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Insurance Models for Joint Life and Last Survivor Benefits

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Abstract. Three kinds of the insurance policies for the net premium calculation for married couples are considered. The net premium equation principle is used in all premium calculations. The particular quality of the additional pension assurance is the individual form of its undertaking and the limitation of annual (monthly) pension payments. Due to this fact the biggest interest to the individual insurance show people after age 40–45 years, when the insurance premium rates are so high that most people can't buy such policies. So, the discussed form of the joint life insurance could be proposed to the participant of the pension plan when he or she is reached the pension age and wants to buy the life insurance policy for the accumulated capital of pension.

Key words: joint life assurance, pension benefits, surrender value, last survival assurance.

1. Introduction

Nowadays the family insurance contracts have a wide application when, as against individual insurance, the contract covers minimum two persons. In theory it is quite possible to have more lives assured if insurable interest exists. These types of joint life contracts are either based on the first death or the second death. A joint life first death policy pays out on the death of the first of the two lives assured. First death term assurance and family income policies are used for family protection properties, and first death endowments are commonly used in connection with house purchase arrangements. A joint life second death policy, sometimes referred to as a joint life last survivor policy, is often used to plan for provision against inheritance tax and sometimes for investment. Where retirement provision is required, joint life and last survivor annuities have been developed to ensure that the annuity payments continue to the surviving partner after the death of their spouse. These annuities can be in advance or arrears, with or without proportion and with or without guarantee, as for single life annuities.

The formation of the proportions between premiums and benefits in the life assurance contracts of two persons is based on the general actuarial theory. The specificity here consists that the value of the net premium depends on probability of certain events, relating to cumulative life of two persons. The symbol ${}_np_{xy}$ designates probability that two persons, one of age – x, and other – of y, will live simultaneously – n years, i.e., that their cumulative life will not be disturbed during n years of a moment of achievement of age

x and y correspondently. The considered event will come, if there will be two events: the person in the age of x will live up to age $x+n$, and person in the age y – up to age $y+n$. Probabilities of each of these events are ${}_np_x$ and ${}_np_y$ correspondently. These probabilities are independent, so ${}_np_{xy} = {}_np_x \cdot {}_np_y$.

This paper explores the value of joint life annuities for married couples. It describes the existing market for joint life annuities, and summarises the range of annuity products that are currently available to couples. It then considers the value that married couples would place on access to an actuarially fair annuity market. This calculation differs from the analogous one for a single individual for two reasons. First, joint-and-survivor life tables differ from individual life tables. The life expectancy of the second-to-die in a married couple is substantially greater than that for a single individual. Second, joint life annuities provide time-varying payouts, because survivor benefit options permit the payout when both members of a couple are alive to differ from that when one member has died. The paper develops a new annuity valuation model and applies it to evaluate a married couple's utility gain from annuitization. The findings suggest that previous estimates of the utility gain from annuitization, which applied to individuals, overstate the benefits of annuitization for married couples. Since most potential annuity buyers are married, these findings may help to explain the limited size of the private market for single premium annuities.

While the treatment of married couples is an important issue in Social Security program design, virtually all of the previous research on annuities has focused on individuals rather than couples as decision-making units. This paper presents new evidence on the structure of joint-life annuity products that insurers currently offer, and it evaluates these annuity contracts from the standpoint of couples.

In paper we explore two issues related to married couples' demand for private annuities. First, we describe the range of joint life annuity products that is currently available. We show that existing annuity products are much more complex than textbook level-premium, single-life annuities, and that these products provide married couples with resource allocation options that cannot be achieved with single-life annuity products alone. Second, we extend our previous work (Matvejevs and Matvejevs, 2000) on the amount those individuals, or in our case married couples, which would be prepared to pay to obtain access to an actuarially fair annuity market.

2. Joint Life and Last Survivor Assurance

The insurance contract consists with a married couple – man of age x and woman of age y. According to the contract the participants of the insurance contract are obliged to pay the annual premium at a rate of P Ls during n years while both are alive.

