INFORMATICA, 2002, Vol. 13, No. 1, 3–22

Modeling and Control – Flexible Structure Spacecraft

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Received: October 2001

Abstract. Installation dynamics, optimal thruster propulsion, and specific impulse thrust for reignitable fuel motors to regulate pointing error is obtained for geosynchronous (e.g., the Indian National Satellite, INSAT–1B), and sunsynchronous (e.g., the Indian Earth Resource Satellite, IRS–1) satellites parked in transfer orbits. Filter dynamics for slow and fast terrestrial system are obtained. Design for inter satellite link server to maximize drop outlets is given. Technique for extension of commutative rings for coordinated control and communication systems with predetermined target signature in global and protected environment is given. Visibility model of flexible structure terrestrial exploration space monitoring station with associated space coordinate system is given. Technique proposed is applied for information acquisition and synchronization of inertial targets. Design of long range navigation in projective space and robust hybrid controller for AAFM auto pilot system is given. AAFM system reliability is analyzed.

Key words: control systems, control applications, space vehicles, satellite control, propulsion control.

1. Introduction

Decentralized systems offer advantages in computational speed, robustness, survivability and modularization. Coordinated control and communication environment in which decentralized system may operate is frequently hostile and in general is not known. Satellites link (e.g., the Indian National Satellite, INSAT_1B – INSTAT_ID) transponder and ground station propagation delay is quite large. Packets connectivity time and slots assignment may be employed to resolve contention due to lack of synchronization, instability, drift, and propagation time. Extension of commutative rings of reachable zone in global and protected environment mode to design coordinated control and communication of multi terrestrial objects with predetermined signature is described.

Hohmann transfer procedure may be employed to install satellite parked in transfer orbit to geostationary, geosynchronous (INSAT–1B), sunsynchronous (e.g., the Indian Earth Resources Satellite, IRS–1, 13.95 orbits/day, 25.80° shift/orbit) orbits. Optimal class of thruster propulsion controls, and specific impulse thrust is given. Kepler's law of Planetary motion for swept area is applied to obtain dynamic representation for Keplerian orbit. From Relativity principle maneuvering acceleration (Kang, 1996) is modeled as first order Gauss Markov process (Berg, 1983) to estimate object parameters. To improve system performance, silence detectors and bit slicing may be employed. Frequently singularities are introduced by dynamic obstacles. Spherical trigonometry and self locator analysis technique is applied to determine spherical coordinates in image plane for expert tuned satellite locator system. Problem of precision index robustness of satellite locator (Fig. 4) with reignitable fuel motors is equivalent to classical augmented L_∞ stability (Kang, 1970) robustness problem. Design procedure for performance robust extremum controller (Kang and Kang, 1997) auto pilot for AAFM system (Cluotier, 1989) is given.

2. Mathematical Formulation

Dynamic system under consideration (Fig. 1) is characterized by sub groups of borel set of ordered abelian time group. Set X, Y are the abelian group of vectors with $k\Delta t \to X$, $k\Delta t \to Y$ of reflexive $x \in X\colon \boldsymbol{R}^n, y \in Y\colon \boldsymbol{R}^m$. X is a homomorphism of abelian group with state mapping transformation $Y \to X$ given by $[(k+1)\Delta t,\ k\Delta t, y, x, u_i, u_0] \to x$. y is a homomorphism of abelian observance process. U is the abelian groups of class of controls. State transformation map is given by $[(k+1)\Delta t, k\Delta t, Y, X, U_i, U_0] \to X$. $\Omega \in \Re$ is the cost space. Vector space state transformation function is given by $x_i((k+1)\Delta t) = \Psi(k\Delta t, k_0\Delta t, y, x) \in X, \Psi\colon \boldsymbol{R}^1 \times \boldsymbol{R}^r \times \boldsymbol{R}^n; r, n \leqslant m.$

2.1. *Structural Descriptor*

Let θ^e_{ci}, θ^a_{ci}; $i = 0, 1$ (Fig. 1) be the elevation and azimuth angles for satellite control and monitoring stations from earth. Let $x^s = [x^{sm}]$; $x^s_{cj} = [x^{sm}_{cj}]$; $j = 0, 1$; $m = i, \ldots, 3$; be the target and controller states, h^s be the satellite altitude. For primary controller define $x^1_{c10} = (x^1_{ci} - x^1_{cj})$; $i, j = 0, 1$; $i \neq j$, Beacon marks (P_h, P_v) are placed on the horizontal and vertical great circles. By applying opposite and adjacency rules in virtual projective

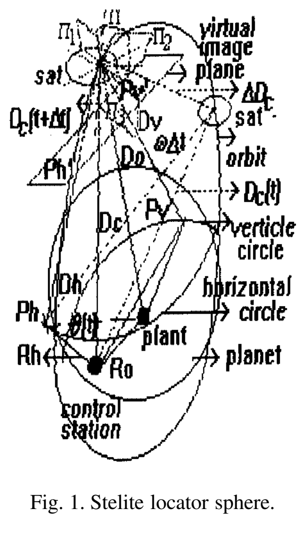

Fig. 1. Stelite locator sphere.

image planes extracted parameters are given by

$$\begin{array}{l} x_{cj}^{s1} = h_{cj}^{s} \cos\theta_{cj}^{e} \cos\theta_{cj}^{a}; \quad x_{cj}^{s2} = h_{cj}^{s} \cos\theta_{cj}^{e} \sin\theta_{cj}^{a}; \\ x_{cj}^{s3} = h_{cj}^{s} \sin\theta_{cj}^{a}; \quad j = 0, 1; \\ h_{cj}^{s} = \left[x_{c10}^{1} \sin\theta_{ci}^{a} - x_{c10}^{2} \cos\theta_{ci}^{a}\right] / \cos\theta_{cj}^{e} \sin(\theta_{cj}^{a} - \theta_{ci}^{a}). \end{array} \tag{1}$$

Coordinate Mapping – Satellite/Earth Link. Let $d_e = (1/cos\theta_1)$ be the conformal stretching factor; θ^1 be the latitude angle, area $(\Delta S_a P_v C_n)$ be the area of spherical triangle (Fig. 2). Assuming origin is located at the satellite and translating with satellite motion, from law of cosines sides and cosines arcs and property of cylindrical area projection, center angles, orientation and translation transformation for the terrestrial target (Fig. 2) are given by

$$\begin{array}{l} \beta_2 = \tan^{-1}\left[\sin\alpha_2 / \{(D_{cv}/D_{pv}) - \cos\alpha_2\}\right]; \\ R_{ov} = d_e D_{pv} (\sin\alpha_2 / \sin\beta_2); \\ \gamma_4 = \sin^{-1}\left[(d_e D_v / R_v) \sin\alpha_4\right]; \\ \beta_4 = \pi - (\alpha_4 + \gamma_4) + \text{area}(\Delta S_a P_v C_n)/(D_{cv})^2; \\ D_{pv} = D_{cv} \sin\beta_2 / \sin(\alpha_2 + \beta_2); \\ D_{cv} = (R_v/d_e)(\sin\beta_4 / \sin\alpha_4). \end{array} \tag{2}$$

Terrestrial Link. Let triple $\Theta^0 = [\varphi_r^0 \varphi_p^0 \varphi_q^0]$ be the roll pitch yaw reference frame euler angles. To reduce propellant requirements, to increase lifetime, available payload mass, and propulsion thrust, systems with bielliptical high latitude launch may be employed.

Multiframe Field Isomorphism. Let X be the Hausdorff Space, $\{R(G_i), +, .\}$ be the field of commutative group rings, $d^c \in D^c$, $d^s \in D^s$ be the position vectors, $\theta^c \in \Theta^c$, $\theta^s \in \Theta^s$; be the orientation of commutative ring frames (Fig. 3). Problem is formulated as isomorphism extension to maximize compounded mapping affine transformation (Kang, 1970) which preserves norm $\|x_i\| = \sup\{|\psi(d_i^s, \theta_i^s)|/\|d_i^s\|\|\theta_i^s\|\}$; $d_i^s, \theta_i^s \in$

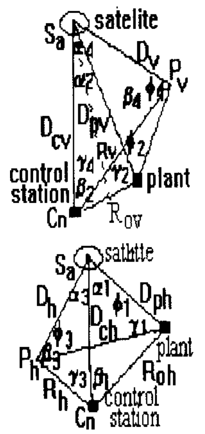

Fig. 2. Horizintal vertical proj.

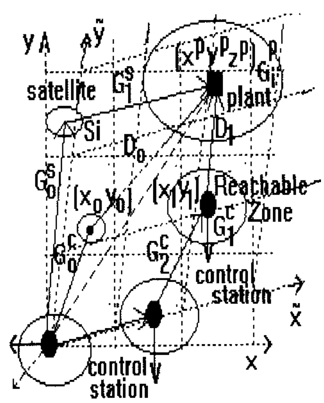

Fig. 3. Inertial coordinate mapping.

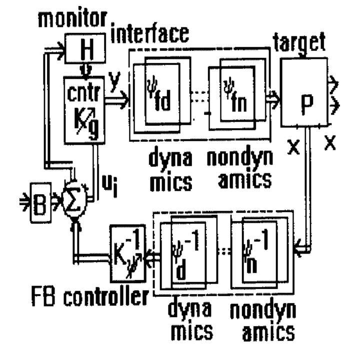

Fig. 4. Satelite locator syst.

