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SOME LOGIC FUNCTIONS REALIZED ON A STATIONARY NONLINEAR DENDRITE. 1. Exciting synapses

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Abstract. The binary logic functions "AND" and "OR" are realized by the model of a nonlinear stationary dendritic branch. The neuron with such dendrites is a complex logic system performing a great number of elementary logic operations.

Key words: Neurocomputer, dendrite, synapse, current-voltage nonlinearity.

1. Introduction. Nets of simple quasineurons become the object of numerous investigations. A rapidly developing field of neurocomputer science has distinquished itself from neurocybernetics. However, real neurons in most cases are not ordinary threshold elements, their dendrites possess nonlinear electric properties. Such dendrites are able to generate impulses in accordance with logic schemes "AND/OR" and are characterized by two stable points of the membrane current-voltage characteristic ($C - V$) (Gutman, 1984).

In the present paper we shall continue the analysis of logic operations of dendrites made previously only on a conceptual level (Gutman, 1984). By means of numerical simulation we show the possibilities of realizing some of binary logic functions of two independent variables on a separate dendritic branch. We shall restrict ourselves to classic synaptic features and linear description of load of the proximal end of dendrite.

2. A mathematical model of the nonlinear dendrite. A mathematical model was constructed for a simplified dendritic branch, presented in Fig. 1. Two stimulating synapses are located on the branch. The distal end of the branch is closed by the same membrane which forms the lateral surface of the branch.

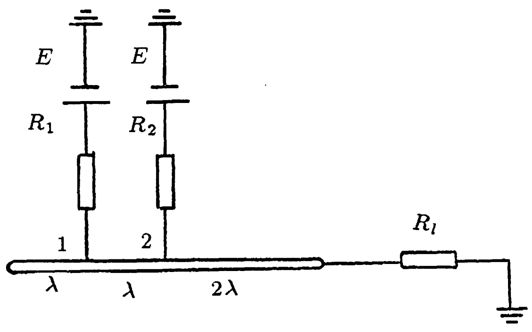

Fig. 1. The scheme of a dendrite branch with ohmic load R_l; R_1, R_2 are the resistances of synapses; E is the electromotive force of synapses.

According to (Gutman, 1984) dynamic processes in a differential segment of the dendrite with nonlinear $C-V$ of the membrane in a general case may be represented by the following equation:

$$\frac{r}{2}\frac{R_m}{R_a}\frac{\partial^2 V}{\partial x^2} - R_m C_m \frac{\partial V}{\partial t} - f(V) = 0, \qquad (1)$$

where $f(V) = I(V) \cdot R_m$; $I(V)$ is a nonlinear membrane current density $[mcA/cm^2]$; R_m is the resistance of the membrane unit area at the rest potential $[om \cdot cm^2]; R_a$ is specific resistance of dendroplasma $[om \cdot cm]; c_m$ is the capacity of the membrane unit area $[mcF \cdot cm^2]; V$ is the potential $[mV]$, dependent, in a general case, on the axial cable coordinate $x[cm]$ and time $t\,[ms]$.

Since, at the given stage of the study, our aim is to consider a stationary nonlinear dendrite, equation (1) is simplified as follows:

$$\frac{d^2V}{dx^2} = \frac{1}{\lambda^2} f(V), \tag{2}$$

where $\lambda = \left[(R_m/R_a) \cdot r/2\right]^{1/2}$ is the characteristic length $[cm]$, i.e., the distance along the axis of the cable up to the point at with the potential of the semi-finite dendrite is reduced e times; r is the radius of the dendrite $[cm]$.

A nonlinear N-shaped $C - V$ of the membrane is approximated by the cubic polynomial (Gutman, Shimoliunas, 1973):

$$f(V) = \frac{V^3 - 3V^2 + 3V(1 - b^2)}{3(1 - b^2)}, \tag{3}$$

where b is the constant "controling" the depth of N-shaping of the curve.

