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A Tool for Modeling Optical Beam Propagation

Raimondas ČIEGIS, Galina ŠILKO
Department of Mathematical Modelling, Vilnius Gediminas Technical University
Saulėtekio al. 11, 2040 Vilnius, Lithuania
e-mail: rc@fm.vtu.lt, gs@sc.vtu.lt

Aleksandr DEMENT'EV
Institute of Physics
Goštauto 12, 2600 Vilnius, Lithuania
e-mail: adement@ktl.mii.lt

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Abstract. A tool for modeling the propagation of optical beams is proposed and investigated. Truncated Laguerre–Gauss polynomial series are used for approximation of the field at any point in free space. Aposteriori error estimates in various norms are calculated using errors for input functions. The accumulation of truncation errors during space transition is investigated theoretically. The convergence rate of truncated LG series is obtained numerically for super-Gaussian beams. An optimization of algorithm realization costs is done by choosing parameters in such a way that the error reaches minimum value. Results of numerical experiments are presented.

Key words: spectral methods, optical beams, Laguerre–Gauss polynomials, software tools.

1. Introduction

In this paper we consider the propagation of axially symmetric laser beams through optical systems. Assuming that the field $u(z,r)$ is specified at the plane $z=0$, the field at any point in free space can be obtained as a solution of the following problem:

$$\begin{aligned} &2ik\frac{\partial u}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0, \quad u(r,0) = u_0(r),\ 0 \leqslant r \leqslant R, \\ &r\frac{\partial u}{\partial r}\Big|_{r=0} = 0, \quad u(R,z) = 0,\ 0 \leqslant z \leqslant L, \end{aligned} \tag{1}$$

where u is a complex valued optical field amplitude, k is the wave number of the field.

In many applications it is important to evaluate the field at a plane $z = L$. Different prescribed fields at $z = 0$ are interesting in applications, e.g., propagation of multi-Gaussian, super-Gaussian, Bessel–Gauss beams in $ABCD$ optical systems (see, Belafhal and Dalil–Essakali, 2000; Caron and Potvliege, 1999; Ibnchaikh and Belafhal, 2001; Ramee and Simon, 2000; Zhao *et al.*, 1997).

The propagation of optical beams in free space is also important for solving nonlinear optics problems. For example, an interaction of two counteracting laser beams in stimulated Brillouin scattering media is described by the following problem (see, Boyd, 1992; Čiegis and Dement'ev, 1992; Girdauskas *et al.*, 1997; Reintjes and Bashkansky, 2001):

$$
\begin{aligned}
&\frac{\partial e_L}{\partial t} + \frac{\partial e_L}{\partial z} - i\mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e_L}{\partial r}\right) + \alpha_L e_L = i\Gamma\sigma_S e_S + i\,\eta_L\left(|e_L|^2 + 2|e_S|^2\right)e_L,\\
&\frac{\partial e_S}{\partial t} - \frac{\partial e_S}{\partial z} - i\mu \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e_S}{\partial r}\right) + \alpha_S e_S = i\Gamma\sigma_S^* e_L + i\,\eta_S\left(2|e_L|^2 + |e_S|^2\right)e_S,\\
&i\gamma_0\left(\frac{\partial^2\sigma_S}{\partial t^2} + \gamma_1\frac{\partial\sigma_S}{\partial t}\right) + \frac{\partial\sigma_S}{\partial t} + a\sigma_S = i\Gamma e_L e_S^*, \qquad (2)
\end{aligned}
$$

where $e_{L,S}, \sigma_S$ are laser, Stokes and sound waves complex amplitudes, respectively, and t, z, r are non-dimensional coordinates. We introduce a difference mesh $z_j = jh, j = 0, 1, \dots, N, z_N = L$, and approximate the system (2) by using the splitting method. Then beam propagation problem (1) is solved for $e_{L,S}$ on each interval $[z_j, z_{j+1}]$.

When investigating the light propagation through an $ABCD$ optical system, one usually uses the diffraction integral of such a system (see, D'Arcio *et al.*, 1994; Barakat *et al.*, 1998). But evaluation of these integrals is a very complicated numerical task.

