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A Multiechelon Repairable Item Inventory System with Lateral Transshipment and a General Repair Time Distribution

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Abstract. This paper discusses the determination of the spare inventory level for a multiechelon repairable item inventory system, which has several bases and a central depot with emergency lateral transshipment capability. Previous research is extended by removing a restrictive assumption on the repair time distribution. A mathematical model that allows a general repair time distribution, as well as an algorithm to find a solution of the model, is developed. Thus, the main focus of this study is to improve the accuracy of previous models and to estimate the gain in accuracy from use of the current methodology. Computational experiments are performed to estimate the accuracy improvement and to determine the managerial implications of the results.

Key words: repair time distribution, repairable item, multiechelon inventory system.

1. Introduction

Repairable items refer to expensive, critically important components which have infrequent failures; these are common in the military and in a variety of commercial settings. Aircraft and warship engines, transportation equipment, and high cost electronics are typical examples of repairable items. While the repairable inventory problem has its roots in military applications, it is extremely relevant today for both the military and commercial sectors. For this reason, researchers study numerous policies on setting the spare inventory stocking levels and on estimating the operating characteristics of the system.

Research reports on single or multi-echelon systems include the works of Sherbrooke (1968), Gross *et al.* (1987), Albright and Gupta (1993), and Kim *et al.* (1996, 2000). The METRIC model, developed by Sherbrooke (1968) assumes infinite repair capacity. Gross *et al.* (1987) consider a two-echelon (two levels of repair, one level of supply) system and present an implicit enumeration algorithm to calculate the capacities of the base and depot repair facilities, as well as the spare levels, which jointly guarantee a specified service rate at a minimum cost. Albright and Gupta (1993) develop an approximation algorithm for the system with added assumptions of finite operating and multi-indentured items; these models have two levels of supply and one level of repair. Kim *et al.* (1996, 2000) introduce algorithms that determine the optimal inventory level under finite repair capacities.

For a lateral transshipment, Das (1975) suggests a periodic review inventory model that consists of only two locations, and where transshipment is allowed at pre-specified times during the review period. Hoadley and Heyman (1977) consider a one-period multiechelon model that allows lateral transshipment between stocking points at the same echelon level. Lee (1987) develops a model that derives an approximation for the expected level of backorders and the quantity of emergency transshipments. Axsäter (1990) suggests a method to estimate the same operating characteristics of a similar system that puts more emphasis on accurately modeling the demand at a base; additionally, it can be applied to the case of nonidentical bases, which is in contrast to (Lee, 1987). Jung *et al.* (2003) consider lateral transshipments with finite repair channels. They develop a model for the determination of the local optimal spare levels that minimizes the total expected cost of the system; the algorithm is tested using examples of various size and format.

Most of the previous research on repairable item systems adopts an exponential distribution for the repair time. A few exceptions include the M/G/c models by Díaz and Fu (1997), the VARI-METRIC models with an M/G/c queueing system by Sleptchenko *et al*. (2002), the Erlang distribution model by Perman *et al*. (2001), and a dual sourcing system by Fong *et al*. (2000). Real world distributions are often approximated by an exponential distribution since it can closely represent the various types of repair times and has high tractability. However, this kind of approximation is often too crude for an acceptable system representation. When this is the case, a more specific type of distribution must be used. For this reason, this study suggests a mathematical model that allows for a variety of repair time distributions and an algorithm is proposed to find the solution of the model.

This paper is organized as follows. In the next sections, Section 2 and Section 3, the model and the probability distributions of the system are described, respectively. In Section 4, the details are provided for the algorithm. Section 5 presents the results of the computational experiments. The paper is concluded with Section 6, which includes managerial implications and comments on the extension.

2. Model Description

The system considered in this study has several bases and a central depot with a single type of repairable item. Each base has its own spare inventories and base repair center. The central depot stocks no spares and only repairs failed items transported from the bases. An infinite number of items are operating at each base. Failures occur according to the Poisson process with a rate of Λ_i at the base i. If an item fails in a base and a spare is available at the base, then it is immediately replaced. However, if there is no spare available at the base, the same type of item is requested as a lateral transshipment to another base that has the item in stock. If an item is not available at any of the bases, the item is backordered until a spare becomes available. The backorder is filled when a repair is completed at the base repair center or a repaired item is returned from the depot.

