INFORMATICA, 1990, Vol.1, No.1, 059–070

# MINIMIZATION ALGORITHM IN THE PRESENCE OF RANDOM NOISE

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**Abstract.** In this paper the problem of optimization of multivariate multimodal functions observed with random error is considered. Using the random function for a statistical model of the objective function the minimization procedure is suggested. This algorithm is convergent on a discrete set. To avoid computational difficulties, the modified algorithm is defined by substituting the parameters of minimization procedure by their estimates.

**Key words:** global optimization, random noise, estimates of parameters.

**1. Introduction. Formulation of problem.** The necessity of global optimization in the presence of noise is very common in different applications (identification, adaptation etc.). To solve such problems, a multimodal generalization of the stochastic optimization algorithm was proposed. However, its efficiency depends very much on a heuristic choice of many parameters. To avoid this, the axiomatic approach for the construction of rational statistical models and optimization algorithms, suggested by Žilinskas (1986), seems prospective. The main idea of the proposed approach is to formulate some simple rational assumptions on available information, implying the structure of statistical model and optimization algorithm. In the papers of Žilinskas, Katkauskaitė (1987) and Žilinskas et al. (1987) this idea was applied to solve the more complicated problem of global minimization in the presence of noise. It may be described as follows below.

Let the unknown continuous function $f(x)$, $x \in A \subset R^n$ be minimized, when the only objective information on this function is noisy observations $z_i = f(x_i) + \varepsilon_i$, $i = \overline{1,k}$, where $\varepsilon_i$–independent Gaussian random variables, $M\varepsilon_i = 0$, $D\varepsilon_i = \sigma^2$, $i = \overline{1,k}$. The objective and subjective information on the objective function $f(\cdot)$ may be formalized by simple rational axioms similar to the axioms of Žilinskas (1986) for the case of exact observations. It may be supposed that the minimal considerable a priori information on $f(\cdot)$ contains the possibility to compare any two intervals of the values $f(x)$, according to their likelihood. So a certain binary relation may be defined, which implies the existence of a family of random variables $\xi(x)$, $x \in A$, compatible with this relation. Consequently, $\xi(x)$ may be accepted for a statistical model of $f(\cdot)$. Finally, a minimization procedure may be defined as a rational choice of the current evaluation of $f(\cdot)$ or, according to the suggested model, as a choice of a certain random variable from the family $\xi(x)$, $x \in A$. It is shown in Žilinskas (1986), that the rationality is compatible with a certain utility function and, in particular, the minimization algorithm may be described by the following relations:

$$x_{k+1} = \arg\min_{x \in A} M\{\min(y_{ok}, \xi(x))/\xi(x_i) + \varepsilon_i = z_i,$$

$$i = \overline{1,k}\}. \tag{1}$$

$$y_{ok} = \min_{x \in A} M\{\xi(x)/\xi(x_i) + \varepsilon_i = z_i,\ i = \overline{1,k}\},$$

where $\xi(x)$ a Gaussian random function, $M\xi(x) = 0$.

The computational realization of the suggested algorithm is rather complicated from the computational point of view, because of a matrix inversion procedure appearing in the evaluations of parameters of the algorithm (1)–a conditional mean value and a conditional variance. The computational difficulties may be reduced by substituting the parameters by their estimates. The minimization procedure may also be simplified by solving the problem on a discrete set, i.e., finding $\min_{x \in \widehat{A}} f(x)$, where $\widehat{A} = \{a_i, i = \overline{1, L}\}$, $a_i = (a_i^1, \dots, a_i^n)$–a site of n-dimensional lattice $\widehat{A} \subset A$. In this paper the convergence of algorithm (1) on a set $\widehat{A}$ is proved and the estimates of parameters are suggested to construct the modified algorithm. Note that a particular one-dimensional version of the algorithm (1) is considered by Žilinskas (1986), using the Wiener process as a statistical model of the objective function.

