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Determination of Stress Strain State Components of Butt Welded Joint with Mild Interlayer Subjected to Elasto-Plastic Tension

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Abstract. Determination of stress strain state components of butt welded joint with a mild interlayer at elasto-plastic tension is presented in this paper. Function of normal transverse stresses distribution on the thickness of mild interlayer and equations for determination of the stress intensity maximum value in hard metal at elasto-plastic deforming are proposed. The longitudinal stresses of mild and hard metals at elasto-plastic loading are determined from their integral equilibrium condition to mean value of external load by estimating equality of hard and mild metal longitudinal strains on their contact plane. Algorithm of stress strain state components calculation in separate zones of this model is presented. The strength of butt welded joint with a mild interlayer and longitudinal strain distribution at static and cyclic elasto-plastic loading on its surface calculated analytically is verified by the results of experimental investigations.

Key words: model of welded joint, stress strain state, non-uniform strain distribution, mild interlayer, elasto-plastic tension, function of stress distribution.

1. Introduction

A great number of welded joints show higher or lower heterogeneity of mechanical properties of their separate zones. In some cases it is formed for constructional and technological reasons by using base metals and welding materials of different strength. In other cases it appears during exploitation of the welded structures. The mechanically heterogeneous welded joint zone which strength properties are smaller than this of a base metal is called mild interlayer, in opposite case – hard interlayer. Theoretically and experimentally is proved that mild interlayer is strongly affecting the strength of welded joint especially under cyclic elasto-plastic straining. In these solutions hard metal H^1 is

[1] Upper indexes H and M denotes stress strain components of hard and mild metals respectively;
* upper index denotes values of the content plane.

assumed to be absolutely rigid and only in some cases it is absolutely elastic. Experiments and calculations which were performed in our works showed that the life time of defect less construction calculated by neglecting non-uniform strain distribution and was found to be $6 \ldots 10$ times longer than the real one. In order to increase reliability of structures at low cyclic loading the stress and strain state at separate welded joints zones must be evaluated. The conventional analytical calculation methods of mechanically heterogeneous welded joints usually consider the mild interlayer stress state neglecting both the effect of mild metal M on a hard one and non uniform stress distribution in the thickness of mild interlayer at elasto-plastic loading. In addition, hardening of material M in the elasto-plastic zone is not evaluated (Prandtle's type diagram of deforming material M is used). Therefore these analytical methods are suitable approximately to evaluate the ultimate monotonic strength, but they can not to evaluate the strain state of welded joints.

In this paper stress strain state of weld joint with a mild interlayer is determined by using these refinements:

- the effect of mild metal M to base metal H is evaluated;
- distribution function of stress σ_x^M on the thickness of a mild interlayer and stress σ_x^H in the direction of longitudinal axis are proposed;
- deforming diagrams of metal M and H in elasto-plastic zone, approximated linearly or by power function, are used;
- stress intensity determination method at the external point of contact plane (fusion line) $\sigma_{i|\xi=1}^{H*} = \sigma_i^{H*}(1)$, when material H at this point is deformed elasto-plastically, is proposed.

Model of welded joint consist of two materials-base metal H and mild interlayer M (Fig. 1, a).

For stress strain determination of weld joint with a mild weld was choused mathematically more correct analytical solution of elasto-plastically deformed thin mild interlayer ($æ = h/l \ll 1$) subjected to tension (compression) at plane deformation obtained by L.Kachanov (1962). This solution was made on dimensionless co-ordinates $\xi = x/l$, $\eta = y/l$ (Fig. 1, a). The problem was solved by making use of the theory of small elasto-plastic deformations by applying the hypothesis of flat cross-sections and assuming that residual stresses are abolished by heat treatment. The modules of elasticity of both materials are equal each to other ($E^H = E^M = E$), and the material is not pressed neither in elastic nor in elasto-plastic zones. Additionally, when solving this problem it was assumed that the base metal H is ideally elastic and is not influenced by mild metal ($\sigma_x^H = 0$) and

$$\tau_{xy}^M = \tau_{xy}^* \frac{\eta}{æ}, \tag{1}$$

where $\tau_{xy}^* = \tau_{xy}^*(\xi)$ is unknown function of shear stress distribution on the contact plane. Shear stress distribution expressed by Eq. 1 is equivalent to presumption $(\partial\sigma_x/l)\partial\eta$. Also is assumed that tensile curve of mild material M is approximated linearly.

When mild metal is deformed elastically ($\sigma_i^M \leqslant \sigma_e^M$) and Poisson's ratio $\nu = \nu_p = 0.5$ follows that under the plane deformation ($\varepsilon_z = 0$) stresses $\sigma_x = \tau_{xy} = 0$, $\sigma_y = p$,

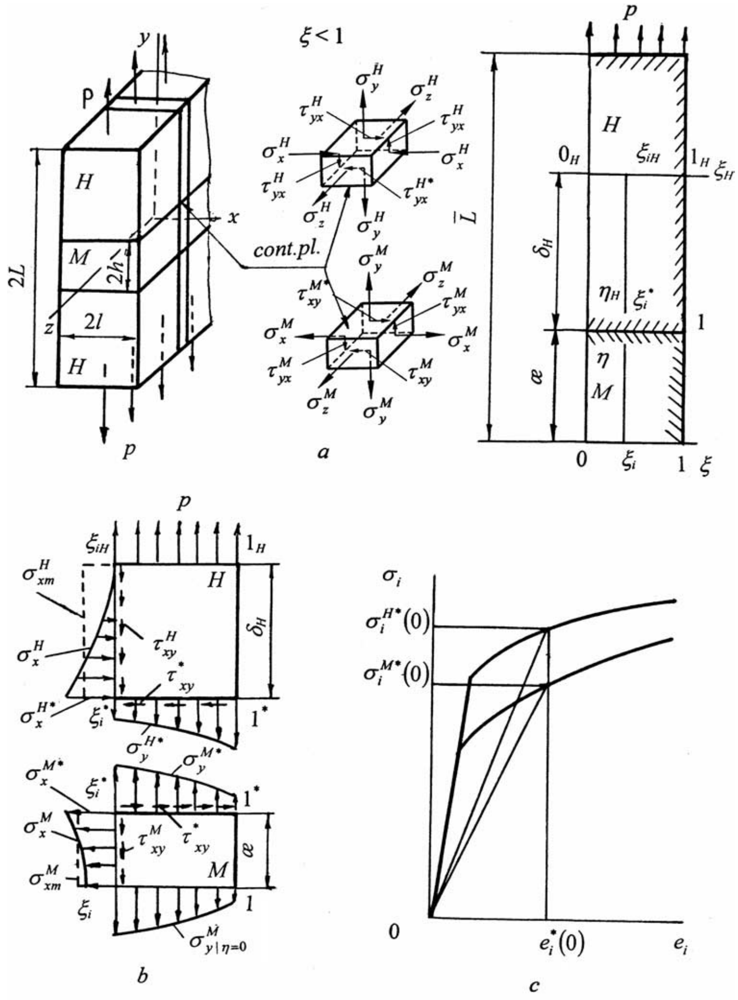

Fig. 1. Scheme for the calculation of welded joint with a mild interlayer at plane deformation.