The failed item is inspected and assorted into a base-repairable classification with a probability of α_i, where it is brought to the base repair center. Otherwise, it is classified into depot-repairable with a probability of $1-\alpha_i$. It is then transported to the depot repair center. Upon receipt of the failed item from a base, the depot adds the base to the waiting list, which contains the base numbers in the order of arrival. When a repair job is finished at the depot repair center, a repaired item is sent to a base on a first come, first served (FCFS) basis and the waiting list is updated.

3. The Probability Distribution of the Number of Unavailable Items

This section discusses the steady-state probability distribution obtained for the total number of *unavailable* items at each base i. The unavailable item denotes the items that are not ready due to three reasons: they are at the base repair center, on the depot's waiting list, or in transit between the depot and base i.

The probabilistic behaviors of the base and depot repair centers follow the M/G/c queueing system. However, it is well known that the M/G/c queue with a general service time does not permit a simple analytical solution for the distribution of the number of customers in the system, including the average waiting time (see, (Tijms, 1994)). Instead, useful approximations have been obtained by many researches, including Kimura (1996) and Miyazawa (1986). Hur and Lee (2000) extend their results and obtain a better approximation. They compare their algorithm with simulation and find that it shows remarkable performance in terms of accuracy. Their approximation is within 6% of the simulation result, especially with use of a small number of servers ($c < 5$) and light traffic ($\rho < 0.7$), which is a commonly observed situation in the system evaluated here; their result is briefly described in this subsection.

First, the M/G/c queueing system is considered with an arrival rate of Λ and a service rate of μ. Here, the service time is denoted by A, so that $E(A) = 1/\mu$. Let $\pi_n(\Lambda, A, c)$ be the steady-state probability distribution that there are n customers in this system. Then, for $0 \leqslant n \leqslant c-1$:

$$\pi_n(\Lambda, A, c) = \frac{(\Lambda/\mu)^n}{n!} P_0(M), \tag{1}$$

where $P_0(M) = \left(\sum_{n=0}^{c-1} \frac{(\Lambda/\mu)^n}{n!} + \frac{(\Lambda/\mu)^c}{c!(1-\rho)}\right)^{-1}$ and $\rho = \Lambda/c\mu$.

That is, the solution of M/M/c is used because the probabilistic behavior of M/G/c when $0 \leqslant n \leqslant c-1$ is very similar to that of a M/M/c system.

For the $n = c$ case:

$$\pi_n(\Lambda, A, c) = \frac{(\Lambda/\mu)^c}{c!} \cdot \frac{1-\nu}{1-\rho} P_0(M), \tag{2}$$

where $\nu = \frac{\rho Q}{1-\rho+\rho Q}$.$Q$ is given by $\frac{1}{4} \cdot \bigl(1 + \frac{3E(A^2)}{2E^2(A)}\bigr)$ for light traffic and by $\frac{E(A^2)}{2E^2(A)}$ for heavy traffic (Kimura, 1996).

Finally, when $n \geqslant c$:

$$\pi_n(\Lambda, A, c) = \Bigl(\frac{\Lambda E(A) + 3\Lambda E(A^+)}{4c - 3\Lambda E(A) + 3\Lambda E(A^+)}\Bigr)^{n-c} \cdot \pi_c(\Lambda, A, c), \tag{3}$$

where A^+ is the remaining service time, so that $E(A^+) = E(A^2)/2E(A)$.

Next, in order to obtain the steady-state probability distribution of the total number of unavailable items at each base, some probability distributions are derived.

The following describes some of notations used:

$P(b_i)$ – probability distribution that there are n items at the repair center of base i,
$P(D)$ – probability distribution that there are N items in the waiting list of the depot repair center,
$P(k_i)$ – probability distribution that there are k_i depot-shortage items with respect to base i at the depot,
$P(l_i)$ – probability distribution that there are l_i items in transit from/to base i,
$P(z_i)$ – probability distribution that the total number of unavailable items of base i is z_i.

A complete listing of notations can be found in the Appendix.