**2. Convergence of the minimization algorithm.** First of all, consider the asympthotic properties of the parameters of the algorithm (1)–a conditional mean value and a conditional variance:

$$m_k(x, x_i, z_i, i = \overline{1, k}) = M\{\xi(x)/\xi(x_i) + \varepsilon_i = z_i,\ i = \overline{1, k}\},$$

$$s_k(x, x_i, z_i, i = \overline{1, k}) = M\{(\xi(x) - m_k(x, x_i, z_i, i = \overline{1, k}))^2/ \\ /\xi(x_i) + \varepsilon_i = z_i,\ i = \overline{1, k}\}.$$

Let a continuous Gaussian function $\xi(x)$, $x \in R^n$ for a statistical model of $f(\cdot)$ be chosen. Denote $\eta(x_i) = \xi(x_i) + \varepsilon_i$, $i = \overline{1, k}$, $M^k$–a determinant of the covariance matrix of the vector $(\xi(x), \eta(x_1), \dots, \eta(x_k))$, $r_{00} = M\xi(x)\xi(x)$, $r_{0i} = M\xi(x)\xi(x_i)$, $r_{ij} = M\xi(x_i)\xi(x_j)$, $M_{0j}^k$–the cofactor of the element $r_{0j}$ of $M^k$. Let $M^k$ be positively definite. It is well known that for Gaussian variables

$$m_k(x, x_i, z_i, i = \overline{1, k}) = \sum_{i=1}^{k} w_i^k(x, x_j, j = \overline{1, k}) z_i, \qquad (2)$$

$$s_k(x, x_i, z_i, i = \overline{1,k}) = s_k(x, x_i, i = \overline{1,k}) =$$
$$= r_{00} - \sum_{i=1}^{k} w_i^k(x, x_j, j = \overline{1,k}) r_{0i}, \quad (3)$$

where
$$w_i^k(x, x_j, j = \overline{1,k}) = -\frac{M_{0i}^k}{M_{00}^k}.$$

Denote $X^k = \{x_i, i = \overline{1,k}\}$, $I_j^k$ - a set of indices of the observations performed at the point $a_j, n_j^k$ - a number of indices belonging to $I_j^k$. It is easy to check that under the assumptions $\min\limits_{i=\overline{1,L}} n_i^k > 0$, $x, x_i \in \widehat{A}$, the following relations are true:

$$m_k(x, x_i, z_i, i = \overline{1,k}) = \widehat{m}_k(x, a_i, \widehat{z}_i, i = \overline{1,L}) = -\sum_{i=1}^{L} \frac{\widehat{M}_{0i}^L}{\widehat{M}_{00}^L} \widehat{z}_i,$$

$$s_k(x, x_i, i = \overline{1,k}) = \widehat{s}_k(x, a_i, i = \overline{1,L}) = \widehat{r}_{00} + \sum_{i=1}^{L} \frac{\widehat{M}_{0i}^L}{\widehat{M}_{00}^L} \widehat{r}_{0i},$$

where $\widehat{z}_i = \frac{1}{n_i^k} \sum\limits_{j \in I_i^k} z_j$, $\widehat{r}_{0j} = M\xi(x)\xi(a_j)$, $\widehat{r}_{ij} = M\xi(a_i)\xi(a_j)$, $\widehat{M}_{0i}^L$ - the cofactor of the element $\widehat{r}_{0i}$ in $\widehat{M}^L$,

$$\widehat{M}^L = \begin{vmatrix} \widehat{r}_{00} & \widehat{r}_{01} & \widehat{r}_{02} & \cdot & \widehat{r}_{0L} \\ \widehat{r}_{10} & \widehat{r}_{11} + \frac{\sigma^2}{n_1^k} & \widehat{r}_{12} & \cdot & \widehat{r}_{1L} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \widehat{r}_{L0} & \widehat{r}_{L1} & \widehat{r}_{L2} & \cdot & \widehat{r}_{LL} + \frac{\sigma^2}{n_L^k} \end{vmatrix}.$$

It is easy to prove

**Lemma.** If $n_i^k \to \infty$, $k \to \infty$, then $\widehat{m}_k(a_i, a_j, \widehat{z}_j, j = \overline{1,L}) \to f(a_i)$ in probability, $\widehat{s}_k(a_i, a_j, j = \overline{1,L}) \to 0$.

