Hypothesis of Normality in the Context of the Market Model

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Abstract. This paper discusses the normality assumption of the market model errors, conventionally accepted. Some other possible specifications are proposed and their performance is testing using a test statistic based on the empirical distribution function of the residuals of the model and assuming that the null distribution can depend on some unknown parameters. The parametric bootstrap method is used. Empirical evidence is provided using a sample of thirty companies of the Spanish Stock Market.

Key words: market model, test statistics based on the empirical distribution function, parametric bootstrap.

1. Introduction

In the financial literature, the available approaches to calculate the normal return of a given security are grouped into two categories: statistic and economic. In this paper, we are interested on the first category, which are the models that follow from statistical assumptions concerning the behaviour of asset returns and do not depend on any economic arguments. In particular, we focus on the market model, which relates the return of any given security to the return of the market portfolio.

For the statistical models, it is conventional to assume that asset returns are jointly multivariate normal and identically distributed through time. The main objective of this paper is to test the hypothesis of normality of the market model errors and, if it can not be accepted, other possible specifications will be tested. To realize these goodness of fit tests we propose to use a well-known test statistic based on the empirical distribution function, the Crámer von Mises test statistic (CVM). We use this type of test instead of the Log-Likelihood criterion or the graphical analysis since the CVM test allows determining which functional form fits the data better than any alternative distribution. Moreover, we assume that the null distribution is a completely known continuous distribution function except by a vector $\theta \in \mathbb{R}^s$ of unknown parameters, that must be estimated. Since the error terms are not observable variables, it is necessary to study their behaviour using the market model residuals. We focus on the following possible specifications: (I) the Student's t, (II) the Logistic and (III) a mixture of two normals.

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The tabulated critical values used for the Crámer-von Mises test (CVM) are valid only when testing whether a set of independent observable random variables are from a completely specified continuous distribution (Shorack and Wellner, 1986). However, these asymptotic critical values are not longer valid when the null distribution depends on some unknown parameters and when the random variables are unobservable. The reason is that, in these cases, the limit distribution of the CVM test statistic is different from the standard Brownian motion process and it has been shown that it depends on the postulated null distribution (e.g., Sukhatme, 1972 and Loynes, 1980). A comprehensive analysis of the basic theory about the goodness of fit test when parameters are estimated has been given by Durbin (1973).

In this paper, we are in the case in which the set of observations are unobservable random variables (market model errors) and we assume that the null distribution can depend on a k-dimensional vector of unknown parameters (incomplete specification). We consider a context of incomplete specification since the hypothesis that the null distribution of the errors is completely known is very restrictive because, for example, this implies that the variance of the errors can not be unknown. Asymptotic results are already given for the problem of testing normality of the errors assuming (1) the regression model is a linear regression model, (2) the errors are independent and identically distributed random variables with variance equals to $\sigma^2 > 0$ and (3) σ^2 is an unknown parameter (see, for example, Koul, 1991). However, there are not asymptotic critical values to use with CVM when the null distribution is other different to the normal depending on unknown parameters. We propose to use the parametric bootstrap to approximate the distribution of CVM, to obtain the bootstrap p-value of the test and to choose the functional form that better fits the behaviour of the market model errors.

The structure of the paper is as follows. In Section 2, we introduce the assumed statistical model (market model) and we review some different possible functional forms to model the behaviour of the market model errors. Moreover, the designed bootstrap procedure is described. In the Section 3, we present the empirical results based on a sample of companies of the Spanish Stock Market and on the General Index of the Madrid Stock Market. Finally, the conclusions appear in Section 4.

2. The Statistical Model and the Bootstrap Procedure

The market model is a statistical model which relates the return of any given security to the return of the market porfolio. For any security i in the period t we have

$$R_{it} = \delta_i + \beta_i R_{mt} + \varepsilon_{it},$$

$$E[\varepsilon_{it}] = 0, \quad \text{Var}[\varepsilon_{it}] = \sigma_{\varepsilon_i}^2$$

where R_{it} and R_{mt} are the period-t returns, t = 1, ..., T, on security i, i = 1, ..., N, and the market portfolio, respectively. The error terms of the models are denoted ε_{it} . We assume that $\varepsilon_{1t}, ..., \varepsilon_{Nt}$ are independent and identically distributed unobservable random variables from a population with a continuous distribution function wich depends on some unknown parameters $\theta \in \mathbb{R}^s$. Therefore, we assume that, for each security i, there exists a vector $\gamma_i = (\delta_i, \beta_i, \theta)'$ of k = (2 + s) unknown parameters, that must be estimated.