$\psi(d_i^c, \theta_i^c, d_{i-1}^s, \theta_{i-1}^s)$. It is assumed that system descriptor group G^c is isomorphic and description frame is decomposable to subgroups $\Pi_{i-1}^n \Pi_{j-1}^n G_j^c u_i^c$; $j, i = 1, 2, \ldots$ components of molnya, geo synchronous and sun synchronous satellites link (Fig. 4). From terrestrial object Kinematics equations, field of controls may be expressed as $u_1^c = I_{11}\varphi_r$, $u_2^c = I_{22}\varphi_p$. $u_3^c = I_{33}\varphi_q$. Contiguous connected multiframe segments of satellite frame, intermediate frames, base frame and controller $\{\Theta_{1,0}^c(\theta^c, d^c)\}$ reference frames can be expressed as $G_{n+1,0}^s() = G_{n+1,nb}^s \ldots G_{n,0}^c = \Theta_{n+1,n}^S(\theta^s, d^s) \ldots \Theta_{1,0}^c(\theta^c, d^c)$. Compounded mapping affined transformation of Kinematics decomposable ring, and frame orientation transformation Θ^s of rotational operators about satellite moving frame principle axis can be expressed as:

$$G_{i+1,i}^s = \begin{bmatrix} \Theta_{i+1,i}^s & D_{i+1,i}^s \\ 0 & 1 \end{bmatrix} \begin{matrix} \ldots \\ \ldots \end{matrix} \begin{bmatrix} \Theta_{i+1,i}^c & D_{i+1,i}^c \\ 0 & 1 \end{bmatrix} \begin{bmatrix} G_{i,0}^s \\ 1 \end{bmatrix};$$

$$\Theta_{i+1} = \begin{bmatrix} \cos\varphi_q & -\sin\varphi_q & 0 \\ \sin\varphi_q & \cos\varphi_q & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\varphi_p & 0 & \sin\varphi_p \\ 0 & 1 & 0 \\ -\sin\varphi_p & 0 & \cos\varphi_p \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi_r & -\sin\varphi_r \\ 0 & \sin\varphi_r & \cos\varphi_r \end{bmatrix} \Theta_i \quad (3)$$

Long Range Navigator. Let G_i^c be the group of control stations and transponders of terrestrial positioning system. From the properties of perspective triangles, state of the target extracted from set of spherical coordinates may be expressed as $G_{il}^p = G_i^c G_i^p \cap G_{i-1,1}^p$; i: $\{1, \ldots, n\}$. Let $\Delta t_i, \Delta t_{i+1}$ be the navigation intervals of states $(G_i^p G_{i+1}^p), (G_{i+1}^p G_{i+2}^p)$. State of object in extended measure space is given by G_{i+1}^p: $H_i \cap H_{i+1}$, here H_i: $|G_i^p G_i^c - G_{i+1}^c G_i^p| = c\Delta t_i$, and H_{i+1}: $|G_i^p G_i^c - G_{1+2}^c G_i^p| = c\Delta t_{i+1}$ are the fields of hyperbola, c is the velocity of light, and G_i^p are the collection of indices points generated by generator projective plane observation process.

2.2. *Relativity Model – Terrestrial Objects*

Rotation matrix present difficulties in computation and interpretation. Consider class of controls $u(k) = [\Delta\theta_{ck}^a \Delta\theta_{ck}^e \Delta h_{cj}^s]^t$; $j = 0, 1$; $k = 0, 1, \ldots$. Let w_k be the zero mean white noise, σ: $R \rightarrow \boldsymbol{R}$ is Lipschitz continuous with σ^2 bounded away from zero on every compact subset of $\boldsymbol{R}$. System descriptor dynamics may be expressed as

$$x^s_{k+1} = x^s_k + F_{\theta,d} u_k + \sigma w_k, \tag{4}$$

$$F_{\theta,d} = \begin{bmatrix} -h^s_{c1}\cos\theta^e_{c1}\sin\theta^a_{c1} + h^s_{c0}\cos\theta^e_{c0}\sin\theta^a_{c0} & -h^s_{c1}\sin\theta^e_{c1}\cos\theta^a_{c1} + h^s_{c0}\sin\theta^e_{c0}\cos\theta^a_{c0} \\ h^s_{c1}\cos\theta^e_{c1}\cos\theta^a_{c1} - h^s_{c0}\cos\theta^e_{c0}\cos\theta^a_{c0} & -h^s_{c1}\sin\theta^e_{c1}\sin\theta^a_{c1} + h^s_{c0}\sin\theta^e_{c0}\sin\theta^a_{c0} \\ 0 & h^s_{c1}\cos\theta^e_{c1} - h^s_{c0}\cos\theta^e_{c0} \end{bmatrix.$$
$$\begin{matrix} k_1\cos\theta^e_{c1}\cos\theta^a_{c1} - k_0\cos\theta^e_{c0}\cos\theta^a_{c0} \\ k_1\cos\theta^e_{c1}\sin\theta^a_{c1} - k_0\cos\theta^e_{c0}\sin\theta^a_{c0} \\ -k_1\sin\theta^e_{c1} - k_0\sin\theta^e_{c0} \end{bmatrix}.$$

Class of Controls. Consistent with reasonable satellite life, field of control actions consists of set of controls to install satellite in the lowest possible orbit. Let Δv_n be the impulse thrust normal to satellite trajectory support plane.

Satellite movement at latitude equilibrium may be expressed as

$$d^2\theta^l_{cj}/dt^2 = -A_e(w_{sl}, h^s)\sin 2\theta^l_{cj} \tag{5}$$

here w_{sl} is the angular velocity. Inclination angle can be expressed as $\Delta\theta^i_{cj} \approx (1/k_{sj})\Delta v_n$; $k_{sj} = f(v_s^{-1})$, here $v_s = \sqrt{g_e}[(2/r_s) - 1/(r_p + r_a)]$ is the satellite velocity, where g_e is the gravitational constant, r_s is the satellite radius vector, r_p is the perigee radius vector, r_a is the apogee radius vector. Tangential impulse thrust control is given by $\Delta v_t = v_{pr}\log(m_s + m_p/m_s)$, here m_s is the satellite mass, v_{pr} is velocity of ejection, $\Delta\theta^1_{ck} = k_1|\Delta v_t|$. Let $\theta_c = [\Delta\theta^1_{ck}\ \Delta\theta^i_{ck}\ \Delta\theta_{ph}]^t$, let I^{ii}, $i = 1, 2, 3$ be the moment of inertia of terrestrial object with respect to principal axis. It is assumed that for introduced perturbations, interaction for principal axis moment inertia is small. Let T^i_d be the external disturbance moment vectors, ω^i, ω^i_r be the angular and relative angular velocities of space craft and reaction wheel with respect to axis of rotation. Dynamics of stabilized geosynchronous satellite and reaction wheels (e.g., the Indian Earth Resources Satellite, and the Indian National Satellite, INSAT_B, IRS_1), pitch momentum, yaw reaction wheels may be expressed as

$$I^{ii}(d\omega^1/dt) = I^1_w + u^i + T^i_d;\ \ i = 1, 2, 3;\ \ J^j\big((d\omega^j/dt) + (d\omega^j_r/dt)\big) = -u^j; \tag{6}$$

here $l^1_w = (I^{22} - I^{33})\omega^2\omega^3$, $I^2_w = (I^{33} - I^{11})\omega^3\omega^1$, $I^3_w = (I^{11} - I^{22})\omega^1\omega^2$. Let $u^i = [\pi_t\pi_n I_{sp}]^t$ be the Impulse thrust controls, here $I_{sp} = (\pi_t/m_{pr}g_e)$ is the specific impulse thrust, where m_{pr} is the ejected mass flow rate, π_t is the external thrust, Specific impulse may also be expressed as $I_{sp} = k_h h^s$. Let $\zeta^{Ij}(F_\theta(n_r), k\Delta t) = H^j(x^{Ij}(k/k - 1, x_{nr}))$ be the observance process, here $H(\ )$ is the measurement matrix. For terrestrial positioning system with thruster fuel motors, bipropellant reignitable motors, fast and slow dynamics for inner (I) and outer (II) loops, and class of controls $u^i = [\pi_t \pi_n I_{sp}]^t$ may be expressed as

$$\begin{aligned} &x^s_{k+1} = x^s_k + F_{\theta,d} B^I u_k + F_{\theta,d} B^{II} u_k + \sigma w_k; \quad T^i_d = \sigma^i w^i_k; \\ &u^I_c = K^I \theta_c = \operatorname{diag}(k_1 I^{ii}\ k_{sj} I^{ii}\ 0)\theta_c; \\ &u^{II}_c = K^{II}\theta_c = \operatorname{diag}(0\ 0\ k_h c/2\pi f)\theta_c. \end{aligned} \tag{7}$$

Acquisition of Synchronization. Ground propagation time via satellite link may be 0.238 sec to 0.278 sec. Variations in incident signal, and relative satellite distance may result in problems of link synchronisation. Phase shift for transmitted and received packets (e.g., s/s-s/x band, IRS -1) from global positioning system is given by $\theta_{ph} = (1/k_p)h^s_c$, here $k_p = c/2\pi f$, f is transmission frequency (Hz). Let $k_d = (f_d/f_u)$, here f_d, f_u are the up link and down link frequencies. Let Δf be the frequency shift due to Doppler's effect. Terrestrial object velocity can be expressed by observation process $v_s = (c/2)(1/k_d)(\Delta f/f_u)$, here $c = 3 \times 10^8$m/s is the velocity of light. $(\delta\Delta t/\Delta t)$ is the Doppler shift and is function of number of orbits completed per day, and for $f_u = 6$ GHz Doppler shift is 18 kHz (for $h^s = 11000$ km and period 6 Hrs). Observation process $\zeta^{Ij}(\ )$ is decomposable switching process, and may be identified as r groups taking values in finite set $S = \{1, 2, \ldots\}$ with no absorbing group.