Further we'll consider the convergence of the minimization algorithm taking a more general form than (1):

$$\begin{aligned} x_{k+1} &= \arg\max_{x\in\widehat{A}} M\{u_k(\xi(x))/\eta(x_i) = z_i,\ i = \overline{1,k}\}, \\ y_{0k} &= \min_{x\in\widehat{A}} M\{\xi(x)/\eta(x_i) = z_i,\ i = \overline{1,k}\}, \end{aligned} \tag{4}$$

where the utility function $u_k(y)$ is continuous and nonincreasing in the neighbourhood of $y_{0k}$ and equal to zero for $y > y_{0k}$; $u_k(y)$ is finitely integrable with respect to a standard Gaussian distribution.

**Theorem 1.** Let the function $f(\cdot)$ be minimized by the algorithm (4) under the assumptions presented above. Then $y_{0k}$ converges to $\min\limits_{x\in\widehat{A}} f(x)$ in probability as $k \to \infty$.

**Proof.** Denote $I^k = \{j : a_j \in X^k\}$ (i.e., $I^k$–a set of indices of the points belonging to $\widehat{A}$ at which at least one observation is performed), $L_k = \text{card } I^k$. Then for any $x \in \widehat{A}$

$$\widehat{m}_k(x, a_i, \widehat{z}_i, i \in I^k) = \sum_{i\in I^k} \widehat{w}_i^k(x,\cdot)(f(a_i) + \frac{1}{n_i^k}\sum_{j\in I_i^k} \varepsilon_j)$$

and, consequently,

$$\begin{aligned} \left|\widehat{m}_k(x,\cdot)\right| \leqslant \sum_{i\in I^k} \left|\widehat{w}_i^k(x,\cdot)\right| \Big\{ \max_{j\in I^k} \left|f(a_j)\right| + \\ + \max_{p\in I^k} \left|\frac{1}{n_p^k} \sum_{j\in I_p^k} \varepsilon_j\right| \Big\}. \end{aligned} \tag{5}$$

Note, that the random value $\zeta = \max\limits_{i\in I^k} \left|\frac{1}{n_i^k} \sum\limits_{j\in I_i^k} \varepsilon_j\right|$ is bounded in probability by a certain constant independent of

$k$, i.e., for all $k$ large enough and for any $\Delta > 0$ there exists a number $c_\Delta > 0$, such that

$$P\Big\{\max_{i\in I^k}\frac{1}{n_i^k}\Big|\sum_{j\in I_i^k}\varepsilon_j\Big| < c_\Delta\Big\} > 1-\Delta. \tag{6}$$

It may be shown that the inequality $\zeta < c_\Delta$ yields $\min\limits_{i\in I^k} n_i^k \to \infty$ as $k \to \infty$. Indeed, let it be just the contrary: $\zeta < c_\Delta$ and there exists the point $a_r \in \widehat{A}$, such that $\lim\limits_{k\to\infty} n_r^k < \infty$. Let us consider two cases: $n_r^k > 0$ and $n_r^k = 0$. Since for Gaussian variables $\xi(x), \varepsilon_i,\ i = \overline{1,k}$

$$\begin{aligned}\widehat{s}_k(a_r,\cdot) &= M\Big\{\xi(a_r) - \sum_{j=\overline{1,k}} w_j^k(a_r,\cdot)\eta_j\Big\}^2 = \\ &= M\Big\{\xi(a_r) - \sum_{j\in I^k}\widehat{w}_j^k(a_r,\cdot)\xi(a_j)\Big\}^2 + \\ &+ M\Big\{\sum_{j\in I^k}\frac{1}{n_j^k}\widehat{w}_j^k(a_r,\cdot)\sum_{i\in I_j^k}\varepsilon_i\Big\}^2,\end{aligned} \tag{7}$$

for $n_r^k > 0$ (i.e. $r \in I^k$)

$$\widehat{s}_k(a_r,\cdot) \geqslant (\widehat{w}_r^k(a_r,\cdot))^2\frac{\sigma^2}{n_r^k}. \tag{8}$$

