INFORMATICA, 2006, Vol. 17, No. 3, 393–406

Algorithms for Inner Magic and Inner Antimagic Labelings of Some Planar Graphs

Auparajita KRISHNAA, Manohar Singh DULAWAT
Department of Mathematics and Statistics College of Science, Mohan Lal Sukhadia University Udaipur 313 002 (Rajasthan) India
e-mail: godseeking1@yahoo.com

Received: May 2005

Abstract. In this work labeling of planar graphs is taken up which involves labeling the p vertices, the q edges and the f internal faces such that the weights of the faces form an arithmetic progression with common difference d. If $d = 0$, then the planar graph is said to have an Inner Magic labeling; and if $d \neq 0$, then it is Inner Antimagic labeling. Some new kinds of graphs have been developed which have been derived from Wheels by adding vertices in a certain way and it is proposed to give new names to these graphs namely Flower-1 and Flower-2. This paper presents the algorithms to obtain the Inner Magic and Inner Antimagic labeling for Wheels and the Inner Antimagic labeling for Flower-1 and Flower-2. The results thus found show much regularity in the labelings obtained.

Key words: vertex label, edge label.

1. Introduction

An Antimagic labeling is an edge labeling of the graph with the integers $1, 2, \ldots, e$ so that the weight at each vertex is distinct. The *weight* of a vertex under an edge labeling is the sum of all edge labels incident on that vertex. Variations of Antimagic labeling have been studied by different authors. Hartsfield and Ringel (1989) conjectured that simple, finite and connected graph other than K_2 admits an Antimagic edge labeling and by dualization, this gives a conjecture on vertex labeling of a special class of hypergraphs. Sonntag (2002) discusses existence of Antimagic vertex labeling of classes of hypergraphs like Cacti, Cycles, Wheels and the existence and nonexistence of the Antimagic vertex labeling of Wheels have been discussed in theorems. The method to obtain Antimagic labeling for trees has been given in Krishnaa (2004) where the trees can be drawn as bipartite graph such that there is almost one edge between any two vertices of the two partite sets, and this labeling has also been analysed for Cycles, Paths and Wheels in Krishnaa *et al.* (2004).

Bodendiek and Walther (1993) defined an (a, d) – Antimagic labeling in which the vertex weights form an arithmetic progression starting from a and having a common difference d. If we consider a plane graph G drawn such that no edges cross each other then for $G = (V, E, F)$, its faces are considered including the unique face of unbounded area where f is the number of faces. Baca, Lin and Miller (2001) opted that a bijection

$g: VUEUF \rightarrow \{1, 2, \ldots, v + e + f)$ will be called a labeling of type (1,1,1), where V, E and F are the sets of vertices, edges and faces respectively and the bijection $h: F \rightarrow \{1, 2, \ldots, f\}$ is called a face labeling. The *weight* $w(x)$ of a face x under such a labeling is the sum of labels of the surrounding vertices, edges and the face label.

A labeling of type (1,1,1) of plane graph is called d-Antimagic if for every number s the set of s-sided face weights is $W_s = \{a_s, a_s + d, a_s + 2d, \ldots, a_s + (f_s - 1)d\}$ for some integers a_s and $d,\ d \geqslant 0$ where f_s is the number of s-sided faces. Different sets of W_s allowed for different s. If $d = 0$, then it is Magic. In other words, the problem of labeling the edges, vertices and faces for a planar graph is such that the sequence of weights of the faces formed by adding the labels of vertices, edges and face forms an arithmetic progression with a common difference d. Magic (0-Antimagic) labelings of type (1,1,1) for Grids and Honeycombs are given in Baca (1992), Baca and Miller (1992). Ko-Wei-Lih (1983) gives the Magic labelings for Wheels, Prisms and Friendship graphs.

In the present work the Inner Antimagic and Inner Magic labelings have been defined where the weight of each internal face is found by adding the labels of the surrounding vertices, edges and the internal face labels. The unbound face is not considered as in the structure of the planar graph it is the internal faces which are more of practical significance particularly so when all the internal faces are bound by the same number of edges. These weights form an arithmetic progression with a common difference d. If $d \neq 0$ then the labeling is Inner Antimagic and if $d = 0$ then the labeling is Inner Magic. Some new kinds of graphs have been developed which have been derived from Wheels by adding vertices in a certain way and it is proposed to give new names to these graphs namely Flower-1 and Flower-2. These labelings have been studied for Wheels, Flower-1 and Flower-2 and much regular patterns have been shown in both kinds of labelings by all the graphs studied.