$$\zeta^{Ij}(F_\theta(n_r), k\Delta t) = H^J x^{Ij}\big(F_\theta(n_r), k\Delta t(1 - \delta\Delta t/\Delta t)\big). \tag{8}$$

3. Compounded Locator Filter

Target signature access technique may be employed to access multiple targets with in satellite beam scanning. Let $\{\theta^e_{ck}, \theta^a_{ck},\ \text{range}\ h^s_{cj}\}$; $i = 0, 1, \ldots$ be the set of triangulated measurements process (Fig. 3). Satellite locator system (Fig. 4) consists of group of $m = \{\sum_{j=1}^m n^j_i\}$ fast mode states. Let $S_i = \{n^m_i\}$; $i = 2, \ldots, r$; $n^m_r = \sum_{i=1}^r n^m_i$; $r \geqslant 2$, be set of r slow mode states, a subset of set $\cup_{i=1}^r S_i \subset S = \{n^1_1, n^2_1, \ldots, n^m_1, n^1_2, \ldots, n^m_2, \ldots, n^1_r, \ldots, n^m_r\}$. Aggregation of connectivity time is represented by indices of sequence of slow and fast mode states. $\theta^a_j(n_r, k)$, $\theta^e_j(n_r, k)$; $j > r$; $r \neq j$ are the Brownian motion process. Let $\Phi^c \in \Phi$ be the closed subspace from field of group rings with coset $[\theta^s_{r,0}] \in \Phi/\Phi^c$, characterized as quotient space Φ modulo Φ^c, and norm $\|[\theta^s_{m,0}]\| = \inf \theta^a$, $\theta^e \in \Phi^c$; $\|\theta^j_{m-1}{}^a \oplus \theta^j_{m-1}{}^e \oplus \theta^s_{m-1,0}\|$, is the Banach space characterizing switching modes.

3.1. *Terrestrial Positioning Filter Dynamics*

Rotation of axis and poles movement for earth may be decomposed in periodic terms (amplitude 20 sec of arc), nutation, and non periodic terms with cumulative precession, resulting in equator plane variations 50" per year. Sun eclipse and satellite conjuction may modify satellite dynamics. Let $[.]_{r(\omega\theta)}$ be periodic matrix of integer period with $S_i \in S$, ω_θ be rotational velocity of the base frame. The observation process

$\{\theta^e(k)\theta^a(k)\}$, $n_r \leqslant t_k \leqslant n_{r+1}$ is separable martingale. Let $L(w_{i0}(t)) \sim \boldsymbol{N}(0, W_{i0})$, and $L(v_{i0}(t)) \sim \boldsymbol{N}(0, V_{io})$; $i = i, 2, \ldots$. In the following extension field is employed to design performance robust satellite and ground locator system. Multiple extended filter (satellite) dynamics can be expressed as.

Fast (Inner Loop) Dynamics.

$$\begin{aligned}
&E[x^{lj}(k/k, x_{nr})] = E\big[x^{Ij}(k/k-1, x_{nr})\big] \\
&\qquad\qquad +M^{Ij}(k)\Big[\zeta^{Ij}\big(F_\theta(n_r)\big) - H^j E\big(x^{Ij}(k/k-1, x_{nr})\big)\Big]; \\
&M^{Ij}(k) = P^{Ij}(k/k-1)H^{jt}\big[H^j P^{Ij}(k/k-1)H^{jt} + V^I(k)\big]^{-1};\;\; r-1 \leqslant j < r; \\
&E\big[x^{Ij}(k+1/k, x_{nr})\big] = \big[I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij}\big]_{r-1(q)} E\big[x^{Ij}(k/k, x_{nr})\big]; \\
&\qquad\qquad i, r = 1, 2, \ldots; \\
&P^{Ij}(k/k) = \big[P^{Ij}(k/k-1) - M^{Ij}(k)H^{jt}P^{Ij}(k/k-1)\big]; \quad n_{r-1} \leqslant k \leqslant n_r; \\
&P_0^{Ij} = P^{Ir}(n_r); \quad \theta_r^e(t_0) = \theta_{r-1}^e(t_{nr}); \quad \theta_r^a(t_0) = \theta_{r-1}^a(t_{nr}); \\
&P^{Ij}(k/k-l) = \big[I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij}\big]_{r-1(q)} P^{Ij}(k-1/k-1) \\
&\qquad\qquad \times\big[I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij}\big]_{r-1(q)}^t + W^I(k).
\end{aligned} \tag{9}$$

Slow (Outer Loop) Dynamics.

$$\begin{aligned}
&E\big[x^{IIj}(k/k, x_{nr})\big] = E\big[x^{IIj}(k/k-1, x_{nr})\big] + M^{IIj}(k)\big[\zeta^{IIj}(F_\theta(n_r), k) \\
&\quad -H^{IIj}E(x^{IIj}(k/k-1, x_{nr}))\big]; \\
&E\big[x^{IIj}(k+1/k, x_{nr})\big] = \big[I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij} - F_\theta^j B^{IIj}K^{IIj} \\
&\quad -M^{Ij}(k+1)H^j(I - F_\theta^j(n_{r-1})B^{Ij}K^{IIj})\big]_{r-1(q)} E\big[x^{IIj}(k/k, x_{nr})\big]; \\
&M^{IIj}(k) = P^{IIj}(k/k-1)H^{jt}\big[H^j P^{IIj}(k/k-1)H^{jt} + V^{II}(k)\big]^{-1}; \\
&P^{IIj}(k+1/k+1) = \big[P^{IIj}(k+1/k) - M^{IIj}(k)H^{jt}P^{IIj}(k+1/k)\big]; \\
&P_0^{IIj} = P^{IIr}(n_r); \\
&P^{IIj}(k+1/k) = \big[I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij} - F_\theta^j B^{IIj}K^{IIj} \\
&\quad -M^{IIj}(k+1)H^{Ij}(I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij})\big]_{r-1(q)} P^{Ij}(k/k) \\
&\quad \times\big[I - F_\theta^j(n_{r-1})B^{Ij}K^{Ij} - F_\theta B^{IIj}K^{IIj} \\
&\quad -M^{Ij}(k+1)H^j(I - F_\theta^j(n_{r-1})B^{Ij}K_i^{Ij})\big]_{r-1(q)} + W^{II}(k).
\end{aligned} \tag{10}$$

To compensate for earth rotation, and for satellite stabilization steering control action may be employed.

3.2. *Satellite Time Optimal Regulator*

Variations in the velocity of rotation may cause stabilized axis pointing error. Start sensor, sun sensor, infra-red horizon sensor and rate integrating gyro may be employed to generate attitude (IRS-1), and roll, pitch, yaw error sensing and error correction. For Geosynchronous, Molnya or Tundra orbit satellites link, frequent switching of tracking satellites may be necessary. The class of controls consists of control action normal to the orbit support plane for inclination control and tangential control action to modify long term drift and the eccentricity (Lemma 3.1). It is assumed that periodic perturbations are small as compared to the window size. The time optimal control for propellant consumption is given in Theorem 3.1. To combat disturbing torques, and for gyroscopic stabilization pairs of low power hybrid thrusters, Unified bipropellant propulsion (oxidant fuel pair) reignitable motors, and liquid propellant may be employed. Impulse control action $u^j(k)$ to generate regulating action may be expressed by the following Lemma.

Lemma 3.1 (Gyroscopic stabilization). *Let $x^j(0)$ be independent of driving motion. Given set of policies*

$$u(k) = \left[\Delta v_t \Delta v_n I_{sp}\right]^t = \mathrm{diag}[k_l\ k_{sj}\ k_h]\mathrm{sgn}(x^j(k)). \tag{11}$$

Terrestrial object trajectory has an upper bound.

Class of controllers for gyroscopic stabilization consisting of low power thrusters characterized by low power level, large number of operating cycles, long lifetime and long cumulative operating time.

Reachable Set. The orbit corrections can not be made during the periods when the value of right ascension falls in the prohibited regions. Reachable set may be expressed as $\cup_{(\theta^{er}_{mx}, -\theta^{er}_{mn})}\{(T_d)t/H_s \subset \boldsymbol{R};\ t^d_{i-1} \leqslant t \leqslant t^d_i$. T_d is the yaw axis disturbing torque, H_s is satellite angular momentum. For sun synchronous orbit, error introduced (0.9856 °/day) is a function of mean angular velocity of sun. Let $\theta^{er} = (\theta^{er}_{t+s} - \theta^{er}_s)$ be the error introduced by the ground station relocation, factors as drift, and oscillatory orbital parameters. The curve representing drift $(d\theta^l/dt)$ is function of θ^l and is a parabola defined by nominal longitude. Let ε be the deviation determined by the dimensions of the mission longitude window, margins of the orbit restoration errors, maneuver inaccuracies and short period oscillations. Let δ_h be the discontinuous function. The Time optimal error regulation problem may be stated as to control the progression of the orbital parameters under the effect of perturbations by periodic orbit corrections in the most economic manner so that the satellite remains within the mission window ($\pm 0.1°$ in longitude and latitude) as determined by the radio communication regulations. Let t: $[t_0, s_0, s_1, \ldots)$ be the open switching time set. The dimensions of mission window are determined by the considerations of design of steerable antenna, its mobility and mounting.