We wish to test the null hypothesis H_0 : "the distribution of ε_{it} is $F(x, \theta), x \in \mathbb{R}$ " versus the alternative H_1 : "the distribution of ε_{it} is not of this type" using the CVM test statistic based on the residuals of the model. Under incomplete specification, its expression is given by

$$\widetilde{W}_N^2 = \int_{\mathbb{R}} N \left[\widetilde{F}_N(x) - \widehat{F}(x) \right]^2 \mathrm{d}\, \widehat{F}(x), \tag{1}$$

where, $\widehat{F}(\cdot) = F(\cdot, \widehat{\theta})$, $\widehat{\theta}$ is an efficient estimate of θ , \widetilde{F}_N is the empirical distribution function of the residuals $e_i = (R_{it} - \widehat{\delta}_i - \widehat{\beta}_i R_{mt})$, given by $\widetilde{F}_N(x) = N^{-1} \sum_{i=1}^n I(e_i \leq \widehat{x})$, $\widehat{x} = \widehat{F}^{-1}(s)$, for $s \in (0, 1)$ and $I(\cdot)$ is the indicator function.

We use the following alternative expression of the CVM test statistic,

$$\widetilde{W}_n^2 = \sum_{i=1}^n \left(\widetilde{F}_n(e_i) - \widehat{F}(e_i) \right)^2,\tag{2}$$

to test that the market model errors are from a normal distribution with mean 0 and unknown variance $\sigma^2 > 0$. If the normal distribution does not provide a good fit of the market model errors, then we propose the following three possible specifications: (I) the Student's t distribution with mean 0, variance $\sigma^2 > 0$ and degrees of freedom (g > 2)as unknown parameters, (II) the Logistic distribution with mean 0 and unknown variance $\sigma^2 > 0$, (III) a mixture of $N(0_1, \sigma_1^2)$ and $N(0, \sigma_2^2)$ with a probability λ , being $\sigma_1^2, \sigma_2^2, \lambda$ the unknown parameters. We evaluate \widetilde{W}_n^2 using the ordinary least squares residuals of the models $e_{iOLS} = (R_{it} - \hat{\delta}_{iOLS} - \hat{\beta}_{iOLS} R_{mt})$ and the maximum likelihood estimates of θ . The possible density functions of the error terms in period t are, respectively,

$$f(\varepsilon_t, \sigma_t^2) = \frac{\lambda}{\sqrt{2\pi\sigma_t^2}} \exp\left\{-\frac{\varepsilon_t^2}{\sigma_t^2}\right\}, \quad \theta = (\sigma_t^2), \tag{3}$$

$$\begin{split} f(\varepsilon_t; \sigma_t^2, g) &= \frac{\Gamma\left(\frac{g+1}{2}\right)}{\Gamma\left(\frac{g}{2}\right)\Gamma\left(\frac{1}{2}\right)} \frac{1}{\sqrt{\sigma_t^2(g-2)}} \left[1 + \frac{\varepsilon_t^2}{\sigma_t^2(g-2)} \right]^{-\frac{(g+1)}{2}}, \quad \theta = (\sigma_t^2, g)', \\ f(\varepsilon_t, \sigma_t^2) &= \frac{\pi}{\sqrt{3\sigma_t^2}} \frac{1}{\left[1 + \exp\left\{\frac{\pi\varepsilon_t}{\sqrt{3\sigma_t^2}}\right\} \right]^2} \exp\left\{\frac{\pi\varepsilon_t}{\sqrt{3\sigma_t^2}}\right\}, \quad \theta = (\sigma_t^2), \\ f(\varepsilon_t, \sigma_{1t}^2, \sigma_{2t}^2, \lambda) &= \frac{\lambda}{\sqrt{2\pi\sigma_{1t}^2}} \exp\left\{-\frac{\varepsilon_t^2}{\sigma_{1t}^2}\right\} + \frac{1-\lambda}{\sqrt{2\pi\sigma_{2t}^2}} \exp\left\{-\frac{\varepsilon_t^2}{\sigma_{2t}^2}\right\}, \\ \theta &= (\sigma_{1t}^2, \sigma_{2t}^2, \lambda)'. \end{split}$$