The shape of the curve (3) for $b = 0.55$ is shown in Fig. 2. As we see, such $f(V)$ has two stable points: $V = 0$ (rest potential) and $V = H$ (stable depolarization), and one unstable point, $V = h$.

For the semi-infinite cable the input resistance equals $w[om]$, where $w = \left[R_m \cdot R_a/(2\pi^2\, r^3)\right]^{1/2}$. Introducing a dimensionless length $\widetilde{x} = x/\lambda$ into equation (2) we obtain:

$$\frac{d^2V}{d\widetilde{x}^2} = f(V); \tag{4}$$

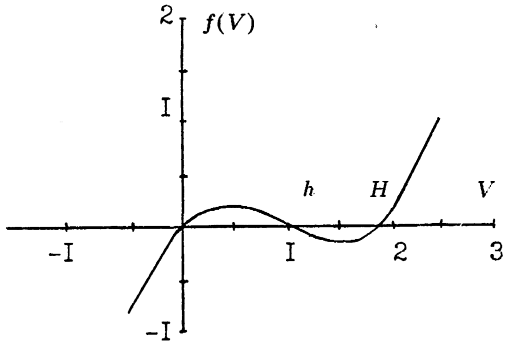

Fig. 2a. The current-voltage characteristic $f(V)$ of the membrane. The curve is described by a cubic polynomial (3), $b = 0.55$.

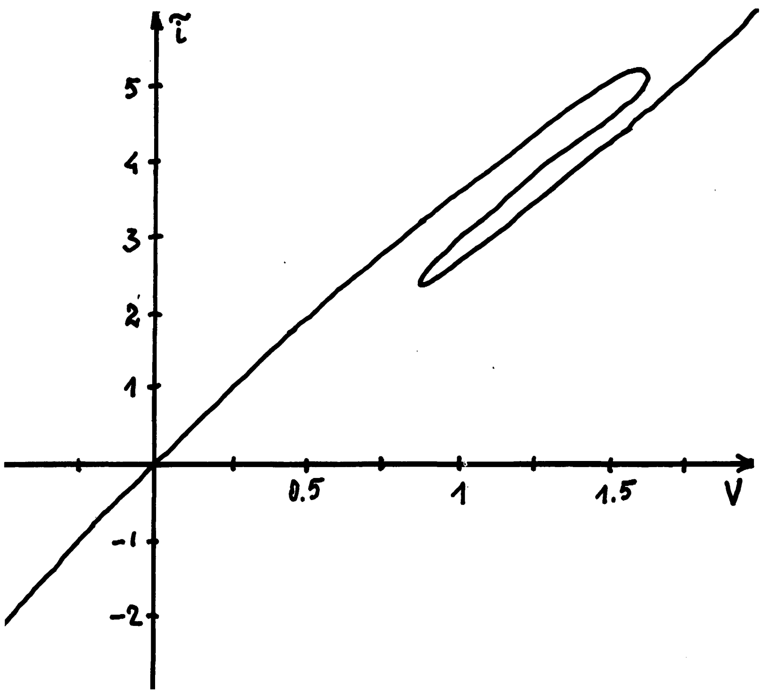

Fig. 2b. The output current-voltage characteristic of the cable, $R_l = w/3$; $R_1 = R_2 = \infty$.

Since $\frac{dV}{dx} = -\frac{R_a}{\pi r^2}\, i = -\frac{w}{\lambda}\, i$, the corresponding current is equal to $\frac{dV}{dx} = -wi = -\tilde{i}$, where i is the axial current of the cable.

Units of the measurment of $\tilde{i}$ are the same as the units of the potential $[mV]$. Correspondingly, the dimensionless resistance $\tilde{R} = R/w$ is determined by the equality $i = V/R = V/(\tilde{R}w)$.

The basic stage of numerical simulation of the described above dendritic branch is mapping of the final cable segment and its synaptic currents into the output $C-V$ of the proximal end of the segment. The computing procedure is based on the usual numerical solution of differential equation (2).