3.1. *Derivation of* $P(b_i)$

There are c_i repair channels at the repair center of base i, where the repair times at each channel are assumed to be a independent and identically distributed (IID) random variable A_i with mean $1/\mu_i$. Since an infinite population is assumed, the base repair center can be modeled as an M/G/$c_i$queueing model, where the arrival (base-repairable failure) rate is $\alpha_i\Lambda_i$ and the repair time is A_i. Using the results from equations (1) through (3), the steady-state probability distribution that there are b_i items at the base repair center i, $P(b_i)$, is given by the following (4):

$$P(b_i) = \pi_{b_i}\bigl(\alpha_i(\Lambda_i + \beta_i - \delta_i), A_i, c_i\bigr). \tag{4}$$

3.2. *Derivation of* $P(z_i)$

The probability distribution of the unavailable items of base i, $P(z_i)$, can be obtained by convolution of the probability distributions, as shown in Eq. 5. The rightmost distribution on the right-hand side of (5) is the probability distribution of the number of items at the base repair center.

$$P(z_i) = \sum_{l_i=0}^{z_i} \sum_{k_i=0}^{z_i-l_i} P(l_i) \cdot P(k_i) \cdot P(z_i - k_i - l_i). \tag{5}$$

Derivation of $P(l_i)$ and $P(k_i)$ can be found in (Jung *et al.*, 2003) and is omitted from this paper.

3.3. *Probability Distribution of Lateral Transshipments to Other Bases*

The probability that a lateral transshipment is requested to base i from base j is

$$B_{ij} = \begin{cases} P(z_1 \geqslant s_1)P(z_2 \geqslant s_2)\ldots P(z_{i-1} \geqslant s_{i-1})P(z_i < s_i)P(z_j \geqslant s_j) & \text{if } i < j, \\ P(z_1 \geqslant s_1)P(z_2 \geqslant s_2)\cdots P(z_{i-1} \geqslant s_{i-1})P(z_i < s_i) & \text{if } i > j. \end{cases} \tag{6}$$

For example, the probability that a transshipment is requested to base number 3 when a failure occurs at base number 5 is the P (no positive stock at base 1 and 2) $\times P$ (positive stock at base 3)$\times P$ (no positive stock at base 5). The probability that a transshipment is requested to base number 5 when a failure occurs at base number 3 is the P (no positive stock up to base 4) $\times P$ (positive stock at base 5).

3.4. *Fill-Rate and Cost Function*

The probability that a demand at base i is met by a lateral transshipment is the probability that base i has no operational item and at least one of the other bases has a positive stock value. Thus, the probability is as follows:

$$R_i = P(z_i \geqslant s_i)\big[1 - P(z_1 \geqslant s_1)\cdots P(z_{i-1} \geqslant s_{i-1})P(z_{i+1} \geqslant s_{i+1})\cdots P(z_I \geqslant s_I)\big]. \tag{7}$$

Actual fill-rate, which can be interpreted as the probability that a failed item is replaced immediately by on hand stock or by lateral transshipment, is the sum of O_i and R_i. If no spare item is available in any of the bases, it is backordered. Thus, the probability that a demand at base i is backordered is shown in Eq. 8.

$$E_i = P(z_1 \geqslant s_1)P(z_2 \geqslant s_2)\cdots P(z_I \geqslant s_I). \tag{8}$$

When assuming linear holding, backorder and transshipment costs, the total expected cost of the system can be expressed as shown in Eq. 9, which is the sum of the costs of holding, backorder and transshipment at the bases.

$$TC(S) = \sum_{i=1}^{I}\{h_i s_i + e_i E_i \Lambda_i + v_i R_i \Lambda_i\}. \tag{9}$$

4. The Algorithm

An algorithm that finds the spare inventory level to operate a system at a minimum cost is presented in this section. The algorithm is as follows:

Step 1. Verify that the following steady-state conditions are satisfied.

$$\rho_d = \sum_{i=1}^{I}(1-\alpha_i)\Lambda_i/c_d\mu_d < 1 \quad \text{and} \quad \rho = \sum_{i=1}^{I}\alpha_i\Lambda_i/c_i\mu_i < 1.$$

If the conditions are met, go to Step 2. Otherwise, stop since the system cannot reach the steady-state.