Theorem 3.1 (pointing error regulator). *Time optimal impulse thrust (hybrid reignitable fuel motors) is given by*

$$\begin{aligned} &u^0_t(\eta_t) = \left(T_d - \eta_t u(t_k)\right)/H_s; \quad \eta_t = 1_{(td,t)}; \quad \delta_h = (-\varepsilon/2); \quad t^d_i \leqslant t \leqslant t^u_i; \\ &\eta_t = 0; \quad \delta_h = (+\varepsilon/2); \quad t^u_{i-1} \leqslant s_i \leqslant t \leqslant t^d_i. \end{aligned} \tag{12}$$

Here η_t is the outward normal to the support plane. Non negative switching impulse thrust optimal sequence $[t_i^d, t_i^u, \ldots)$; $i = 0, 1, \ldots$ is given by

$$\begin{aligned} L\big(t_i^d, \theta^{er}(\omega_{sl})\big) &= \mathrm{meas}_{si \leqslant s \leqslant t}\Big(|W_\theta(t_d) - \theta_{av}^{er}| - \big\{|W_\theta(s_i) - \theta_{av}^{er}| \\ &\quad -|W_\theta(t_i - t_{i-1}^u) - \theta_{av}^{er}|\big\} + f_{si}^t 1_{(si,t)}\Big(\big(T_d - \eta_t u(t_k)\big)/H^s\Big) ds \leqslant (\varepsilon + \delta_h)\Big); \\ &\quad W_\theta(s_i) = \theta_{mn}^{er}; \quad t_{i-1}^u \leqslant s_i \leqslant t \leqslant t_i^d; \end{aligned} \tag{13}$$

$$\begin{aligned} L\big(t_i^u, \theta^{er}(\omega_{sl})\big) &= \mathrm{meas}_{tdi \leqslant s \leqslant t}\Big(- |W_\theta(t_u) - \theta_{av}^{er}| - \big\{|W_\theta(t_d) - \theta_{av}^{er}| \\ &\quad -|W_\theta(t_i - t_{i-1}^u) - \theta_{av}^{er}|\big\} + f_{td}^t 1_{td,t}\Big(\big(T_d - \eta_t u(t_k)\big)/H^s\Big) ds \leqslant (\varepsilon + \delta_h)\Big); \\ &\quad t_i^d \leqslant t \leqslant t_i^u; \quad W_\theta(t_d) = \theta_{mx}^{er}; \quad W_\theta(t_u) = \theta_{mn}^{er}. \end{aligned} \tag{14}$$

Proof. Let $\{W_\theta, \Im;\ 0 \leqslant t < \infty\}$, $(\Omega, \Im)$ be the Brownian family on canonical space $\Omega = C[0, \infty)$ with finite set $\theta^{er} \in \Omega$. It is assumed that the Levys measure $L(t, \theta^{er}(\omega_{sl})) = \lim(1/\varepsilon)$ meas $\{0 \leqslant s \leqslant t;\ |W_\theta - \theta_{av}^{er}| \leqslant \varepsilon\};\ t \in [0, \infty)$ is jointly singularly continuous in (t, θ_t^{er}) on each optimal switching impulse thrust sequence, with Brownian time intervals t^u, t_i^d, and the Brownian path for satellite is modulus continuous. For every Borel set $B \in \boldsymbol{B}$ we define Brownian switching time as Lebesgue measure $f_{si}^t 1_{(si,t)}(W_\theta) ds = \mathrm{meas}\,\{0 \leqslant s \leqslant t;\ W_\theta \in \boldsymbol{B}\};\ t \in [0, \infty)$. It is also assumed that $d(L(t, \theta_t^{er}))/dt$ exists and is zero for Lebesgue almost every where in the interval. It can be verified that for P-a.e. $\{(t, \omega_{sl})[0, \infty) \times \Omega\}$, has a finite Lebesgue measure. Consider the switching time sequence representation $L(t, \theta^{er}(\omega_{sl})) = (1/2) f_s^t \delta(W_\theta(\omega_{sl}) - \theta_t^{er})\, ds$. The continuous Process $(0 \leqslant s \leqslant t;\ |W_\theta - \theta^{er_t}| \leqslant \varepsilon\};\ t \in [0, \infty))$ is sub martingale; and admits a unique Doob–Meyer decomposition.

$$|W_\theta(t_d) - \theta_{av}^{er}| = |W_\theta(s_i) - \theta_{av}^{er}| - |W_\theta(t_i - t_{i-1}^u) - \theta_{av}^{er}| + M_t(\theta_{av}^{er}) + A_t(\theta_{av}^{er}). \tag{15}$$

Here $M_t(\theta_{av}^{er})$ is a martingale, and $A_t(\theta_{av}^{er})$ is a monotonic continuous process. Tanaka formula may be employed to identify $A_t(\theta_{av}^{er}) = L(t, \theta^{er}(\omega_{sl})) = (1/2) f_s^t \delta(We(\omega_{sl}) - \theta_t^{er}) ds$ and $M_t(\theta_{av}^{er}) = f_{si}^t 1_{si,t}\big((T_d - \eta_t u(t_k)\big)/H^s)\, ds;\ t \in [0, \infty)$. The integral $E^u \big||W_\theta(t_d) - \theta_{av}^{er}| - \big\{|W_\theta(s_i) - \theta_{av}^{er}| - |W_\theta(t_i - t_{i-1}^u) - \theta_{av}^{er}|\big\} - f_{td}^t \big|1_{(td,t)}\big((T_d - \eta_t u(t_k))/H^s\big)\big|^2\, ds \leqslant (\varepsilon_{mx} + \delta_h)$ converges to a bounded set in quadratic mean. By applying Ito rule for convex $(W_\theta - \theta_t^{er})$ density with respect to Lebesgue measure for switching interval can be expressed as

$$\begin{aligned} L(t_i^d, \theta^{er}(\omega_{sl})) &= \mathrm{meas}_{si \leqslant s \leqslant t}\Big(|W_\theta(t_d) - \theta_{av}^{er}| - \big\{|W_\theta(s_i) - \theta_{av}^{er}| \\ &\quad -|W_\theta(t_i - t_{i-1}^u) - \theta_{av}^{er}|\big\} + f_{si}^t 1_{(si,t)}\big((T_d - \eta_t u(t_k))/H^s\big)\, ds \\ &\quad \leqslant (\varepsilon_{mx} + \delta_h)\Big); \quad W_\theta(s_i) = \theta_{mn}^{er}; \quad t_{i-1}^u \leqslant s_i \leqslant t \leqslant t_i^d. \end{aligned} \tag{16}$$

Equation for $L(t_i^u, \theta^{er}(\omega_{sl})$ follows by employing symmetry equivalence transformation property.

It may be verified that the cycle duration and the cost of control depends on the position of the satellite with respect to stable equilibrium point, longitudinal dimension, size of the window, and S_a/m_s ratio of the satellite, here S_a is the satellite surface area and m_s is the satellite mass.

3.3. *Hierarchical Tuned – Navigator/Locator*

Coordinated adaptive periodic control and communication operation is characterized by hierarchical structure with maneuvering targets sharing common workspace tracked by satellite and ground locator system performing in global environment. Define terrestrial object precision index measure over manifold in volume form $\Omega(V_i) = \|V_i\|$; $V_i \in \Re^n$; $\Omega^j = \|x^j_{k+1} - x^j_d\|^2_\kappa + \|u^j_k\|^2_\rho$; here ρ_{t-tk} is the shape function. It may be shown that optimal adaptive control action over subset of manifold is given by $u^{j0}(k) = -\big[\rho_{t-tk}I + F^j_{\theta d}(n_r,k)^t B^t \kappa^j B F^j_{\theta d}(n_r,k)\big]^{-1} B^t F^j_{\theta d}(n_r,k)^t \kappa^j [x^j_k - x^j_d]$. Let $\Upsilon^j_k = [x^{jt}_k u^{jt}_k]^t$, and let $E\big[\{\Upsilon^j_k - E(\Upsilon^j_k)\}\{\Upsilon^j_k - E(\Upsilon^j_k)\}'\big] = P^\Upsilon_k$ be the covariance with $P^{\Upsilon^j}_0 = P^r(n_r)$; $j > r$; $j \neq r$; $j, 1 = 1, 2, \ldots$; let C_a be the weight sequence. It may be shown that optimal state trajectory is given by $x_{k+1} = \Gamma^{jt}_k \Upsilon^j_k$, here $\Gamma^j k = \big[I\rho_{t-tk} B^t (F^j \theta_d(n_r,k))\big]^t$. Weighted least square estimate $\Gamma^{j\wedge}_k$ of plant dynamics may be expressed as

$$\Gamma^{j\wedge}_k = \Gamma^j_{k-1} + C_a P^{\Upsilon j}_{k-1} \Upsilon^j_k \{x^j_k - \Gamma^j_{k-1}{}^{\wedge t} \Upsilon^j_{k-1}\}^t [I + C_a \Upsilon^{jt}_k P^{\Upsilon j}_{k-1} \Upsilon^j_k]^{-1}, \quad (17)$$

$$P^{\Upsilon j}_k = P^{\Upsilon j}_{k-1} - C_a P^{\Upsilon j}_{k-1} \Upsilon^j_k \Upsilon^j_k{}^t P^{\Upsilon j}_{k-1} [I + C_a \Upsilon^j_k{}^t P^{\Upsilon j}_{k-1} \Upsilon^j_k]^{-1}. \quad (18)$$