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We realize these tests using the parametric bootstrap (Giné and Zinn, 1991) because we are assuming the process that generates the data belongs to a parametric family. That is, $F(\cdot) = F(\cdot, \theta)$, where F is completely specified except by a vector of unknown parameters θ that must be estimated. We distinguish the following stages:

- We consider a sample R_{1t}, \ldots, R_{Nt} of N securities in the period t. We calculate the vector of ordinary least square residuals $e_{iOLS} = (R_{it} - \hat{\delta}_{iOLS} - \hat{\beta}_{iOLS} R_{mt})$, $i = 1, \ldots, N$. Under H₀, we estimate θ by maximum likelihood using e_{iOLS} and we evaluate \widetilde{W}_N^2 using the standarized residuals $(e_{iOLS}/\hat{\sigma}_t)$ and substituting θ by $\hat{\theta}$.
- We draw B = 500 bootstrap samples of i.i.d. random variables $e_{bt}^* = (e_{b1t}^*, \dots, e_{bNt}^*)'$ from $F(\cdot, \hat{\theta})$, $b = 1, \dots, B$. We use them to compute $R_{bit}^* = \hat{\delta}_{iOLS} + \hat{\beta}_{iOLS} R_{mt} + e_{bt}^*$, $b = 1, \dots, B$. We calculate new ordinary least squares of $\hat{\delta}_{iOLS}$ and $\hat{\beta}_{iOLS}$, which are denoted $\hat{\alpha}_{iOLS}^*$ and $\hat{\beta}_{iOLS}^*$. In this way, we obtain $e_{bt}^{**} = (e_{b1t}^{**}, \dots, e_{bNt}^*)'$, where $e_{bit}^{**} = R_{bit}^* \hat{\delta}_{iOLS}^* \hat{\beta}_{iOLS}^* R_{mt}$. Using e_{bit}^{**} we compute new maximum likelihood estimates $\hat{\theta}_b^{**}$ and we evaluate \widehat{W}_{bN}^2 . Hence, we obtain $\widetilde{W}_{1N}^2, \dots, \widetilde{W}_{BN}^2$ for the period t.
- We order $\widetilde{W}_{1N}^2 \leq \ldots \leq \widetilde{W}_{BN}^2$ and we calculate the bootstrap *p*-value as $p_B = \operatorname{card}(\widetilde{W}_{bN}^2 \geq \widetilde{W}_N^2)/B$.

This procedure is made for each of the postulated null distributions. Comparing the bootstrap *p*-values we examine if it is possible to accept that the data are generated from one or several of the possible functional forms.

To asses for the validity of the designed procedure we realize the following simulation experiment: we draw a sample $\varepsilon_{1t}, \ldots, \varepsilon_{Nt}, N = 995$, from a mixture $\lambda N(0,3) + (1 - \lambda)N(0,1), \lambda \in [0,1]$. We consider the simple linear regression model $R_{it} = \delta_i + \beta_i R_{mt} + \varepsilon_{it}$, where, for each security i, R_{mtT} is a vector composed by T = 300 daily observations of the General Index of the Madrid Stock Market for the period 1997–2000 and the initial values of $\eta = (\delta, \beta)'$ are $\eta_0 = (1, 1)'$. For different significance levels α , we test H_0 : " $\varepsilon_{1t}, \ldots, \varepsilon_{Nt}$ are distributed as a normal with mean 0 and variance σ^2 , both of them unknown" versus H_1 : "the distribution of $\varepsilon_{1t}, \ldots, \varepsilon_{Nt}$ is not of this type", using the designed bootstrap procedure with B = 200. If $\lambda = 0$ or $\lambda = 1, R_1, \ldots, R_T$ is generated under H_0 . We repeat the test R = 1000 times and we estimate the power function of the test, $P(\lambda)_{\alpha}$.