Step 2. Let $\beta_i = \delta_i = 0$ and $S = (s_1, s_2, \cdots, s_I) = 0$; calculate $P(b_i)$ and $P(z_i)$.

Step 3. For each base with $h_i/e_i \leqslant 1$, set s_i to the value that satisfies

$$\sum_{k=1}^{\infty} P(z_i = s_i + k) < h_i/e_i < \sum_{k=0}^{\infty} P(z_i = s_i + k).$$

Step 4. For each base with$s_i \geqslant 1$, obtain the probability values for S and for S^+ and S^-, which are the same as S except s_i is increased by one unit for S^+ and decreased by one unit for S^- by; these are solved using the subroutine below.

Step 5. *Step* 5.1. For each base with$s_i \geqslant 1$, calculate

$$\Delta TC(S|s_i) = \max\left\{TC(S) - TC(S^+), TC(S) - TC(S^-)\right\}.$$

Step 5.2. If $\rho_i = \alpha_i(\Lambda_i + \beta_i - \delta_i)/c_i\mu_i < 1, \Delta TC(S|s_i) \leqslant 0$ or $s_i = 0$ for all i, go to Step 6.

Otherwise, set $i^* = \arg\max_i \Delta TC(S|s_i)$ and $s_{i^*} \leftarrow s_{i^*} + 1$ if the maximum in Step 5.1 is from S^+ or $s_{i^*} \leftarrow s_{i^*} - 1$ if the maximum in Step 5.1 is from S^-. If S^+ and S^- have the same value then randomly select one of the two.

Step 5.3. Go to Step 4.

Step 6. S is the solution of the algorithm. The expected total cost of the system is $TC(S)$.

Subroutine

Step 1. For the current values of β_i, δ_i, and S, calculate $P(b_i)$ and $P(z_i)$.

Step 2. Calculate B_{ij} and R_i.

Step 3. $\beta_i^{new} = \sum_{j \neq i} B_{ij}\Lambda_i, \delta_i^{new} = R_i\Lambda_i$.

Step 4. If $|\beta_i^{new} - \beta_i| \leqslant \varepsilon$ and $|\delta_i^{new} - \delta_i| \leqslant \varepsilon$ for all i or if the iteration exceeds a limit, then calculate $R_i, E_i, TC(S)$. Return the values and stop.

Step 5. $\beta_i = \beta_i + \omega(\beta_i^{new} - \beta_i),\ \delta_i = \delta_i + \omega(\delta_i^{new} - \delta_i)$.

Step 6. Go to Step 1.

In the algorithm, Step 2 calculates the previously introduced probability distributions. In Step 3, the starting point of the search is chosen to be the minimum cost spare levels of the same system but with no lateral transshipment (Kim *et al.*, 1996) and sets the vector of spare levels, $S = (s_1, s_2, \cdots, s_I)$, to the corresponding values.

In Steps 4 and 5, the base is searched, which gives the largest decrease in the objective function by the unit change of s_i, and sets the stock level accordingly. Thus, the search procedure is same as the steepest descent method. The error limit is specified by ε, which is a small number, e.g., 10^{-2}. If there is no more cost decrease possible, the algorithm stops and generates the current point as a solution to the problem. The subroutine is used to find the converged values of β_i and δ_i for a given S and to calculate $TC(S)$. A control parameter for the movement speed to the next point is represented by ω and is set to a value between 0.01 and 0.3. Since the method is based on the well known steepest descent method, it is ensured to find a local optimum solution with guaranteed convergence.

5. Computational Experiments

This section briefly introduces the results of the computational experiments. The main purpose of the experiments is to compare the result of the current method to the result of the most recent development on the same subject, but with an exponential repair time (Jung *et al.*, 2003). Comparing the result from the two methods, the amount of improvement in accuracy could be estimated.

To perform the test, the proposed algorithm was written in C++ and run on an IBM compatible personal computer with an Intel Pentium IV processor (2.0 GHz), 512 MB memory, and Windows XP operating system. The input parameters were prepared from the actual data values for the repairable items on an aircraft in the US Air Force (Sherbrooke, 1992). Cost data, which was unavailable, was constructed so that the system had a realistic fill-rate; i.e., about 90%. The prepared data is shown in Table 1 and the solution and related information generated by the algorithm are summarized in Table 2.

