

Variable Gain Takagi–Sugeno Fuzzy Logic Controllers

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Abstract. Most of the Takagi–Sugeno Fuzzy (TSF) systems found in the literature have only used linear functions of input variables as rule consequent and can be called as TSF Models with Fixed Coefficient (TSFMFC). This paper presents TSF model with variable coefficient (TSFMVC) which can more closely approximate a class of nonlinear systems, nonlinear dynamic systems, and nonlinear control systems. It is also shown that TSFMFC is a special case of TSFMVC. Moreover Variable Gain TSF Controller (VGTSFC) is defined and it performs better, as shown by the simulation results, when compared with Fixed Gain TSF Controller (FGTSFC).

Key words: fuzzy controller, Takagi–Sugeno fuzzy models with fixed coefficient, Takagi–Sugeno fuzzy models with variable coefficient, fixed gain Takagi–Sugeno fuzzy controller, variable gain Takagi–Sugeno fuzzy controller.

1. Introduction and Problem Statement

Takagi–Sugeno fuzzy systems have widely been employed in the fields of control and modeling. In the context of control, the question is whether a fuzzy controller can be constructed to approximate any desired control solution with enough accuracy, whereas, in the context of modeling the issue is whether a fuzzy model can be established which is capable of approximating any physical system as accurately as one desires. Model-Based control design assumes the existence of a mathematical model of the plant. The controller is designed to modify the behavior of the plant and achieve some desired performance. To this end, a systematic way to construct a model, mapping the inputs to the outputs is needed.

Most of the Takagi–Sugeno fuzzy systems found in the literature (Joh *et al.*, 1998; Li *et al.*, 2003; Takagi and Sugeno, 1985; Tanaka and Sugeno, 1992; Wang *et al.*, 1996; Wang *et al.*, 2003; Wang *et al.*, 1995; Ying, 1998) have only used linear functions of input variables as rule consequent (i.e., linear rule consequent) and can be called as Takagi–Sugeno Fuzzy Models with Fixed Coefficient (TSFMFC). It simply means that the coefficients of state variables in the consequents of each rule are fixed constants. This

paper presents an extended Takagi–Sugeno fuzzy model named as Takagi–Sugeno fuzzy model with variable coefficient (TSMVC) and it is proved that it can approximate a class of nonlinear systems, nonlinear dynamic systems, and nonlinear control systems. It is also shown that TSMFC is a special case of TSMVC.

Takagi–Sugeno fuzzy controllers designed using parallel distributed technique found in the literature (Wang *et al.*, 1995) uses linear functions of input variables as rule consequents and may be called as Fixed Gain Takagi–Sugeno Fuzzy Controller (FGTSFC). This paper also presents an extended T-S fuzzy controller called as Variable Gain Takagi–Sugeno Fuzzy Controller (VGTSFC) using Parallel distributed Compensation. A nonlinear benchmark problem corresponding to the physical system that has been built by Dr. Bernstein and his co-workers (Bernstein, 1998; Bupp *et al.*, 1995) at the University of Michigan namely the Translational Oscillation by a Rotational Actuator (TORA) (Alleyne, 1998; Jankovic *et al.*, 1996; Li *et al.*, 2000; Kanellakopoulos and Zhao, 1995; Margaliot and Langholz, 2004; Rand *et al.*, 1992) is simulated and the results show that VGTSFC performs better than FGTSFC.

This paper is organized as follows. Section 2 deals with the review of T-S fuzzy models with fixed coefficients and the corresponding fixed gain T-S fuzzy controller. Section 3 presents the proposed Takagi–Sugeno Fuzzy Models with Variable Coefficient (TSMVC). Section 4 deals with the analysis of approximation of a class of nonlinear systems using TSMVC. Section 5 deals with the proposed VGTSFC, its approximation and comparison with FGTSFC. In Section 6 TORA system is simulated using FGTSFC, VGTSFC, and linear controller (linearized around the origin) and the results are compared.

2. Review of T-S Fuzzy Models with Fixed Coefficients (TSMFC) and the Corresponding Fixed Gain T-S Fuzzy Controller (FGTSFC)

A general representation of T-S model (Wang *et al.*, 2003) with fixed coefficient (TSMFC) is presented in this section.

Suppose that the nonlinear function $f(x): R^n \rightarrow R$ is defined over a compact region $D \subset R^n$ with the following assumptions:

$$A_1: f(0) = 0$$

$A_2: f \in C_1^2$. Therefore, f , $\frac{\partial f}{\partial x}$ and $\frac{\partial^2 f}{\partial x^2}$ are continuous and therefore bounded over D .

$A_3: f(x)$ can be approximated in the form of $f_{j_1 j_2 \dots j_n}(x) = a_{j_1 j_2 \dots j_n} x$ in each local region $D_{j_1 j_2 \dots j_n} = \{x \mid x \in D, j_i \epsilon \leq x_i \leq (j_i + 1)\epsilon \forall i\}$, where j_i are integers and ϵ is a small chosen positive number.

Then $f(x)$ can be expressed by linear Takagi–Sugeno fuzzy systems. The main feature of linear Takagi–Sugeno fuzzy systems is to express the local properties of each fuzzy implication (rule) by a linear function. The overall fuzzy system is achieved by fuzzy “blending” of these linear functions.

Then the linear Takagi–Sugeno fuzzy rules are as follows:

$$R_{j_1 j_2 \dots j_n}: \text{IF } x_1 \text{ is about } j_1 \epsilon \dots \text{ and } x_n \text{ is about } j_n \epsilon \text{ THEN } \hat{f} = a_{j_1 j_2 \dots j_n} x,$$

where $x = [x_1, x_2, \dots, x_n]^T$ are the function variables and $\hat{f} = a_{j_1 j_2 \dots j_n} x$ is the consequence of the $j_1 j_2 \dots j_n$ th IF-THEN rule. [It should be noted that, “ x_i is about $j_i \epsilon$ ” is same as telling that “ x_i takes the membership function $M_{j_i}(x_i)$.”]