Fig. 1 presents the empirical approximations of the power function of W_T^2 for the bootstrap case and for the asymptotical case (Stephens, 1976), given $\alpha = .05$. The curves are drawn by joining the points $(\lambda, P(\lambda)_{.05})$ by straight lines for the 11 different values of the parameter λ , where $P(\lambda)_{.05}$ denotes the percentage of times that H₀ is rejected.

It can be seen that the estimated power functions are very similar for all the possible values of λ .

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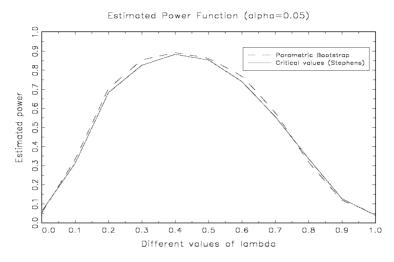


Fig. 1. Estimated power function (alpha=0.05).

3. Empirical Results

The data set consists on 300 daily observations coming from 30 companies of the Spanish Stock Market, randomly chosen in the period 1990–2000, and from the General Index of the Madrid Stock Market (which represents R_{mt})¹. The series of daily stock-returns are computed as variation rates, taking the natural logarithmic differences of the daily closing price series; that is, the daily observations are computed as $R_t = \log(S_t/s_{t-1})$, where S_t is the closing price in the day t. The series are adjusted by capital expansions, payment of dividends and splits. Table 1 exhibits some descriptive statistics on the distribution of the series².

It can be seen that the values of the observations are very concentrated around the mean: the distance between the minimum value and the maximum value is very short and the standard deviation is very small. Slight leptokurtosis appears in most of the cases.

The performance of the normal distribution and of the other postulated functional forms is assessed applying the bootstrap procedure designed in the Section 2. We compute the bootstrap *p*-values and we choose the functional form that better fits the market model errors (that functional form with the biggest *p*-value). This appears summarized in the Table³ 2.

The hypothesis of normality of the market model errors is only accepted in four cases (for $\alpha = 0.05$). Given our data set, the Student's t distribution works well. It is accepted that it fits the behaviour of the market model errors in twenty one cases given $\alpha = 0.01$. It must be noted that, in several cases, more than one of the postulated null distributions can

¹See Appendix for the list of the chosen Companies.

²In Table 1, the descriptive measures are computed under normality of the observations. SD:standard deviation; MIN:minimum value; MAX:maximum value; SK:skewness; K:kurtosis; M:mean.

 $^{{}^{3}}p_{BSt}$, p_{BLg} , p_{BN} y p_{BMix} denote the bootstrap *p*-value under the Student's *t*, the Logistic, the Normal and the Mixture of two normals, respectively.