Considering that $\widehat{M}^{L_k}$ is of a finite dimension ($L_k \leqslant L$) for $k \to \infty$ and is positively definite, it is easy to show that $\widehat{w}_r^k(a_r,\cdot) = -\frac{\widehat{M}_{0r}^{L_k}}{\widehat{M}_{00}^{L_k}} \geqslant c_w > 0$ , $c_w$ being a certain constant independent of $k$. Consequently, by (8) for $n_r^k > 0$

$$\widehat{s}_k(a_r, a_j, j \in I^k) \geqslant \frac{c_w^2}{n_r^k}\sigma^2 = c_1 > 0. \tag{9}$$

Let $n_r^k = 0$ (i.e., at the point $a_r$ no observation is performed). Taking into account (7), we get

$$\widehat{s}_k(a_r,\cdot\,) \geqslant M\big\{\xi(a_r) - \sum_{j\in I^k} \widehat{w}_j^k(a_r,\cdot\,)\xi(a_j)\big\}^2 \geqslant$$

$$\geqslant \min_{b_j} M\{\xi(a_r) - \sum_{j\in I^k} b_j\xi(a_j)\}^2 = M\big\{\xi(a_r) - \sum_{j\in I^k} b_j^*\xi(a_j)\big\}^2.$$

It is known that for Gaussian variables $\xi(a_i)$, $i \in I^k$, $b_j^* = -\frac{\overline{M}_{0j}^{L_k}}{\overline{M}_{00}^{L_k}}$, where $\overline{M}_{0j}^{L_k}$ is a cofactor of the element in the zeroth row and the j-th column of the covariance matrix of the vector $(\xi(a_r), \xi(a_j), j \in I^k)$. Denote by $\overline{M}^{L_k}$ the determinant of this matrix. Then

$$M\big\{\xi(a_r) - \sum_{j\in I^k} b_j^*\xi(a_j)\big\}^2 = \frac{\overline{M}^{L_k}}{\overline{M}_{00}^{L_k}}.$$

It may be shown that $\frac{\overline{M}^{L_k}}{\overline{M}_{00}^{L_k}}$ is bounded from the below by a constant $c_2 > 0$ independent of $k$. Hence,

$$\widehat{s}_k(a_r, a_j, j \in I^k) \geqslant c_2. \tag{10}$$

Further, according to the procedure (4), we have

$$U_{kr} = M\big\{u_k(\xi(a_r))/\eta(x_i) = z_i, i = \overline{1,k}\big\} =$$

$$= \frac{1}{\sqrt{2\pi\widehat{s}_k(a_r,\cdot\,)}} \int_{-\infty}^{+\infty} u_k(y) \exp\left\{-\frac{(y - \widehat{m}_k(a_r,\cdot\,))^2}{2\widehat{s}_k(a_r,\cdot\,)}\right\} dy =$$

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} u_k\big(t\sqrt{\widehat{s}_k(a_r,\cdot\,)} + \widehat{m}_k(a_r,\cdot\,)\big) \exp\left\{\frac{-t^2}{2}\right\} dt.$$

Consequently, $U_{kr}$ is a nondecreasing function of $\widehat{s}_k(a_r,\cdot)$ and a nonincreasing function of $\widehat{m}_k(a_r,\cdot)$. By this and (9), (10), we get

$$U_{kr} \geqslant \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} u_k(t\sqrt{c_s}+D_\Delta)\exp\left\{\frac{-t^2}{2}\right\}dt = \delta > 0, \quad (11)$$

where $c_s = \min\{c_1, c_2\}$, $D_\Delta$ is a certain constant for which $|\widehat{m}_k(a_r,\cdot)| \leqslant \max\limits_{x\in\widehat{A}}|f(x)| + c_\Delta = D_\Delta$ with probability exceeding $(1-\Delta)$.