4. Satellite Launch – Installation

Cost of launching and installation of satellite depends on thrust performance of launcher (e.g., bi-liquid propulsion for poplar SLV launch). For solid fuel propellant system with small combustion time, propulsion may be characterized as impulse thrust system. The trajectory of terrestrial object launched can be expressed as $\big\{(r_0 V^2_n / g_e m_p) - 1\big\} = e$, here g_e is the gravitational constant, m_p is the mass of the base planet, r_0 is the radius vector, V_n is the velocity normal to the minimum radius vector. The case $e < 1$ correspond to trajectory of a satellite. The case $e > 1$ correspond to trajectory which lead to probes. Propellant for launch is selected from desired trajectory conic section. $m_{pr} = \pi_t t_c / g_e I_{sp}$ is the mass of solid propellant consumed, here π_t is the thrust, I_{sp} is the specific impulse, t_c is the propellant ejection time. Let x^a, x^p be the satellite transfer state at apogee and perigee, F_v, F_h be the drift matrices with associated borel measure functions, $\Delta b_i(t)$ be the inclination and altitude perturbations, $\Delta v_p(s_1, s_2) = \Delta v_p \delta(t - s_1)$ be the impulse thrust introduced at perigee by reignitable fuel motor pairs, σ_t be the dispersion matrix. Let w_t be the family of perturbation and constraints introduced by longitudinal and transverse acceleration, vibrations, shocks characterized by Brownian motion. Let $g_m(s_0, s_1), g_a(s_1, , s_2), g_b(s_2, \Delta b_j(x,t))$ be the parameters characterizing launch, transfer, and positioning phase dynamics, $k_w(s_2, t, w)$ be the parameter characterizing drift orbit and satellite dynamics uncertainty, k_v, k_g be the constant coefficients, π_t be the thrust impulse normal to satellite axis requiring regulation. The set of measurements process $\zeta\colon\ x(\zeta) < h^s$ is dependent on characteristics of environment during launch.

PROPOSITION 4.1 (installation dynamics). Given bounded domain functions closure $F_k\colon D \to [0,\infty)$, $(g_a(s_1,s_2)\Delta v_p(s_1,s_2) + g_m(s_0,s_1)m_{pr} + g_b(s_2,\Delta b_j(x))\pi_t\}\colon D \to R$. Let $p(\zeta,t)$ be the process probability density for the process $x^v(\zeta,t)$. Installation dynamics state $x^v\colon\ [0,t_f]\times R^d \to R^d$ of vehicle admits following stochastic representation on $[0,t_f]\times R^d$.

$$\begin{aligned}
dx^v(\zeta,t) = &\Big\{\big(-F_v(t) - F_h(d/d\zeta,\zeta,t)\big)x^v(\zeta,t) + g_m(s_0,s_t)m_{pr}\\
&+g_a\big(\Delta v_p(s_1,s_2)\big)\pi_t + g_b(s_2,\Delta b_j)\pi_t\Big\}dt + \big(\sqrt{}k_w(s_2)\big)\pi_t\sigma_t dw_t;\\
&x^v(\zeta,t_f) = x^v_{tf};
\end{aligned} \tag{19}$$

$$\begin{aligned}
x^v(\zeta) = &f_{\zeta\in Rd}p(\zeta,t,t_j)x^v_{tf}(\zeta)d\zeta + f_t^{tf} f_{\zeta\in Rd}\Big\{g_a\big(\Delta v_p(s_1,s_2)\big)\pi_s\\
&+g_m(s_0,s_1)m_{pr} + g_b\big(s_2,\Delta b_j(x^s,s)\big)\pi_s\Big\}p(\zeta,s)d\zeta ds\\
&+f_t^{tf}(\sqrt{}k_w(s_2))\pi_t\sigma_t(x^v)dw_t;\\
\partial p(\zeta,s)/\partial s = &-\sum_{l=1}^{d}(\partial/\partial\zeta)F_h(d/d\zeta,\zeta,t)p(\zeta,s) - F_v p(\zeta,s).
\end{aligned} \tag{20}$$

Proof. It is assumed that $x^v(\zeta,t)$ system dynamics solution satisfies uniform elasticity, Bounded ness, Holder continuity conditions. If in addition the function $(\partial/\partial\zeta)F_h$ is bounded and Holder continuous, then $p(\zeta,s)$ is of the class $C^{1,2}, ((t,T]\times R^d)$, and satisfies $\partial p(\zeta,s)/\partial s = -\sum_{l=1}^{d}(\partial/\partial\zeta)F_h(d/d\zeta,\zeta,t)p(\zeta,s) - F_v p(\zeta,s)$ in forward variables (ζ,s). Let $p(\zeta,s)\colon\ R^d \to \boldsymbol{R}$ be the collection of Borel-measurable functions. For existence of a unique solution, assume that satellite launch dynamics admits two finite measures $\mu_1(d\zeta)$ and $\mu_2(d\zeta)$ on $\boldsymbol{B}(R^d)$. The identity $f_{\zeta\in Rd}p(\zeta,s)\mu_1(d\zeta) = f_{\zeta\in Rd}p(\zeta,s)\mu_2(d\zeta)$ characterizing satellite launch implies the unique ness of the solution. From polynomial growth condition $\max_{0<t\leqslant tf}|x^v(\zeta,t)| \leqslant \boldsymbol{M}(1+\|\zeta\|^{2\mu})$; $x\in\boldsymbol{R}^d$, $\boldsymbol{M}>0$, $\mu\geqslant 1$, admits following stochastic representation for $x^v(\zeta,t)$.

$$\begin{aligned}
x^v(\zeta,t) = &E^{t,\zeta}\Big[x^v(\zeta,t_f)\exp\{-f_{\zeta\in Rd}F_v(\zeta)d\zeta\} + f_t^{tf}\Big\{g_m(s_0,s_1)m_{pr}\\
&+g_a\big(\Delta v_p(s_1,s_2)\big)\pi_t + g_b(s_2,\Delta b_j)\pi_t\Big\}\exp\big\{-f_{\zeta\in Rd}F_v(\zeta)d\zeta\big\}ds\Big]\\
&+f_t^{tf}(\sqrt{}k_w(s_2))\pi_t\sigma_t(x^v)dw_t;\quad x^v(\zeta,t_f) = x^v_{tf}.
\end{aligned} \tag{21}$$

By applying Ito rule to the process $x^v(\zeta,s)\exp\{-f_{\zeta\in Rd}F_v(\zeta)d\zeta\}$; $s\in[t,t_f]$, and by application of dominated monotone convergence theorem, and Chebyshev inequality, it may be verified that Fundamental solution admits following stochastic representation

$$x^v(\zeta) = -f_{\zeta\in Rd}p(\zeta,t,t_f)x^v_{tf}d\zeta + f_t^{tf}f_{\zeta\in Rd}\Big\{g_a\big(\Delta v_p(s_1,s_2)\big)\pi_s$$

$$+g_m\big(s_0,s_1\big)m_{pr}+g_b\big(s_2,\Delta b_j(x^s,s)\big)\pi_s\Big\}p(\zeta,s)d\zeta ds$$
$$+f_t^{tf}(\surd k_w(s_2))\pi_t\sigma_t(x^v)dw_t;\quad x^v(\zeta,t_f)=x^v_{tf}. \tag{22}$$

Orbit Index. Let (r^z,θ_z), (r^y,θ_y) be coordinates of beacon marks (P_v,P_h) placed on vertical and horizontal great circles, $h^p = h^s + r_e$ be the radial distance. Satellite (geostationary SLV, Fig. 3) entry coordinates for the P_v beacon visibility horizon are given by

$$z^p=\surd\{h^{p2}-(r_e\cos\theta_e+d_y\sin\theta_e)^2\};\quad x^p=(r_e\cos\theta_e+d_y\sin\theta_e);$$
$$d_y=\surd(h^{p2}-r_e^2). \tag{23}$$

4.1. *Optimal Thruster Propulsion Control*

Let $\Delta x^v(s_1,s_2)$ be introduced impulse thrust from propellant thruster perigee and apogee fuel motors, where $x^v = (g_e/2r_e^3)x^s$ is the transfer orbit velocity. Let $\{\beta(s),\Im_s;\ 0\leqslant s\leqslant t\}$ be the measurable adapted uniformly bounded discount process. It is desired to select set of admissible control triplet $(m_p,\Delta x^v(s_1,s_2),\pi)$ to minimize concave utility function $PI(x^v) = Ef_0^t(1/2)k_\beta\exp(-f_0^s\beta(\upsilon)d\upsilon)(\Delta x^v)^tQ(\Delta x^v)ds$. Let $v(x^v,t)=(1/2)k_\beta\exp(-f_0^s\beta(\upsilon)d\upsilon)(\Delta x^v)^tS(\Delta x^v)$. Define differential operator $\boldsymbol{A}=(1/g_m(s_0,s_1))\{k_v+\sum_{i=1}^d F_h(\partial/\partial\zeta)\}$; $k_v = ((1/2)QS^{-1}+F_v)$. Using Stochastic Hamilton Jacobi Bellman equation impulse system optimal solid drop off fuel is given by $m_{pr}=\boldsymbol{A}x_p^v$; and optimal propulsion thrust for reignitable motors is given by

$$\pi_t^0=-\Big[\big(g_b(s_2,\Delta b_j(x^v,t)\big)+g_a(\Delta x^v(s_1,s_2))\big)/\big(k_w(s_2,w)\|\sigma_t\|^2\big)\Big]h_p^s. \tag{24}$$

Unified bipropellant propulsion with single set of reservoirs enable dividing of optimal propulsions into several intervals in satellite launch and installation phase.