If after the expiration period of n years both participants of the contract are alive, i.e., have reached the age of $x+n$ and $y+n$ accordingly, the lump sum Q Ls is paid and therefore the insurance contract is finished.

If one of the participants has died before the termination period in n years (period of payments), the insurance company undertakes to pay the life rent at a rate of R Ls to other spouse annually. There are two possibilities to start payment of benefits:

– at the end of year of death of the spouse (or in the beginning of the next year);
– upon termination of the period of payments, i.e., after the spouse is reached by the age of $x + n$ years (or, accordingly, $y + n$ years).

If the death of the second spouse is occurred, the action of the insurance contract stops also and any additional payments are not made. If both participants of the insurance contract have died within one year before termination of the expiration period, then depending on conditions of the contract:

– the action of the contract is finished and any additional payments are not made;
– the premiums had paid without percents are paid to the legatee of the participants and after that the insurance contract is finished;
– the lump sum will be return to the legatee by the insurance company in case of death of the insured person before the benefit period.

The insured person has the right to terminate the contract at any time before to the beginning of the benefit period. In this case the surrender value will be return to the insured person by the insurance company.

3. The Formation of the Mutually Advantageous Tariffs in the Life Assurance Contracts of Two Persons

There are much more various situations in the life assurance contracts of group of the persons, than in individual insurance. It is displayed in a variety of possible demographic condition for group of the persons.

Let group G, consisting from m persons of various age, is available $x_1, x_2, \dots x_m$, where x_i – the age of i person. We shall consider, that each person in group has individual number, so a vector $< x_1, x_2, \dots, x_m >$ completely describes age structure of group.

After a few years some members of group can die and the number of group will decrease. Beforehand usually it is impossible to tell, who will die in the given group. In this situation it is possible only to speak about probability of distribution of death. Let δ_i is the indicator of the demographic status, which in any moment of time accepts meaning 1 ($\delta_i = 1$), if the i-th member of group is still alive, and meaning 0 ($\delta_i = 0$), if he is not alive. Distribution of death, described by the binary cortege $< x_1, x_2, \dots, x_m >$ completely defines a demographic condition all group in the given moment of time. It is natural, that in an initial moment of time all persons of group are alive and its initial condition is described by a vector $\delta_{0i} =< 1, 1, \dots 1 >$. Below us will interest probabilities of various demographic condition of group. At calculation of these probabilities we shall consider carried out a condition of independence, meaning, that the demographic events for the various persons in group are in pairs independent. From this condition follows, that probability for all persons in group to live n years, designated by ${}_np_{x_1 x_2,\dots,x_m}$, is equal to product of survival probabilities of each person separately, i.e.,

$${}_np_{x_1 x_2 \dots x_m} = {}_np_{x_1} \cdot {}_np_{x_2} \cdot \dots \cdot {}_np_{x_m}.$$

