OPTIMAL IDENTIFICATOR OF DYNAMIC SYSTEMS

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Abstract. Identification problems of linear dynamic systems in the class of parametric mathematical models are considered. A method of calculating of guaranteed estimates of indefinite parameters is proposed. The method is based on the specific semiinfinite extremal problems solution.

Key words: synthesis, identificator, support, plan.

1. Introduction. Identification of perturbations and dynamic object parameters composes an important part of general optimal control theory (Leondes, 1976; Eykhoff, 1975).

A new approach to the problem of control systems identification is proposed in the paper. It consists in receiving of the concrete numeric characteristics of the possible parameters changes sets by means of the solution of extremal problem constructed in special way. Two types of such problem solutions are investigated. They are: program type and feedback one. Algorithm of program solution construction is based on specific extremal problems methods elaborated by authors earlier (Gabasov et. al., 1986, 1991). Program solution is the base for optimal identificator construction processing in the mode

of real time incomplete and inexact information on current system states coming from measuring device.

Perturbations acting on the control system and the measurement errors also are considered to be uncertain and may take the values from the given sets. Opposite to stochastic methods of information processing assigned for getting the probability characteristics of unknown values in the centre of the proposed approach there are a structure of sets of possible values of perturbations and parameters and the reasons of the necessity of characteristics received for them. Just for these reasons the refusal from classical methods of conditional optimization and involving the constructive theory of extremal problems are explained (Gabasov et. al., 1986, 1991).

In the paper the concrete solutions of a number of problems are given on the base of a general approach to the identification problems (Gabasov and Kirillova, 1991).

2. The perturbation identification problem.

Let the dynamic system on the interval $T = [t_*, \ t^*]$ is described by the equation

$$\dot{x} = A(t)x + \omega(t), \quad x(t_*) = x_0, \tag{1}$$

where x is a n-vector of state, $A(t)$, $t \in T$, is a piece-wise continuous $n \times n$ matrix function, $\omega(t)$ is a n-vector function of unknown perturbations.

Assume that a priori information on perturbations has the form

$$\omega(t) = \omega_0(t) + \sum_{i=1}^{q} w_i \omega_i(t), \tag{2}$$

$$w = (w_1, w_2, \ldots, w_q) \in \check{W} = \{w \in R^q : Gw = f,$$

$$w_* \leqslant w \leqslant w^*\},$$

where $\omega_0(t)$, $\omega_1(t), \ldots, \omega_q(t)$, $t \in T$, are known piece-wise continuous functions, G is a known $n \times q$ matrix, f, w_*, w^* are known vectors.

The efficiency of control can be reduced considerably while fulfilling optimization of system (1) in the presence of perturbations (2). Therefore, to decrease the uncertainty of perturbations it is appropriate that the control procedure should be supplemented by the identification procedure.

Assume that there is the measuring device

$$y = C(t)x + \xi, \quad y \in R^m, \tag{3}$$

writing the output signal with the measurement error $\xi(t)$, $t \in T$. A piece-wise continuous $m \times n$ matrix function $C(t)$, $t \in T$, is considered to be known. Relative to the measurement errors $\xi(t)$, $t \in T$, we shall assume that any piece-wise continuous m-vector function satisfying the unequality

$$\xi_* \leqslant D(t)\xi(t) \leqslant \xi^*, \quad t \in T, \tag{4}$$

can given such error.

Let the measuring device (3) wrote the signal $y(t)$, $t \in T_\theta = [t_*, \theta]$, where θ is some moment from the interval of control. This information allows to delete from the a priori distribution $\overset{\lor}{W}$ of perturbation parameters the elements that were not realized automatically in the situation considered. The set $\overset{\land}{W}$ consisting of those and only those elements $w \in \overset{\lor}{W}$, which together with some errors $\xi(t)$, $t \in T_\theta$, are able to generate the observed signal $y(t)$, $t \in T_\theta$, will be called a posteriori parameter distribution.

Intending to apply the results of identification for linear problems of optimal control (Gabasov and Kirillova, 1991) we'll consider the problem of identification of perturbations, consisting of calculation of the linear estimate

$$\hat{\alpha}_\theta = \max h'x(t^* \mid w), \quad w \in \overset{\land}{W}, \tag{5}$$

where $x(t^* \mid w)$ is a terminal state of system (1), corresponding to the value w of the vector of perturbation parameters (2).