An initial observation is that the solutions given by the different methods are not identical. Although the optimal cost of the current method is lower than that of the previous method, the fill-rate achieved by the current method is higher. This is an interesting phe-

Table 1
Input data for the first test problem

Base/Depot	Parameter							
	Λ_i	α_i	c_i	μ_i	t_i	h_i	e_i	v_i
Base 1	0.04	0.5	3	0.05	16.0	10.0	900.0	2.0
Base 2	0.04	0.5	2	0.05	16.0	10.0	900.0	2.0
Base 3	0.04	0.5	2	0.05	16.0	10.0	900.0	2.0
Base 4	0.04	0.5	1	0.05	16.0	10.0	900.0	2.0
Base 5	0.04	0.5	1	0.05	16.0	10.0	900.0	2.0
Depot	–	–	10	0.0167	–	–	–	–

Table 2
Output of the first test problem ($h_i = 10$, $e_i = 900$, $v_i = 2$)

	New algorithm			Previous algorithm		
	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$
Base 1	3		0.922	2		0.902
Base 2	3		0.922	2		0.902
Base 3	2	96.0	0.922	5	117.9	0.902
Base 4	1		0.922	1		0.902
Base 5	1		0.922	0		0.902
Average	2			2		

nomenon which suggests that it is possible to achieve the same performance with less cost when a system is controlled by the new method.

To investigate a more congested system, the input parameters are changed so that the repair capacities reach around 90% utilization. This relatively heavy traffic situation represents a commercial industry system rather a military one, which usually has ample repair capacities. The heavy traffic data is shown in Table 3 and the results are summarized in Table 4. The result is similar to the previous case. The cost for the same service rate is reduced by 79.1 or 13.6% by adopting the policy of the proposed method.

To test some extreme cases, the holding cost is multiplied by 1/10 and the transshipment cost by 200, which makes them 1 and 400, respectively; the result is shown in Table 5. For this system, two algorithms generate the same solutions or spare levels. The fill-rate of 1.00 implies that the system has light traffic. The result implies that as the system becomes more congested the accuracy of the repair time distribution has a larger effect on the accuracy of the methods. It may be thus interpreted that it is acceptable to use an approximate distribution for a light traffic system.

When the transshipment cost is restored to its original value, the results are as shown in Table 6. The fill-rate values show that the corresponding situation is slightly more

Table 3

Input data for the heavy traffic problem

Base/Depot	Parameter							
	Λ_i	α_i	c_i	μ_i	t_i	h_i	e_i	v_i
Base 1	0.27	0.5	3	0.05	16.0	10.0	900.0	2.0
Base 2	0.18	0.5	2	0.05	16.0	10.0	900.0	2.0
Base 3	0.18	0.5	2	0.05	16.0	10.0	900.0	2.0
Base 4	0.09	0.5	1	0.05	16.0	10.0	900.0	2.0
Base 5	0.09	0.5	1	0.05	16.0	10.0	900.0	2.0
Depot	–	–	10	0.045	–	–	–	–

Table 4

Output of the heavy traffic problem ($h_i = 10$, $e_i = 900$, $v_i = 2$)

	New algorithm			Previous algorithm		
	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$
Base 1	15		0.920	16		0.914
Base 2	11		0.920	11		0.914
Base 3	11	504.4	0.920	12	583.5	0.914
Base 4	6		0.920	6		0.914
Base 5	6		0.920	7		0.914
Average	9.8			10.4		

Table 5

Output of the problem with low holding and high transshipment costs ($h_i = 1$, $e_i = 900$, $v_i = 400$)

	New algorithm			Previous algorithm		
	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$
Base 1	5		1.000	5		1.000
Base 2	5		1.000	5		1.000
Base 3	6	32.8	1.000	6	32.8	1.000
Base 4	6		1.000	6		1.000
Base 5	6		1.000	6		1.000
Average	5.6			5.6		

Table 6

Output of the problem with low holding costs ($h_i = 1$, $e_i = 900$, $v_i = 2$)

	New algorithm			Previous algorithm		
	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$	Optimal spare level	Optimal total cost	Fill-rate $(O_i + R_i)$
Base 1	4		0.938	2		0.994
Base 2	3		0.938	3		0.994
Base 3	2	12.9	0.938	5	16.3	0.994
Base 4	1		0.938	3		0.994
Base 5	1		0.937	2		0.994
Average	2.2			3.0		

congested than the previous case. The discrepancy of the results of the two methods now becomes noticeable once again, showing the cost difference of 20.9%.