$\sigma_z = 0.5p$ and $\sigma_i = \sqrt{3}p/2$ (p is mean stress caused by external load). In this case strains are distributed uniformly ($e_y = -e_x = 3p/4E$). When $p > 2\sigma_e^M/\sqrt{3}$, elasto-plastic deforming begins in all volume of the mild interlayer. But on the contact plane of metals M and H the mild metal strains are constrained by hard metal H. Shear stress appears and near the contact plane there is a triaxial stress state. In the mild metal M tension stress σ_x^M appears. Therefore the effect of contact hardening of mild metal takes place.

From equation

$$\frac{\sigma_y - \sigma_x}{\varepsilon_y - \varepsilon_x} = 2\frac{\tau_{xy}}{\gamma_{xy}} \tag{2}$$

which describes condition that directions of stress and strains vectors are the same. By Eq. 1 written to contact plane ($\eta = æ$) and hypothesis of flat sections ($\frac{\partial v}{\partial x} = \frac{1}{l}\frac{\partial v}{\partial \xi} = 0$) was obtained function

$$\tau_{xy}^{M*}(\xi) = \pm\Big[\frac{\sigma_e^M}{\sqrt{3}} \cdot \frac{1-\bar{E}_1^M}{\sqrt{1+C^{M*2}\xi^2}}\xi + \frac{2}{3}E_1^M\frac{v'(æ)}{l}C^{M*}\xi\Big], \tag{3}$$

where E_1^M is modulus of mild metal hardening at elasto-plastic zone when curve $\sigma_i^M - \varepsilon_i^M$ is approximated linearly; $\bar{E}_1^M = E_1^M/E$; $v'(æ)/l = 3p/(4E)$.

After integration differential condition of equilibrium it was obtained

$$\sigma_x^M = \sigma_x^M(\xi) = \frac{1}{æ}\int_\xi^1 \tau_{xy}^{M*}(\xi)\,\mathrm{d}\xi + \beta, \tag{4}$$

where β is the constant of integration.

Stress

$$\sigma_y^M = \sigma_x^M + \frac{2\tau_{xy}^M(\xi)}{C^{M*}\xi} \tag{5}$$

is obtained by condition (2) and hypothesis of flat sections $\partial v/(l\partial\xi) = 0$.

By integral equilibrium condition

$$p = \int_0^1 \sigma_y\,\mathrm{d}\xi \tag{6}$$

and Eq. 5 is obtained the dependence

$$\frac{C_1^M}{4æC^{M*}} + \frac{1}{C^{M*}}\Big(1 - \frac{1}{4æC^{M*}}\Big)\ln(C^{M*} + C_1^M) = \frac{\frac{\sqrt{3}}{2}\big[p - \frac{4}{3}E_1^M e_y^*\big(1 + \frac{C^{M*}}{6æ}\big) - \beta\big]}{\sigma_e^M(1-\bar{E}_1^M)} \tag{7}$$

for determining of constant C^{M*}; where $C_1^M = \sqrt{1+(C^{M*})^2}$. For comparison equations of work (Kachanov, 1962) with presented in this subchapter these expressions are a little changed. When $E_1^M = \bar{E}_1^M = 0$ constant $\beta = \frac{\sigma_e^M}{\sqrt{3}}\big(\frac{C_1^M}{C^{M*}}\arcsin\frac{C^{M*}}{C_1^M} - \frac{1}{C_1^M}\big)$. In this case stress at the cross-sections $0 \leqslant \eta \leqslant æ$ can be calculated from condition $\sigma_i^M = \sigma_e^M$.

Because determination of coefficient C^{M*} is very complicated $E_1^M = 0,\ \partial\sigma_y^M/\partial\eta = 0,\ C^{M*} \to \infty$ were assumed. Then limit mean stress was determined from formula

$$p_{ut} = \frac{2}{\sqrt{3}}\sigma_e^M\Big(\frac{\pi}{4} + \frac{1}{4æ}\Big). \tag{8}$$

From Eq. 8 follows that $p_{ut} \to \infty$ when $æ \to 0$. Limit strength of real welded joint with a mild interlayer $p_{ut\,\max} = \sigma_{ut}^H$ when $æ \leqslant æ_{\min}$ and $p_{ut\,\min} = \sigma_{ut}^M$ when $æ \leqslant æ_{\max}$

($æ_{\min}$ and $æ_{\max}$ values can be determined from Eq. 8 when $p = p_{ut\,\max}$ and $p = p_{ut\,\min}$ respectively).

Because hardening of material M at elasto-plastic zone is not evaluated ($E_1^M = 0$) this method is suitable to evaluate approximately the ultimate strength of welded joint when $æ_{\min} \leqslant æ \leqslant æ_{\max}$ but it is not applicable for determination of strain distribution. For determination lifetime of welded joint at cyclic elasto-plastic loading the separate zone strain distribution must be known (Bae, 2003; Dexter, 1999; Rudolph, 2001).

From Saint–Venant'e principle follows that with increasing of a relative distance from the contact plane of M and H materials stress σ_x^M rapidly decreases.

Therefore the presumption that stresses σ_x^M and σ_y^M at the thickness of interlayer are distributed uniformly ($\sigma_x^M/\partial\eta = \partial\sigma_y^M/\partial\eta = 0$) for real mild welds ($æ \geqslant 0.6 \ldots 0.75$) is not acceptable.

2. Analytical Calculation of Stresses and Strains

Solution presented in this paper made more accurate by estimating influence of metal M on stress state of hard metal H ($\sigma_x^{H*} < 0$), real strength characteristics of M and H determined from their tensile curves and non-uniform σ_x^M distribution on the direct of longitudinal axis of welded joint. At elasto-plastic straining of material M ($p > 2\sigma_e^M/\sqrt{3}$) when stress σ_x^{M*} on the contact plane is known in other cross-sections

$$\sigma_x^M = \sigma_x^{M*} f_1(\eta). \tag{9}$$

Examination of stress distribution in a mild interlayer (Fig. 1, b) shows, that function $f_1(\eta)$ must have such peculiarities:

$$\begin{aligned}
&\lim_{æ\to 0} f_1(\eta) = 1,\\
&\lim_{æ\to æ_l} f_1(\eta) = 0,\\
&f_1(\eta) = f_1(-\eta),\\
&f_1(æ) = 1,
\end{aligned}$$

which leads to the equation

$$f_1(\eta) = 1 - \Big[1 - \frac{1}{F(æ)}\Big]\Big(1 - \frac{\eta^2}{æ^2}\Big). \tag{10}$$

Function $F(æ) = 1 + M_1 æ^{n_1}$ increases with increasing of $æ$ and mechanical heterogeneity of welded joint. Determination of a function $F(æ)$ is described in work (Bražėnas, 1995).