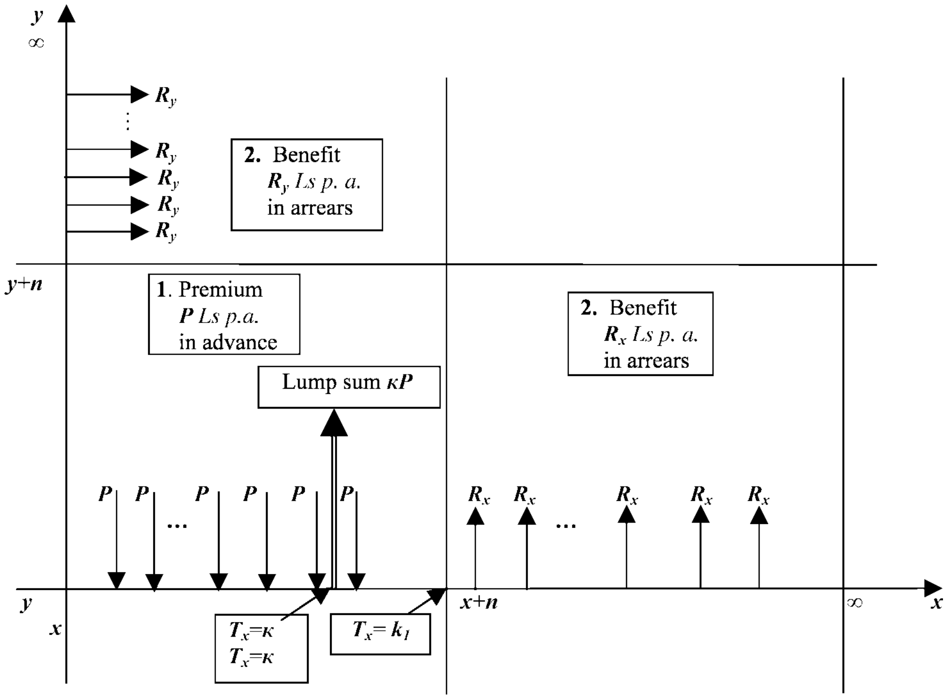
As the usually individual rent, paid at the end of each year provided that all members of group are still alive, and, designating its current cost by a symbol $a_{x_1x_2\dots x_m:\overline{n}|}$, it is possible to write down

$$a_{x_1x_2\dots x_m:\overline{n}|} = \sum_{t=1}^{\omega} v^t \cdot {}_tp_{x_1x_2\dots x_m}.$$

Completely similarly, considering the life insurance of group for sum assured $S = 1$, paid at the end of a year, in which there will be the first death of which or person from group, it is possible to receive actuarial cost of such contract, i.e.,

$$A^1_{x_1x_2\dots x_m:\overline{n}|} = \sum_{t=1}^{\omega} v^{t+1} \cdot \left({}_tp_{x_1x_2\dots x_m} - {}_{t+1}p_{x_1x_2\dots x_m}\right).$$

With respect to an insurance policy we define the total loss L to the insurer to be the difference between the present value of the benefits and the present value of the premium



R_x – man annual benefit, if woman is died; $R_x = 1$ Ls;
R_y – woman annual benefit, if man is died; $R_y = 1$ Ls;
x – man's age;
y – woman's age;
n – the term of the contract.

Fig. 1. The financial obligations of joint-life annuity contract.

payments. This loss must be considered in the algebraic sense: an acceptable choice of the premiums must result in a range of the random variable L that includes negative as well as positive values.

A premium is called a net premium if it satisfies the equivalence principle

$$E(L) = 0, \tag{1}$$

i.e., if the expected value of the loss is zero.

The fundamental rule to be followed in calculation of net premium in insurance practice is to specify the insurer's loss L, and then to apply the equivalence principle. Thus, the premium P and the benefits R in formulated above model can be calculated from the condition of equality of the financial obligations between the insurer and the insured person on a moment of the contract.

Expected present value of future premiums:

$$P \cdot \left(1 + v \cdot p_{xy} + v^2 \cdot {}_2p_{xy} + \cdots + v^{n-1} \cdot {}_{n-1}p_{xy}\right) = P \cdot \ddot{a}_{xy:\overline{n}|}. \tag{2}$$

Component $P \cdot \ddot{a}_{xy:\overline{n}|}$ means the payments of the annual premium P in advance during n of years with the annual interest rate i.

${}_tq_x$ – probability, that (x) died during t years;

${}_{t|}q_x$ – probability, that (x) survive t years and died during the next year;

$\ddot{a}_{xy:\overline{n}|}$ – n-year joint life annuity, if joint life status is not change.