Although it may be too soon to draw a solid conclusion from the small scale experiments performed here, it is shown that the developed method is a better control policy that results in an enhanced performance with less cost. Thus, it is expected that it will contribute to curtailing operating costs when applied to a real world system.

Finally, to test the speed of the algorithm for realistic problems, 10 different problems are solved for base cases of 5, 10, 15 and 20; the obtained results are shown in Table 7. The algorithm is capable of producing a solution in an average of less than two hours. In all cases, the speed of the proposed algorithm is better than that of the previous one.

6. Concluding Remarks

In this paper, a model was presented for the multiechelon repairable inventory system with emergency lateral transshipments, which extended the scope of research to the system with a general repair time distribution. A method was developed to calculate the

Table 7

Average time to solve the problems

Number of base	New algorithm				Previous algorithm			
	5	10	15	20	5	10	15	20
Time in seconds	46	427	1458	4553	54	428	2819	5315

appropriate initial spare inventory levels to optimally control the system. Experimental results showed that the solution from the current method was different from the previously suggested algorithm. It was also implied that an equal service rate with less operating cost could be achieved with the control policy of the current method.

Managerial implications drawn from this research are that it is worthwhile to apply the method to real world systems to try to achieve better performance. Additionally, the diverse use of the current method can easily answer other 'what-if' type of questions, especially for finding the desired service rate of a military system. A suggestion for future study would be to relax the assumption of an infinite number of items operating at each base, which has enabled the use of formulas from the M/G/c model.

Appendix

List of notations

Λ_i – failure rate at base i,
α – probability that a failed item is base-repairable,
c_i – number of repair channels at base i repair center,
μ_i – repair rate per repair channel at base i repair center,
b_i – number of items at base i repair center,
c_d – number of repair channels at the depot repair center,
μ_d – repair rate per repair channel at the depot repair center,
β_i – failure rate increase at base i due to transshipments to other bases,
δ_i – failure rate decrease at base i due to transshipments from other bases,
D – number of items at the depot repair center,
k_i – number of items at the depot repair center owed to base i,
t_i – transit time from base i to the depot repair center,
l_i – number of items in transit between base i and the depot repair center,
z_i – number of unavailable items of base i,
B_{ij} – probability that a lateral transshipment is requested to base i from base j,
R_i – probability that a demand at base i is met with a lateral transshipment,
E_i – probability that a demand at base i is backordered,
s_i – spare inventory level at base i, $S = (s_1, s_2, \dots, s_I)$,
h_i – unit holding cost per unit time of base i,
e_i – unit backorder cost of base i,
v_i – unit transshipment cost of base i.

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Daugiaešaloninė pakeičiamų įrenginių inventorizavimo sistema su papildomu perkrovimu ir bendru atstatymo laiko pasiskirstymu

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Šiame straipsnyje nagrinėjama, kaip nustatyti reikalingą atsarginių įrenginių lygį daugiaešaloninei pakeičiamų įrenginių inventorizavimo sistemai, kurioje yra keletas bazių ir centrinis sandėlys su avarinio papildomo perkrovimo galimybe. Ankstesnis tyrimas yra išplėstas atsisakant apribojančios atstatymo laiko pasiskirstymo prielaidos. Matematinis modelis su bendru atstatymo laiko pasiskirstymu ir algoritmas modelio sprendinio paieškai yra sukurti. Pagrindinis šio tyrimo akcentas yra skiriamas ankstesnių modelių tikslumo pagerinimui ir esamos metodologijos panaudojimo įvertinimui. Tikslumo pagerinimas yra įvertintas eksperimentiškai.