Equilibrium condition of element $\xi_i\xi_i^* 1^* 1$ of metal M (Fig. 1, b) makes it possible to calculate mean value of stress σ_x^M on the thickness of interlayer

$$\sigma_{xm}^M = \frac{1}{æ}\int_0^{æ} \sigma_x^M \,\mathrm{d}\eta = \frac{1}{æ}\int_{\xi}^{1} \tau_{xy}^{M*}\,\mathrm{d}\xi. \tag{11}$$

Then from Eqs. 9–11 is found

$$\sigma_x^M = f_2(\eta)\int_\xi^1 \tau_{xy}^{M*}(\xi)\,\mathrm{d}\xi + C_1, \tag{12}$$

where

$$f_2(\eta) = 3\frac{1-\left[1-\frac{1}{F(æ)}\right]\left(1-\frac{\eta^2}{æ^2}\right)}{æ\left[1+\frac{2}{F(æ)}\right]}. \tag{13}$$

Constant $C_1 = 0$ because $\sigma_{x|\xi=1}^M = \sigma_{x1}^M = 0$.

Shear stress τ_{xy}^M in cross-section $\eta = \text{const}$ may be calculated from equilibrium condition

$$\tau_{xy}^M = -\int_0^\eta \frac{\partial\sigma_x^M}{\partial\xi}\,\mathrm{d}\eta + C_2, \tag{14}$$

where $C_2 = 0$ because $\tau_{.xy|\eta=0}^M = \tau_{xyc}^M = 0$. After designating

$$\Phi(\eta) = \int_0^\eta f_2(\eta)\,\mathrm{d}\eta = 3\frac{\eta}{æ}\frac{1-\left[1-\frac{1}{F(æ)}\right]\left(1-\frac{\eta^2}{3æ^2}\right)}{1+\frac{2}{F(æ)}} \tag{15}$$

from Eqs. 11–15 is calculated shear stress

$$\tau_{xy}^M = \Phi(\eta)\tau_{xy}^{M*}(\xi). \tag{16}$$

Shear stress τ_{xy}^* is determined from condition of interaction materials H and M at the contact plane. An initial point of material H coordinate system O_H is chosen in a section which is remote from the contact plane at relative distance δ_H in which $\sigma_x^H \approx 0$ (Fig. 1, b). When metal H is deformed elastically the relationship between stress strains intensities may be expressed by equation

$$\sigma_i^H = \sqrt{\frac{3}{4}\left(\sigma_y^H - \sigma_x^H\right)^2 + 3\tau_{xy}^{H^2}} = E\sqrt{\frac{4}{3}e_y^{H^2} + \frac{1}{3}\gamma_{xy}^{H^2}}. \tag{17}$$

From Eq. 3 for elastically deformed material H ($E_1 = E$, $\bar{E}_1 = E_1/E = 1$) was determined

$$\tau_{xy}^{H*} = \frac{2}{3}Ee_y^*C_H^*\xi = pC_p^*\xi, \tag{18}$$

where pC_p^* is a parameter of shear stress.

In other cross-sections

$$\tau_{xy}^H = \Phi(\eta_H)pC_p^*\xi. \tag{19}$$

Expression of σ_x^H is obtained in the same way as σ_x^M calculated by Eq. 9

$$\sigma_x^H = -\frac{1}{2} f_2(\eta_H) p C_p^* (1-\xi^2). \tag{20}$$

From the hypothesis of flat sections and the generalized Hooke's law when $\nu = 0.5$ follows that on the contact plane

$$e_y^{H*} = \frac{3}{4E}(\sigma_y^{H*} - \sigma_x^{H*}) = \text{const.} \tag{21}$$

Stress

$$\sigma_y^{H*} = \frac{2}{\sqrt{3}} \sigma_i^{H*}(0) + \sigma_x^{H*} \tag{22}$$

may be determined from Eqs. 17, 21 by estimating that $\tau^*_{xy|\xi^*=0} = \tau^*_{xy}(0) = 0$.

Functions $f_2(\eta_H)$ and $\Phi(\eta_H)$ are determined from presumption that $f_2(\eta_H) = f_2(\eta)$ and $\Phi(\eta_H)$ $\Phi(\eta)$ when $æ = \delta_H$. Therefore $f_2(\eta_H)$ and $\Phi(\eta_H)$ are determined from Eqs. 12 and 15 by substituting δ_H, η_H and $F(\delta_H)$ instead of $æ$, η and $F(æ)$.

Shear stress on the contact plane of materials H and M are equal$\tau_{xy}^{M*} = \tau_{xy}^{H*} = \tau_{xy}^* = pC_p^*\xi$. Then from Eqs. 9, 10, 12 and 13 follows

$$\sigma_x^{M*} = \frac{1}{2} f_2(æ) p C_p^* (1-\xi^2), \tag{23}$$

$$\sigma_x^{M} = \frac{1}{2} f_2(\eta) p C_p^* (1-\xi^2). \tag{24}$$

From equality of linear strains of metals M and H on the contact plane is determined stress

$$\sigma_y^{M*} = \sigma_x^{M*} + \frac{2}{\sqrt{3}} \sigma_i^{M*}(0) - F^{M*}, \tag{25}$$

where F^{M*} is a correction function, obtained from equality of linear strains of metals M and H on the contact plane ($e_y^{M*} \approx e_y^{H*}$). Determining of F^{M*} is related with the considerable mathematical difficulties. From condition $e_y^{M*} \approx e_y^{H*}$ and by estimating that $E'^M = \sigma_i^M / e_i^M$ and strain e_y^{M*} depends on τ^*_{xy} and this function rapidly decrease with increasing distance form the contact plane point $\xi^* = 1$ it may be expressed by equation

$$F^{M*} = D^M \left|\tau_{xy}^{M*}(1)\right| \xi^n = D^M p C_p^* \xi^n. \tag{26}$$

Coefficient D^M is determined from the condition $e_y^{H*}(1) = e_y^{M*}(1)$ at an external point of the contact plane $\xi^* = 1$ by approaching method. Power index n is calculated from the condition $e^{H*}_{y|\xi=0.5} = e^{M*}_{y|\xi=0.5}$. Then strains e_y^{H*} and e_y^{M*} are equal at the three points of the contact plane $\xi^* = 0;\ 0.5;\ 1$. At the other points of contact plane the

Table 1

Values of longitudinal strains on the contact plane of welded joint when $\gamma_e = \sigma_e^H/\sigma_e^M = 1.2$, $æ = 0.8$, $m_0^M = 0.102$, $m_0^H = 0.125$, $p/\sigma_e^M = 1.4$, when precision of calculations is 1%