Expected present value of future benefits:

$$\begin{aligned} Q \cdot v^n \cdot {}_np_{xy} &+ R_x \sum_{m=0}^{n-1} \sum_{k=n}^{\infty} v^k \cdot {}_kp_x \cdot {}_{m|}q_y \\ &+ R_y \sum_{k=0}^{n-1} \sum_{m=n}^{\infty} v^m \cdot {}_mp_y \cdot {}_{k|}q_x - P \cdot (IA)^1_{xy:\overline{n}|} \\ &= Q \cdot A_{xy:\frac{1}{\overline{n}|}} + R_x \cdot {}_{n|}\ddot{a}_x \cdot {}_nq_y + R_y \cdot {}_{n|}\ddot{a}_y \cdot {}_nq_x - P \cdot (IA)^1_{xy:\overline{n}|}. \end{aligned} \tag{3}$$

$A_{xy:\frac{1}{\overline{n}|}}$ – n-year pure endowment joint life assurance contracts;

${}_{n|}\ddot{a}_x$ – n-year deferred annuity;

$(IA)^1_{xy:\overline{n}|}$ – n-year increase joint life assurance contract.

Net premium equation:

$$P = \frac{Q \cdot A_{xy:\overline{n}|}^{\;\;1} + R_x \cdot {}_{n|}\ddot{a}_x \cdot {}_nq_y + R_y \cdot {}_{n|}\ddot{a}_y \cdot {}_nq_x}{\ddot{a}_{xy:\overline{n}|} - (IA)^1_{xy:\overline{n}|}}. \tag{4}$$

4. Calculation Example

Today computers are used in all branches of science and also in insurance. Therefore, to use developed fares in insurance companies as a new kind of insurance all calculations

Table 1

The constants are used in the Helligmann-Pollard formula

Constants	Men	Women
A	0.00194	0.00115
B	0.05093	0.03310
C	0.14249	0.12811
D	0.00607	0.00029
E	1.61992	23.44606
F	57.83349	21.11713
G	0.00005	0.00006
H	1.10715	1.09116

should be automated. Due to simplicity of usage and visualization of outcomes Microsoft Excel is one of the most popular programs for compilation of the financial reports. Thus, the database for joint life and last survival benefits calculations was generated in Microsoft Excel 97 environment.

The Helligmann-Pollard formula (5) for calculation the rates of mortality is used with the dates obtained by the Latvian Central Statistical Bureau.

$$\frac{q_x}{p_x} = A^{(x+B)^C} + D * \exp\big\{ - E * (\ln x - \ln F)^2 \big\} + G * H^x, \tag{5}$$

where, x – the age of the individual;

q_x – a probability to die within one year for age x;

p_x – a probability to survive within one year for age x;

A, B, C, D, E, F, G, H – constants.

The Table 1 presents the constants, which are used in the Helligmann-Pollard formula.

The sum up to 80 is used instead of infinity in all the previous formulas where it necessary. Interest rate is equal to 5% in the monitoring example.

The table of fares is calculated for each kind of the insurance contract, using that:

The lump sum, when both insured persons are still alive after n years, Q	1 Ls
The man's rent in case when the woman is dead, R_x	1 Ls
The woman's rent in case when the man is dead, R_y	1 Ls

For visualization all the tables are calculated for 10 insured couples in the monitoring example:

All calculations are collected in 8 spreadsheets:

1) Fares – joint life and last survival benefits calculations. Net premiums are calculated according with 3 kinds of the insurance contracts.
2) Premiums – current values of the future premium payments.
3) Payments - current values of the future premium discounts.
4) Men – the rates of mortality for men.

5) Women – the rates of mortality for women.
6) Joint Probability - joint probabilities for man and woman.
7) Men: Mortality Table – the men's mortality table.
8) Women: Mortality Table – the women's mortality table.

5. Conclusions

The conditions of the joint life insurance agreement can be different. There are 3 kinds of the joint life insurance contracts in the article. Term of insurance for all of them is n years. There is similar type of the annual insurance premium in each kind of the contracts: premiums occur in the beginning of each year during whole contribution period and interrupts, if one of the marital partners will die during this period.