The whole computing procedure consisted of the mapping of N-shapped $C-V$ of three segments of the cable. Here the input $C-V$ of the considered part of the cable includes the linear $C-V$ of the synapse:

$$\tilde{i} = \frac{E - V}{\tilde{R}_j}, \tag{5}$$

where R_j is resistance of the $j^{\underline{\text{th}}}$ synapse, E is its electromotive force. The output $C-V$ of the given segment is the input $C-V$ for the next segment. The input $C-V$ for a first segment of the cable is the N-shaped $C-V$ of the membrane of distal end of the branch.

Completing the computing proccedure we subtract the $C-V$ of the load $\tilde{i} = V/\tilde{R}_l$ from the output $C-V$ of the whole cable; here $\tilde{R}_l$ denotes load resistance. Zero values of the current of this obtained $C-V$ express physically realizable states of the dendritic cable in these conditions.

The load was selected in accordance with the branching low of the dendrite introduced by (Rall, 1959): $D^{3/2} = d_1^{3/2} + d_2^{3/2}$, here D is the diameter of the parent branch and d_1 and d_2 diameters of daughter branches. In accordance with that ohmic loading per daughter branch is equal to $R_l = w/3$ if $d_1 = d_2$.

3. Results. The selected parameters of the cable correspond either to two stable and one unstable point of the output potential (Fig. 4a,b) or to one stable point (Fig. 3a-c). The equilibrium output potential under the conditions of natural functioning of a dendrite means a zero output current under the given load (Gutman, 1984).

If one restricts oneself to the binary logic, then the stable point, close to the rest potential, corresponds to the logic variable's value "0", and the stable point, at which the distal end of a dendrite is stably depolarized, corresponds to the logic variable's value "1".

For the beginning we restrict ourselves to the demonstration of easily reproducable logic functions "AND" and "OR".

The unconditional realization of logic multiplication is presented in Fig. 3. Under separate activation of the first or second synapse ($R_1 = 15w$; $R_2 = 15w$), (Fig. 3a; b) a dendritic branch always remains in the state of equilibrium rest ("0"); under simultaneous activation of synapses ($R_1 = R_2 = 15w$) the branch always transfers to the state of stable depolarization ("1"), (Fig. 3c).

Fig. 4a; b illustrates clearly that both stable states of a dendritic branch are possible :"0" (rest state) and "1" (stable depolariration); here either synapse 1 ($R_1 = 10w$) or synapse 2 ($R_2 = 10w$) are active. It is the prehistory of the dendritic branch that determines which of the states will be realized: if the rest state was initial then "0" will be realized, and if the branch was stably depolarized initially, then "1" will be realized. Activation of both synapses ($R_1 = R_2 = 10w$) always transfers the branch to the state of stable depolarization (state "1", Fig. 4c) not depending of the initial state. If the initial