2. Main Results

In this section, algorithms are being presented which have been developed to obtain the Inner Antimagic and Inner Magic labelings for Wheels and Inner Antimagic labelings for the proposed new kinds of graphs namely Flower-1 and Flower-2. The Wheels exhibit definite patterns in the labelings obtained so much so that the same algorithm with minor modification yields both Inner Magic as well as Inner Antimagic labelings. The algorithms that follows dubbed as ALGORITHM W describe a method to compute Inner Magic and Inner Antimagic labeling for Wheels:

ALGORITHM W

/* Inner Magic and Inner Antimagic labelings for Wheels with p vertices, q edges and f internal faces */

BEGIN

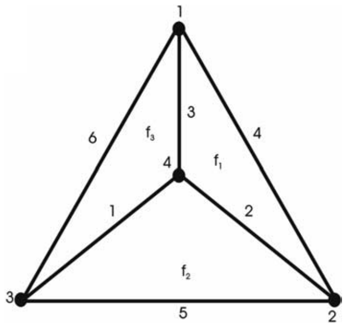
/* label the outer vertices */

1. for $ov = 1$ to $p - 1$ /* ov is the outer vertex */

2. label the outer vertices clockwise with ov /* label the outer vertices clockwise with $1, 2, \ldots, p-1$. */
3. end
4. label the central vertex with p
/* label the inner edges */
5. set edge e connected by vertices labeled p and $p-1$
6. for $i = 1$ to $q/2$ /* i is the inner edge */
7. start with e and label inner edges anticlockwise with i /* start with e and label the inner edges anticlockwise with $1, 2, \ldots, q/2$; q is even in Wheels */
8. end
/* label the outer edges */
9. set the edge e' which is formed by joining the vertices labeled 1 and 2.
10. for $oe = q/2$ to q /* oe is the label for outer edge */
11. start from e', label the outer edges clockwise with oe. /* start with e' and label the outer edges clockwise with $(q/2+1), (q/2+2), \ldots q$. */
12. end
13. for $i = 1$ to $f/*i$ is the internal face */
14. sum the vertex and edge labels
15. arrange the sums in ascending order
16. in this order designate the respective internal faces as $f_1, f_2, \ldots, f_f$.
17. end
/* Inner Antimagic labeling*/
/* label the internal faces */
18. for $i = 1$ to f
19. label the internal face f_i with i /* label the internal faces $f_1, f_2, \ldots, f_f$ with $1, 2, 3, \ldots, f$. respectively */
20. end.
21. Add the labels of vertices, edges and internal faces for each face to get Inner Antimagic labeling.
/* Inner Magic labeling */
/* label the internal faces */
22. in_label= f
23. $i = 0$
24. while $i < f$
25. $i = i + 1$
26. in_label $= f - (i-1)$
27. label internal face f_i with in_label /*label the internal faces, $f_1, f_2, f_3 \ldots, f_f$ with $f, f-1, \ldots, 3, 2, 1$ respectively */
28. end.
29. Add the labels of vertices, edges and internal faces for each face to get Inner Magic labeling.

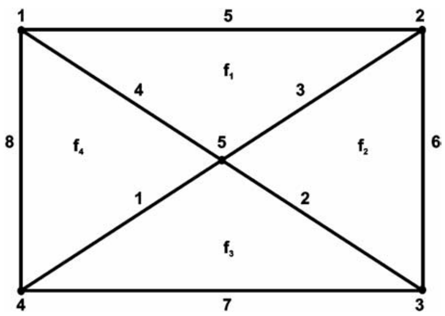
END

Remark that all Wheels differ by 2 in the Inner Antimagic labeling. Moreover the Inner Magic weight number is the middle number of the sequence of Inner Antimagic



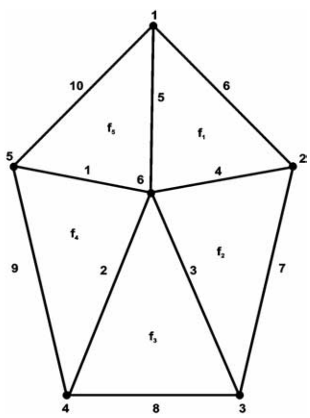
Inner magic internal face labels:
$f_1 = 3$, $f_2 = 2$, $f_3 = 1$.
Inner Magic weight number = 19.
Inner magic internal face labels:
$f_1 = 1$, $f_2 = 2$, $f_3 = 3$.
Inner Antimagic internal face weights:
17, 19, 21.