For solving nonlinear optics problems finite-difference (FD), finite-element (FE) or finite-volume (FV) methods are used (see, Sanz–Serna, 1984; Akrivis *et al.*, 1991; Čiegis, 1992). These algorithms are fast, stable, but they become non-efficient in the case of focused laser beams.

For many beam propagation problems spectral methods are used for finding the numerical solution. We take a set of functions, which are complete in L_2, and expand a field into a truncated series of these functions. The efficiency of spectral methods depends on approximation properties of basic functions and on the existence of fast transform algorithms, e.g., Fast Fourier Transform (FFT). Laguerre–Gauss (LG) beams afford a simple means for studying propagation of any paraxial field (see, Siegman, 1986). In order to evaluate the field at some plane $z = L$, we simply sum up the propagated LG beams with the same expansion coefficients as at $z = 0$.

Extensive comparison of different methods for solving linear problem (1) is done by Čiegis and Čiegis (1991).

Numerical methods which are used in scientific computations and mathematical modeling must be *robust* and *efficient*. Both of these properties depend essentially on the quality of aposteriori error estimators. Firstly, similar to physical experiments, it is not sufficient to find a discrete solution, we also need to know the boundaries of the error of the obtained discrete solution. The key ingredient of such methodology is a reliable method for assessing the quality of computed approximation. An aposteriori error estimator must be computed using the data for the given problem and the discrete approximation itself. Such method is efficient if the costs of obtaining the estimator are small compared with the computation of the discrete solution. Secondly, efficient numerical algorithms use adaptive approximations. Error estimation and mesh and parameter adaptation goes hand-in-hand leading to economical discrete schemes. The robustness of such strategy depends on the quality of aposteriori error estimation procedures.

In this paper we shall propose a tool, which enables an investigator to estimate the error of approximation at the initial plane $z = 0$. An adaptation of parameters of spectral scheme can be done by using this information. We shall investigate the hypothesis, that the error at any plane $z = L$ remains of the same order in most cases. Thus the tool presents an aposteriori error estimation procedure.

The paper is organized as follows. In Section 2, we describe our tool for numerical approximation of optical beams propagation. Laguerre–Gauss expansion algorithm is presented and the error of truncated series is studied. The accuracy of LG series is investigated numerically in Section 3. Different beams are used in these experiments. We also test the accuracy of our hypothesis on accuracy of the aposteriori error estimator. In Section 4, we use the tool to find optimal value of spot-size w_0 of Laguerre–Gauss functions, which minimize the truncation error of truncated series. Some final conclusions are given in Section 5.

2. Description of the Tool

In this section we describe the main features of the developed tool. Using this tool we solve the following tasks:

1. Given the complex-valued function $u_0(r)$, compute the corresponding truncated Laguerre–Gauss series approximation $U_0(r; P)$.
2. Compute the errors $||u_0 - U_0(P)||$ in various norms.
3. If the required accuracy is not achieved the number of LG polynomials can be increased and then proceed to $\sharp 1$.
4. Compute the field at given $z = L$ by simply summing up the propagated LG beams.
5. Choose optimal values of parameters, which lead to minimal algorithm realization costs.

Our main hypothesis is that the error estimate at $z = 0$ can be used as an aposteriori error estimate for any plane $z = L$. We note that the exact solution of (1) satisfies the beam power conservation property

$$\int_0^\infty r\,|u(r,z)|^2 \mathrm{d}\,r = \int_0^\infty r\,|u_0(r)|^2 \mathrm{d}\,r.$$

The same property is also valid for the beam quality factor M^2 that is defined as (see, Buzelis *et al.*, 1996; Porras *et al.*, 1992; Vicalvi *et al.*, 1998)

$$M^2(u) = \frac{\left(\int_0^\infty \left|\dfrac{\partial u}{\partial r}\right|^2 r \mathrm{d}\,r \int_0^\infty r^3 |u|^2 \mathrm{d}\,r - \dfrac{1}{4}\left|\int_0^\infty \left(u\dfrac{\partial u^*}{\partial r} - u^*\dfrac{\partial u}{\partial r}\right) r^2 \mathrm{d}\,r\right|^2\right)^{1/2}}{\int_0^\infty |u|^2 r \mathrm{d}\,r}. \quad (3)$$