The possibility of firing for the $j_1 j_2 \dots j_n$ th rule is given by the product of all membership functions associated with the $j_1 j_2 \dots j_n$ th rule.

$$h_{j_1 j_2 \dots j_n}(x) = \prod_{i=1}^n M_{j_i}(x_i), \quad (1)$$

where the membership function for x_i is given as

$$M_{j_i}(x_i) = \begin{cases} 1 - \frac{|x_i - j_i \epsilon|}{\epsilon}, & |x_i - j_i \epsilon| < \epsilon, \\ 0, & \text{else where.} \end{cases} \quad (2)$$

It is noted that $h_{j_1 j_2 \dots j_n}(x)$ have already been normalized, i.e., $h_{j_1 j_2 \dots j_n}(x) \geq 0$ and $\sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) = 1$.

Then by using center of gravity method for defuzzification, we can represent the T-S model as

$$f(x) = \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) a_{j_1 j_2 \dots j_n} x. \quad (3)$$

Theorem 1 (Wang *et al.*, 2003). *For a smooth nonlinear function $f(x): R^n \rightarrow R$ satisfying Assumptions A_1 , A_2 and A_3 , defined on a compact region, it can be approximated, to any degree of accuracy, by a linear T-S fuzzy system.*

2.1. TFSMFC for Nonlinear Dynamic Systems

The Takagi–Sugeno fuzzy model with fixed coefficient (TFSMFC) is used to describe dynamic systems. It is of the following form:

$$R_i: \text{IF } x_1(t) \text{ is } M_{1i} \dots \text{ and } x_n(t) \text{ is } M_{ni} \text{ THEN } \dot{x}(t) = A_i x(t),$$

where $x^T(t) = [x_1(t), x_2(t), \dots, x_n(t)]$ are the system states. $i = 1, 2, \dots, r$ and r is the number of IF-THEN rules. M_{ij} are fuzzy sets and $\dot{x}(t) = A_i x(t)$ are the consequence of the i -th IF-THEN rule.

By using center of gravity method for defuzzification, the T-S model can be represented as

$$\dot{x} = \hat{f}(x) = \sum_{i=1}^r h_i(x) A_i x, \quad (4)$$

where $h_i(x)$ is the possibility for the i -th rule to fire.

Consider the nonlinear system:

$$\dot{x} = f(x), \quad (5)$$

where $f(x)$ is a vector field defined over compact region $D \subset R^n$ with the following assumptions:

B_1 : $f(0) = 0$, i.e., the origin is an equilibrium point.

B_2 : $f \in C_n^2$. Therefore, f , $\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial x^2}$ are continuous and bounded over D .

COROLLARY 1 (Wang *et al.*, 2003). For any smooth nonlinear system (5) satisfying the assumptions B_1 and B_2 , it can be approximated, to any degree of accuracy, by a T-S model (4).

Similarly, smooth nonlinear control system $\dot{x} = f(x) + g(x)u$ can also be approximated using a T-S fuzzy model $\dot{x} = \sum_{i=1}^r h_i(x)(A_i x + B_i u)$. By treating u as extraneous system state, we can also approximate the smooth nonlinear control system $\dot{x} = f(x, u)$ by T-S fuzzy model $\dot{x} = \sum_{i=1}^r \hat{h}_i(x, u)(A_i x + B_i u)$. In this case, the fuzzy rule is of the following form:

Rule i : IF $x_1(t)$ is M_{i1} , ..., and, $x_n(t)$ is M_{in} ,
 $u_1(t)$ is N_{i1} , ..., and $u_m(t)$ is N_{im}
 THEN $\dot{x} = A_i x(t) + B_i u(t)$

where $x^T(t) = [x_1(t), x_2(t), \dots, x_n(t)]$ are the system states and $u^T(t) = [u_1(t), u_2(t), \dots, u_m(t)]$ are system inputs. $i = 1, 2, \dots, r$ and r is the number of IF-THEN rules. M_{ij} , N_{ij} are fuzzy sets and $\dot{x}(t) = A_i x(t) + B_i u(t)$ is the consequence of the i -th IF-THEN rule. $\hat{h}_i(x, u) = \prod_{j=1}^n M_{ij}(x_j(t)) \prod_{k=1}^m N_{ik}(u_k(t))$ is the possibility for the i -th rule to fire.

2.2. FGTSFC Using PDC Technique

The structure of FGTSFC using parallel distributed compensation (PDC) (Wang *et al.*, 1996; Wang *et al.*, 1995) consists of the fuzzy rules of the given below:

Rule i : IF $x_1(t)$ is M_{i1} , ..., $x_n(t)$ is M_{in} , THEN $u(t) = K_i x(t)$, (6)

where $i = 1, 2, \dots, s$. The output of the PDC controller is

$$u = \sum_{j=1}^r h_j(x) K_j x. \quad (7)$$

Theorem 2 (Wang *et al.*, 2003). Any smooth nonlinear state feedback controller $u = K(x)$ where x is defined over a compact region can be approximated, to any degree of accuracy, by a PDC controller (7).

3. T-S Fuzzy Models with Variable Coefficient (TSFMVC)

A general representation of TSFMVC is as described below. Suppose that the nonlinear function $f(x): R^n \Rightarrow R$ is defined over a compact region $D \subset R^n$ with the following assumptions:

$$P_1: f(0) = 0.$$

$P_2: f \in C_1^2$. Therefore, f , $\frac{\partial f}{\partial x}$ and $\frac{\partial^2 f}{\partial x^2}$ are continuous and therefore bounded over D .

$P_3: f(x)$ can be approximated as $f_{j_1 j_2 \dots j_n}(x) = a_{j_1 j_2 \dots j_n}(x)x$ in the region $D_{j_1 j_2 \dots j_n} = \{x / x \in D, j_i \epsilon \leq x_i \leq (j_i + 1)\epsilon \forall i\}$, where $a(x)$, $a_{j_1 j_2 \dots j_n}(x)$ are continuous, $x = [x_1, x_2, \dots, x_n]^T$, j_i are integers, and ϵ is a small positive number.