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Descriptive measures

COMPANIES	SD	MAX	MIN	SK	К	М
DRC	0.0224	0.1149	-0.0943	0.4343	7.8590	-0.000052
AUM	0.0171	0.0373	-0.0435	-0.0524	2.6424	-0.00023
TEF	0.0257	0.0780	-0.0988	-0.1802	3.8521	0.0015
BBV	0.0177	0.0790	-0.1353	-0.9786	15.243	0.00092
ELE	0.0155	0.0691	-0.0953	-0.3111	9.2331	0.00190
ZNC	0.0336	0.1397	-0.1612	0.0708	6.1459	-0.0034
HSB	0.0280	0.0923	-0.0699	0.4247	3.2146	0.00097
TUD	0.0306	0.1395	-0.1103	1.0250	7.5984	0.00353
AMP	0.0277	0.0983	-0.1049	0.0964	4.5988	0.00202
FFR	0.0332	0.1389	-0.1537	0.1188	6.1730	-0.00204
BAM	0.0252	0.0992	-0.0785	0.5258	4.1848	-0.00043
ACS	0.0265	0.1395	-0.1289	0.2459	7.1658	0.0048
AGR	0.0331	0.1290	-0.1625	-0.3444	6.4858	-0.0015
VDR	0.0232	0.1237	-0.0962	0.3355	7.0939	0.000041
UFE	0.0322	0.1372	-0.1406	0.6180	6.7263	0.00274
STG	0.0293	0.1381	-0.1120	0.3484	6.6254	0.00024
REP	0.0106	0.0374	-0.0495	-0.4519	6.0496	0.00038
PRY	0.0181	0.0674	-0.0794	-0.2386	5.2704	0.00025
MVC	0.0187	0.0675	-0.7460	-0.2553	4.1562	-0.0015
LAI	0.0234	0.0956	-0.0677	0.4066	4.5041	0.00033
CUB	0.0280	0.1090	-0.1533	-0.0728	7.7411	0.0040
AZC	0.0327	0.1315	-0.1102	0.2365	5.0728	-0.0026
BKT	0.0119	0.0451	-0.0433	0.0765	4.5514	0.00013
CFR	0.0188	0.0768	-0.0495	0.2513	3.9679	0.00092
CEP	0.0159	0.0718	-0.0657	0.4246	5.3516	-0.000065
CTF	0.0230	0.0702	-0.0585	0.3520	3.5360	0.00071
FNZ	0.0272	0.1046	-0.1142	0.0174	5.6965	-0.00026
GSW	0.0335	0.1392	-0.1552	0.7709	7.5300	0.00058
ECR	0.0244	0.1009	-0.1412	0.0929	8.0231	-0.00174
SAR	0.0240	0.1397	-0.0584	1.3337	7.6596	0.00090

be chosen to fit the errors (for $\alpha = 0.05$). The choice of which of them is the functional form that fits the market model errors is a subjective opinion of the researcher. The Fig. 2 exhibits the results for the residual vectors of **AUM** and **MVC**.

It can be observed that, for AUM, the Student's t captures the peak at zero of the distribution of the residuals better than the others, while the mixture of two normals is the one that fits better the behaviour of the tails. In the case of **MVC**, the logistic and the Student's t provide the same fit of the data and, the mixture of two normals presents a very similar p-value but this is because it captures the tails quite well.

However, there are cases in which neither of the postulated null distributions fit the market model errors. This is represented in the Fig. 3, where the behaviour of the residual vector of **AMP** is presented.

It can be seen that the histogram of **AMP** presents a big peak at zero that it can not be captured by any of the proposed functional forms. In this case, the value of the estimate of the degrees of freedom of the Student's t distribution is very small ($\hat{g} = 3.6904$) to

COMPANIES	p_{BLG}	p_{BST}	p_{BN}	p_{BM}
DRC	0.352	0.272	0.056	0.274
AUM	0.338	0.650	0.320	0.290
TEF	0.008	0.018	0.000	0.068
BBV	0.156	0.134	0.000	0.000
ELE	0.028	0.204	0.000	0.000
ZNC	0.038	0.042	0.000	0.234
HSB	0.376	0.346	0.044	0.196
TUD	0.000	0.004	0.000	0.000
AMP	0.000	0.004	0.000	0.004
FFR	0.032	0.156	0.000	0.326
BAM	0.054	0.136	0.000	0.114
ACS	0.020	0.116	0.000	0.068
AGR	0.000	0.004	0.000	0.012
VDR	0.000	0.120	0.000	0.592
UFE	0.000	0.002	0.000	0.000
STG	0.000	0.008	0.000	0.230
REP	0.226	0.226	0.000	0.000
PRY	0.012	0.080	0.000	0.230
MVC	0.742	0.784	0.160	0.702
LAI	0.038	0.054	0.000	0.024
CUB	0.000	0.008	0.000	0.004
AZC	0.038	0.056	0.000	0.008
BKT	0.728	0.698	0.022	0.002
CFR	0.086	0.122	0.002	0.020
CEP	0.114	0.396	0.000	0.000
CTF	0.002	0.044	0.000	0.000
FNZ	0.000	0.180	0.000	0.118
GSW	0.000	0.010	0.000	0.000
ECR	0.238	0.210	0.000	0.542
SAR	0.000	0.016	0.000	0.000

 Table 2

 Bootstrap *p*-values of the goodness of fit tests

try to capture the peak at zero; moreover, this is the reason for which the estimates of the variances of the mixture of the normals are small too ($\hat{\sigma}_1^2 = 0.00019$ and $\hat{\sigma}_2^2 = 0.0013$).