Let $F(t, \tau)$, t, $\tau \in T$, be a fundamental matrix of solutions of homogeneous part of system (1),

$$z(t) = y(t) - \int\limits_{t_*}^{t} C(t)F(t,\tau)\omega_0(\tau)d\tau,$$

$$M(t) = [m_i(t), \; i = \overline{1,q}], \quad \hat{h}' = (\hat{h}_i, \; i = \overline{1,q}),$$

$$m_i(t) = \int\limits_{t_*}^{t} C(t)F(t,\tau)\omega_i(\tau)d\tau,$$

$$\hat{h}_i = y(t) - \int\limits_{t_*}^{t^*} h'F(t^*,\tau)\omega_i(\tau)d\tau, \quad i = \overline{1,q}.$$

In new designations problem (5) takes the form:

$$\hat{\alpha}_\theta = \max \hat{h}'w,$$

$$\xi_* \leqslant D(t)\big(z(t) - M(t)w\big) \leqslant \xi^*, \quad t \in T_\theta; \qquad (6)$$

$$Gw = f, \quad w_* \leqslant w \leqslant w^*.$$

The constructive theory and the results of numerical experiments of semiinfinite extreme problem solution are given in (Gabasov et. al., 1986; Kirillova et. al., 1986).

For the synthesis of optimal systems one needs to calculate the estimates (5) in the mode of real time. It can be done by means of an optimal identificator generating the elements of solution for the problem (6) continuously. In calculations cited below let us restrict ourselves to the case $G = 0$, $f = 0$, $m = 1$, $D(t) = 1$, $t \in T$.

According to (Gabasov et. al., 1986; Kirillova et. al., 1986) the optimal support plan $\{w(\theta), \; S_{\mathrm{sup}}(\theta)\}$, is the solution of simple problem (6). The support $S_{\mathrm{sup}}(\theta) = \{\tau_{\mathrm{sup}}(\theta), J_{\mathrm{sup}}(\theta)\}$ consists of the totality $\tau_{\mathrm{sup}}(\theta)$ of time moments $\tau_i(\theta)$,

$i = \overline{1,l}$ $(t_* \leqslant \tau_1(\theta) < \tau_2(\theta) < \ldots < \tau_l(\theta) \leqslant \theta)$ and the set of indexes $J_{\text{sup}}(\theta) \subset J$, $J_{\text{sup}}(\theta) = l$. The support $l \times l$ – matrix

$$M_{\text{sup}}(\theta) = M(\tau_{\text{sup}}(\theta),\ J_{\text{sup}}(\theta))$$

$$= \begin{bmatrix} M_j(\tau_i(\theta)),\ j \in J_{\text{sup}}(\theta), \\ i = \overline{1,l} \end{bmatrix},\quad \det M_{\text{sup}}(\theta) \neq 0,$$

the vector of potentials

$$v'(\theta) = v'(\tau_{\text{sup}}(\theta)) = \hat{h}'_{\text{sup}} M_{\text{sup}}^{-1}(\theta),$$

and the vector of estimates

$\Delta'_N = \Delta'(J_N \mid \theta) = v'(\theta) M(\tau_{\text{sup}}(\theta),\ J_N) - \hat{h}'_N$,

$J_N = J \backslash J_{\text{sup}}(\theta)$, $\hat{h}_{\text{sup}} = \hat{h}(J_{\text{sup}}(\theta))$, $\hat{h}_N = h(J_N)$, correspond to it.

By the support S_{sup} and its accompanying vectors $v(\tau_{\text{sup}}(\theta))$, $\Delta(J_N \mid \theta)$, with the help of the optimality criterion (Gabasov et. al., 1986; Kirrilova et. al., 1986) we can test the plan $w(\theta)$ for optimality.

Let θ be a moment of time such that

1) functions $A(t)$; $C(t)$; $\omega_i(t)$, $i = \overline{0,q}$, $t \in T_\theta$, are continuous together with second derivatives in the neighbourhood of points $\tau_i(\theta)$, $i = \overline{1,l}$;

2) $w_{*\text{sup}} < w_{\text{sup}}(\theta) < w^*_{\text{sup}}$;

3) $\xi_* < z(t) - M(t)w(\theta) < \xi^*$, $t \in T_\theta \backslash \tau_{\text{sup}}(\theta)$;

4) $v(\tau_{\text{sup}}(\theta)) \neq 0$, $\Delta(J_N \mid \theta) \neq 0$;

5) $\tau_l(\theta) = \theta$, $\dot{z}(\theta) - \dot{M}(\theta)w(\theta) \neq 0$;

6) $\ddot{z}(\tau_i(\theta)) - \ddot{M}(\tau_i(\theta))w(\theta) \neq 0$, $i = \overline{1,l-1}$.