Fig. 1. Inner Magic and Inner Antimagic labelings for Wheel $W(4, 6)$.



Inner Magic internal face labels:
$f_1 = 4$, $f_2 = 3$, $f_3 = 2$, $f_4 = 1$.
Inner Magic weight number = 24.
Inner Antimagic internal face labels:
$f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$.
Inner Antimagic internal face weights:
21, 23, 25, 27.

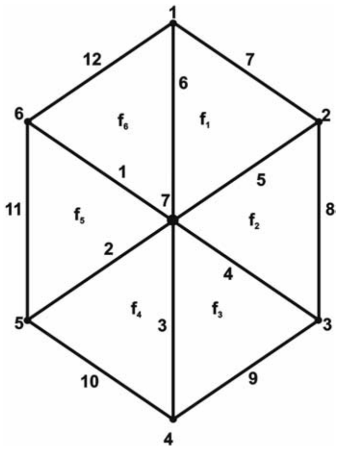
Fig. 2. Inner Magic and Inner Antimagic labelings for Wheel $W(5, 8)$.



Inner Magic internal face labels:
$f_1 = 5$, $f_2 = 4$, $f_3 = 3$, $f_4 = 2$, $f_5 = 1$.
Inner Magic weight number = 29.
Inner Antimagic internal face labels: $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$.
Inner Antimagic internal face weights: 25, 27, 29, 31, 33.

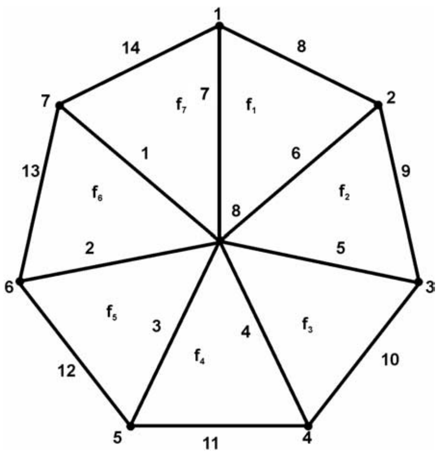
Fig. 3. Inner Magic and Inner Antimagic labelings for Wheel $W(6, 10)$.

labels arranged in ascending order for a particular Wheel. For example, the Inner Magic weight number for the Wheel (4,8) is 19 where the Inner Antimagic labels are 17, 19, 21. For the Wheel (7,12), which has the Inner Antimagic labels of 29, 31, 33, 35, 39, the Inner Magic weight number is 34.



Inner Magic internal face labels: $f_1 = 6$, $f_2 = 5$, $f_3 = 4$, $f_4 = 3$, $f_5 = 2$, $f_6 = 1$.
Inner Magic weight number = 34.
Inner Antimagic internal face labels: $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$, $f_6 = 6$.
Inner Antimagic internal face weights: 29, 31, 33, 35, 37, 39.

Fig. 4. Inner Magic and Inner Antimagic labelings for Wheel $W(7, 12)$.



Inner Magic internal face labels: $f_1 = 7$, $f_2 = 6$, $f_3 = 5$, $f_4 = 4$, $f_5 = 3$, $f_6 = 2$, $f_7 = 1$.
Inner Magic weight number = 39.
Inner Antimagic internal face labels: $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$, $f_6 = 6$, $f_7 = 7$.
Inner Antimagic internal face weights: 33, 35, 37, 39, 41, 43, 45.

Fig. 5. Inner Magic and Inner Antimagic labelings for Wheel $W(8, 14)$.

The next larger Wheel with one more vertex has 5 more for the Inner Magic weight number (i.e., the Inner Magic weight numbers of the Wheels in themselves also form an arithmetic progression of 19, 24, 29, 34 . . . with a common difference of 5). Moreover, the same scheme of vertex and edge labels give Inner Magic and Inner Antimagic labelings where only the face labels need to be reversed. It is to be noted that the sums of vertex and edge labels form a sequence of consecutive integers which makes it result in both Inner Magic and Inner Antimagic labelings just by reversing order of the face labels.

Let us define Flower-1 as a planar graph with one central vertex and rest all being outer vertices and all the internal faces are bound by four edges. All the internal faces in a Wheel are bound by three edges. By adding one vertex to each face of the Wheel, Flower-1 is obtained. The following algorithms ALGORITHM F1-A (for odd no. of internal faces) and ALGORITHM F1-B (for even no. of internal faces) present the methods to label the vertices, edges and internal faces so as to obtain two distinct sets of Inner Antimagic labelings for Flower-1 with p vertices, q edges, f internal faces.