2.1. *Laguerre–Gauss expansion*

Let use the Laguerre–Gauss functions

$$
\begin{aligned}
W_p(r, z; w_0, f) &= \sqrt{\frac{2}{\pi}} \frac{1}{w(z)} L_p\left(\frac{2r^2}{w^2(z)}\right) \\
&\quad \times \exp\left(-\frac{r^2}{w^2(z)}\Big(1 - i\frac{z - z_f}{z_w}\Big) - i(2p+1)\arctan\frac{z - z_f}{z_w}\right), \\
w^2(z) &= \frac{2z_w}{k}\left(1 + \left(\frac{z - z_f}{z_w}\right)^2\right), \quad z_f = \frac{f z_R^2}{f^2 + z_R^2}, \\
z_R &= \frac{k w_0^2}{2}, \quad z_w = \frac{z_R f^2}{f^2 + z_R^2},
\end{aligned} \tag{4}
$$

where w_0 is the radius of the fundamental ($p = 0$) Gaussian beam at $z = 0$, and f is the modulus of the wavefront curvature $R(z)$ of a LG mode at $z = 0$. Here L_p denotes the pth Laguerre polynomial (see, Abramowitz and Stegun, 1972). The values of these polynomials can be computed by the recurrence relation

$$
\begin{aligned}
&L_0(x) = 1, \quad L_1(x) = 1 - x, \\
&pL_p(x) = (2p - 1 - x)L_{p-1}(x) - (p-1)L_{p-2}(x).
\end{aligned}
$$

The set of LG functions is complete in L_2 space, as a consequence any prescribed initial function $u_0(r)$ can be expanded into a series of LG functions

$$
u_0(r) = \sum_{p=0}^{\infty} c_p \, W_p(r, 0; w_0, f). \tag{5}
$$

The basis functions obey the ortho-normality condition

$$
(W_i, W_j) = 2\pi \int_0^{\infty} r \, W_i \, W_j^* \mathrm{d}\, r = \delta_{ij},
$$

where δ_{ij} is the Kronecker symbol, and $(\cdot, \cdot)$ denotes the inner product. Then coefficients c_p in (5) are defined as follows

$$
c_p = (u_0, W_p).
$$

We approximate the exact function by a truncated LG series

$$
U_0(r; P) = \sum_{p=0}^{P} C_p \, W_p(r, 0; w_0, f), \tag{6}
$$

where C_p are numerical approximations of c_p, that are obtained by using the Simpson numerical quadrature.

In order to evaluate the field at some plane $z = L$, we simply sum up the propagated LG beams with the same expansion coefficients as at $z = 0$:

$$U(r, z; P) = \sum_{p=0}^{P} C_p \, W_p(r, z; w_0, f).$$

It follows from (6) that

$$\|U(z; P)\|^2 = \sum_{p=0}^{P} |C_p|^2,$$

where we denote the norm:

$$\|U\|^2 = 2\pi \int_0^{\infty} r \, U(r) \, U(r)^* \mathrm{d}\, r.$$

Thus the approximate solution obeys the conservativity property:

$$\|U(z; P)\| = \|U(0; P)\|.$$

The beam quality factor M^2 of the beam $u(r, z)$ with the radius of curvature $R(z = 0) = f$ can be computed by the following formula (see, Martinez–Herrero and Mejias, 1998):

$$M^2(u) = \left[\left(\sum_{p=0}^{\infty} (2p+1)|c_p|^2 \right)^2 - \left(2 \sum_{p=0}^{\infty} (p+1) c_p^* c_{p+1} \right)^2 \right]^{1/2} \Bigg/ \sum_{p=0}^{\infty} |c_p|^2.$$

We again obtain that the beam quality factor $M^2(U)$ of the approximated solution $U(r, z; P)$ is constant for any plane $z = L$:

$$M^2(U) = \left[\left(\sum_{p=0}^{P} (2p+1)|C_p|^2 \right)^2 - \left(2 \sum_{p=0}^{P} (p+1) C_p^* C_{p+1} \right)^2 \right]^{1/2} \Bigg/ \sum_{p=0}^{P} |C_p|^2.$$

2.2. *Accuracy Analysis*

The quadrature error of the composite Simpson method can be estimated as

$$|c_p - C_p| \leqslant M_4 H^4, \tag{7}$$

where $H = \max |r_{i+1} - r_i|$, r_i are quadrature points and M_4 is some constant. Aposteriori error estimators are well-known for numerical quadrature methods and adaptive numerical integration algorithms are constructed on the basis of these estimators (see, Eriksson *et. al.*, 1996). Thus this source of error in mathematical modeling of beam propagation can be estimated and controlled very efficiently.