Then $f(x)$ can be approximated by Takagi–Sugeno Fuzzy Model with Variable Coefficient (TSFMVC). The rules of TSFMVC is of the form:

$$\begin{aligned} \text{Rule } j_1 j_2 \dots j_n : \quad & \text{IF } x_1 \text{ is about } j_1 \epsilon \dots \text{ and } x_n \text{ is about } j_n \epsilon \\ & \text{THEN } \hat{f}(x) = a_{j_1 j_2 \dots j_n}(x)x \end{aligned}$$

The possibility of firing for the $j_1 j_2 \dots j_n$ th rule is given by the product of all membership functions associated with the $j_1 j_2 \dots j_n$ th rule and is $h_{j_1 j_2 \dots j_n}(x(t)) = \prod_{i=1}^n M_{j_i}(x_i(t))$ where the membership function for x_i is given by (8). It is assumed that $h_{j_1 j_2 \dots j_n}(x)$ have already been normalized, i.e., $h_{j_1 j_2 \dots j_n}(x) \geq 0$ and $\sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) = 1$.

$$M_{j_i}(x_i) = \begin{cases} 1 - \frac{|x_i - j_i \epsilon|}{\epsilon}, & |x_i - j_i \epsilon| < \epsilon, \\ 0, & \text{else where.} \end{cases} \quad (8)$$

Then by using center of gravity method for defuzzification, the TSFMVC can be represented as:

$$y = \hat{f}(x) = \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) a_{j_1 j_2 \dots j_n}(x)x. \quad (9)$$

4. Analysis of Approximation

4.1. Approximation of a Function by T-S Fuzzy Model with Variable Coefficient (TSFMVC)

In this section, it is proved the fact that any smooth nonlinear function satisfying the assumptions P_1 , P_2 , and P_3 can be approximated, to any degree of accuracy, using the TSFMVC. In the following discussions, only one of such regions $D_{j_1 j_2 \dots j_n}$ is concentrated by assuming that $x \in D_{j_1 j_2 \dots j_n}$. In the following, for simplicity, ϕ is substituted instead of $j_1 j_2 \dots j_n$.

Consider $e(x)$, the approximation error between $f(x)$ and $\hat{f}(x)$

$$\begin{aligned}
\|e(x)\| &= \left\| f(x) - \sum_{\phi} h_{\phi}(x) a_{\phi}(x) x \right\| \\
&= \left\| f(x) - \sum_{\phi} h_{\phi}(x) a_{\phi}(x_{\phi}) x_{\phi} + \sum_{\phi} h_{\phi}(x) (a_{\phi}(x_{\phi}) x_{\phi} - a_{\phi}(x) x) \right\| \\
&\leq \left\| f(x) - \sum_{\phi} h_{\phi}(x) f(x_{\phi}) \right\| + \left\| \sum_{\phi} h_{\phi}(x) (a_{\phi}(x_{\phi}) x_{\phi} - a_{\phi}(x) x) \right\| \\
&\leq \sum_{\phi} h_{\phi}(x) \|f(x) - f(x_{\phi})\| + \sum_{\phi} h_{\phi}(x) \|(a_{\phi}(x_{\phi}) x_{\phi} - a_{\phi}(x) x)\| \\
&\leq \max_{\phi} \|f(x) - f(x_{\phi})\| + \max_{\phi} \|(a_{\phi}(x_{\phi}) x_{\phi} - a_{\phi}(x) x)\|.
\end{aligned}$$

Since $x \in D_{j_1 j_2 \dots j_n}$, the distance between x and any vertex point of $D_{j_1 j_2 \dots j_n}$ is less than $\sqrt{n}\epsilon$, i.e., $|x - x_{j_1 j_2 \dots j_n}| \leq \sqrt{n}\epsilon$, and since $f(x)$ and $a(x)$ are continuous over x , it is possible to make $e(x)$ arbitrarily small by just reducing ϵ .

Therefore, the following theorem is obtained by summarizing the results above.

Theorem 3. For a smooth nonlinear function $f(x): \mathbb{R}^n \rightarrow \mathbb{R}$ satisfying assumptions P_1 , P_2 , and P_3 can be approximated, to any degree of accuracy, by a T-S Fuzzy Model with Variable Coefficient (TSFMVC).

4.2. Approximation of Nonlinear Dynamic Systems by TSFMVC

TSFMVC in the following form can be used to describe dynamic systems.

$$\begin{aligned}
\text{Rule } j_1 j_2 \dots j_n: \quad &\text{IF } x_1 \text{ is about } j_1 \epsilon \dots \text{ and } x_n \text{ is about } j_n \epsilon \\
&\text{THEN } \dot{x} = A_{j_1 j_2 \dots j_n}(x) x,
\end{aligned}$$

where $x = [x_1, x_2, \dots, x_n]^T$ are the function variables and $\dot{x} = A_{j_1 j_2 \dots j_n}(x) x$ is the consequence of the $j_1 j_2 \dots j_n$ th IF-THEN rule.

By using weighted average defuzzification, the TSFMVC can be represented as (10), where $h_i(x)$ is the possibility for the i th rule to fire.

$$\dot{x} = \hat{f}(x) = \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) A_{j_1 j_2 \dots j_n}(x) x. \quad (10)$$

Consider the nonlinear system:

$$\dot{x} = f(x), \quad (11)$$

where $f(x)$ is a vector field defined over compact region $D \subset \mathbb{R}^n$ with the following assumptions:

S_1 : $f(0) = 0$, i.e., the origin is an equilibrium point.

S_2 : $f \in C_n^2$. Therefore, f , $\frac{\partial f}{\partial x}$ and $\frac{\partial^2 f}{\partial x^2}$ are continuous and therefore bounded over D .

Suppose $f(x)$ can be written as $[f_1(x) \dots f_n(x)]^T$. What is meant by approximation is finding a T-S fuzzy model $\hat{f}(x) = [\hat{f}_1(x) \dots \hat{f}_n(x)]^T$ such that $\|f(x) - \hat{f}(x)\|$ is small. Since $\|f(x) - \hat{f}(x)\|$ is small iff each of its components (which are nonlinear functions) is small, then by applying Theorem 3 proved in the previous section, the following corollary is obtained.