Due to the conditions of the first version of the joint life insurance contract the insurance benefits start at the end of year of death, if one of the insured marital partners will die during the contribution period. All three versions imply insurance benefits Q Ls in a case pure endowment.

The first version of the insurance agreement envisions payment of the rent to the surviving spouse (Rx Ls annually – in case of the woman death, Ry Ls annually – in case of the man death) at the end of year of death of the first marital partner. On conditions of the second and third versions of the insurance agreements the payment of the rent is made through n of years from the moment of a conclusion of a contract (at once after termination of the contribution period). As against the first and second version the third version of the agreement envisions return of the premiums without percents in case both insured marital partners will die within one year for n of years from the moment of a conclusion of a contract.

Values of the insurance net – premiums are designed in environment MS Excel 97, guessing values Q, Rx and Ry equal 1 Ls. In addition to this the Helligmann-Pollard formula is applied for the calculation of the mortality.

Now we shall discuss alone outcomes and conclusions made ground of numerical calculations.

Table 2

The person's age at the moment when the insurance contract is subscribed

Man age	Woman age
50	45
51	46
52	47
53	48
54	49
55	50
56	51
57	52
58	53
59	54

At first – about apparent outcomes. In each age-grade the value of the annual net – premiums decreases, if term of insurance n is augmented, and is augmented, if the age of the insured persons on the moment of a conclusion of the contract increases. The relation of value of the premium to term of insurance and age of the insured persons is illustrated in the reduced diagrams (see Fig. 2. and Fig. 3.). The reduction of the annual premiums is explained due the principle of equivalence, the insurance cost, i.e., the financial liabilities of the insured person before the insurer, are divided on n of equal payments, and, therefore, than more n, the less value of the annual premium.

The increase of the annual premiums is explained to that at increase of age of the life its probability is augmented to die within following year (see Fig. 3 and Table 3.). Therefore, as the premiums are paid only while both insured are living, the obtained premiums cannot "suffice" for fulfilment of the insurance obligations, therefore value of the contributions augment at increase of age of the person entering the insurance agreement.

Let's compare the considered versions of the insurance contracts (see Fig. 3). From the diagram it is visible that the second kind of the insurance contract is more attractive for

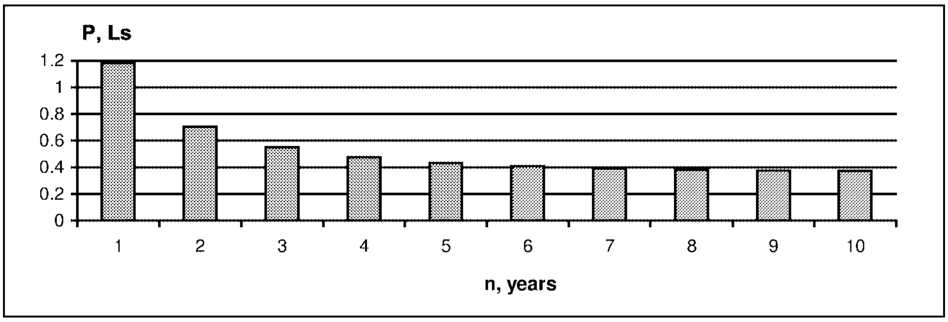


Fig. 2. Annual net premium as function of the term of insurance; age of spouse: 45 (woman) and 50 (man); first version of the contract.

