost presentations for two channels.

In this paper we concentrate on energy cost optimization under the assumptions that the delay is controlled in the tolerant range for each queue. The delay guarantee for wireless network is beyond the scope of our study, reader has interest can refer (Coutras *et al.*, 2000; Liu and Wu, 2000; Jain, 1989).

5. Conclusions

In wireless environments, energy consumption is an important problem since energy is the most precious resource in wireless networks. In this paper, the optimization of energy cost is achieved by employing the convex function characteristics. The optimization of energy cost is derived by Lagrange multiplier method. We present an easy method on service rate dividing according to variant packet size for a multi-queue single server system. Our contributions are focused on optimization of energy conservations by assuming the delay tolerance is within the threshold value.

The conditions that satisfy both of energy cost optimization and delay constraint induce a non-linear programming issue. The tradeoff between delay guarantee and energy saving will be a challenge in future work. Our technique depicted in this paper can be widespread used in wireless system for energy saving by efficiently resource control.

Appendix

Proof of Lemma 1

From (6), without loss of generality, the channel bandwidth W and noise power N_0 can be assumed to be constant, then for fixed values of transmission reliable probability ϵ and cost per unit of energy d_i , we search for the relations of packet size and transmission rate for energy cost optimization. Therefore our object is to minimize the following equation, i.e.,

$$\text{Min} \sum_{i=1}^N \frac{b_i}{k_i} \left(2^{\frac{k_i \mu}{W}} - 1 \right).$$

Using the Lagrange multiplier methods as follows:

$$\text{let } f(k_1, \dots, k_N) = \sum_{i=1}^N \frac{b_i}{k_i} \left(2^{\frac{k_i \mu}{W}} - 1 \right) \text{ with the auxiliary equation}$$

$$g(k_1, \dots, k_N) = k_1 + \dots + k_N - 1 \quad (7)$$

set $F(k_1, \dots, k_N) = f(k_1, \dots, k_N) + rg(k_1, \dots, k_N)$ for optimal solution we have $\frac{\partial F}{\partial k_i} = 0$, where i is a positive integer with $0 < i < N + 1$, this implies

$$\frac{b_i}{k_i^2} \left[\frac{\mu}{W} k_i (\log 2) 2^{\frac{k_i \mu}{W}} - 2^{\frac{k_i \mu}{W}} + 1 \right] = -r = \frac{b_j}{k_j^2} \left[\frac{\mu}{W} k_j (\log 2) 2^{\frac{k_j \mu}{W}} - 2^{\frac{k_j \mu}{W}} + 1 \right].$$

in which $1 \leq i, j \leq N$.

Proof of Lemma 2

From (6), let $k_i\mu = x$ and take μ equal to unity, then $0 < x \leq 1$. Assume the rest parameters in (6) are constant, to prove the Lemma, we only need check the following equation, i.e.,

$$h(x) = \frac{1}{x}(2^{2x} - 1). \quad (8)$$

Then we have

$$\frac{d h(x)}{d x} = \frac{x2^x \log 2 - 2^x + 1}{x^2}. \quad (9)$$

Hence x indicates the fraction of the allocated transmission rate under the normalization that the total service rate is equal to one.

Applying L'Hospital's rule to (9), we have the following:

$$\begin{aligned} \lim_{x \rightarrow 0^+} \frac{x2^x \log 2 - 2^x + 1}{x^2} \\ = \frac{(\log 2)^2 2^x + (\log 2)^2 (2^x + x2^x \log 2) - 2^x (\log 2)^2}{2} > 0, \end{aligned}$$

then (9) is proved to be positive.

By applying L'Hospital's rule once again, we obtain the following relation after some algebra calculations.

$$\lim_{x \rightarrow 0} \frac{d h^2(x)}{d x^2} = \frac{1}{3x^2} > 0.$$

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Energijos sąnaudų optimizavimas bevielių komunikacijų sistemoje naudojant tinkamą perdavimo dažnumo padalijimą

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Išlaidos energijos sąnaudoms yra svarbiausias šiuolaikinių bevielių komunikacinių sistemų apribojimas. Šiame straipsnyje siūlomas vertingas sprendimas optimaliam energijos sąnaudų parinkimui, kai ryšiui užtikrinti naudojamas vienas serveris. Bevielėse komunikacijose bendra paketų perdavimo energija gali būti sumažinta reguliuojant perdavimo dažnumą priklausomai nuo perduodamų paketų ilgio. Šiame darbe optimizavimui yra siūloma taikyti apibendrintą Lagranžo daugiklių metodą. Parodoma, kad energijos sąnaudų tikslo funkcija yra išskila ir todėl lengvai optimizuojama. Akcentuojamas bendras perduodamos energijos minimizavimas dirbant vienam serveriui bei esant daugelio vartotojų eilei. Šiame straipsnyje pasiūlytas sprendimo įgyvendinimas gali efektyviai sumažinti energijos sąnaudas bevielėse komunikacijų sistemose.