Theorem 4. For any smooth nonlinear system (11) satisfying the assumptions S_1 to S_2 , it can be approximated, to any degree of accuracy, by a TFSMVC (10).

Similarly, smooth nonlinear control system

$$\dot{x} = f(x) + g(x)u \quad (12)$$

can also be approximated using a T-S fuzzy model with variable coefficient (TFSMVC) and is given by

$$\dot{x} = \sum_{j_1 j_2 j_3 \dots j_n}^r h_{j_1 j_2 j_3 \dots j_n}(x) (A_{j_1 j_2 j_3 \dots j_n}(x) x + B_{j_1 j_2 j_3 \dots j_n}(x) u), \quad (13)$$

where the rules are of the form:

$$\begin{aligned} R_{j_1 j_2 \dots j_n}: \quad & \text{IF } x_1 \text{ is about } j_1 \epsilon \dots \text{ and } x_n \text{ is about } j_n \epsilon \\ & \text{THEN } \dot{x} = A_{j_1 j_2 \dots j_n}(x) x + B_{j_1 j_2 \dots j_n}(x) u, \end{aligned}$$

where $x = [x_1, x_2, \dots, x_n]^T$ are the system states and $u = [u_1, u_2, \dots, u_n]^T$ are the system inputs. $\dot{x} = A_{j_1 j_2 \dots j_n}(x) x + B_{j_1 j_2 \dots j_n}(x) u$ is the consequences of the $j_1 j_2 \dots j_n$ th IF-THEN rule. The possibility for the $j_1 j_2 \dots j_n$ th rule to fire is given by $h_{j_1 j_2 j_3 \dots j_n}(x) = \prod_{p=1}^n M_{j_p}(x_p)$.

4.3. Comparison of Approximations Done in TFSMFC and TFSMVC

Consider, for example, the function

$$f(x_1, x_2) = \frac{\sin x_1}{1 + \cos x_1} + \frac{x_2}{1 + \sin x_1},$$

where $0 \leq x_1, x_2 \leq \pi/4$.

The function approximated using the Takagi–Sugeno fuzzy model with fixed coefficient (TFSMFC) [also called as Takagi–Sugeno fuzzy model with linear coefficient (Takagi and Sugeno, 1985)] using two rules approximated at $x_1 = 0$ and $x_1 = \pi/4$ is as

follows:

$$\text{if } x_1 = 0, \text{ then } f(x_1, x_2) = A|_{x_1=0} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$\text{if } x_1 = \pi/4, \text{ then } f(x_1, x_2) = A|_{x_1=\pi/4} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where $A|_{x_1=0}$ and $A|_{x_1=\pi/4}$ are coefficients of the function approximated at $x_1 = 0$ and $x_1 = \pi/4$ respectively and is given by

$$\begin{aligned} A|_{x_1=0} &= \left[\frac{1}{2} \quad 1 \right] \text{ since } \sin(x_1) \approx x_1 \text{ when } x_1 \rightarrow 0, \\ A|_{x_1=\pi/4} &= \left[\frac{4}{\pi(1+\sqrt{2})} \quad \frac{\sqrt{2}}{1+\sqrt{2}} \right] \\ &\quad \text{since } \sin(x_1) \approx \frac{4x_1}{\pi\sqrt{2}} \text{ when } x_1 \text{ is about } \pi/4. \end{aligned} \quad (14)$$

At the same time the function approximated using the proposed Takagi–Sugeno fuzzy model with variable coefficient (TSFMVC) using two rules approximated at $x_1 = 0$ and $x_1 = \pi/4$ is as follows:

$$\text{if } x_1 = 0, \text{ then } f(x_1, x_2) = A(x)|_{x_1=0} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$\text{if } x_1 = \pi/4, \text{ then } f(x_1, x_2) = A(x)|_{x_1=\pi/4} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

where $A(x)|_{x_1=0}$ and $A(x)|_{x_1=\pi/4}$ are coefficients of the function approximated at $x_1 = 0$ and $x_1 = \pi/4$ respectively and is given by

$$\begin{aligned} A(x)|_{x_1=0} &= \left[\frac{1}{1+\cos x_1} \quad \frac{1}{1+\sin x_1} \right] \text{ since } \sin(x_1) \approx x_1 \text{ when } x_1 \rightarrow 0, \\ A(x)|_{x_1=\pi/4} &= \left[\frac{4/(\pi\sqrt{2})}{1+\cos x_1} \quad \frac{1}{1+\sin x_1} \right] \\ &\quad \text{since } \sin(x_1) \approx \frac{4x_1}{\pi\sqrt{2}} \text{ when } x_1 \text{ is about } \pi/4. \end{aligned} \quad (15)$$

It can be seen that (14) is again an approximation of (15) at x_1 , i.e.,

$$A(x)|_{x_1=0} \text{ at } (x_1 = 0) = A(x|_{x_1=0})|_{x_1=0} = A|_{x_1=0},$$

$$A(x)|_{x_1=\pi/4} \text{ at } (x_1 = \pi/4) = A(x|_{x_1=\pi/4})|_{x_1=\pi/4} = A|_{x_1=\pi/4}.$$

In general, consider $j_1 j_2 \dots j_n^{th}$ rule of TSFMVC given by (3) and $j_1 j_2 \dots j_n^{th}$ rule of TSFMVC given by (9). Then the approximation of $f(x)$ using fuzzy models

TFSMVC (9) and TFSMFC (3) is as follows:

$$f(x) \xrightarrow{\text{1st approx.}} \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) a_{j_1 j_2 \dots j_n}(x) x$$

$$\xrightarrow{\text{2nd approx.}} \begin{cases} \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) a_{j_1 j_2 \dots j_n}(x_{j_1 j_2 \dots j_n}) x \\ \text{or} \\ \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) a_{j_1 j_2 \dots j_n} x. \end{cases}$$

It can be seen that the first approximation gives our proposed Takagi–Sugeno Fuzzy Model with Variable Coefficient (TFSMVC) and the second approximation leads to Takagi–Sugeno Fuzzy Model with Fixed Coefficient (TFSMFC, also called Takagi Sugeno Fuzzy Model with linear consequence).