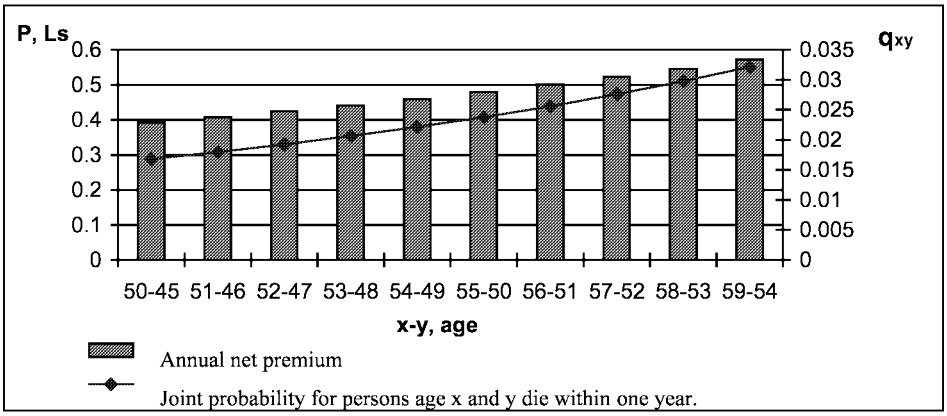


Fig. 3. Annual net premium as function of the age for married couples (at the day of the signing of the agreement): x (man) and y (woman); term of the insurance is 7 years; first version of the contract.

potential clients if to compare the agreements for annual values of the net – premiums. So happens because in matching with the first version of the agreement, the rent payments of the second version starts after n of years, beginning from the start moment of the agreement, instead of at the end of year of death. There is a case of return of the premiums in the third version, if both insured will die within one year.

Analysing the second and third version of the agreement difficultly uniquely to say, why on the third version of value of the net premiums it is more in matching with the second version, however augmenting a valid time of the agreement from some moment of the premium in the third version become less. This fact, and also that fact, that even if the married couple is insured for the term of less, than n of years, results in increase of the net – premium, can is explained to that, at first, life times of the men and women are considered independent, and, the woman life time in the given age is more than a life time the man in the same age (see Table 4.).

The specific weight of the net premium greatest in case of the pure endowment and in case of the rent insurance, which one is received by the woman after the man death. Still it is necessary to say, that at increase of term of insurance n of years, the specific weight of the net premium in case pure endowment decreases, and in case of the rent insurance, to the contrary, is augmented (see Fig. 4.).

This fact is due to with the increase of age the probability to die during one year is also increases. As contrasted to the insurance rent of the surviving spouse, the insurance

Table 3

A probability for (x) or (y) to die within one year

Man age	q_x	Woman age	q_y
50	0.01381	45	0.00305
51	0.01470	46	0.00332
52	0.01567	47	0.00362
53	0.01674	48	0.00395
54	0.01791	49	0.00431
55	0.01919	50	0.00470
56	0.02059	51	0.00512
57	0.02213	52	0.00558
58	0.02382	53	0.00609
59	0.02569	54	0.00664

Table 4

The net premium is increasing for the first version

Insured person's age		Annual net premium as function of the term of insurance	
Men	Women	9 years	10 years
58	53	0.53747	0.53760
59	54	0.56366	0.56424

of the pure endowment, considered in this article is more expensive. It is visible from the diagram (see Fig. 5.).

However the pure endowment makes the agreement is more attractive, as in this case the insured marital partners will receive even part from the paid premiums, and the value of the premiums for the pure endowment decreases at increase of term of insurance.

Considering all above said it is possible to draw a conclusion, that for the age-grades, indicated in activity, the considered kinds of insurance are effective when term of insurance more than 5 years. If the contribution period is longer, then the value of the annual net premiums is smaller, which one increases also with increase of age of the individual.