ALGORITHM F1–A

/* Algorithm for two distinct sets of Inner Antimagic labeling for Flower-1 with odd number of internal faces; p vertices, q edges, f internal faces */

BEGIN

/* label outer vertices */

1. for $ov = 1$ to x /* ov is the number of outer alternate vertex and x is the no. of alternate outer vertices */
2. label outer alternate vertex clockwise with ov /* label outer alternate vertices

clockwise with $1, 2, \dots, x$ */

3. end
4. label the central vertex with $x + 1$
5. set the vertex as v' which is just right of the vertex labeled 1
6. for $v = (x + 2)$ to p /* v is label of remaining alternate outer vertices */
7. start from v' and label remaining alternate outer vertices clockwise with v
/* label remaining alternate outer vertices clockwise with $(x + 2),\ (x + 3), \dots, p$ */
8. end
/* label inner edges */
9. set the edge e which is formed by joining vertices labeled $(x + 1)$ and x.
10. for $i = 1$ to y /* y is the number of inner edges */
11. start from e and label the consecutive inner edges anticlockwise with i
/* start from e and label the consecutive inner edges anticlockwise with $1, 2, \dots, y$ */
12. end
/* label outer edges */
13. set e' as the edge formed by joining the vertices labeled with 1 and $x + 2$.
14. for $e = (y + 1)$ to q /* e is the outer edge */
15. start from e' and label consecutive outer edges clockwise with e
/* start from e' and label consecutive outer edges clockwise with
$(y + 1), (y + 2), \dots, q$ */
16. end
17. for $i = 1$ to f /* i is the internal face */
18. sum the vertex and edge labels
19. arrange the sums in ascending order
20. in this order designate the respective internal faces as $f_1, f_2, \dots, f_f$.
21. end
/* First set of Inner Antimagic labeling */
/* label the internal faces */
22. for $i = 1$ to f

23. label the internal face f_i with i /* label the internal faces $f_1, f_2, \ldots, f_f$ with $1, 2, 3, \ldots, f$. respectively */
24. end.
25. Add the labels of vertices, edges and internal faces for each face to get first set of Inner Antimagic labeling.
/* Second set of Inner Antimagic labeling */
/* label the internal faces */
26. in_label $= f$
27. $i = 0$
28. while $i < f$
29. $i = i + 1$
30. in_label $= f - (i - 1)$
31. label internal face f_i with in_label /* label the internal faces, f_1, f_2, $f_3, \ldots, f_f$ with $f, f - 1, \ldots, 3, 2, 1$ respectively */
32. end.
33. Add the labels of vertices, edges and internal faces for each face to get second set of Inner Antimagic labeling.

END

ALGORITHM F1–B

/* Two distinct sets of Inner Antimagic labelings for Flower-1 with even number of internal faces; p vertices, q edges, f internal faces */

BEGIN

/* label outer vertices */
1. for $ov = 1$ to x /* ov is the outer vertex and x is the no. of alternate outer vertices */
2. label alternate outer vertices clockwise with ov and label alternate outer vertices clockwise with $1, 2, \ldots, x$
3. end
4. set the vertex v which is immediate right to the vertex labeled 1
5. for $ov = (x + 1)$ to z /* z is the number of remaining alternate vertices */
6. start with v and label remaining alternate outer vertices clockwise with ov
/* start with v and label remaining alternate outer vertices clockwise with $(x + 1), (x + 2), \ldots, z$ */
7. end
8. this labeling ends at the central vertex with label p.
/* label inner edges */
9. set the edge e which is formed by joining the central vertex labeled by p and vertex labeled x
10. for $i = 1$ to f /* i is the inner edge */
11. start with e and label inner edges anticlockwise with i
/* start with e and label the inner edges anticlockwise with $1, 2, \ldots, f$ */
12. end