	Strains are equal at three points of contact plane $\xi = 0.0;\ 0.5;\ 1.0$			Strains are equal at two points of contact plane $\xi = 0.0;\ 1.0$		
	$p/\sigma_e^M = 1.398$; $e_i^*(0)/e_e^M = 1.684$; $\gamma_N = 1.187$; $pC_p^*/\sigma_e^M = 0.161$; $D^M = -0.738$; $D^H = -1.234$; $n = 1.987$; the total count of iterations: 142			$p/\sigma_e^M = 1.390$; $e_i^*(0)/e_e^M = 1.621$; $\gamma_N = 1.186$; $pC_p^*/\sigma_e^M = 0.159$; $D^M = -0.726$; $D^H = -1.020$; $n = 2.000$; the total count of iterations: 94		
ξ	$\frac{e_y^{M*}(1)}{e_e^M}$	$\frac{e_y^{H*}(1)}{e_e^M}$	disagreement, %	$\frac{e_y^{M*}(1)}{e_e^M}$	$\frac{e_y^{H*}(1)}{e_e^M}$	disagreement, %
0.0	1.458	1.458	0.000	1.404	1.404	0.000
0.1	1.477	1.477	0.000	1.421	1.421	0.000
0.2	1.534	1.535	0.065	1.474	1.474	0.000
0.3	1.634	1.635	0.061	1.567	1.567	0.000
0.4	1.782	1.783	0.056	1.705	1.705	0.000
0.5	1.990	1.990	0.000	1.897	1.898	0.053
0.6	2.27	2.271	0.044	2.156	2.159	0.139
0.7	2.645	2.644	0.038	2.501	2.506	0.200
0.8	3.14	3.138	0.064	2.955	2.965	0.337
0.9	3.795	3.791	0.106	3.553	3.571	0.504
1.0	4.661	4.655	0.129	4.339	4.370	0.709

condition $e_y^{H*} = e_y^{M*}$ is content approximately (Table 1). But determination of n is more complicated. Therefore it is recommended to determinate n at strain loading for making more accurate determination of mean longitudinal strain e_{ym}. At first approaching is assumed $n = 2$. In this case strains e_y^{H*} and e_y^{M*} are equal at the two points of the contact plane $\xi^* = 0; 1$.

In cross-sections $\eta < æ$

$$\sigma_y^M = \sigma_x^M + \frac{2}{\sqrt{3}}\sigma_i^M(0) - \Phi(\eta)F^{M*}. \tag{27}$$

For determining of stress strain state components of M and H metals the strain intensity $e_i^*(0)$ at the centre of the contact plane and coefficient of mechanical heterogeneity $\gamma_N = \sigma_i^{H*}(0)/\sigma_i^{M*}(0)$ must be known (Fig. 1, c).

Stress intensity of metal H on the contact plane is calculated from Eq. 17 by estimating (18) and (22). Then

$$\sigma_i^{H*} = \sqrt{\left[\sigma_i^{H*}(0)\right]^2 + 3(pC_p^*\xi)^2}. \tag{28}$$

The largest it value is at external point of contact plane $\xi^* = 1$, maximum stress intensity of elastically deformed metal H

$$\sigma^H_{i\max} = \sigma^{H*}_{i|\xi=1} = \sigma^{H*}_i(1) = \sqrt{\left[\sigma^{H*}_{ie}(0)\right]^2 + 3(p_e C^*_{pe})^2} = \sigma^H_e, \tag{29}$$

where $\sigma^{H*}_{ie}(0)$ and $p_e C^*_{pe}$ are values of $\sigma^{H*}_i(0)$ and pC^*_p which corresponds to $\sigma^{H*}_i(1) = \sigma^H_e$. Their values is determined from condition (28) by approaching method.

When $\sigma^{H*}_i(0) > \sigma^{H*}_{ie}(0)$ metal H at the contact plane point $\xi^* = 1$ is deformed elasto-plastically ($\sigma^{H*}_i(1) > \sigma^H_e$). Stress state near this point and stress concentration zone under elasto-plastic loading is similar. Therefore strain e^{H*}_y near of the point $\xi^* = 1$ increases and hypothesis of plane section is not valid.

Stress intensity $\sigma^{H*}_i(1)_{SCZ}$ at the stress concentration zone, caused by interaction of materials H and M, under elasto-plastic loading is determined from equation

$$\left[\bar{\sigma}^{H*}_{if}(1)\right]^2 = \bar{\sigma}^{H*}_i(1)_{SCZ} \cdot \bar{e}^{H*}_i(1), \tag{30}$$

where $\sigma^{H*}_{if}(1)$ is fictitious stress intensity at the point $\xi^* = 1$, when material H is ideally elastic; $\bar{\sigma}^{H*}_{if}(1) = \sigma^{H*}_{if}(1)/\sigma^H_e$; $\bar{\sigma}^{H*}_i(1)_{SCZ} = \sigma^{H*}_i(1)_{SCZ}/\sigma^H_e$; $\bar{e}^{H*}_i(1) = e^{H*}_i(1)/e^H_e$.

This equation was used for determination of $\bar{\sigma}_{i\max}$ in stress concentration zone at elasto-plastic straining (Neuber, 1961). When tensile curve of material H at elasto-plastic loading is approximated by power function

$$\sigma^{H*}_i(1)_{SCZ} = \sigma^{H*}_e\left[\bar{\sigma}^{H*}_{if}(1)\right]^{2m^H_0/(m^H_0+1)}, \tag{31}$$

where m^H_0 is power index of material H hardening at elasto-plastic zone.

When tensile curve of material H in elasto-plastic zone is approximated linearly

$$\sigma^{H*}_i(1)_{SCZ} = \sigma^{H*}_e\left[\frac{1-\bar{E}^H_1}{2} + \sqrt{\left(\frac{1-\bar{E}^H_1}{2}\right)^2 + \bar{E}^H_1\frac{\sigma^{H*}_{if}(1)}{\sigma^{H*}_e}}\right]. \tag{32}$$

Intensity of fictitious elastic stress $\sigma^{H*}_{if}(1)$, which corresponds to $e^*_i(0)$, may be calculated from Eq. 30 when material H is absolutely elastic and $\xi^* = 1$, then

$$\sigma^{H*}_{if}(1) = \sqrt{\sigma^{H*}_{if}(0)^2 + 3(p_f C^*_{pf})^2}, \tag{33}$$

where $\sigma^{H*}_{if}(0) = \gamma_{Nf}\sigma^{M*}_i(0)$ is intensity of fictitious elastic stress on the centre of contact plane. Fictitious coefficient of mechanical heterogeneity $\gamma_{Nf} = \gamma_e$, when $e^{H*}_i(0) \leqslant e^H_e$ and $\gamma_{Nf} = \bar{e}^{M*}_i(0)$ when $e^{H*}_i(0) > e^H_e$. Parameter $p_f C^*_{pf}$ is calculated by substituting γ_{Nf} instead of γ_N.