The error of truncated LG series is investigated by Martinez–Herrero and Mejias (1998). They estimated the relative error in the L_2 norm

$$\|e\|^2 = \frac{\int\limits_0^R |u(r) - U(r;P)|^2 r \mathrm{d}\, r}{\int\limits_0^R |u|^2 r \mathrm{d}\, r}$$

and proved the inequality, which gives the bound of error from above:

$$\|e\|^2 \leqslant \frac{C_{MH}}{2P} \tag{8}$$

with the constant $C_{MH} = M^2(u) - 1$, where $M^2(u)$ is the beam quality factor.

Let α be the convergence order of truncated LG series with respect to P, i.e., the relation $||e|| = cP^{-\alpha}$ is satisfied. Then it follows from (8) that the convergence order of truncated LG series is at least 1/2. It is important to estimate the convergence rate, when $u(r)$ is a smooth function. The constant C_{MH} in (8) also can be improved for many beams.

The estimate (8) bounds the error in the L_2 norm. In many applications it is desirable to measure the error in different norms. It is noted by Eriksson *et al.* (1996) that specifying the quantities to be approximated and the norm in which to measure the error is a fundamentally important part of modeling. It directly affects the choice of approximation and error control algorithm. For example the maximum norm is most important when problem (1) is a part of a more general algorithm for solving nonlinear problem (2).

2.3. *Accumulation of the Truncation Error*

We can solve problem (2) by using the splitting method. Then on each interval (z_j, z_{j+1}), $z_j = jh, j = 0, 1, \ldots, N-1, z_N = L$ two problems are considered. Firstly we calculate the beam propagation in the free space, then we consider the nonlinear interaction of optical waves. We note that both processes are approximated in different functional spaces, i.e., the spectral method is used for diffraction problem and direct physical unknowns are used to approximate a nonlinear interaction.

Hence we consider the following prototype splitting scheme

$$\begin{aligned}
&2ik\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v}{\partial r}\right) = 0, \quad && z_j \leqslant z \leqslant z_{j+1}, \\
&v(z_j, r) = w^j(r), && 0 \leqslant r \leqslant R, \\
&\frac{w^{j+1} - W^j}{h} = f\left(w^{j+1}\right), && 0 \leqslant r_l \leqslant R, \\
&W_l^j = v(z_{j+1}, r_l).
\end{aligned} \tag{9}$$

It can be proved that global error of the solution satisfies the inequality (see, Samarskij (1988) for the analysis of splitting schemes):

$$||u(z_{j+1}) - w^{j+1}|| \leqslant ||u(z_j) - w^j|| + C_1 h^2 + C_2 H^4 + C_3 P^{-\alpha}, \tag{10}$$

where the second term on the right hand side of (10) describes the splitting error and the last two terms describe the error of truncated LG series. By summation of (10) over j we find that

$$||u(z_N) - w^N|| \leqslant C_1 Lh + C_2 NH^4 + C_3 NP^{-\alpha}. \tag{11}$$

In view of (11) we deduce that the splitting error decreases linearly when N is increased, but the error of truncated LG series grows up. Thus in order to reduce the global error we need to change also H and P.

3. Numerical Experiments

In this section we report some numerical experiments with the proposed tool. We investigate the convergence order of truncated LG series and test the validity of our hypothesis on the aposteriori error estimator.