On the other hand, if $k\to\infty$, then there exists at least one point $a_l$ for which $n_l^k \to \infty$. Therefore, as it follows by Lemma, $\widehat{s}_k(a_l,\cdot)\to 0$ as $k\to\infty$ and, consequently, $U_{kl} < \delta$ for all $k$ large enough. But if (11) is held, then, according to the definition of algorithm (4), inequality $U_{kl} < \delta$ is possible only for bounded $n_l^k$. The obtained contradiction shows that the assumption $\lim\limits_{k\to\infty} n_r^k < \infty$, yielding (11), is incorrect.

Now it will be shown that if $n_i^k \to \infty$, then $\widehat{w}_i^k(a_i,\cdot)\to 1$, while $\widehat{w}_j^k(a_i,\cdot)\to 0$ for $j\neq i$. Indeed, denoting by $\widehat{M}_{00}^{L_k}(i)$ the cofactor of the element in the i–th row and the i–th column of $\widehat{M}_{00}^{L_k}$, we get $\widehat{M}_{0i}^{L_k} = \widehat{M}_{00}^{L_k} - \frac{\sigma^2}{n_i^k}\widehat{M}_{00}^{L_k-1}(i)$ for $r_{00}=r_{ii}$. Hence,

$$\widehat{w}_i^k(a_i,\cdot) = -\frac{\widehat{M}_{0i}^{L_k}}{\widehat{M}_{00}^{L_k}} = 1 - \frac{\sigma^2}{n_i^k}\frac{\widehat{M}_{00}^{L_k}(i)}{\widehat{M}_{00}^{L_k}} \to 1,\ k\to\infty.$$

Analogously, it may be shown that $\widehat{w}_j^k(a_i,\cdot)\to 0$ as $k\to\infty$ for $j\neq i$. Thus, for all $k > K_{\Delta_1}$ large enough $\min\limits_{i=\overline{1,L_k}} \widehat{w}_i^k(a_i,\cdot) > 1-\Delta_1$ with probability exceeding (1-$\Delta$). Following the ideas used to prove the theorem 5.4 in Žilinskas (1986), it may be shown that for any $a_i\in\widehat{A}$ and $k > K(\Delta,\delta_1)$ $|\widehat{m}_k(a_i,\cdot) - f(a_i)| < \delta_1$ with probability exceeding $(1-\Delta)$.

The latter inequality implies $P\{|y_{0k} - \min\limits_{x \in \widehat{A}} f(x)| < \delta_1\} \to 0$ as $k \to \infty$. The theorem is proved.

**3. Construction of the modified algorithm.** As mentioned above, a minimization algorithm (1) is rather complicated from the computational point of view because of a matrix inversion procedure. To avoid these difficulties, the more simple formulae of the parameters $m_k, s_k$ may be used. A theoretical support for such a substitution follows from the axioms, basing the statistical model, presented in Žilinskas, Katkauskaitė (1987) and Žilinskas et al. (1987). Some axioms imply the formula of a mean value (2). Refusing of them (notice, that they are less justified), one can get a more simple expression than (2)–the weighted mean extrapolator

$$m_k^*(x, x_i, z_i, i = \overline{1,k}) = \sum_{i=1}^{k} v_i^k(x, x_j, j = \overline{1,k}) z_i \,, \qquad (12)$$

where $v_i^k(x, \cdot)$-continuous weight functions, satisfying the following conditions:

$C1.$ $\sum\limits_{i=1}^{k} v_i^k(x, x_j, j = \overline{1,k}) = 1,$

$C2.$ $v_i^k(x, x_1, \ldots, x_k) = v_p^k(x, x_1, \ldots, x_{j-1}, x_l, x_{j+1}, \ldots, x_{l-1}, x_j, x_{l+1}, \ldots, x_k), p = i \ for \ j \neq i, l \neq i; p = j \ for \ i = l; p = l \ for \ i = j.$

The additional assumptions imply a more specific form of weight functions (see Katkauskaitė, 1986):

$$\overline{v}_i^k(x, x_j, j = \overline{1,k}) = \frac{d_k(x, x_i)}{\sum\limits_{j=1}^{k} d_k(x, x_j)} \,,$$

where $d_k(x, x_i)$ are the monotonically decreasing functions of $|x - x_i|$. Further the extrapolator $\overline{m}_k^*(x, \cdot)$ will be considered for which

$$d_k(x, x_i) = (|x - x_i| + c_k)^{-l}, \ l > n, c_k > 0. \tag{13}$$

The extrapolator $\overline{m}_k^*(x, \cdot)$ generalizes the well known Shepard's formula applied in the extrapolation under exact observations. (see Shepard, 1965; Farwig, 1986; Frank, 1982).