At elasto-plastic deforming of material H the same expression of σ^H_y as in mild metal σ^M_y is accepted. On the contact plane

$$\sigma^{H*}_y = \sigma^{H*}_x + \frac{2}{\sqrt{3}}\sigma^{H*}_i(0) - F^{H*} \tag{34}$$

and in other cross-sections

$$\sigma_y^H = \sigma_x^H + \frac{2}{\sqrt{3}}\sigma_i^H(0) - \Phi(\eta_H)F^{H*}, \tag{35}$$

where $F^{H*} = D^H pC_p^* \xi^2$. When $\sigma_i^{H*}(1) > \sigma_e^H D^H$ is calculated from dependency

$$D^H = \frac{2\{\sigma_i^{H*}(0) - \sqrt{[\sigma_i^{H*}(1)_{SCZ}]^2 - 3(pC_p^*)^2}\}}{\sqrt{3}pC_p^*}. \tag{36}$$

Mean stress p is calculated from equilibrium condition (6) by estimating (25) or (34) equation, i.e.:

$$p = \frac{2}{\sqrt{3}}\sigma_i^{M*}(0) + \frac{1}{3}pC_p^*\Big[f_2(æ) - \frac{3D^M}{n+1}\Big]$$

or

$$p = \frac{2}{\sqrt{3}}\sigma_i^{H*}(0) - \frac{1}{3}pC_p^*\big[f_2(\delta^H) + D^H\big].$$

Parameter of shear stress pC_p^* is determined from equality of these equations, then

$$pC_p^* = \frac{2\sqrt{3}(\gamma_N - 1)\sigma_i^{M*}(0)}{f_2(\delta_H) + f_2(æ) - (3D^M/(n+1) - D^H)}.$$

Stress intensity $\sigma_i^M(0)$ at the cross-section $0 \leqslant \eta \leqslant æ$ is determined from Eq. 6 by estimating dependence (27). Analogically when $0 \leqslant \eta_H \leqslant \delta_H$ from Eq. 6 by estimating Eq. 35 is determined $\sigma_i^H(0)$. When material H is deformed elastically ($\sigma_i^{H*}(1) \leqslant \sigma_e^H$) in equations of σ_y^H, pC_p^* and $K_æ$ must be accepted $D^H = 0$.

Coefficient of the contact hardening of mild metal can be determined by equation

$$K_æ = \frac{p}{\sigma_{i\,\max}^M K_{sh}}, \tag{37}$$

where $K_{sh} = p/\sigma^H_{i|\eta_H=0}$ is shape coefficient which depends on stress state triaxiality on the cross-section of hard metal, which is remote from contact plane at relative distance more than δ_H. At the plane deformation $K_{sh} = 2/\sqrt{3}$ and $K_{sh} = 1$ at axial symmetrical deformation. $K_æ$ is found due to interaction of materials H and M. This coefficient is determined by estimating that $\sigma_{i\,\max}^M = \sigma_{i|\eta=0}^M(0) = \sigma_{ic}^M(0)$, then

$$K_æ = \frac{1}{1 - \frac{f_2(0)(\gamma_N - 1)}{f_2(\delta_H) + D^H + \gamma_N[f_2(æ) - D^M]}}. \tag{38}$$

When it is necessary to obtain mean stresses $p = p(\sigma_{i\,\max}^M)$ maximum value of coefficient $K_æ$ must be determined by Eqs. 37 and 38 by evaluating$\sigma_{i\,\max}^M = \sigma_{ut}^M$. Then ultimate strength of welded joint with the mild flat interlayer

$$p_{ut} = \sigma_{ut}^M K_{sh} K_æ. \tag{39}$$

In the first approaching may be assumed $\gamma_N \approx \sigma_{ut}^H/\sigma_{ut}^M - æ(\sigma_{ut}^H/\sigma_{ut}^M - \sigma_e^H/\sigma_e^M)$ and $D^H \approx \gamma_e D^M$.

Distribution of stresses $\tau_{xy}^M, \tau_{xy}^H, \sigma_x^M$ and σ_x^H in longitudinal direction of joint depends on $F(æ)$. Function $F(æ)$ was obtained by presumption that it depends on coefficient of mechanical heterogeneity γ_N, relative thicknesses $æ$, δ_H and for materials M and H is the same. Also it was assumed that $F(æ)$ does not depend on cross-section of welded joint when $æ = \delta_H$. These presumptions were verified by the results of experimental investigations of mechanically heterogeneous welded specimens with rectangular, disk and ring shape cross-sections. From results of theoretical and experimental investigations (Bražėnas, 1995) where obtained

$$F(æ) = 1 + M_1 æ^{n_1} \quad \text{and} \quad F(\delta_H) = 1 + M_1 \delta_H^{n_1},$$

where $M_1 = 6.23 + 0.758(\gamma_N - 1)$ and $n_1 = 2.53 - 0.15(\gamma_N - 1)$. The values of contact hardening coefficient $K_{æ} = \sqrt{3}p/(2\sigma_e^M)$, when $\gamma_N > 2$ and $0.2 \leqslant æ \leqslant 0.8$, calculated from Eq. 8 showed a good agreement with the experimental data (Bakshi, 1965). Expression of $F(æ)$, upper value of M_1 and lower value of n_1 were determined by conditions (8) and (38) when $\gamma_N = 3.33$ (Bražėnas, 1995). Also from this condition $f_2(\delta_H) = 2.13$ and $\delta_H = 1.2$ are determined. This value of δ_H showed a good agreement with results obtained by FEM.

Solution presented in this paper makes it possible to calculate strains in each point of welded joint. Longitudinal strains of the base metal H

$$e_y^H = -e_x^H = \frac{3(\sigma_y^H - \sigma_x^H)}{4E}, \quad \text{when } \sigma_i^H \leqslant \sigma_e^H,$$
$$e_y^H = -e_x^H = \frac{3(\sigma_y^H - \sigma_x^H)}{4E'^H}, \quad \text{when } \sigma_i^H > \sigma_e^H, \tag{40}$$

and of mild metal

$$e_y^M = -e_x^M = \frac{3(\sigma_y^M - \sigma_x^M)}{4E'^M}, \tag{41}$$

where $E'^H = \sigma_i^H/e_i^H$ and $E'^M = \sigma_i^M/e_i^M$ are secant modules of tensile curves.

Strain intensity at plane deformation

$$e_i = \sqrt{\frac{4}{3}e_y^2 + \frac{1}{3}\gamma_{xy}^2}.$$

Cyclic stresses and strains of separate zones of welded joint are calculated referring to V. Moskvitin theorem. In simple loading they can be calculated from the dependence of static loading by substituting the parameters of cyclic loading diagrams instead of static loading characteristics (Moskvitin, 1965).