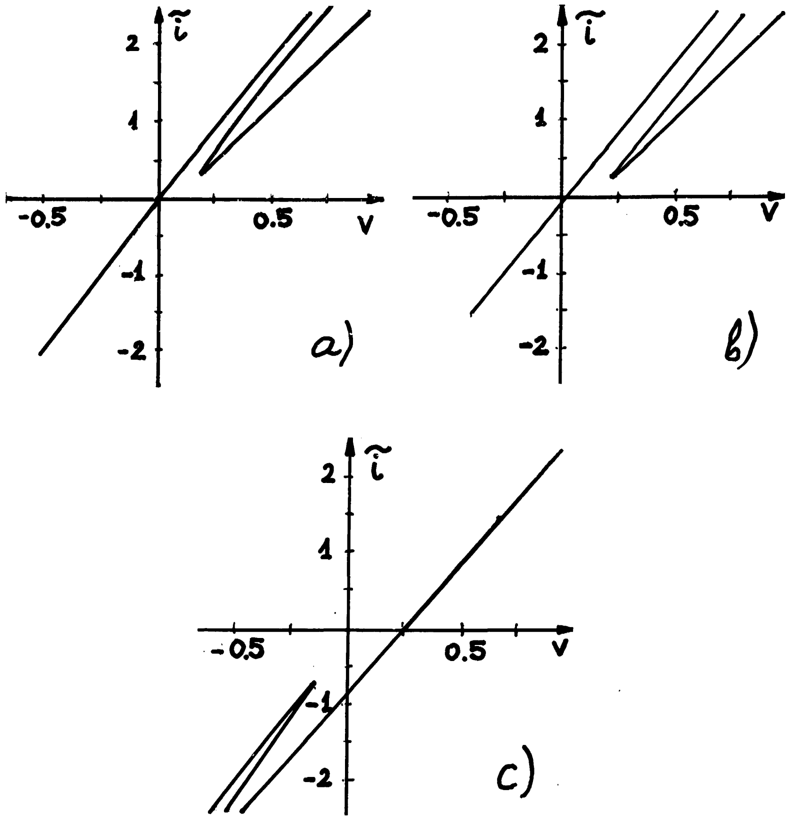

Fig. 3. The example of the unconditional function "AND". Output current-voltage characteristics of the cable are presented with one or two active stimulating synapses:
a) $R_1 = 15w$;
b) $R_2 = 15w$;
c) $R_1 = R_2 = 15w$;
The lefthand branch on a) and b) corresponds to the vicinity of the rest potential, the right-hand branch on c) corresponds to the vicinity H (stable depolarization) at the distal end.

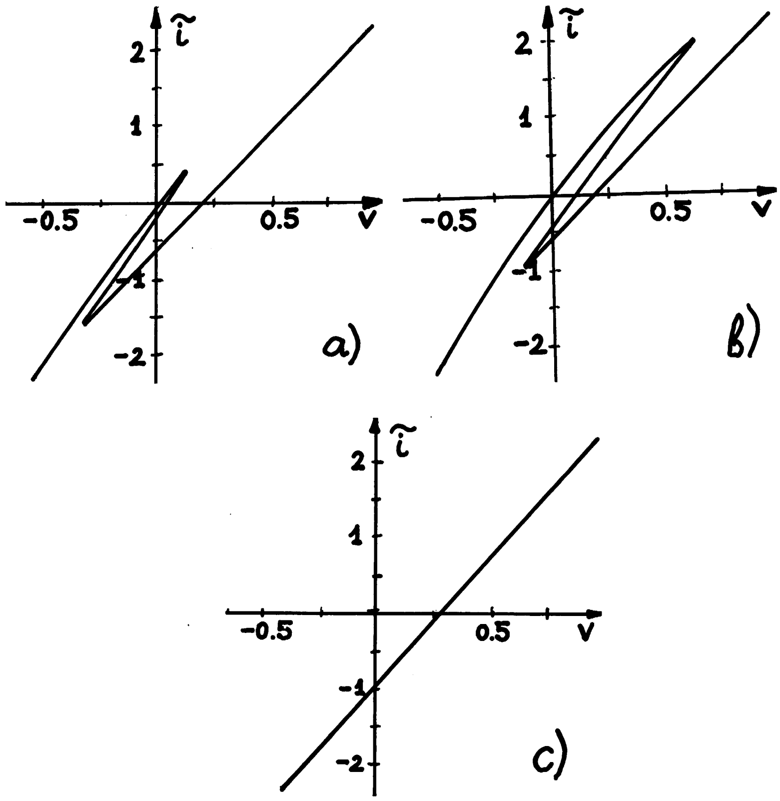

Fig. 4. The example of conditional functions "*AND*" and "*IDENTITY*" :

a) $R_1 = 10w$;
b) $R_2 = 10w$;
c) $R_1 = R_2 = 10w$;

If the initial condition of the dendrite is a rest potential, then function "*AND*" is realized; if the stable depolarization is at the beginning, then the function "*IDENTITY*" is realized. The lefthand branch on a) and b) corresponds to the vicinity of the rest potential; the righthand branches on a) and b) and the whole characteristic on c) corresponds to the vicinity of H ,(stable depolarization) at the distal end.