The main lack of the personal additional pension insurance consists of individuality of the form of its implementation and limitation of annual payments. The greatest insurable interest in case of individual insurance is watched after 45–50 years, when the volumes of payments of the insurance premiums become almost unavailable to the majority of the insured persons. This kind of insurance supplements pension of the inhabitants to the full then, when the age of the insured employees will be till 30 years, i.e., right at

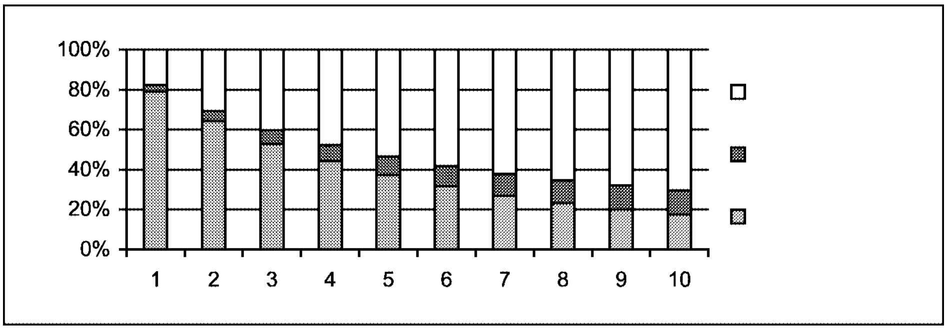


Fig. 4. The specific weight of the annual net premium as function of the term of insurance; age of spouse: 45 (woman) and 50 (man).

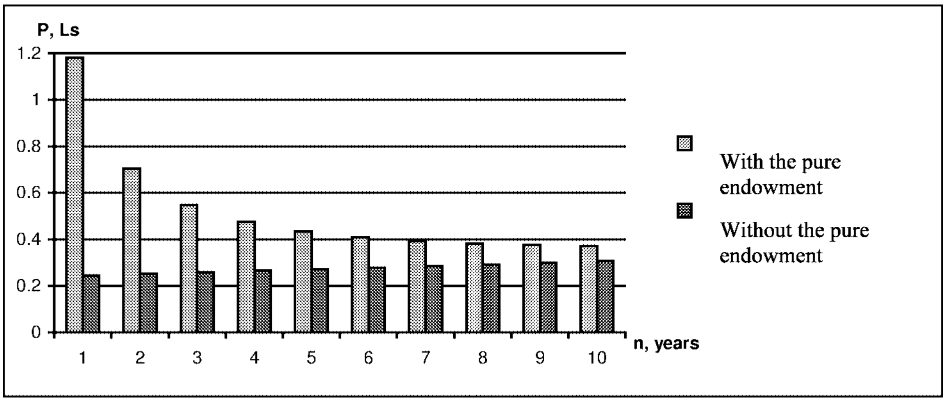


Fig. 5. The value of the annual net premium as function of the term of insurance with the pure endowment assurance and without it; first version of the contract.

the beginning opencasts. It can be achieved only at collective insurance, when the chiefs conclude the insurance agreements on the employees and the insurance premiums are paid from funds of an enterprise. Then also it is possible to increase volumes of the future pensions.

There is one way how to apply the joint life and last survivor benefits considered in our article. It is a case, when the participant, who has achieved a pension age, according to the pension plan should gain the life insurance policy using accumulated additional pension capital conformity with the law "The private pension funds" in Latvia.

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Dviejų asmenų bendrojo draudimo modeliai

Andrejs MATVEJEVS, Aleksandrs MATVEJEVS

Nagrinėjami trys dviejų asmenų draudimo modeliai – vyro ir moters, apskaičiuojant grynąsias premijas pagal kiekvieną draudimo polisą. Visų premijų apskaičiavimas remiasi grynosios premijos lygties principu. Atskiros papildomos priemonės pensijai užtikrinti yra individualios metinių (mėnesinių) pensijinių įmokų pradžios ir jų dydžio ribojimo formos. Ryšium su tuo labiausiai individualiu draudimu domisi 40–45 metų asmenys, kai draudimo įmokos yra pernelyg aukštos, kad dauguma žmonių galėtų tokius polisus nusipirkti. Tad aptariama bendro gyvenimo draudimo forma gali būti pasiūlyta pensijų plano dalyviams, kai ji arba jis sulaukia pensijinio amžiaus ir ketina pirkti gyvenimo draudimo polisą sukauptam pensijos kapitalui.