At strain loading a mean longitudinal strain $e_{ym}(e_{zm})$ on the relative base of deformation $\bar{L} = L/l$ must be known. Because solution of the problem is approximate mean longitudinal strain at the base deformation $\bar{L}$ is calculated at the longitudinal section $\xi = \xi_c = 0.5$. Under plane deformation a mean longitudinal strain

$$e_{ym} = \frac{1}{\bar{L}}\left[e_{yl}^{H}(\bar{L} - æ - \delta_H) + \int_0^{\delta_H} e^H_{y|\xi=0.5}\,\mathrm{d}\eta_H + \int_0^{æ} e^M_{y|\xi=0.5}\,\mathrm{d}\eta\right], \quad \text{when } \bar{L} > æ + \delta_H,$$

$$e_{ym} = \frac{1}{\bar{L}}\left[\int_{1.2+æ-\bar{L}}^{1.2} e^H_{y|\xi=0.5}\,\mathrm{d}\eta_H + \int_0^{æ} e^M_{y|\xi=0.5}\,\mathrm{d}\eta\right], \quad \text{when } \bar{L} \leqslant æ + \delta_H, \tag{42}$$

where e_{yl}^H is strain of metal H at the section in which disappears interaction between M and H materials ($\sigma_x^H = 0$). These integrals were calculated by numerical method.

3. Analytical Calculation of Stresses and Strains Distribution

For determination of stress strain state in welded joint the mechanical properties of M and H materials at elasto-plastic deformation, $e_i^*(0)$, $æ$, δ_H and precision of calculations must be known. Sufficient precision of calculations for practical requirements is 1...3%.

The basic parameters of calculations are $\gamma_N, pC_p^*, D^M, D^H$ and n. When these parameters are known the stress strain state at any point of welded joint can be determined by changing ξ and η(or η_H) in (16), (19), (20), (24), (27), (35), (40) and (41) equations.

When welded joint is subjected to tension may be two cases of loading: stress loading ($p = \text{const}$) strain loading ($e_{ym} = \text{const}$). Therefore the approaching method for determination of stress strain state in welded joint must be used.

Algorithm for determination of welded joint stress strain state components under stress or strain loading is presented in Fig. 2. Where and in Fig. 3 algorithm the function f describes dependency between stress intensity σ_i and strain intensity e_i at elasto-plastic deforming. For materials approximated by power function

$$\begin{cases} \sigma_i = f(e_i) = e_i E, & \text{when } \sigma_i \leqslant \sigma_e, \\ \sigma_i = f(e_i) = \sigma_e\left(\frac{e_i}{e_e}\right)^{m_0}, & \text{when } \sigma_i > \sigma_e, \end{cases}$$

and

$$\begin{cases} e_i = f(\sigma_i) = \frac{\sigma_i}{E}, & \text{when } e_i \leqslant e_e, \\ e_i = f(\sigma_i) = e_e\left(\frac{\sigma_i}{\sigma_e}\right)^{1/m_0}, & \text{when } e_i > e_e, \end{cases}$$

and for materials approximated linearly

$$\begin{cases} \sigma_i = f(e_i) = e_i E, & \text{when } \sigma_i \leqslant \sigma_e, \\ \sigma_i = f(e_i) = \sigma_e + E_T(e_i - e_e), & \text{when } \sigma_i > \sigma_e, \end{cases}$$

and

$$\begin{cases} e_i = f(\sigma_i) = \frac{\sigma_i}{E}, & \text{when } e_i \leqslant e_e, \\ e_i = f(\sigma_i) = e_e + \frac{\sigma_i - \sigma_e}{E_T}, & \text{when } e_i > e_e, \end{cases}$$

where m_0 and E_T are material hardening parameters under elasto-plastic loading for power and linear approximation respectively.

The strain intensity $e_i^*(0)$ of first approaching in Fig. 2 item 5 may be calculate in this manner:

for stress loading

$$\sigma_i^{M*}(0) = p\Big[\frac{\sqrt{3}}{2} - \frac{(\gamma_N - 1)f_2(æ)}{f_2(\delta_H) + \gamma_N f_2(æ)}\Big] \text{ and } e_i^*(0) = f(\sigma_i^{M*}(0));$$

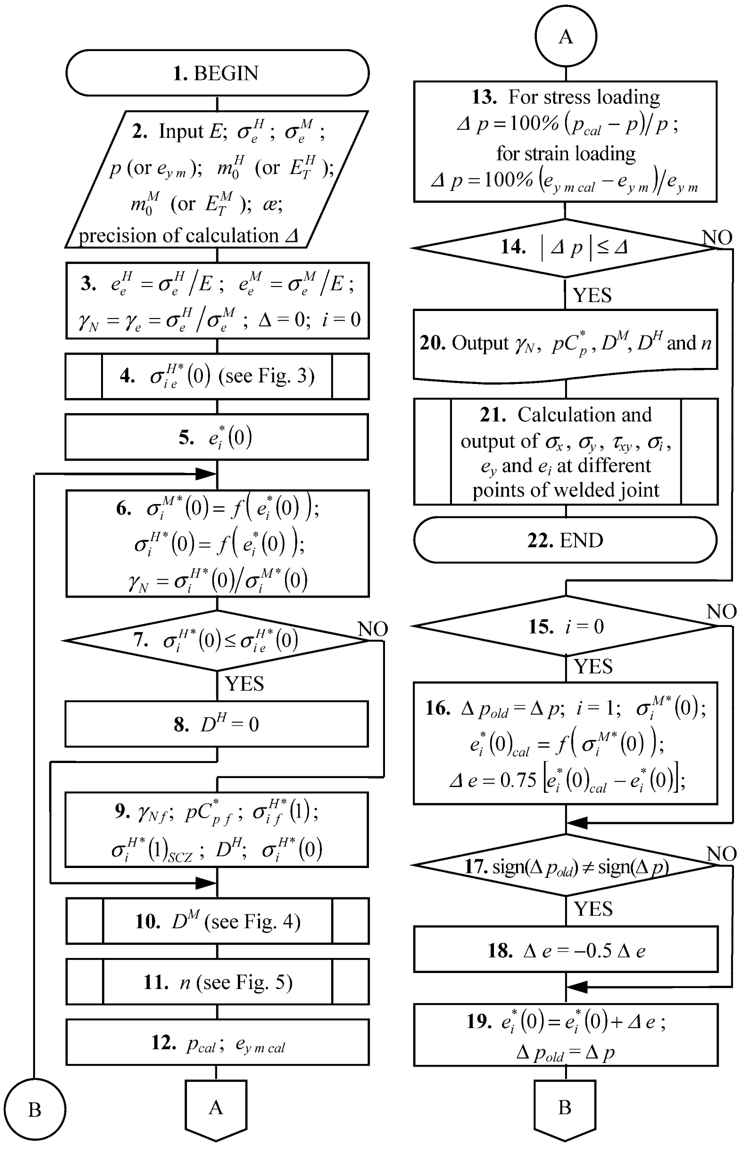

Fig. 2. Algorithm for determination of stress strain state components in welded joint at stress or strain loading.