Than the totality $J_{\text{sup}}(\theta)$ remains unchanged in the neighbourhood of the point θ, the components $\tau_{\text{sup}}(\theta)$, $w_{\text{sup}}(\theta)$, of solution $\{w(\theta),\ S_{\text{sup}}(\theta)\}$ of problem (6) satisfy the following system of differential equations

$$M_{\text{sup}}(\tau_i(\theta))\dot{w}_{\text{sup}} = 0,\ i = \overline{1,l-1};$$

$$M_{\text{sup}}(\theta)\dot{w}_{\text{sup}} = \dot{z}(\theta) - \dot{M}(\theta)w(\theta);\ \tau_l = 1; \qquad (7)$$

$$(\ddot{z}(\tau_i) - \ddot{M}(\tau_i)w)\dot{\tau}_i = M_{\text{sup}}(\theta)\dot{w}_{\text{sup}},\ i = \overline{1,l-1}.$$

Correlations (7) will be called the equations of optimal identificator of perturbations. The initial conditions for (7) can be obtained for $\theta = t_*$ from a priori distribution of parameters $\check{W}$. The form of equations (7) undergoes qualitative changes in the moments of violation of conditions 1)-6).

3. The input device identification problem. Consider the control system

$$\dot{x} = A(t)x + b(t)u, \quad x(t_*) = x_0, \tag{8}$$

with incomplete given input device $b(t)$, $t \in T$.

Assume that function $b(t)$, $t \in T$, has the form

$$b(t) = b_0(t) + \sum_{i=1}^{q} w_i b_i(t),$$

where $b_0(t), b_1(t), \ldots, b_q(t)$, $t \in T$, are the known piece-wise continuous functions, $w = (w_1, w_2, \ldots, w_q)$ is q-vector from the set $\check{W}$.

The piece-wise continuous control $u(t)$, $t \in T$, being known, the system (8) converges to (1), that allows to use the results of the previous item for solution of problem of input device identification.

4. The control object identification problem. Let us now investigate the system

$$\dot{x} = A(t, w)x, \quad x(t_*) = x_0, \tag{9}$$

with $n \times n$ - matrix function $A(t, w)$, $t \in T$, in the form

$$A(t) = A_0(t) + \sum_{i=1}^{q} w_i A_i(t),$$

where $A_0(t), A_1(t), \ldots, A_q(t)$, $t \in T$, are the known piece-wise continuous $n \times n$ matrix functions, $w = (w_1, w_2, \ldots, w_q)$ is q-vector from the set $\check{W}$.

Supply the system (9) by the measuring device (3), (4). The problem of identification of object parameters (9) formally remains the same as in item 2. But now the solution is connected with nonlinear semiinfinite extreme problem

$$\hat{\alpha}_\theta = \max h' x(t^* \mid w),$$

$$\xi_* \leqslant D(t)\big(y(t) - C(t)x(t \mid w)\big) \leqslant \xi^*, \quad t \in T_\theta; \qquad (10)$$

$$Gw = f, \quad w_* \leqslant w \leqslant w^*.$$

We obtain an optimal identificator for the case $G = 0$, $f = 0$, $m = 1$, $D(t) = 1$, $t \in T$.

Let $w^0(\theta)$ be an optimal plan of the simple problem (10), $x^0(t \mid w^0)$, $t \in T$, is the corresponding to it solution of problem (9).

Generalizing the technique for investigation of the semiinfinite linear problems (Gabasov et. al., 1986) and the constructive approach to solution of nonlinear problems (Gabasov et. al., 1991) let us write the necessary support conditions of optimality in the problem (10) for $w^0(\theta)$.