3.1. *Super-Gaussian Beams*

In the numerical experiments we have used super-Gaussian input profiles

$$u(r) = \exp\left(-\left(\frac{r}{w_{sg}}\right)^{2n}\right), \quad n \geqslant 2.$$

Our goal has been to estimate numerically the convergence rate of truncated LG series in the L_2 norm. In Table 1 we present errors $||e||$ and convergence rates α for three different super-Gaussian beams, letting $w_{sg} = 1$. The solution was computed at plane $z = 0$.

Table 1

The errors and convergence rates of truncated LG series for super-Gaussian beams with $n = 2, 4, 7$

P	$n = 2$		$n = 4$		$n = 7$	
	$\|\|e\|\|$	α	$\|\|e\|\|$	α	$\|\|e\|\|$	α
8	0.204e-2	2.06	0.126e-1	0.84	0.221e-1	0.52
16	0.177e-3	3.53	0.519e-2	1.29	0.130e-1	0.77
32	0.480e-5	5.20	0.115e-2	2.17	0.555e-2	1.23
64	0.142e-7	8.40	0.234e-3	2.30	0.221e-2	1.32

We see that the convergence rate depends on the smoothness of input function, and it decreases for increased values of n. Note, that α is an increasing function of P. In all cases the obtained convergence rates are larger than 1/2.

3.2. *Convergence in Different Norms*

Since basis functions of LG series are orthogonal and complete with respect to the L_2 norm, we obtain that the error decreases monotonically in the L_2 norm. In this section we will estimate the accuracy of approximation in the modified maximum norm

$$\|e\|_\infty = \max_{0 \leqslant r_j \leqslant R} \frac{|u(r_j) - U_P(r_j)|}{1 + \max\limits_{0 \leqslant r_j \leqslant R} |u(r_j)|} .$$

Fig. 1 plots the errors $\|e\|_\infty$ (solid line) and $||e||$ (dashed line) as functions of P for the super-Gaussian beams with $n = 4, 7$.

We see that the error $\|e\|_\infty$ changes non-monotonically, while $||e||$ decreases monotonically. The proposed tool is very useful in such situations. Note that the results given above impose some restriction on the selection of the number of basis functions P. In order to guarantee that the approximation error is decreasing in a series of computations we should increase P by some factor $\nu > 1$. Simple adding of P_1 polynomials can decrease the accuracy.

3.3. *Accuracy of Aposteriori Estimators*

In this section we shall test the hypothesis on aposteriori error estimation rule by computing numerically the errors of representative examples of test functions. We remind, that our hypothesis leads to to the following rule: "*The approximation error estimate of the truncated LG series expansion at the initial plane $z = 0$ can be used as aposteriori error estimator for any plane $z = L$*".

As examples of test functions we shall use the Bessel–Gauss beam, a linear combination of two purely Gaussian beams and super-Gaussian beams.

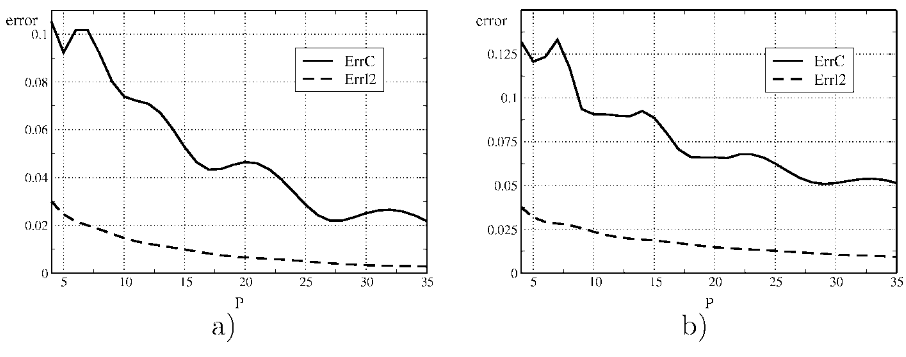

Fig. 1. Errors $\|e\|_\infty$ (solid line) and $\|e\|$ (dashed line) for the super-Gaussian beams: (a) $n = 4$, (b) $n = 7$.