$$\text{for strain loading } e_i^*(0) = \frac{6\gamma_N e_{ym} L}{4[3(L-æ)-\delta_H] + 2\gamma_N(æ+\delta_H) + \gamma_N^2(3æ-\delta_H)}.$$

Algorithm for determination of $\sigma_{ie}^{H*}(0)$, i.e. loading state when hard base metal H is deformed only elastically, is presented in Fig. 3. For cyclic loading may be taken $D^H = 0$ (Bražėnas, 1995). The stress intensity $\sigma_{ie}^{H*}(0)$ of first approaching in Fig. 3 item 4.2 may be calculate in this manner:

$$\sigma_{ie}^{H*}(0) = \sqrt{(\sigma_e^H)^2 - 12\Big[\frac{(\gamma_e - 1)\sigma_e^M}{f_2(\delta_H) + f_2(æ)}\Big]^2}.$$

Algorithm for determination of parametre D^M is presented in Fig. 4. The ΔD of Fig. 4 item 10.9 may be calculate in this manner:

$$\Delta D = \frac{2[\sigma_i^{M*}(0) - 2e_i^{H*}(1)E'^M(1)/(\sqrt{3}\gamma_N)]}{\sqrt{3}pC_{pe}^*}\operatorname{sign}(\Delta e^*).$$

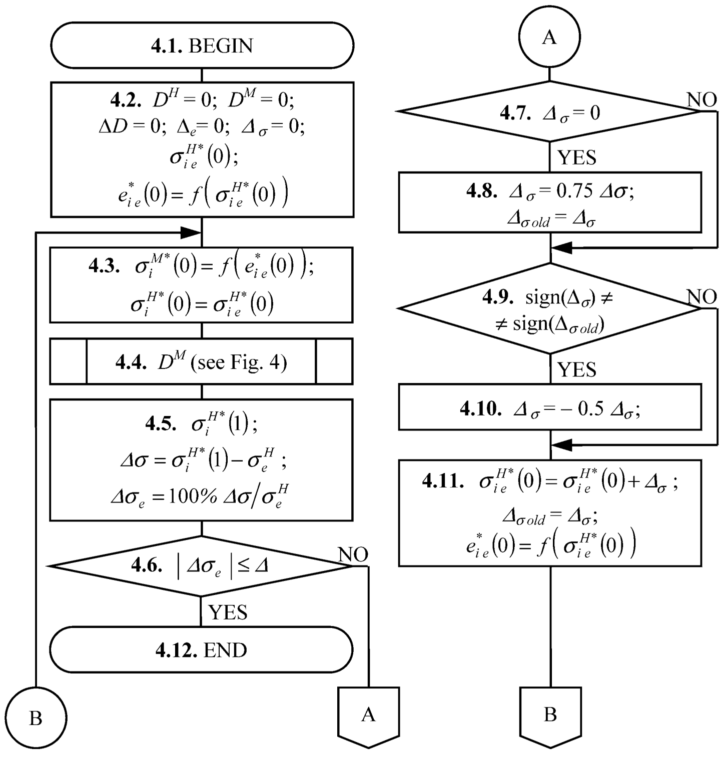

Fig. 3. Algorithm for determination of stress intensity $\sigma_{ie}^{H*}(0)$.

Algorithm for determination of parametre n is presented in Fig. 5. The Δn of Fig. 5 item 11.8 may be calculate in this manner:

$$\Delta n = n - \frac{\lg\left[\frac{2\sigma_i^{M*}(0)/\sqrt{3} - e_y^{H*}(0.5)E'^{M}(0.5)}{pC_p^* D^M}\right]}{\lg 0.5}.$$

Values of longitudinal strains in H and M materials on the contact plane of welded joint, calculated by method presented in this work, are shown in Table 1. More accurate solution, for the same calculation precision, is then longitudinal strains are smoothed in three points. But in this case calculations are more complex and take more time. Therefore it is recommended to use the constant value of parameter n ($n = 2$).

Distribution of stresses and longitudinal strains in H and M materials on the contact plane of welded joint, calculated analytically, are shown in Fig. 6.

Analogical solution, when $\sigma_r = \sigma_\varphi$, is obtained for welded joint with the mild interlayer at axial symmetrical deformation. In this case co-ordinate x must be changed by r, y – by z and differ dependencies which are connected with shape of welded joint (Bražėnas, 1995). Algorithm of calculation at plane and axial symmetrical deformation is the same.

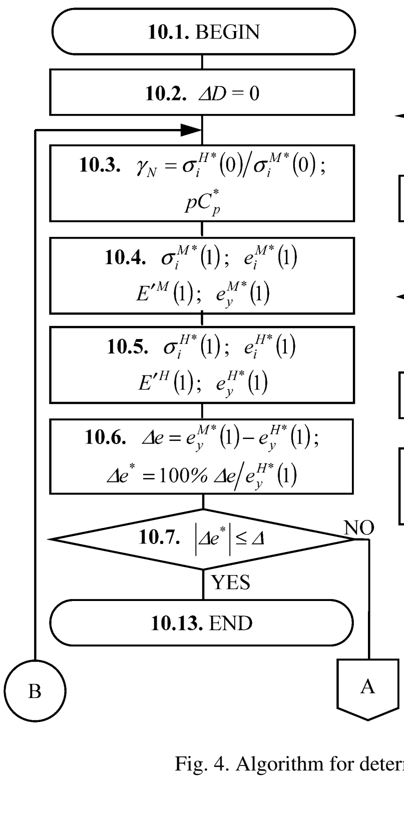

Fig. 4. Algorithm for determination of D^M.

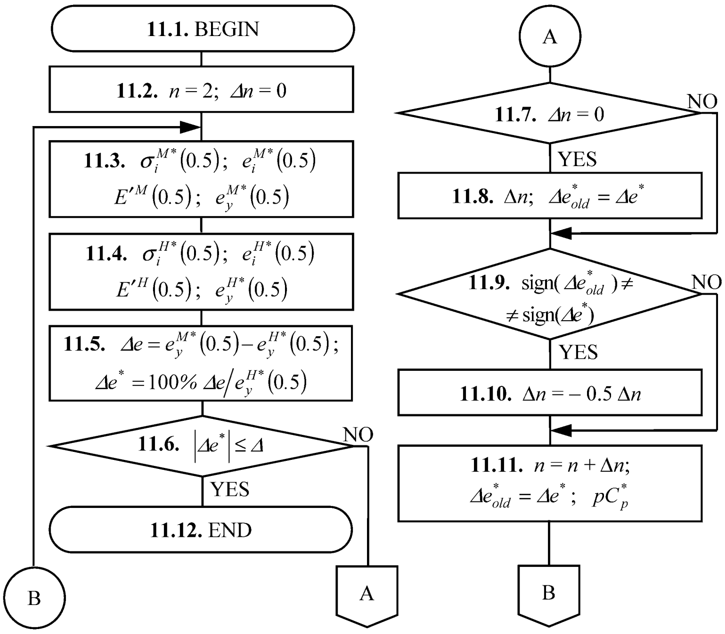

Fig. 5. Algorithm for determination of n.