Now consider the approximation of dynamic system of the form $\dot{x} = A(x) + B(x)u$ using fuzzy models.

$$\dot{x} = A(x) + B(x)u$$

$$\xrightarrow{\text{1st approx.}} \dot{x} = \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) (A_{j_1 j_2 \dots j_n}(x) x + B_{j_1 j_2 \dots j_n}(x) u)$$

$$\xrightarrow{\text{2nd approx.}} \dot{x} = \begin{cases} \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) (A_{j_1 j_2 \dots j_n}(x_{j_1 j_2 \dots j_n}) x + B_{j_1 j_2 \dots j_n}(x_{j_1 j_2 \dots j_n}) u) \\ \text{or} \\ \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) (A_{j_1 j_2 \dots j_n} x + B_{j_1 j_2 \dots j_n} u). \end{cases}$$

In this case also it can be seen that the first approximation leads to our proposed TFSMVC. It is to be noted that TFSMFC is obtained only after the second approximation.

Thus TFSMFC are special cases of TFSMVC. Since the approximation error between the original system and the approximated system increases by the increase in the number of approximation, TFSMVC can better approximate systems than TFSMFC.

5. Variable Gain Takagi–Sugeno Fuzzy Controller (VGTSFC)

Parallel distributed compensation (Wang *et al.*, 1995) can be used to design controller for Takagi–Sugeno fuzzy model with variable coefficient (TFSMVC) (13) and the variable gain PDC controller thus obtained is named as Variable Gain Takagi–Sugeno Fuzzy Controller (VGTSFC).

The rules of the VGTSFC are of the form:

$$\text{Rule } j_1 j_2 \dots j_n: \quad \text{IF } x_1 \text{ is about } j_1 \epsilon \dots \text{ and } x_n \text{ is about } j_n \epsilon$$

$$\text{THEN } u = K_{j_1 j_2 \dots j_n}(x) x.$$

The output of the controller is

$$u = \sum_{j_1 j_2 \dots j_n} h_{j_1 j_2 \dots j_n}(x) K_{j_1 j_2 \dots j_n}(x) x. \quad (16)$$

5.1. Comparison of Algorithms for FGTSFC and VGTSFC

VGTSFC is designed based on TSFMVC (1st approximated model), while FGTSFC is based on FGTSFC(2nd approximated model). Since VGTSFC is designed based on the 1st approximated model (TSFMVC) it can better control any system than FGTSFC. The comparison of algorithms for FGTSFC and VGTSFC are given in Table 1. It can be seen from the table that in FGTSFC the gain values (K_i s) are fixed constants for each region i and all the K_i s are determined before the simulation. While in VGTSFC, the gain values are not fixed and it changes inside each region based on the current state of the system (or plant) and hence K_i s are determined only during simulation.

6. Translational Oscillation by a Rotational Actuator (TORA) – an Application

TORA (translational oscillation by a rotational actuator) (Alleyne, 1998; Jankovic *et al.*, 1996; Li *et al.*, 2000; Kanellakopoulos and Zhao, 1995; Margaliot and Langholz, 2004; Rand *et al.*, 1992) is a nonlinear benchmark problem corresponding to the physical system that has been built by Dr. Bernstein and his co-workers (Bernstein, 1998; Bupp *et al.*, 1995) at the University of Michigan. It is illustrated in Fig. 1. The problem considers a translational oscillator with an attached eccentric rotational proof mass actuator, where the nonlinear coupling between the rotational motion of the actuator and the translational motion of the oscillator provides the control mechanism. The behavior TORA is similar to that of a dual-spin spacecraft exhibiting the resonance phenomenon.

Table 1
Algorithm for FGTSFC and VGTSFC

Algorithm for FGTSFC	Algorithm for VGTSFC
\vdots $K_i = lqr(A_i, B_i, Q, R), \forall i$ begin $t = 1$ $x_t = \text{current state}$ repeat \vdots $u = -\sum_{i=1}^r h_i(x_t)K_i x_t$ $x_{t+1} = \text{plant}(x_t, u)$ \vdots $t = t + incr$ until $t = \text{maxtime}$ end	\vdots begin $t = 1$ $x_t = \text{current state}$ repeat \vdots $K_i = lqr(A_i(x_t), B_i(x_t), Q, R), \forall i$ $u = -\sum_{i=1}^r h_i(x_t)K_i x_t$ $x_{t+1} = \text{plant}(x_t, u)$ \vdots $t = t + incr$ until $t = \text{maxtime}$ end

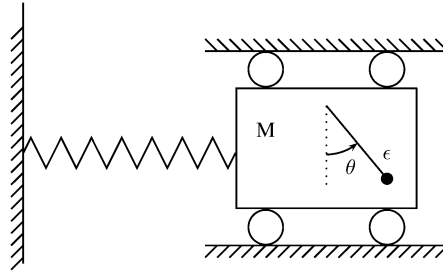


Fig. 1. Nonlinear benchmark problem – Translational Oscillation by a Rotational Actuator (TORA).

Let x_1 be the normalized displacement of the platform from the equilibrium position, $x_2 = \dot{x}_1$, $x_3 = \theta$ be the angle of the rotor, and $x_4 = \dot{x}_3$. Then, the system dynamics can be expressed by the following equation:

$$\dot{x} = f(x) + g(x)u, \quad (17)$$

where u is the torque applied to the eccentric mass and

$$f(x) = \begin{bmatrix} x_2 \\ \frac{-x_1 + \epsilon x_4^2 \sin(x_3)}{1 - \epsilon^2 \cos^2(x_3)} \\ x_4 \\ \frac{\epsilon \cos(x_3)(x_1 - \epsilon x_4^2 \sin(x_3))}{1 - \epsilon^2 \cos^2(x_3)} \end{bmatrix}, \quad g(x) = \begin{bmatrix} 0 \\ \frac{-\epsilon \cos(x_3)}{1 - \epsilon^2 \cos^2(x_3)} \\ 0 \\ \frac{1}{1 - \epsilon^2 \cos^2(x_3)} \end{bmatrix},$$

where $\epsilon = 0.5m$, $-\pi/2 \leq x_3 \leq \pi/2$ and $-1 \leq x_4 \leq 1$. The equilibrium point of this system could be any point $[0, 0, x_3, 0]$ among which only the point $[0, 0, 0, 0]$ is the desired equilibrium point. The linearization around the point $[0, 0, 0, 0]$ has two eigenvalues $\pm i$, which means that the TORA system is a critical nonlinear system.