/* label outer edges */
13. set the edge e' which is formed by joining the vertices labeled with 1 and $(x+1)$
14. for $oe = (f+1)$ to q /* oe is the outer edge */
15. start from e' and label consecutive outer edges clockwise with oe
/* start from e' and label consecutive outer edges clockwise with $f+1, f+2, \ldots, q$ */
16. end
17. for $i = 1$ to f /* i is the internal face */
18. sum the vertex and edge labels
19. arrange the sums in ascending order
20. in this order designate the respective internal faces as $f_1, f_2, \ldots, f_f$.
21. end
/* First set of Inner Antimagic labeling */
/* label the internal faces */
22. for $i = 1$ to f
23. label the internal face f_i with i /* label the internal faces $f_1, f_2, \ldots, f_f$ with $1, 2, 3, \ldots, f$. respectively */
24. end.
25. Add the labels of vertices, edges and internal faces for each face to get first set of Inner Antimagic labeling.
/* Second set of Inner Antimagic labeling */
/* label the internal faces */
26. in_label $= f$
27. $i = 0$
28. while $i < f$
29. $i = i + 1$
30. in_label $= f - (i-1)$
31. label internal face f_i with in_label /* label the internal faces, $f_1, f_2, f_3, \ldots, f_f$ with $f, f-1, \ldots, 3, 2, 1$ respectively */
32. end.
33. Add the labels of vertices, edges and internal faces for each face to get second set of Inner Antimagic labeling.

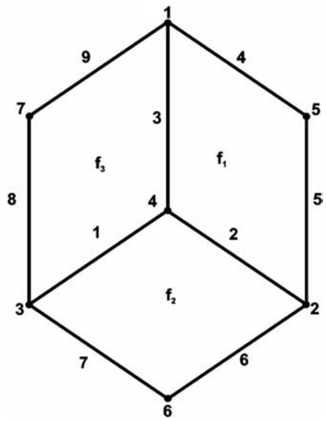
END

Now, let us define Flower-2 as a planar graph with one central vertex and rest all being outer vertices and all the internal faces are bound by five edges. All the internal faces in a Wheel are bound by three edges. By adding two verteices to each face of the Wheel, Flower-2 is obtained. The following algorithm gives the method to obtain two distinct Inner Antimagic labelings for Flower-2 with p vertices, q edges, f internal faces:

ALGORITHM F2

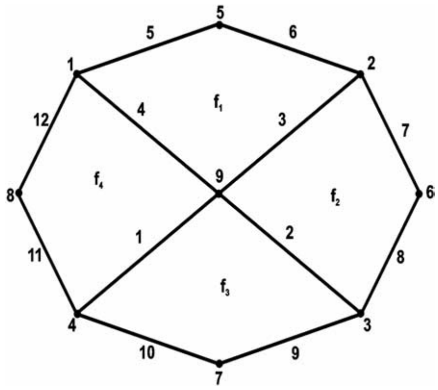
/*Two distinct sets of Inner Antimagic labelings for Flower-2 with p vertices, q edges and f internal faces */

1. Start at an outer vertex which has three edges incident on it. Label it with 1 and and then label every such consecutive outer vertex clockwise with $2, 3, \ldots, x$



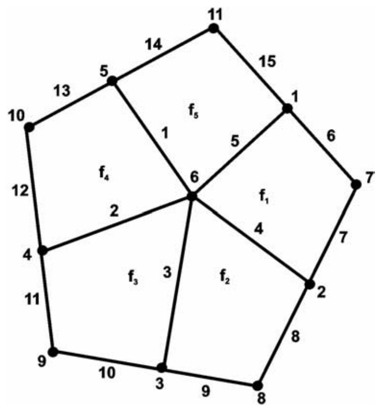
Inner Antimagic internal face weights
and internal face labels are as follows:
(1) 27, 33, 39; $f_1 = 1$, $f_2 = 2$, $f_3 = 3$
(2) 29, 33, 37; $f_1 = 3$, $f_2 = 2$, $f_3 = 1$

Fig. 6. Flower-1 (7,9).



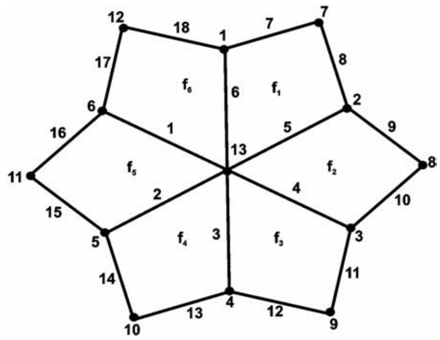
Inner Antimagic internal face weights
and internal face labels are as follows:
(1) 36, 42, 48, 54; $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$.
(2) 39, 43,47, 51: $f_1 = 4$, $f_2 = 3$, $f_3 = 2$, $f_4 = 1$.

Fig. 7. Flower-1 (9,12).



Inner Antimagic internal face weights &
internal face labels are as follows:
(1) 39, 45, 51, 57, 63
$f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$,
(2) 43, 47, 51, 55, 59
$f_1 = 5$, $f_2 = 4$, $f_3 = 3$, $f_4 = 2$, $f_5 = 1$,

Fig. 8. Flower-1 (11, 15).