For the estimate of a conditional variance $s_k$,the following formula will be used

$$\overline{s}_k^*(x, x_i, i = \overline{1, k}) = r_{00} - \sum_{i=1}^{k} \overline{v}_i^k(x, x_j, j = \overline{1, k}) r_{0i} \ ,$$

introduced in Žilinskas, Katkauskaitė (1987) and Žilinskas et al. (1987).

Consider the asymptotic properties of $\overline{m}_k^*(x, \cdot), \overline{s}_k^*(x, \cdot)$ on a discrete set $\widehat{A}$.

**Theorem 2.** If $c_k \to 0, n_i^k \to \infty, i = \overline{1, L}$ when $k \to \infty$ then for any $a_i \in \widehat{A}$ and $x_j \in \widehat{A}, j = \overline{1, k}$

$$\overline{m}_k^*(a_i, x_j, z_j, j = \overline{1, k}) \to f(a_i) \quad (\text{mod P}),$$
$$\overline{s}_k^*(a_i, x_j, j = \overline{1, k}) \to 0.$$

Proof. Really, if $c_k \to 0, n_i^k \to \infty, i = \overline{1, L}$,then by the law of large numbers it follows that

$$\overline{m}_k^*(a_i, x_j, z_j, j = \overline{1, k}) = \sum_{j=1}^{k} \frac{(|a_i - x_j| + c_k)^{-l}}{\sum_{p=1}^{k} (|a_i - x_p| + c_k)^{-l}} z_j =$$

$$= \left[ n_i^k + \sum_{\substack{x_p \neq a_i \\ x_p \in X^k}} \left( \frac{|a_i - x_p| + c_k}{c_k} \right)^{-l} \right]^{-1} \sum_{j \in I_i^k} z_j +$$

$$+ \sum_{\substack{x_j \neq a_i \\ x_j \in X^k}} \frac{(|a_i - x_j| + c_k)^{-l}}{\sum\limits_{x_p \in X^k} (|a_i - x_p| + c_k)^{-l}} z_j \to$$

$$\to f(a_i) + \frac{1}{n_i^k} \sum_{j \in I_i^k} \varepsilon_j \to f(a_i) \text{ (mod P)}.$$

Analogously, as $r_{00} = r_{ii}$ ,then

$$\overline{s}_k^*(a_i, x_j, j = \overline{1,k}) = r_{ii} - \sum_{x_p \in X^k} \overline{v}_p^k(a_i, x_j, j = \overline{1,k}) r_{ip} =$$

$$= r_{ii} \cdot (1 - n_i^k \overline{v}_i^k(a_i, x_j, j = \overline{1,k}) -$$

$$- \sum_{\substack{x_p \in X^k \\ x_p \neq a_i}} \overline{v}_p^k(a_i, x_j, j = \overline{1,k}) r_{ip}.$$

Further, since under the assumptions of the theorem $\overline{v}_i^k(a_i, \cdot) \to \frac{1}{n_i^k}, \overline{v}_p^k(a_i, \cdot) \to 0$ for $x_p \neq a_i$ ,then $\overline{s}_k^*(a_i, \cdot) \to$ $\to 0, \quad k \to \infty$. The theorem is proved.

Substituting the parameters $m_k, s_k$ of the algorithm (1) by their estimates $\overline{m}_k^*, \overline{s}_k^*$,one can get the modified algorithm. Due to Lemma and Theorems 1 and 2, the latter algorithm is convergent in probability on a discrete set $\widehat{A}$.

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Received December 1989

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