In the following subsections, linear modeling and its controller, TSFMFC and FGTSMFC, TSFMVC and VGTSMFC and their simulations are presented.

6.1. Linear Modeling and its Linear Controller

The system is linearized around $x_3 = 0$ and $x_4 = 0$ is given by

$$R_1: \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon}{1-\epsilon^2} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon}{1-\epsilon^2} \\ 0 \\ \frac{1}{1-\epsilon^2} \end{bmatrix} u.$$

Linear controller given by

$$u = Kx. \quad (18)$$

Linear quadratic regulator method is used to find out the value K . The simulation and results are given in Subsection 6.4.

6.2. T-S Modeling with Fixed Coefficient and its Fixed Gain Controller

The system is approximated by the following 4-rule Takagi Sugeno fuzzy model with fixed coefficient (TSFMFC).

R_1 : If x_3 is about 0 and x_4 is about 0, then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2} & 0 & \frac{(0.01)^2 \epsilon}{1-\epsilon^2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon}{1-\epsilon^2} & 0 & \frac{-(0.01)^2 \epsilon^2}{1-\epsilon^2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon}{1-\epsilon^2} \\ 0 \\ \frac{1}{1-\epsilon^2} \end{bmatrix} u,$$

R_2 : if x_3 is about 0 and x_4 is about ± 1 , then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2} & 0 & \frac{\epsilon}{1-\epsilon^2} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon}{1-\epsilon^2} & 0 & \frac{-\epsilon^2}{1-\epsilon^2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon}{1-\epsilon^2} \\ 0 \\ \frac{1}{1-\epsilon^2} \end{bmatrix} u,$$

R_3 : if x_3 is about $\pm \pi/2$ and x_4 is about 0, then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2 \beta^2} & 0 & \frac{2(0.01)^2 \epsilon}{\pi(1-\epsilon^2 \beta^2)} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon \beta}{1-\epsilon^2 \beta^2} & 0 & \frac{-2(0.01)^2 \epsilon^2 \beta}{\pi(1-\epsilon^2 \beta^2)} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon \beta}{1-\epsilon^2 \beta^2} \\ 0 \\ \frac{1}{1-\epsilon^2 \beta^2} \end{bmatrix} u,$$

R_4 : if x_3 is about $\pm \pi/2$ and x_4 is about ± 1 , then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2 \beta^2} & 0 & \frac{2 \epsilon}{\pi(1-\epsilon^2 \beta^2)} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon \beta}{1-\epsilon^2 \beta^2} & 0 & \frac{-2 \epsilon^2 \beta}{\pi(1-\epsilon^2 \beta^2)} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon \beta}{1-\epsilon^2 \beta^2} \\ 0 \\ \frac{1}{1-\epsilon^2 \beta^2} \end{bmatrix} u,$$

where $\beta = \cos(88^\circ)$.

Note: Note that each rule consequent is of the form $\dot{x} = A_i x + B_i u$.

By utilizing the concept of Parallel Distributed Compensation (PDC), the following four rules are designed for the FGTSFC.

R_1 : if x_3 is about 0 and x_4 is about 0, then $u_1 = K_1 x$,

R_2 : if x_3 is about 0 and x_4 is about ± 1 then $u_2 = K_2 x$,

R_3 : If x_3 is about $\pm \pi/2$ and x_4 is about 0, then $u_3 = K_3 x$,

R_4 : if x_3 is about $\pm \pi/2$ and x_4 is about ± 1 , then $u_4 = K_4 x$.

Linear quadratic regulator method is used to find out the values of K_i 's. The simulation and results are given in Subsection 6.4.

6.3. T-S Modeling with Variable Coefficient and its Variable Gain Controller

In this subsection, VGTSFC with 2 rule model is presented. In two rule model, two of the state variables in the coefficient matrix are kept varying, i.e., x_3 and x_4 .

R_1 : if x_3 is about 0, then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2 \cos^2(x_3)} & 0 & \frac{x_4^2 \epsilon}{1-\epsilon^2 \cos^2(x_3)} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon \cos(x_3)}{1-\epsilon^2 \cos^2(x_3)} & 0 & \frac{-x_4^2 \epsilon^2 \cos(x_3)}{1-\epsilon^2 \cos^2(x_3)} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon \cos(x_3)}{1-\epsilon^2 \cos^2(x_3)} \\ 0 \\ \frac{1}{1-\epsilon^2 \cos^2(x_3)} \end{bmatrix} u,$$

R_2 : if x_3 is about $\pm \pi/2$, then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-1}{1-\epsilon^2 \cos^2(x_3)} & 0 & \frac{2x_4^2 \epsilon}{\pi(1-\epsilon^2 \cos^2(x_3))} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\epsilon \cos(x_3)}{1-\epsilon^2 \cos^2(x_3)} & 0 & \frac{-2x_4^2 \epsilon^2 \cos(x_3)}{\pi(1-\epsilon^2 \cos^2(x_3))} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{-\epsilon \cos(x_3)}{1-\epsilon^2 \cos^2(x_3)} \\ 0 \\ \frac{1}{1-\epsilon^2 \cos^2(x_3)} \end{bmatrix} u.$$

By utilizing the concept of Parallel Distributed Compensation (PDC), the following two rules are designed for the variable gain T-S fuzzy controller.

R_1 : if x_3 is about 0, then $u_1 = K_1(x)x$,

R_2 : if x_3 is about $\pm \pi/2$, then $u_2 = K_2(x)x$.

Linear quadratic regulator method is used to find out the values of $K_i(x)$'s. The simulation and results are given in Subsection 6.4.