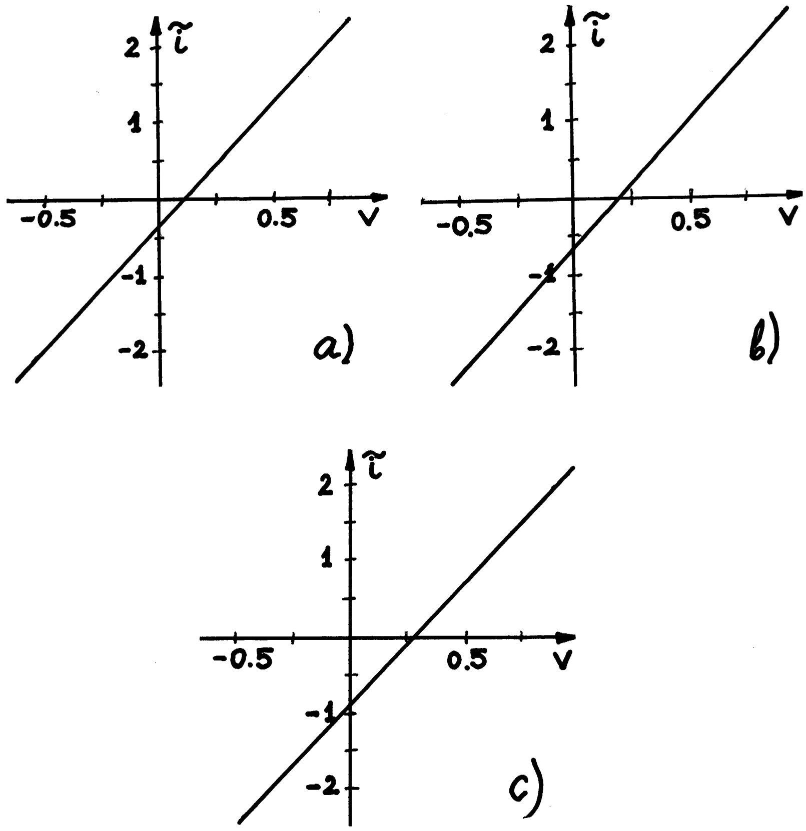

Fig. 5. The example of the conditional functions "OR" and "$IDENTITY$":
a) $R_1 = 5w$;
b) $R_2 = 5w$;
c) $R_1 = R_2 = 5w$;
The pictures illustrates the righthand branch of current-voltage characteristics. If the initial condition of the dendrite is a rest potential, then function "OR" is realized; if a stable depolarization is at the begining, then the function "$IDENTITY$" is realized.

state is "0", then described situation realizes the conditional logic multiplication ("AND").

Fig. 5 presents the realization of the logic function "OR". In this case both a separate activation of synapses ($R_1 = 5w$) and ($R_2 = 5w$) and their simultaneous activation ($R_1 = R_2 = 5w$) ensures the state of stable depolarization of the dendritic branch ("1" in the output).

4. Discussion. We have shown using the numerical model that the simplest logic functions are easily realizable on a separate nonlinear dendritic branch. This possibility is of some interest to neurophysiologists (Miller, 1990) since it expands functional possibilities of the nerve all. For example, a classical object of neurophysiology – a motor neuron of spinal cord of the cat (Eccles, 1966) possesses hundreds of dendritic branches, therefore, it may perform a lot of logic operations. If the dendritic branch is not distal, then not only synapses but also daughter branches may serve as its inputs.

The complexity of a neuron even at the stage of the given above preliminary consideration can be compared to the complexity of a small integral scheme, while the classic neuron with linear dendrites is only a spatial threshold integrator of thousands of independent inputs. A developed conception of a neuron is also important for a better understanding of the complexity of brain on the whole.

On the other hand, introduction of quasineuron elements, performing more complicated logic functions, into neuronetworks enables one to extend the possibilities of quasineuron networks and to achieve a greater efficiency of associative storage and data processing.

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