Molnya Keplerian Orbit. To establish repetitive satellite communication link, satellite may symmetrically return to apogee in the same region. To establish permanent links, it is necessary to provide several suitably phased Molnya orbit (visible to the apogee region for orbit period $\geqslant$ 8 hrs) satellites with different link frequencies. From Keller's law of areas, mission injection velocity thrust for molnya orbit (period = 11 hrs 58 min 2sec, half sidereal day), can be expressed as

$$\Delta x_p^v(t)=(x_p^s-x_{p0}^s)\surd(g_e/2r_e^3);\quad x(t_0)=x_{t0}. \tag{25}$$

Here g_e is gravitational constant. For Molnya (high elevation angle) orbit, noise captured by earth station antenna, from ground and terrestrial objects is low. Stochastic representation of Molnya transfer scaled orbit admits following Brownian oscillation solution

$$\begin{bmatrix}dx_t^s\\ dx_t^v\end{bmatrix}=\begin{bmatrix}0 & I\\ F_a & -F_v\end{bmatrix}\begin{bmatrix}x_t^s\\ x_t^v\end{bmatrix}dt+\begin{bmatrix}0\\ g_a\big(\Delta x_p^v(s_1,s_2)\big)+g_b(s_2,\Delta b_j(x^p,x^a))\end{bmatrix}\pi_t dt$$

$$+\begin{bmatrix} 0 \\ (\sqrt{k_w})\pi_t\sigma_t \end{bmatrix} dw_t. \tag{26}$$

From Brownian oscillation dynamics it may be verified that the effect of Sun-satellite eclipses and conjunction will be minimal for Molnya orbit. Terrestrial potential at the perigee of the orbit has minimal effect on stations situated under the apogee. The precession rate for Sun synchronous orbit can be expressed as $d\theta^p/dt = -9.95(r_{lt}/(r_{lt} + r_s))^{3.5}\cos\theta^l$ (°/day), here $r_{lt} = r_e(0.99832{+}0.0016835\cos 2\theta^l)$ is the base frame radius at latitude θ^l, r_e is the base frame radius. It may be verified that as altitude is increased, inclination is increased, resulting in increased uncovered region near poles.

5. Acceleration Maneuvering

To increase the capacity and elevation angle of the satellite link, and to reduce orbital constraints an inter satellite link may be employed. A large capacity satellite launcher may be replaced by cluster of satellites serviced by mobile stations. Let A_s be the satellite cross sectional area normal to the velocity plane. The aerodynamic drag coefficient c_d is function of mass, shape, and atmospheric density. Due to high satellite velocity, aerodynamic drag is significant factor in satellite performance. Atmospheric density ζ_a is function of orbital altitude and solar activity. The aerodynamic drag may be expressed as $g_{cd} = -0.5\zeta_a c_d A_s x^{v2}/m_s$, here x^v is the velocity of the satellite, m_s is the mass of satellite. For an elliptical orbit, aerodynamic drag results in breaking at the perigee. Let k_{gd} be the acceleration coefficient, spectral density $w_t \sim \boldsymbol{N}\big(0, W(F^i_{\theta d}, x^{ac})\big)$ characterizes the propagation of covariance and nonlinearity introduced by the acceleration during thrust period. Maneuvering target acceleration $x^{ac} = (\partial^2 x^s/\partial t^2)$ measurements are not available to satellite locator system, and dynamic observer may be constructed for the purpose. Markov switching kinematics of dynamic observer in launch thrusting phase may be expressed as

$$\begin{aligned} &\partial(x^{ac})/\partial t = k_{gd}g_{cd}(x^{v2})^{ac}x^{ac} + g_m(s_0,s_1)^{ac}m_{pr};\\ &x^{ac}(t) \sim \boldsymbol{N}(0, 2(k_{gd}g_{cd}(x^{v2})^{ac}){x^{ac}_{\max}}^2(1 - p_{co} + 4p_{cmx}/3). \end{aligned} \tag{27}$$

Here p_{cmx} is the probability that object maneuvers with maximum acceleration, p_{c0} is the probability that object may not accelerate, m_{pr} is the bipropellant consumed, with $[x^{ac}]_{t=t^0} \sim \boldsymbol{N}(0, X_{a0})$. Thrusting phase switching kinematics may be expressed as

$$\begin{aligned} &x^{ac}_{k+1} = g_{cd}(x^{v2})x^{ac}_k + g_m(s_j,s_i)m_{pr};\quad g_{cd}(x^{v2}) = \exp(k_{gd}g_{cd}(t)^{ac}\Delta t);\\ &g_m(s_j,s_i) = \Big(\exp\big(k_{gd}g_{cd}(t)^{ac}\Delta t\big) - I\Big)\Big(\exp\big(k_{gd}\ g_{cd}(t)^{ac}\big)^{-1}g_m(s_0,s_1)^{ac}\Big). \end{aligned} \tag{28}$$

Let $\Upsilon_k = [x^{ac}\ m_{pr}]^t$, let c_a be the weight sequence. In Banach space worst case convex set membership idendification algorithm for terrestrial object drift process $x^{ac}_{k+1} = \Gamma^t_k\Upsilon_k$; $\Gamma_k = \big[g_{cd}(\theta(w_t))\ g_m(s_0,s_1)\big]^t$; $\theta^p \in \Theta^p$, is given by n width spectral radius information (in Kolmogorov sense) Vol S^n: $\{x_k^{ac} - E[\Gamma_{k-1}]\}^t\{x_k^{ac} - E[\Gamma_{k-1}]\}$. Technique given in Section 3.3 may be applied for identification of parameters for target acceleration dynamics.

5.1. *Visibility Model – Space Monitoring Station*

Define quotient space Φ_{xt} modulo $\Phi_{xs}(\Phi_{xt}/\Phi_{xs})$; $s = [0, \ldots, \{2\pi(h^s + r_\theta)^{1.5}\}/\sqrt{g_e})$, characterizing orbit modes as sidereal day with coset $[x(t)] \in X(s)$; here r_θ is the earth radius at latitude θ. Let $[x_e^i]$; $i = 1, \ldots, 3$, be the earth station coordinate system (Fig. 3), $[x_s^i]$ be the state of terrestrial exploration space monitoring station. Let $[x^{\sim i}]$ be inertial coordinate system translating with exploration space station. From the Kepler's law of relative movement of two bodies, Velocity components of j-th terrestrial object to exploration space monitoring station are given by

$$\begin{aligned} &v_j^1 = h_g \cos\theta^e \cos\theta^a; \quad v_j^2 = h_g \cos\theta^e \sin\theta^a; \\ &v_j^3 = h_g \sin\theta^e; \quad h_g = \sqrt{(g_e/(h^s + r_{\theta j}))}. \end{aligned} \tag{29}$$

Here $h^s = -r_p + g_e m_p r_s^2/(2g_e m_p r_s - H_e^2)$ is the satellite altitude, where H_e is the angular momentum of the planet, g_e is the geocentric gravitational constant, r_p is the planet radius, m_p is the mass of the planet, r_s is the orbit radius vector. Let c be the velocity of light. Doppler shift and interferometry may be employed for observance of angular and distance measurements of terrestrial objects.

Lemma 5.1. *Coordinates of the j-th satellite, terrestrial exploration space monitoring station are given by*

$$x_j^{\sim l} = -x_s^l + \{1 - (v_j^l)^2/c^2\}^{-1/2}(x_j^l - (v_j^l)t); \quad l = 1, 2, 3. \tag{30}$$

5.2. *Identification of Active Taps (Droplets)*

The complexity of the connectivity management grows exponentially with network nodes. Satellite transponder searches, examines and merges available opportinities to transmit command packet by employing anticipative policy. System performance improvement (Kang, 1996) can be achieved by servicing only active outlet taps in the network. For markovian switching system, active taps with given signature are identified by extracting collection of event Set $S_l = \{s_{l1}, \ldots, s_{ln}$; $l = 1, \ldots, n$ from the Set $S = \{s_{11}, \ldots, s_{1n}, s_{21}, s_{22}, \ldots, s_{n1}, \ldots, s_{nn}$, such that correlation activity index $PI_{ac}(s) = \sum_{j=1}^n \|E\Big[\{x_{ij}(k)/x_{ij}(k-1)\}^t x_{ij}(k-1)\Big] / \sum_{j=1}^n E\big[x_{ij}(k-1)^t x_{ij}(k-i)\big]\| = \|\Big[I - F_{i\theta}^j B_i K_{iu}^j + K_{ip}^j(k)\{\zeta_i^j(F_{i\theta}, k) - H_i^j(I - F_{i\theta}^j B_i K_{iu}^j)\} - F_{0\theta} B_0^j K_0^j\Big]\|$; $i = 1, \ldots, n$; is optimized. For poisson connectivity, anticipative scheduling (Fig. 1, 2) virtual image plane policy algorithm is given in the following. Proposed algorithm employs subgroup decomposition without degradation of throughput.