Designate $r(w^0) = h'f(t^*, w^0)$, $M(t, w^0) = C(t)f(t, w^0)$, where the function $f(t, w^0)$, $t \geqslant t_*$, is the solution of inhomogeneous linear differential equation

$$\dot{f} = A(t, w^0)f + B(t, x^0), f(t_*, w^0) = 0^{n \times q},$$

$$B(t, x^0) = \frac{\partial}{\partial w}(A(t, w)x^0) = [A_i(t)x^0, \quad i = \overline{1, q}].$$

Let $S_{\text{sup}}(\theta) = \{\tau_{\text{sup}}(\theta), \ J_{\text{sup}}(\theta)\}$, $\tau_{\text{sup}}(\theta) = \{\tau_i(\theta) \in T_\theta, \ i = \overline{1, l}\}$, $J_{\text{sup}}(\theta) \subset J$, $|J_{\text{sup}}(\theta)| = l$.

We shall call the totality $S_{\text{sup}}(\theta)$ a local support of problem (10), if the matrix

$$M_{\text{sup}}(\theta, w^0) = \begin{bmatrix} M_j(\tau_i(\theta), w^0), & j \in J_{\text{sup}}(\theta), \\ & i = \overline{1, l} \end{bmatrix}$$

is nongenerate.

Vector of potentials

$$v'(\theta, \ w^0) = r'(J_{\text{sup}}(\theta) \mid w^0) M_{\text{sup}}^{-1}(\theta, w^0),$$

and vector of estimates
$$\Delta'_N(\theta, w^0) = v'(\theta, w^0) M(\tau_{\text{sup}}(\theta), \ J_N \mid w^0) - r'_N(J_N \mid w^0),$$
$J_N = J \backslash J_{\text{sup}}(\theta)$, are constructed by the support $S_{\text{sup}}(\theta)$.

Assume that θ is such moment of time that for the totality $\{w^0(\theta), \ S_{\text{sup}}(\theta)\}$ the conditions are fulfilled
1) functions $y(t)$; $C(t)$; $A_i(t)$, $i = \overline{0, q}$, $t \in T_\theta$, are continuous together with second derivatives in the neighbourhood of points $\tau_i(\theta)$, $i = \overline{1, l}$;
2) $w_{*\text{sup}} < w_{\text{sup}}^0(\theta) < w_{\text{sup}}^*$;
3) $\xi_* < y(t) - C(t)x^0(t \mid w^0) < \xi^*$, $t \in T_\theta \backslash \tau_{\text{sup}}(\theta)$.

Then for optimality of the plan $w^0(\theta)$ in problem (10) the fulfillment of correlations is necessary:

$$\Delta_j(\theta, w^0) \leqslant 0 \quad \text{for } w_j^0 = w_{*j};$$

$$\Delta_j(\theta, w^0) \geqslant 0 \quad \text{for } w_j^0 = w_{*j};$$

$$\Delta_j(\theta, w^0) = 0 \quad \text{for } w_{*j} < w_j^0(\theta) < w_j^*; \quad j \in J_N.$$

$$v_i(\theta, w^0) \geqslant 0 \quad \text{for } y(\tau_i) - C(\tau_i)x^0(\tau_i \mid w^0) = \xi^*;$$

$$v_i(\theta, w^0) \leqslant 0 \quad \text{for } y(\tau_i) - C(\tau_i)x^0(\tau_i \mid w^0) = \xi_*;$$

$$v_i(\theta, w^0) = 0 \quad \text{for } \xi_* < y(\tau_i) - C(\tau_i)x^0(\tau_i \mid w^0) < \xi^*;$$

$$i = \overline{1, l}.$$

Supply the correlations 1)–3) by conditions
4) $v(\theta, w^0) \neq 0$, $\Delta_N(\theta, w^0) \neq 0$;
5) $\tau_l(\theta) = \theta$, $\dot{y}(\theta) - (\dot{C}(\theta) + C(\theta)A(\theta, w^0))x^0(\theta \mid w^0) \neq 0$;
6) $\ddot{y}(\tau_i) - [\ddot{C}(\tau_i) + 2\dot{C}(\tau_i)A(\tau_i, w^0) + C(\tau_i)(\dot{A}(\tau_i, w^0) + A^2(\tau_i, w^0))]x^0(\tau_i \mid w^0) \neq 0$, $i = \overline{1, l - 1}$.
Conditions 1)–6) in the neighbourhood θ being fulfilled,

equations of optimal identificator have the form:

$$\dot{x} = A(t,w)x; \quad \dot{f} = A(t,w)f + B(t,x);$$

$$M_{\mathrm{sup}}(\tau_i, w)\,\dot{w}_{\mathrm{sup}} = 0, \quad i = \overline{1, l-1};$$

$$M_{\mathrm{sup}}(\theta, w)\,\dot{w}_{\mathrm{sup}} = \dot{y}(\theta) - (\dot{C}(\theta) + C(\theta)A(\theta, w))x(\theta \mid w);$$

$$[\ddot{y}(\tau_i) - [\ddot{C}(\tau_i) + 2\dot{C}(\tau_i)A(\tau_i, w) + C(\tau_i)(\dot{A}(\tau_i, w)+$$

$$A^2(\tau_i, w))]x(\tau_i \mid w)]\dot{\tau}_i = \dot{M}_{\mathrm{sup}}(\tau_i, w)\dot{w}_{\mathrm{sup}}; \quad \dot{\tau}_l = 1.$$

5. Example. Let us illustrate the results for the problem of identification of perturbation $\omega(t) = w_1 t + w_2$ of dynamic system:

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \omega(t), \quad x_1(0) = 0, \quad x_2(0) = 4,$$

$$0 \leqslant w_1 \leqslant 1, \quad -1 \leqslant w_2 \leqslant 0.$$

Use the measuring device $y = x_2 + \xi$, $0 \leqslant \xi \leqslant 1$. Assume that it registered the signal $y(t) \equiv 4$. Calculate the estimate

$$\hat{\alpha}_\theta = \max_{w \in \mathring{W}} h'x(6 \mid w).$$

For $h = (1/2, -2/9)$ the problem has the form:

$$\max w_1 - w_2, \quad -1 \leqslant w_1 t^2/2 + w_2 t \leqslant 0, \quad t \in [0, \theta],$$

$$0 \leqslant w_1 \leqslant 1, \quad -1 \leqslant w_2 \leqslant 0.$$

Conditions 1)–6) are fulfilled for the moment $\theta = 2$. Optimal identificator for $2 \leqslant \theta \leqslant 4$ has the form:

$$w_1 t^2/2 + w_2\theta = \theta, \quad \tau = \theta, \quad w_1(2) = 1, \quad w_2(2) = -1.$$

Hence, $w_2^0(\theta) = -1$, $w_1^0(\theta) = 2/\theta$. Condition 2) is violated in point $\theta = 4$. For $\theta \geqslant 4$ the optimal identificator is described by equations

$$\dot{w}_1 \tau^2/2 + \dot{w}_2\tau = 0, \quad w_1\theta^2/2 + w_2\theta = 0,$$

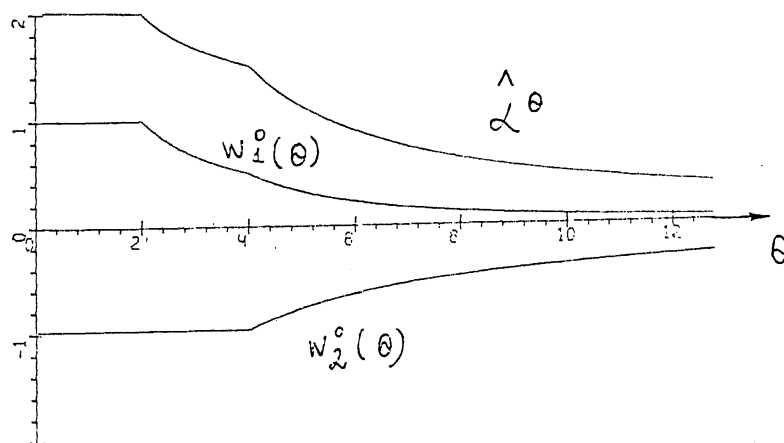$$-w_1\dot{\tau} = \tau\dot{w}_1 + \dot{w}_2, \quad w_1(4) = 1/2, \quad w_2(4) = -1.$$

Fig. 1. The laws of change $W_1^0(\theta)$, $W_2^0(\theta)$, $\hat{a}^\theta$.

The solution of this system has the form $\tau = \theta/2$, $w_1^0(\theta) = 8/\theta^2$, $w_2^0(\theta) = -4/\theta$. The laws of change of the parameters $w_1^0(\theta)$, $w_2^0(\theta)$, and the estimate $\hat{a}_\theta$ are given at the Fig. 1.

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