6.4. Simulation and Results

This subsection presents the simulation results and discussions of linear controller (Subsection 6.1), FGTSFC (Subsection 6.2), and VGTSFC (Subsection 6.3). Linear quadratic regulator method is used to find out the values K_i of above controllers.

$$\text{Let } Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } R = 1.$$

For the above values of Q and R, the linear quadratic method gives the the following value for the gain K of the linear controller described in Subsection 6.1

$$K = [-1.7510 \quad 2.2766 \quad 2.7463 \quad 3.4691]. \quad (19)$$

The gain values of FGTSFC (described in Subsection 6.2) are obtained as

$$K_1 = [-2.6490 \quad 0.5780 \quad 1.0001 \quad 2.1563], \quad (20)$$

$$K_2 = [-1.7510 \ 2.2766 \ 2.7463 \ 3.4691], \quad (21)$$

$$K_3 = [-3.1448 \ 0.0129 \ 1.0001 \ 1.4526], \quad (22)$$

$$K_4 = [-2.6071 \ 1.7641 \ 2.2487 \ 2.1725]. \quad (23)$$

The gain values of VGTSFC (described in Subsection 6.3) is dynamic, not unique and it changes with the state of the system and have to be calculated at runtime.

The simulation of TORA system is carried out on Celeron 2GHz machine using MATLAB 6. To simulate the TORA dynamics, a fourth-order Runge-Kutta method (ode45 (Shampine and Reichelt, 1997)) is used with an integration time step size of 0.01 sec. Observation of the system states were made every time step (integration step). The controllers are assumed to be continuous, therefore, the sampling time of the controller was set equal to the integration time step size.

TORA system is simulated with linear controller, FGTSFC and, VGTSFC for several initial conditions. Figs. 2, 3, 4, and 5 respectively show the plot of displacement (x_1), Velocity (x_2) vs Displacement (x_1), angle (x_3) vs angular velocity (x_4), and control force for the initial state $[0, 0, 85^\circ, 0]$ with each of the above mentioned controllers. Table 2 shows the performance of the Linear controller, FGTSFC and, VGTSFC for the initial state $[0, 0, 85^\circ, 0]$ where the Total Absolute Error (TAE) and Performance index (J) is calculated by (24) and (25).

$$TAE = \sum_{t=0:0.01:maxtime} (|x_1(t)| + |x_2(t)| + |x_3(t)| + |x_4(t)|), \quad (24)$$

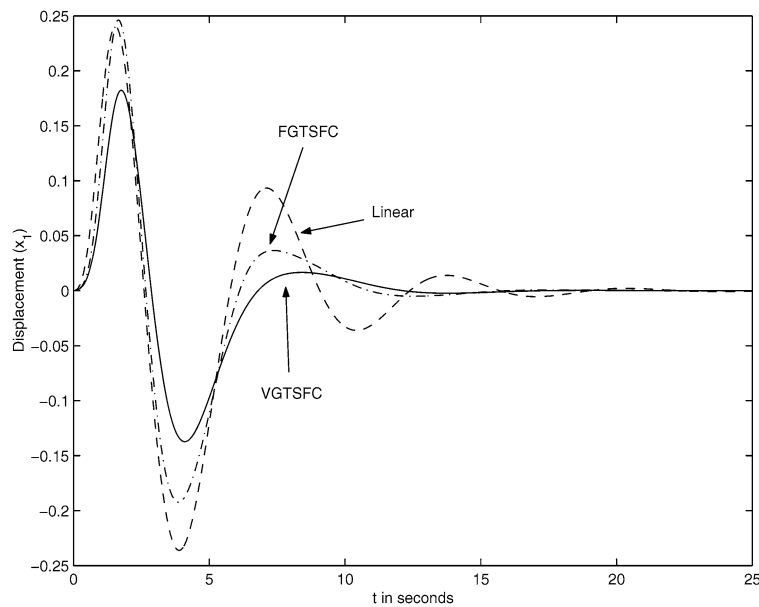


Fig. 2. Displacement (x_1 in meters) of TORA system for the initial state $[0, 0, 85^\circ, 0]$.

$$J = \sum_{t=0:0.01:maxtime} (x^T Q x + u^T R u). \tag{25}$$

It is clear from the Fig. 2 that VGTSFC has lesser displacement compared to Linear controller and FGTSFC. From Fig. 3, it is clear that the area taken by VGTSFC is much lesser when compared to Linear controller and FGTSFC. From Fig. 4, it is clear that the variable x_3 and x_4 also performs well in the case of VGTSFC. It can also be seen from the Fig. 5 that the control force of the VGTSFC is much less when compared to other controllers. It is evident from the Table 2 that Variable Gain Takagi–Sugeno Fuzzy Controllers (VGTSFC) is having lesser error and hence perform much better than the linear controller and Fixed Gain Takagi–Sugeno Fuzzy Controller (FGTSFC).

Simulation results reveals that the Variable Gain Takagi–Sugeno Fuzzy Controller (VGTSFC) can efficiently be used in place of linear controllers and fixed gain Takagi–Sugeno fuzzy controllers with good approximation and control performance.

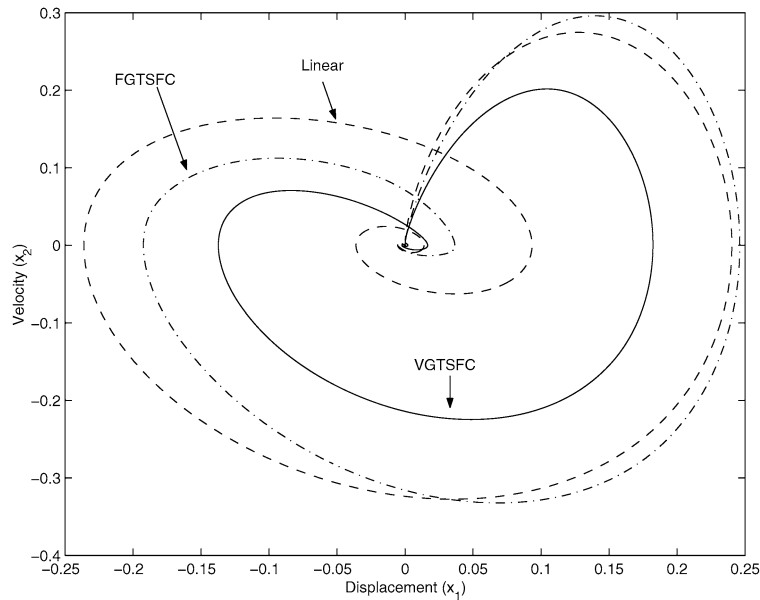


Fig. 3. Velocity (x_2 in m/s) vs Displacement (x_1 in m) for the initial state $[0, 0, 85^\circ, 0]$.