$$\Delta h^s = h^s \omega^s \sin\theta_e \Delta t. \tag{31}$$

6. Flexible Structure Spacecraft

Flexible space structures with large antennas, solar arrays, optical reflectors are characterized by low frequency lightly damped classic modes (INSAT_1B, IRS_1), and may require changing orientation through large angles in minimum time. Torque actuator impulse thrust can be expressed as $\Delta x_t^v = 2\delta t f_{di} l$, here $f_{di} \in F$ is thrust, l is the force arm length, δt is the impulse period. Let $D_e = 2\mathrm{diag}\,(\rho_1\omega_1^e, \ldots, \rho_n\omega_n^e)$ be the elastic modes damping, ρ_i be the damping ratios, ω_i^e be the natural frequency of i-th elastic mode, m_s be the satellite mass, φ be the euler angle vector, $\imath$ be elastic modes vector, J be the moment of inertia, Φ_r, Φ_{tr} be the rotational and transitional mode shapes. Let $x_t = (\varphi d\varphi/dt\ \imath_l(d\imath_l/dt)\ldots\imath_n(d\imath_n/dt))^t$ be the states of flexible structure spacecraft (INSAT_1B, IRS_1). Given the class of torque actuator impulse thrust controls $u_t = \left[\{0\Delta x_s^v\}_i\right]$; $i = i, \ldots, n$. Flexible structure thruster control problem may be stated as, find actuator impulse thrust $\Delta x^v(s_1, s_2)$ to minimize concave utility function (Performance Index).

$$\begin{aligned} PI(x^s) &= E f_0^t (1/2) k_\beta (\exp - f_0^s \beta(v) dv) \big\{(x^s - x_d)^t Q (x^s - x_d) \\ &\quad + (\Delta x^v)^t R (\Delta x^v)\} ds; \quad \beta = \{\beta(s),\ \Im_s;\ 0 \leqslant s \leqslant t\}. \end{aligned} \tag{32}$$

Linearized rigid body elastic motion dynamics and optimal actuator impulse thrust $\Delta x^v(s_1, s_2)$ is given by

$$\begin{aligned} &dx_t = \big\{A(x_t - x_d) + Bu_t\}dt + Gdw_t; \\ &A = \mathrm{diag}\left\{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix},\ \begin{bmatrix} 0 & 1 \\ -\Lambda_1 & -D_{e1} \end{bmatrix}_1, \ldots, \begin{bmatrix} 0 & 1 \\ -\Lambda_i & -D_{ei} \end{bmatrix}_i, \ldots\right\}; \end{aligned} \tag{33}$$

$$\begin{aligned} &B = \left[\{0J^{-1}\}^t \ldots \{0\Phi\}\ldots\right]^t; \\ &\{0J^{-1}\}^t = \left[\{0J^{-1}\}_i^t\right]; \quad \{0\Phi\} = \left[\{0\Phi\}_j\right]; \quad i = 1, 2, \ldots;\ j = 1, 2, \ldots \\ &\Lambda = \mathrm{diag}[\omega_i^2]; \quad \Delta x^v = -R^{-1} B^t S (x^s - x_d); \end{aligned} \tag{34}$$

$$(dS/dt) + \beta_t S + Q - SBR^{-1}B^t S + A^t S + SA = 0; \quad i = l, \ldots, n. \tag{35}$$

To facilitate installing of large antennas, and to install unfolding solar panels, Impulse thrust may be employed to maintain fixed orientation of terrestrial space station in reference to the base frame planet.

7. Network Dynamic Obstacles

For systems with maneuverability constraints satellite locator link may replace ground locator server. Consider switching network mapping with $\boldsymbol{L}_1, \boldsymbol{H}_\infty$ norms for extended reachable zone. Let Set $M_{n0} = \{x\colon\ \boldsymbol{L}^r_{p(t-tk)F\theta d} u(x) = 0\}$ be the collection of fields normal to the dynamical obstacle plane. Let Set $M_{d0} = \{x\colon\ \boldsymbol{L}^{r-1}_{\rho(t-tk)F\theta} u(x) \neq 0\}$ be the collection of fields tangential to dynamic obstacles plane. Let $|x(k)| \leqslant c_{x0}|x(0)| + c_x|x(k)|$ be the state constraints introduced by fields M_{do}, M_{no}. The obstacle membership Set M_{do} and Set M_{no} may include parameter uncertainties introduced under $\boldsymbol{L}_\infty$. For uncertainties with spectral radius less than unity, it can be shown that the class of structured perturbations $\Xi[\Delta F^j_{\theta d}(n_1, k)]\Xi^{-1} = [\delta_{ii}]$; $\delta_{ii} \in \boldsymbol{R}$, $\delta_{ii} > 0$; $\max_i \sup_k |\delta_i(t_k)| < \delta_{mx}$; $i = 1, \ldots, n$; Set $[\Xi_{ij}] \in \Xi$. Hierarchical L_∞ stability (Kang, 1970) problem (Fig. 1) may be formulated as structured norm optimization: $\inf_{Kg} \inf_\Xi \|I - \rho(t - t_k)\Xi^{-1}(F^j_{\theta d}(n_r, k))\{K^j_c\}^{-1}K^j_g\|_{Lp} \leqslant 1$; here $(K^j_c) = \Big[R + (F^j_{\theta d}(n_r, k))^t B^t \kappa^j B F^j_{\theta d}(n_r, k)\Big]$; $K^J_g = B^t[E^j_{\theta d}(n_r, k)^t \kappa^j]\Xi$. To prevent loss of track control, and for smooth recovery K_g, K_ψ, Ξ may be selected for stabilization and track control, from L_∞ performance robustness criterion.

7.1. *Design for Maximum Droplet Resolution*

Let m_s, m_p be the mass of terrestrial object and of base frame planet, $r_s = h^s + r_e$ be the Sun synchronous orbit sub major axis, h^a be the altitude at apogee, $r_p \cong 6378$km be the terrestrial radius for earth frame. From law of constant area swept during planetary motion resolution for service drop outputs may be obtained.

PROPOSITION 7.1. Resolution for service drop outputs in orbit is given by

$$\Big[\big(h^s\surd(g_e m_p)\big)/\big(h^a(1 + m_s/m_p)\surd r_s\big)\Big].$$

7.2. *Geosynchronous Orbit Trajectory*

Frequency reuse may be employed to increase the capacity of Multibeam satellite [IRS-1]. Let x^s_i, $i = 1, \ldots, 3$ be coordinates of terrestrial path, let $h^s \in \boldsymbol{h}^s$, $\theta^a \in \boldsymbol{\theta}^a$, $\theta^e \in \boldsymbol{\theta}^e$. The satellite visibility constraints may be expressed as $\theta^e > 0$, and segment of the orbit may be expressed as arc $(ab)^s = \textstyle\int_a^b \surd \sum_{i=1}^3 (\partial x^s_i/\partial t)^2 dt$. Results for geo_synchronous orbit (Fig. 6) global stabilization is stated in the following proposition.

PROPOSITION 7.2. By application of conformal transformation mapping and spherical coordinates arc stretching, geosynchronous orbit segment $\theta^e \geqslant \theta^{e'}$ (Fig. 5) may be expressed as

$$\begin{aligned} &\textstyle\int_a^b h^{s'} \surd\big\{(\partial\theta^{e'}/\partial t)^2 + (\cos\theta^{e'})^2(\partial\theta^{a'}/\partial t)^2\big\}dt \\ &\quad \geqslant \textstyle\int_a^b h^{s'}\big\{(\partial\theta^e/\partial t) + (\cos\theta^e)(\partial\theta^a/\partial t)\big\}dt. \end{aligned} \tag{36}$$

8. Air to Air Missile System

Design of switching trajectory for non minimum phase planer robust autopilot Preferred Orientation Control AAFM system (Cloutier, 1989) is considered in the following. Body

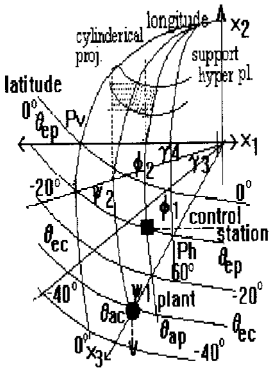

Fig. 5. Spherical trignometry rep.

forces are assumed negligible. Effect of environmental factors as uncertainties in latitude, elevation, and launcher system dynamics is investigated.

Autopilot Design. Define $x_t = [\phi, \psi]^t$; and $x(0) = [x_{10}, x_{20}]^t$, here ψ, ϕ be the angle of attack and sideslip. Let $c_{n\phi}, c_{nup}, c_{uq}, c_{ur}, c_{np}, c_{\psi}, c_r$, be the normal and side aerodynamic force constants, p, q, r be the pitch, yaw, roll rates, ϕ_r, ϕ_p, ϕ_q be the roll pitch yaw angles u_r, u_p, u_q be the roll pitch and yaw control deflection, $\Gamma(g_e, v_m, m_m, a_m, w_p)$ be the coefficient for dynamic pressure, here g_e is the gravitational constant, v_m is the missile speed, m_m is the mass of missile, a_m is the area, w_p is the aerodynamic pressure. Let p_s be the roll rate about stability axis. The problem of design of autopilot may be formulated as minimization of $(\boldsymbol{L}_2)[0, t_f]$ energy performance index to track angle of attack ϕ and bank angle γ in $BV[0, t_f](\boldsymbol{L}_\infty)$ Frechet differentiable Tangent mapping space. For diffeomorphism induced by metric tensor in local neighborhood, Tangent bundle of x can be expressed as $T_{(Fp-KuBp)}x \rightarrow \underset{xi \in X}{\cup} T_t x_i$. Consider the admissible class of variable structure autopilot controls u_t: $[u_p\ u_r\ u_q]^t = -K_u x_t$; with constraint norm $\|u_i\| = \max_j \sup_t |u_i^j(t)| < u_m$; $i = i, \ldots, 3$.