Bessel–Gauss Function. Let us consider the focused Bessel–Gauss beam (see, Belafhal and Dalil–Essakali, 2000)

$$u_{bg}(r, z=0) = J_0(\alpha r) \exp\left(-\left(\frac{r}{w_{bg}}\right)^2 - \frac{ikr^2}{2f}\right), \tag{12}$$

where J_0 is the Bessel function of the first kind and 0th order with $\alpha = k \sin \Theta$. Here $k = 2\pi/\lambda$ is the wave number of the field, and w_{bg} is the spot size of the fundamental Gaussian mode at $z = 0$.

The exact solution of problem (1) is given by

$$\begin{aligned} u_{bg}(r, z) = \frac{w_{bg}}{w_{bg}(z)} J_0\left(\frac{\alpha r}{1 - \dfrac{z}{f} + i\dfrac{z}{z_R}}\right) \exp\left(-i\left(\alpha^2 \frac{z}{2k} - \varphi(z)\right)\right) \\ \times \exp\left(-\left(\frac{1}{w_{bg}^2(z)} - i\frac{k}{2R(z)}\right)\left(r^2 + \frac{\alpha^2 z^2}{k^2}\right)\right), \end{aligned} \tag{13}$$

where

$$z_R = \frac{k w_{bg}^2}{2}, \qquad w_{bg}(z) = w_{bg}\sqrt{\left(1 - \frac{z}{f}\right)^2 + \left(\frac{z}{z_R}\right)^2},$$

$$R(z) = \frac{z\left(1 + \left(1 - \dfrac{z}{f}\right)^2 \left(\dfrac{z_R}{z}\right)^2\right)}{1 - \dfrac{1}{f}\left(1 - \dfrac{z}{f}\right)\dfrac{z_R^2}{z}}, \quad \varphi(z) = \arctan\left(\frac{z}{z_R\left(1 - \dfrac{z}{f}\right)}\right).$$

In Table 2 the errors $\|e\|_\infty$ and $\|e\|$ are presented as functions of different planes $z = L$ for two values of P. The beam parameters are: $w_{bg} = 0.1$, $\lambda = 0.0001064$, $\Theta = 0.001$, $f = 100$, and LG basis functions spot size is $w_0 = w_{bg}$.

We also investigated the case of $f = 30$, when the focusing of the beam is much stronger. The remaining parameters are the same as in the previous experiment. The results are presented in Table 3.

Table 2

The truncation errors $\|e\|_\infty$ and $\|e\|$ for the Bessel–Gauss beam with $f = 100$

P		$z=0$	$z=20$	$z=50$	$z=100$
8	$errc$	0.136594	0.153286	0.217977	0.168731
	$errl2$	0.007551	0.007551	0.007551	0.007552
10	$errc$	0.007025	0.007971	0.012917	0.014247
	$errl2$	0.000474	0.000474	0.000474	0.000470

Table 3

The truncation errors $\|e\|_\infty$ and $\|e\|$ for the Bessel–Gauss beam with $f = 30$

P		$z = 0$	$z = 10$	$z = 20$	$z = 30$
8	$errc$	0.037714	0.045587	0.066236	0.138636
	$errl2$	0.002346	0.002346	0.002346	0.002346
12	$errc$	0.000929	0.001128	0.001693	0.004716
	$errl2$	0.000067	0.000067	0.000067	0.000067

It is seen from the results given in Table 2 and Table 3 that the agreement of computed and predicted errors is good. It should be noted that the aposteriori error estimator in the maximum norm must be multiplied by some factor $\nu \approx 4$.

Linear Combination of Two Gaussian Beams. Let consider the following initial profile

$$u_{gg}(r, z = 0) = \left[\exp\left(-\left(\frac{r}{w_1}\right)^2\right) - \exp\left(-\left(\frac{r}{w_2}\right)^2\right)\right] e^{-\dfrac{ikr^2}{2f}}, \tag{14}$$

where $w_1 = 0.1$ and $w_2 = 0.02$. In all computations we take the wave length $\lambda = 0.0001$. The spot size of LG basis functions has been taken $w_0 = 0.02$, i.e., equal to the spot size of the second (narrow) Gaussian beam.

In Table 4 the errors $\|e\|_\infty$ and $\|e\|$ are presented as functions of different planes $z = L$ for two values of P. The formula for the exact solution $u_{gg}(r, z)$ follows from (13), taking $\Theta = 0$.