Inner Antimagic internal face weights &
internal face labels are as follows:
(1) 50, 56, 62, 68, 74, 80.
$f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$, $f_6 = 6$.
(2) 55, 59, 63, 67, 71, 75.
$f_1 = 6$, $f_2 = 5$, $f_3 = 4$, $f_4 = 3$, $f_5 = 2$, $f_6 = 1$.

Fig. 9. Flower-1 (13,18).

where x is no. of such vertices with three edges incident on it and label the central vertex with $x + 1$.

2. set the vertex as v which is immediately left to the vertex labeled 1.
   /* label outer vertices */
3. for $ov = x + 2$ to p /* ov is the remaining outer vertices */
4. start at v and moving anticlockwise label the remaining outer vertices consecutively with ov

/* start at v and moving anticlockwise label the remaining outer vertices consecutively with $x+2, x+3, \dots, p$ */

5. end

/* label inner edges */

6. set e as the inner edge formed by joining vertices labeled by x and $x+1$.
7. for $i = 1$ to y /* i is the inner edge and y is the number of inner edges */
8. start from e and label inner edges anticlockwise consecutively with i

/* start from e and label inner edges anticlockwise consecutively with $1, 2, 3, \dots, y$ */

9. end

/* label the outer edges */

10. set e' the outer edge formed by joining the vertices labeled by 1 and p.
11. for $oe = y+1$ to q
12. start from e' and moving clockwise, label the outer edges consecutively with oe

/* start from e' and moving clockwise, label the outer edges consecutively with $y+1, y+2, \dots, q$ */

13. end
14. for $i = 1$ to f /* i is the internal face */
15. sum the vertex and edge labels
16. arrange the sums in ascending order
17. in this order designate the respective internal faces as $f_1, f_2, \dots, f_f$.
18. end

/* First set of Inner Antimagic labeling */

/* label the internal faces */

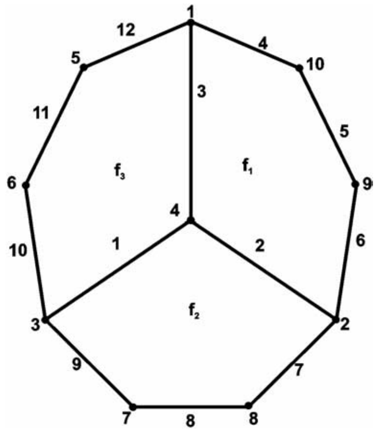
19. for $i = 1$ to f
20. label the internal face f_i with i /* label the internal faces $f_1, f_2, \dots, f_f$ with $1, 2, 3, \dots, f$. respectively */
21. end.
22. Add the labels of vertices, edges and internal faces for each face to get first set of Inner Antimagic labeling.

/* Second set of Inner Antimagic labeling */

/* label the internal faces */

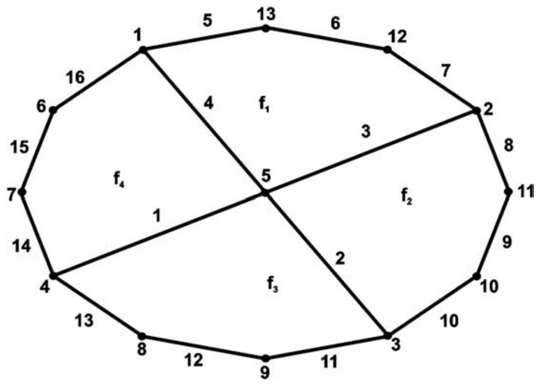
23. in_label $= f$
24. $i = 0$
25. while $i < f$
26. $i = i + 1$
27. in_label $= f - (i-1)$
28. label internal face f_i with in_label /* label the internal faces, $f_1, f_2, f_3, \dots, f_f$ with $f, f-1, \dots, 3, 2, 1$ respectively */
29. end.
30. Add the labels of vertices, edges and internal faces for each face to get second set of Inner Antimagic labeling.

END



Inner Antimagic internal face weights and internal face labels are as follows:
(1) 47, 53, 59; $f_1 = 1$, $f_2 = 2$, $f_3 = 3(2)49, 53, 57$; $f_1 = 3$, $f_2 = 2$, $f_3 = 1$.

Fig. 10. Flower-2 (10,12).