Table 2
Performance of linear controller, FGTSFC and VGTSFC

Methods	Total Absolute Error (TAE)	Performance Index (J)
Linear	702.9854	609.4228
FGTSFC	692.3458	521.8640
VGTSFC	648.5775	432.0092

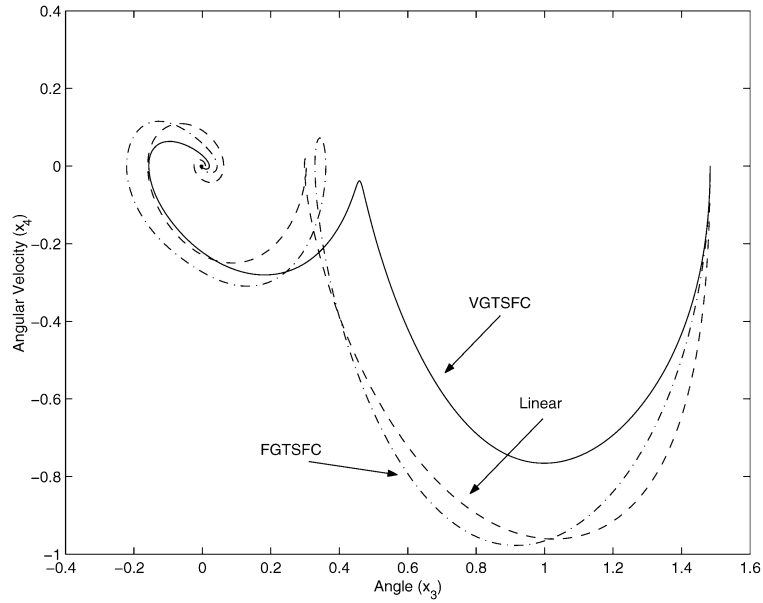


Fig. 4. Angle (x_3) vs Angular velocity (x_4) of TORA system for the initial state $[0, 0, 85^\circ, 0]$.

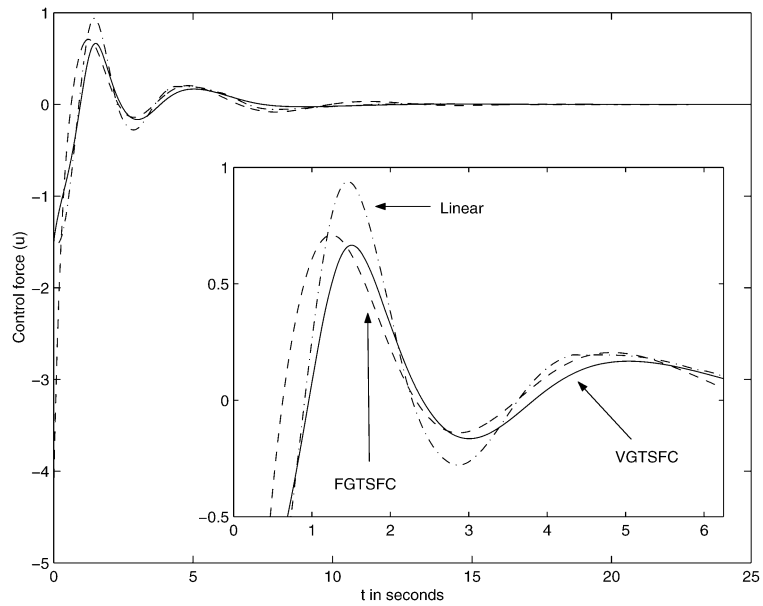


Fig. 5. Control force (u in Newton) applied to the TORA system for the initial state $[0, 0, 85^\circ, 0]$. A portion of the plot is zoomed and is shown inside the figure.

7. Conclusions

A new model namely, Takagi–Sugeno Fuzzy Model with Variable Coefficient (TSFMVC) is presented and it is proved that it can approximate any nonlinear function to any degree of accuracy. A new controller namely, Variable Gain Takagi–Sugeno Fuzzy Controller (VGTSFC) is presented and compared with linear and fixed gain Takagi–Sugeno controllers. The simulation results of TORA system show that the proposed controller is effective and efficient.

The main advantages of the modeling (TSFMVC) and control (VGTSFC) are summarized below.

1. TSFMVC can better approximate a nonlinear dynamic system than TSFMFC and there by reducing the approximation error in modeling.
2. Variable gain Takagi–Sugeno fuzzy controller (VGTSFC), developed on the basis of the Takagi–Sugeno fuzzy modeling with variable coefficient (TSFMVC), shows good control performance when compared to linear controller and fixed gain Takagi–Sugeno fuzzy controllers (FGTSFC).

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Kintamo naudingumo Takagi–Sugeno fuzzy logikos kontroleriai

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Daugelis literatūroje minimų Takagi–Sugeno Fuzzy (TSF) sistemų naudoja tik tiesines įėjimo kintamųjų funkcijas. Jos gali būti vadinamos TSF modeliais su fiksuotais koeficientais (TSFMFC). Šiame straipsnyje pateiktas TSF modelis su kintamu koeficientu (TSFMVC), kuris gali artimiau aproksimuoti netiesinių sistemų, netiesinių dinaminių sistemų ir netiesinių valdymo sistemų klases. Taip pat parodyta, kad TSFMFC yra dalinis TSFMVC atvejis. Be to, yra aprašytas kintamo naudingumo kontroleris (VGTSFC), jis veikia geriau nei fiksuoto naudingumo TSF kontroleris (FGTSFC).