It is seen from the results given in Table 4 that the agreement of computed and predicted errors is very good. It should be noted that the errors are decreasing for large values of z, since only the influence of the second term of (14) is important for such z. The se-

Table 4

The truncation errors $\|e\|_\infty$ and $\|e\|$ for the linear combination of two Gaussian beams

P		$z = 0$	$z = 100$	$z = 400$	$z = 1000$
50	$errc$	0.018943	0.016647	0.013710	0.003709
	$errl2$	0.007291	0.006948	0.004461	0.001530
80	$errc$	0.001716	0.001516	0.001742	0.000393
	$errl2$	0.000661	0.000654	0.000414	0.000160

Table 5
The truncation errors $\|e\|_\infty$ and $\|e\|$ for the super-Gaussian beam with $n = 4$

P		$z = 0$	$z = 10$	$z = 40$	$z = 80$
16	$errc$	0.046364	0.052471	0.081161	0.042039
	$errl2$	0.009025	0.009056	0.009166	0.009166
30	$errc$	0.025069	0.028253	0.052475	0.011079
	$errl2$	0.003294	0.003302	0.003329	0.003347

cond Gaussian beam is described exactly with the first term of the truncated LG series expansion.

Super-Gaussian Beam. In this paragraph we present results obtained for super-Gaussian beam with $n = 4$, $w_{sg} = 0.2, f = 100$. Since exact solutions for planes $z = L$ are not known, we have computed "exact" solutions numerically by using truncated LG series with sufficiently large values of P.

In Table 5 the errors $\|e\|_\infty$ and $\|e\|$ are presented as functions of different planes $z = L$ for two values of P.

It is seen from the results given in Table 5 that the agreement of computed and predicted errors is good, and the proposed rule can be used as aposteriori error estimator.

4. Application of the Tool

In this section we apply the tool to find the optimal value of the parameter w_0, which defines the spot size of LG functions. Let note that the set of LG functions is complete for any choice of w_0. Nonetheless, we expect that the value of w_0 should affect the approximation errors of the truncated LG series. For example, a purely Gaussian beam with a spot size w_G can be approximated by LG series with arbitrarily chosen w_0, but the choice $w_0 = w_G$ ensures a zero truncation error even for the series which is truncated to the first term.

A simple optimization rule is suggested in Borghi *et al.* (1996). They considered the circ function and proved that the optimal value of w_0 should be $1/\sqrt{P}$. We will test this rule by numerical evaluation of the error induced by truncated LG series for important classes of input functions.

First, we approximate super-Gaussian beams with different parameters n. Let note that increasing n we obtain larger gradients of the solution front and for n sufficiently large the super-Gaussian function is close to circ function. Second, we investigate the accuracy of truncated LG series for the Bessel–Gaussian beam. In Fig. 2 for a few values of P the truncation errors $||e||$ are given as functions of w_0: the super-Gaussian beams are used for (a)–(c), and the Bessel–Gaussian beam is used for (d).