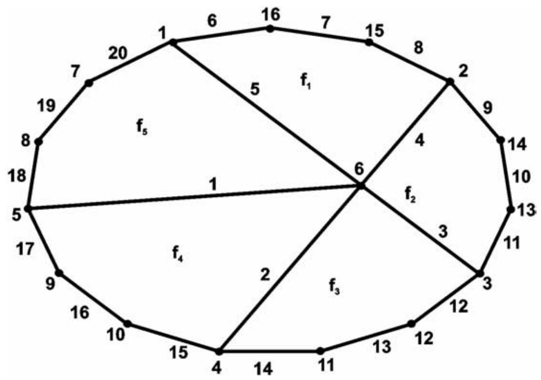


Inner Antimagic internal face weights and internal face labels are as follows:
(1) 59, 65, 71, 77; $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$.
(2) 62, 66, 70, 74; $f_1 = 4$, $f_2 = 3$, $f_3 = 2$, $f_4 = 1$.

Fig. 11. Flower-2 (13,16).

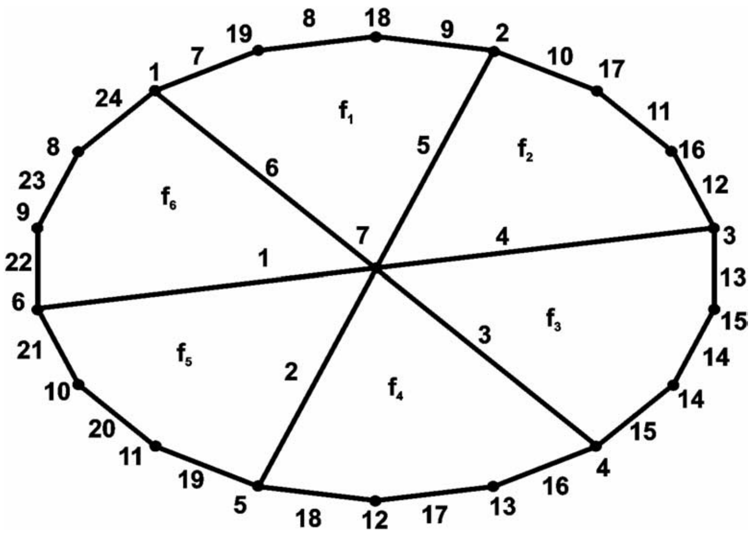
3. Conclusions

In the present work, the Inner Antimagic and Inner Magic labelings for planar graphs have been defined which are obtained by labeling the vertices, edges and internal faces such that the weights of the internal faces form an arithmetic progression with a common difference d. If $d = 0$ then the labeling is Inner Magic labeling otherwise it is Inner Antimagic labeling. No reference of this kind of labeling is found in the available literature. These labelings have particular significance since in the structure of the planar graph it is the internal faces which are more of practical significance and specially when all the internal faces are bound by the same number of edges and hence in these labelings, the



Inner Antimagic internal face weights and internal face labels are as follows:
(1) 71, 77, 83, 89, 95; $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$.
(2) 75, 79, 83, 87, 91; $f_1 = 5$, $f_2 = 4$, $f_3 = 3$, $f_4 = 2$, $f_5 = 1$.

Fig. 12. Flower-2 (16, 20).



Inner Antimagic internal face weights and internal face labels are as follows:
(1) 83, 89, 95, 101, 107, 113; $f_1 = 1$, $f_2 = 2$, $f_3 = 3$, $f_4 = 4$, $f_5 = 5$, $f_6 = 6$.
(2) 88, 92, 96, 100, 104, 108; $f_1 = 6$, $f_2 = 5$, $f_3 = 4$, $f_4 = 3$, $f_5 = 2$, $f_6 = 1$.

Fig. 13. Flower-2 (19, 24).

unbound outer face is not considered.

The Inner Magic and Inner Antimagic labelings have been exhibited by Wheels and the Inner Antimagic labeling has been exhibited by Flower-1 and Flower-2 which are new kinds of planar graphs derived from Wheels by adding more vertices in a particular fashion.

The most significant finding is that even the common differences obtained for Inner

Antimagic labeling has the same value in a particular class of the graph for all the graphs studied. In the Inner Antimagic labelings obtained, all the Flower-1 graphs show the common difference of 4, Flower-2 graphs exhibit the common difference of 4 and 6 and all the Wheels display the common difference of 2. Moreover, two sets of Inner Antimagic labelings are obtained for the Flower-1 and Flower-2 graphs with the same scheme to label the vertex and edge labels and only reversing order of the internal face labels.