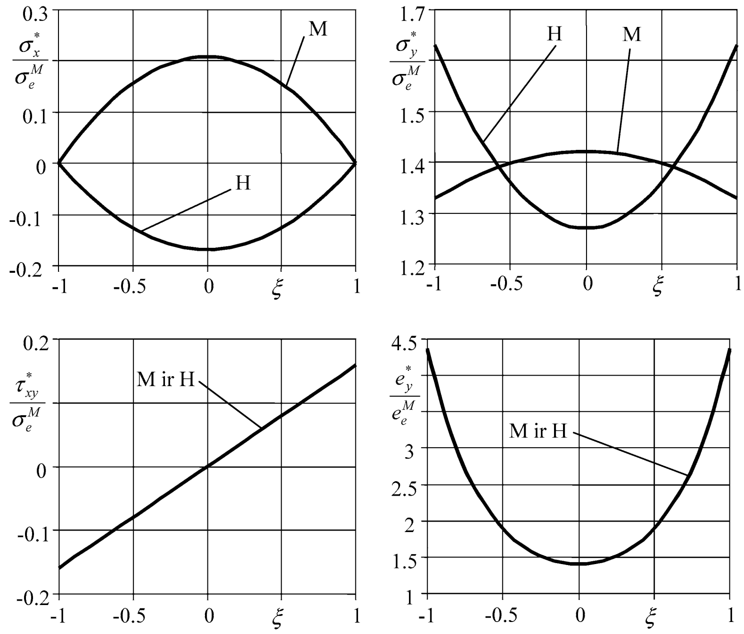

Fig. 6. Distribution of stresses and longitudinal strains on the contact plane of welded joint calculated analytically, when $\gamma_e = 1.2$, $æ = 0.8$, $m_0^M = 0.102$, $m_0^H = 0.125$, $p/\sigma_e^M = 1.4$, $n = 2$.

4. Verification of Obtained Dependencies

Because this solution is approximate the stress state components of considered welded joint calculated by using method presented in this paper were compared with the data of experimental investigations. Longitudinal strain distribution in its separate zones at static

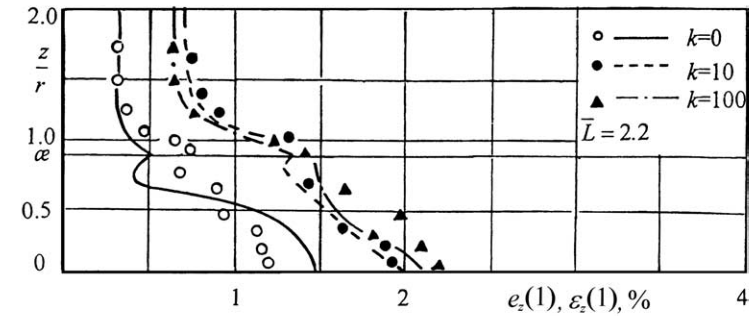

Fig. 7. Analytically calculated (curves) and experimentally obtained (points) longitudinal static ($k = 0$) and cyclic ($k > 0$) strains on the surface of the welded joint at axial symmetrical deformation when $\gamma_N = 1.31$, $æ = 0.82$ and mean longitudinal strain $e_{zm} = 0.55\%$. k is number of semi-cycle.

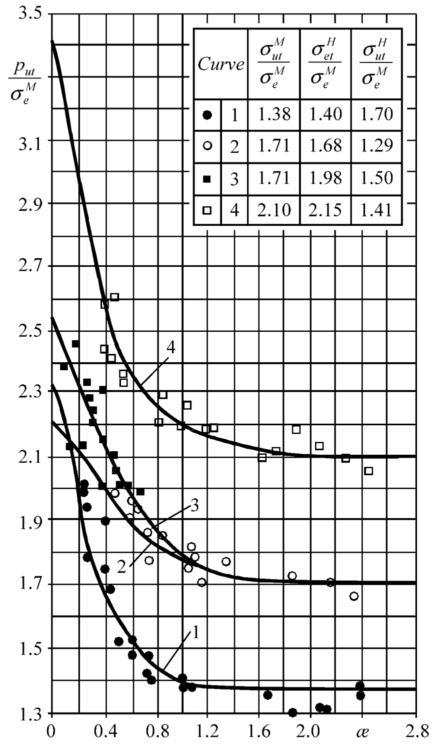

Fig. 8. Strength comparison of welded joint with a mild interlayer calculated analytically and determined experimentally at axial symmetrical deformation. Experimental data were taken from (Bakshi, 1966).

and cyclic elasto-plastic loading (Fig. 7) and strength of welded joint with the flat mild interlayer (Fig. 8) calculated analytically and determined experimentally by photo elastic coatings method showed a good agreement.

5. Conclusions

1. Solution presented in this paper enables calculate stresses and strains in the separate zones of butt welded joint with plane mild interlayer subjected to elasto-plastic tension (compression).
2. The strength of considered welded joint and strain distribution in its separate zones calculated analytically by using obtained dependencies showed a good agreement with the results of experimental investigations at static and cyclic elasto-plastic loading. It means that functions of stress distribution and way of calculation in this solution are chosen enough correctly.

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Įtempimų deformacijų būvio komponenčių nustatymas sandūrinėje suvirintoje jungtyje su minkštu tarpsluoksniu esant tampriai plastiniam tempimui

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Straipsnyje aprašyta įtempimų deformacijų būvio komponenčių nustatymo metodika sandūrinėje suvirintoje jungtyje su minkštu plokščiu tarpsluoksniu esant tampriai plastiniam tempimui. Pasiūlyta skersinių įtempimų pasiskirstymo funkcija išilgai minkšto tarpsluoksnio bei pateikta tampriai plastiškai deformuoto kieto pagrindinio metalo didžiausių įtempimų intensyvumo apskaičiavimo išraiška. Kietos ir minkštos medžiagų išilginiai įtempimai, esant tampriai plastiniam deformavimui, apskaičiuojami iš jų pusiausvyros sąlygos vidutinei išorinei apkrovai, įvertinant minkšto ir kieto metalų išilginių deformacijų lygybę jungties kontakto plokštumoje. Pateiktas įtempimų deformacijų būvio komponenčių atskirose jungties zonose apskaičiavimo algoritmas ir gauti skaičiavimo rezultatai. Analitiškai apskaičiuotas sandūrinės suvirintos jungties su minkštu tarpsluoksniu stiprumas ir išilginių deformacijų pasiskirstymas patikrinti eksperimentiniais tyrimais.