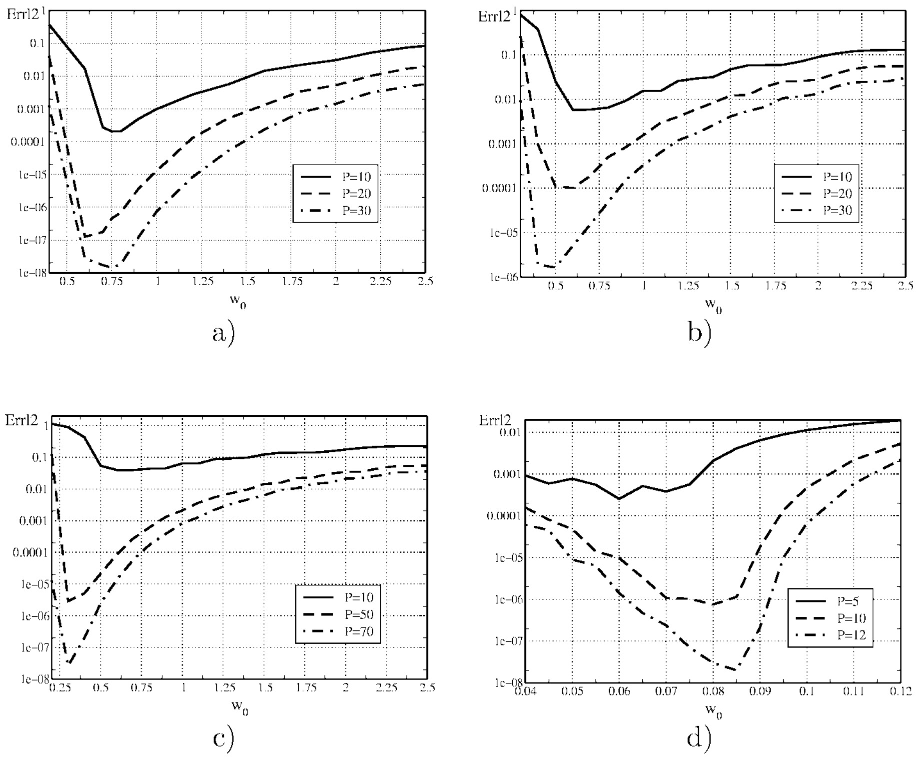

Fig. 2. Behaviour of $||e||$ as a function of w_0 for different values of P: super-Gaussian beams a) $n = 2$, b) $n = 4$, c) $n = 7$, d) Bessel–Gaussian beam.

5. Conclusions

In this paper we developed a tool for modeling the propagation of optical beams. With the help of this tool we can optimize the LG truncated series expansions by minimizing the truncation error for a fixed number of terms. We suggested a simple rule for the definition of aposteriori error estimator at any point $z = L$. The accuracy of this estimator is tested numerically for different beam functions. We investigated the behavior of the truncation error as a function of the number of terms in LG series. It is proved that this error changes non-monotonically in the maximum norm.

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R. Čiegis has graduated from Vilnius University Faculty of Mathematics in 1982. He received the PhD degree from the Institute of Mathematics of Byelorussian Academy of Science in 1985 and the degree of Habil. Doctor of Mathematics from the Institute of Mathematics and Informatics, Vilnius in 1993. He is a professor and the head of Mathematical Modelling Department of Vilnius Gediminas Technical University. His research interests include numerical methods for solving nonlinear PDE, parallel numerical methods and mathematical modelling in nonlinear optics, biophysics, technology.

A. Dement'ev has graduated from the Moscow State University Faculty of Physics in 1971. He received the degree of Candidate of Phys.-Math. Sciences in the same University in 1974 and the degree of Doctor of Phys.-Math. Sc. in optics from the Institute of Physics of Byelorussian Academy of Sciences in 1992. He is a professor and the head of Nonlinear Optics and Spectroscopy Laboratory of the Institute of Physics. His research interests include theoretical modelling of different transient nonlinear optical processes, mainly the generation, and amplification of short laser pulses, pulse compression by stimulated Brillouin and Raman scattering and their use in different applications.

G. Šilko has graduated from Vilnius University Faculty of Mathematics in 1987. She received Master degree in engineering informatics from the Vilnius Gediminas Technical University in 1999. Now she is PhD student in Vilnius Gediminas Technical University. Her research interests include mathematical modelling of non-linear optics and parallel computing.

Optikos bangų sklidimo modeliavimo įrankis

Raimondas ČIEGIS, Galina ŠILKO, Aleksandr DEMENT'EV

Šiame darbe nagrinėjami algoritmai, skirti optinių bangų sklidimo modeliavimui. Sprendinio artinį skaičiuojame panaudodami Lagero–Gauso funkcijų skleidinio baigtines sumas. Sukurtas programinis įrankis, leidžiantis apskaičiuoti bangos artinį bet kuriame taške ir įvertinantis gautojo sprendinio paklaidą pasirinktoje normoje. Parodyta, kad LG funkcijų skleidinio paklaida maksimumo normoje kinta nemonotoniškai polinomų skaičiaus atžvilgiu. Programinis įrankis panaudotas optimizuojant bazinių funkcijų parametrus, tokiu būdu minimizuojami skaičiavimo kaštai.