Some more special features have been displayed by Wheels which are as follows:

1. The Inner Magic weight number is the middle number of the Inner Antimagic labels arranged in ascending order for a particular Wheel. For example, the Inner Magic weight number for the Wheel W(4,8) is 19 where the Inner Antimagic labels are 17, 19, 21. For the Wheel W(7,12), which has the Inner Antimagic labels of 29, 31, 33, 35, 39, the Inner Magic weight number is 34.
2. The next larger Wheel with one more vertex has 5 more for the Inner Magic weight number, i.e., the Inner Magic weight numbers of the Wheels in themselves also form an arithmetic progression of 19, 24, 29, 34, 39, . . . with a common difference of 5.
3. The same scheme of vertex and edge labels give Inner Magic and Inner Antimagic labelings where only the face labels need to be reversed.

These algorithms can be used in applications where graphs serve as models and in applications such as communications network, circuit design etc. In many computer science applications also these algorithms could be used.

Acknowledgement

Authors acknowledge with thanks the help of the referee for valuable comments and suggestions which led to the improvement of the paper.

References

Baca, M. (1992). On magic labelings of grid graphs. *Ars Combinatoria*, **33**, 295–299.

Baca, M. (1992). On magic labelings of honeycomb. *Discrete Mathematics*, **105**, 305–311.

Baca, M., Q. Lin, M. Miller (2001). Antimagic labelings of grids. *Preprint*.

Baca, M., M. Miller (2006). On d-antimagic labelings of type (1,1,1) for prisms, *JCMCC* (to appear).

Bodendiek, R., G. Walther (1993). Arithmetisch anttimagische graphen, In: K. Wagner and R. Bodendiek (Eds.), *Graphentheorie III*. BI-Wiss. Verl., Mannheim.

Gallian, J.A. (1998). A dynamic survey of graph labeling. *The Electronic Journal of Combinatorics*, **5**, 1–43.

Hartsfield, N., G. Ringel (1989). Supermagic and antimagic graphs. *J. of Recreat. Math.*,**21**(2), 107–115.

Krishnaa, A. (2001). Computer modelling of graph labelings. In *Proc. National Conference on Mathematical and Computational Models*, Coimbatore, Allied Publishers. pp. 293–301.

Krishnaa, A. (2004). A study of the major graph labelings of the trees. *Informatica*, **15**(4), 515–524.

Krishnaa, A., M.S. Dulawat, G.S. Rathore (2004). A study on statistical analysis of antimagic graph labelings. *J. of Rajasthan Academy of Physical Sciences*, **3**(2), 111–120.

Koh-Wei, L. (1983). On magic and consecutive labelings of plane graph. *Utilitas Math.*, **24**, 165–197.

Sonntag, M. (2002). Antimagic vertex labelings of hypergraphs. *Disc. Math.*, **247**, 187–199.

A. Krishnaa has masters in mathematical sciences (USA), masters in computer science and doctoral in interdisciplinary work of mathematics and compuer science. She is teaching mathematics in Mohan Lal Sukhadia University, Udaipur. Her research interests include graph labelings and algorithms.

M.S. Dulawat is working as associate professor of statistics in Mohan Lal Sukjadia University, Udaipur. He has done masters and PhD in statistics and is teaching statistics in the university. His research interests include statistical inference and graph theory.

Vidinio magiško ir vidinio antimagiško ženklinimo algoritmai kai kuriems plokštiesiems grafams

Šiame darbe nagrinėjamas plokščiųjų grafų ženklinimas, apimantis p viršūnių, q briaunų ir f vidinių ciklų taip, kad ciklų svoriai suformuotų aritmetinę progresiją su bendru skirtumu d. Jei $d = 0$, tai sakoma, kad plokščiasis grafas turi Vidinį Magišką (Inner Magic) ženklinimą, jei $d \neq 0$, tai Vidinį Antimagišką (Inner Antimagic) ženklinimą. Yra sudaryti nauji grafų tipai, išvesti iš Ratų (Wheels) pridedant tam tikru būdu viršūnes, ir yra siūloma šiuos grafus vadinti Gėlė-1 (Flower-1) ir Gėlė-2 (Flower-2). Šiame straipsnyje pristatomi Vidinio Magiško ir Vidinio Antimagiško ženklinimo Ratams ir Vidinio Antimagiško ženklinimo Gėlei-1 ir Gėlei-2 algoritmai. Rezultatai rodo gautų ženklinimų reguliarumą.