9. Example (Performance Robust Autopilot Design)

Let ∇F: $T_a F \rightarrow F$, $\|\delta F\| < \delta_{am}$, where δ_{am} is uncertainty envelope. $\|\Delta y_1(t)\|_\infty < \delta_{ym}$ be the class of perturbation bounds significant at low altitudes, including introduced perturbations in solar radiation pressure, terrestrial attraction components, air density, aero dynamics coefficients normal to surface plane. $y(t) = \left[y_1(t)\ \Delta y_1(t)\ y_2(t)\right]^t$; state of maneuvering target at command and control center is given by compounded coordinate mapping transform $y_t = \zeta x_t$. From Rigid body motion AAFM system mapped dynamics $X \rightarrow Y$ in space Y can be expressed as

$$T_{(Fp-KuBp)}y \rightarrow T_t y = \underset{yi \in y}{\cup} T_t y_i;$$
$$\partial y_t/\partial t = F^p(y,t)y_t + B^p(y,t)u_t + \zeta\Upsilon;\ \Upsilon = [\Upsilon_\phi, \Upsilon\psi]^t;\ y(t_0) = [y_{10}\ 0\ y_{30}]^t; \quad (37)$$

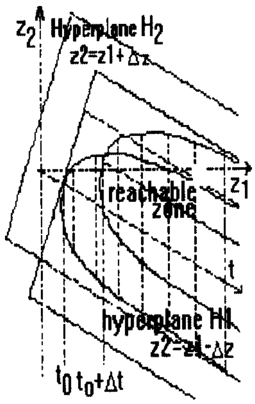

Fig. 6. AAFM syst performance.

$$\begin{aligned}\Upsilon_\phi &= p - p_s \tan\psi + (\Gamma\cos\phi/\cos\psi)c_{np}p \\ &\quad + (\cos\phi\cos\varphi_r\cos\varphi_p + \sin\phi\sin\psi_r)(g_e/v_m\cos\psi); \qquad (38)\\ \Upsilon_\psi &= r\sin\phi - q\cos\phi + (c_r\, r)\Gamma\cos\psi + c_{uq}q\Gamma\cos\psi \\ &\quad + (\sin\varphi_r\cos\varphi_p\cos\psi)(g_e/v_m); \qquad (39)\\ F^p(y) &= \begin{bmatrix} C_{n\phi}(\Gamma\cos\phi/\cos\psi) & c_{n\phi}(\Gamma\cos\phi/\cos\psi) & 0 \\ (\Gamma\cos\phi/\cos\psi)\delta c_{n\phi} & c_{n\phi}(\Gamma\cos\phi/\cos\psi) & 0 \\ 0 & 0 & \Gamma\cos\phi c\psi) \end{bmatrix}; \\ B^p(x) &= \begin{bmatrix} 0 & \zeta c_{nup}(\Gamma\cos\phi/\cos\psi) & 0 \\ 0 & 0 & 0 \\ 0 & \zeta\Gamma\cos\psi c_{ur} & \zeta\Gamma\cos\psi c_{uq} \end{bmatrix}. \qquad (40)\end{aligned}$$

Using transformation $z_t = e^{-Fpt}y_t$; system dynamics can be expressed as $dz_t/dt = e^{-Fpt}\big[B^p(z,t)u_t + \zeta\Upsilon\big]$; $\|\Delta z_1(t)\| < \delta_{zm}$, here $z(t) = \big[z_1(t)\ \Delta z_1(t)\ z_2(t)\big]^t$. The resulting reachable zone is enclosed by hyper planes $H_1 - H_2$ (Fig. 5). Define sliding surfaces $S = (s_\phi^a\ s_d^b)^t$ as

$$S = \Big[c_\phi\big\{(\phi-\phi^d)+(\partial\phi/\partial t-\partial\phi^d/\partial t)\big\},\quad c_d\big\{(\psi-\psi^d)+(r-r^d)\big\}\Big]^t. \qquad (41)$$

Here ϕ^d is adaptive tracking control nonlinear reference trajectory, c_ϕ, c_d, are tracking coefficients. From $\boldsymbol{L}_\infty$ stability design class of inner loop robust controller employing switching trajectory (Fig. 5) to track bank angle and angle of attack in extended fast dynamics (inner loop) and slow dynamics (outer loop) transition zone is given by

$$\begin{aligned} u_t^i &= -sgn\big\{B^p(z,t)^{-1}\zeta\Upsilon\big\}; \\ u_t^i &= -sgnB^p(z,t)^{-1}\big\{F^p(z,t)\exp\big[F^p(z,t)\big]z_t + \zeta\Upsilon\big\}. \end{aligned} \qquad (42)$$

10. Delivery System – Hybrid Reliability

Let $[x_i^j;\ 0 \leqslant t \leqslant \infty$ be the collection of random target locations in cemetery space $P(x_i^j \in dx_i^j) = \prod_{i=1}^n \gamma_i \exp(-\gamma_i x_{tf}^j) dx_{tf}^j$; with γ_i positive parameters. Define limited life time brownian process $(v^{mj} = \Delta)$; $t_{av} \leqslant t < \infty$, here v^{mj} is missile approach velocity, $p_\Delta = \big(1 - exp(-t_{fmx}/t_{f^0})\big)$ is the probability of missile death during maximum mission flight time t_{fmx}, t_{f^0}, is the mean time of first flight mission failure. Let t_d be the time of successful flight to the assigned target.

10.1. *Reliability – Delivery System With Backup Missiles*

Consider subsystem consisting of principle missile assigned to the target, and backup missiles with switching elements. Let the failure rate of principle missile subsystem be Λ_p, the failure rate of backup missile subsystem be Λ_b, and the failure rate of missile switching subsystem be Λ_s. By applying binomial rule it may be verified that extended missile flight time to preidentified target to land on a given target is given by

$$t_b = (1/\Lambda_p) + \Lambda_b/(\Lambda_b + \Lambda_s)^2. \tag{43}$$

10.2. *Confidence Level of Successful Missile Landing*

Limited time Brownian process is modified by introducing dynamic redundancy with backup missile switching. $\aleph^2$ distribution is employed for estimating confidence limit for n number of missile launches required for successful missile landing on preidentified target. Let α be the confidence level. Define a new process in cemetery space $v^{mj} = v^{mj}$; $0 \leqslant t < t_{av}$; $v^{mj} = \Delta$; $t_{av} \leqslant t < \infty$; $t_{av} = \inf\{t \geqslant 0;\ \Delta \geqslant v^{mj}\}$. The upper and lower confidence limits are given by

$$\theta_u{=}2t_d/\aleph^2_{1-\alpha}\Big\{t_d/p_1(1-p_i)p_\Delta t_{f^0}\Big(1+(n-1)/(1+\Lambda_s\, p_1\big(1-p_i)p_\Delta t_{f^0}\big)^2\Big)\Big\}; \tag{44}$$

$$\theta_1{=}2t_d/\aleph^2_{\alpha}\Big\{t_d/p_1(1-p_i)p_\Delta t_{f^0}\Big(1+(n-1)/\big(1+\Lambda_s\, p_1(1-p_i)p_\Delta t_{f^0}\big)^2\Big)\Big\}. \tag{45}$$

Here p_1, is the probability of successful missile launch, p_i is the probability of an intercept. The total number of missile launchers n tracking target is given by

$$n = t_d/p_1(1-p_i)p_\Delta t_{f^0}. \tag{46}$$

11. Conclusions

Positioning filter dynamics for terrestrial objects are derived from relativity model. Impulse controller for gyroscopic stabilization with time optimal regulator employing hybrid reignitable motors is obtained. Observation process is in general dependent on characteristics of environment during launch, installation and correction phases of a satellite. Optimal adaptive control for hybrid Hierarchical tuned navigator locator is obtained.

Brownian oscillation solution to Molnya transfer orbit is obtained. Model for Flexible Structure spacecraft, and visibility model for terrestrial space exploration monitoring station are given. It is shown that optimal impulse thrust for flexible structure space craft is discounted by discount factor. Design of tracking trajectory for system with dynamic obstacles is given. Proposed technique is applied for acquisition of synchronisation information for terrestrial inertial targets. Sat function may be introduced to regulate chattering. System performance may be improved (Kang, 1996) by servicing only active target taps in the satellit link. To increase system throughput design procedure for maximum Droplet resolution is given. Class of robust controller switching trajectory with fast and slow dynamics to track bank angle and angle of attack of AAFM system in extended transition zone is obtained. It is verified that infinite number of linear varieties (strategies) exists. Procedure to obtain required number of missile launches for successful missile landing on pre identified target is given.The upper and lower confidence limit of a successful missile landing on a identified target is determined.

Acknowledgment. Part of the contents of this paper were presented at IF AC (CAO, 2000).

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Kosminių pajėgų lanksčių struktūrų modeliavimas ir valdymas

Harcharan S. KANG

Straipsnyje pateikiamas kosminių pajėgų lanksčių struktūrų modelis, kuomet vykdomas palydovo paleidimas, instaliavimas stacionarioje orbitoje bei jo fazinė korekcija. Duodami sistemos skridimo trajektorijos, esant dinaminėms kliūtims, konstrukcija ir raketų, sėkmingai pataikančių į numatytus objektus, paleidimų skaičiaus ir jo patiklumo rėžių pagrindimas. Analizuojamas navigacinės sistemos patikimumas.