We have checked the data and we could observe that the biggest p_B correspond to those companies of the sample which have high daily trading volume in number of shares.

4. Conclusions

We have focused on the analysis of the behaviour of the market model errors using the Crámer-von Mises test statistic. It is important since this type of test allows to test the

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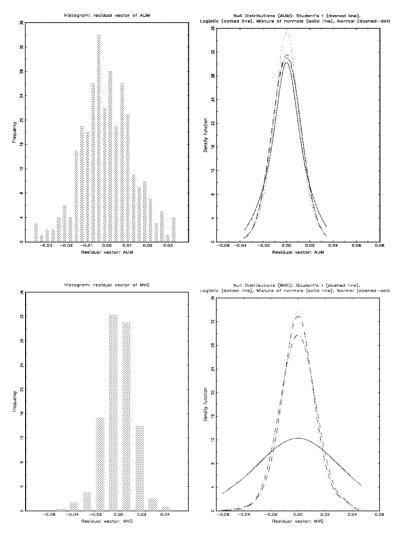


Fig. 2. Histograms and fitted distributions for the residual vector of AUM and MVC.

performance of a functional form versus any alternative one and not versus a specified one like the Log-Likelihood criterion does (very used in the practise). We have demonstrated that the power of the bootstrap test is high and we have compared it with the power of the critical values tabulated by Stephens (1976) (only for the case of the normal distribution). In the Section 3, we have used the proposed bootstrap test with a sample of 30 companies of the Spanish Stock Market and we have obtained the following conclusions: (1) the hypothesis of normality can not be used since the normal distribution does not capture the behaviour of the market model errors; (2) the Student's t distribution seems to be a good alternative to model the distribution of market model errors; (3) it is difficult to fit the behaviour of the companies with less daily trading volume in number of shares.

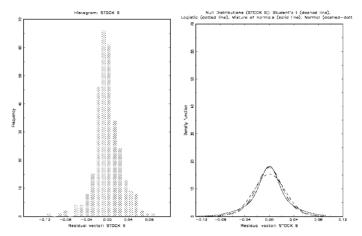


Fig. 3. Histogram and fitted distribution for the residual vector of AMP

Appendix

Abbreviature	Name of the Company
DRC	DRAGADOS Y CONSTRUCCIONES
AUM	AUTOPISTAS DEL MARE NOSTRUM
TEF	TELEFONICA
BBV	BANCO BILBAO VIZCAYA Y ARGENTARIA
ELE	ENDESA
ZNC	ESPAÑOLA DE ZINC
HSB	HORNOS IBERICOS ALBA
TUD	SOCIEDAD ESPAÑOLA ACUMULADOR TUDOR
AMP	AMPER
FFR	GRUPO FOSFORERA
BAM	BAMI
ACS	ACTIVIDADES DE CONSTRUCCION Y SERVICIO
AGR	AGROMAN
VDR	PORTLAND VALDERRIVAS
UFE	AGS-FENIX
STG	SOTOGRANDE
REP	REPSOL
PRY	PRYCA
MVC	METROVACESA
LAI	CONSTRUCCIONES LAIN
CUB	CUBIERTAS Y MZOV
AZC	ASTURIANA DEL ZINC
BKT	BANKINTER
CFR	CORPORACION FINANCIERA REUNIDA
CEP	CEPSA
CTF	CORTEFIEL
FNZ	FINANZAUTO
GSW	GLOBAL STEEL WIRE
ECR	ERCROS
SAR	SARRIO

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Normališkumo hipotezė rinkos modelių kontekste

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Straipsnyje aptariama rinkos modelių normališkumo prielaida. Pasiūlytos kai kurios specifikacijos, testuojant jas paklaidų empirinių skirstinių testais bei darant prielaidą, kad nulinė hipotezė priklauso nuo tam tikrų nežinomų parametrų. Pritaikomas parametrinis but-strepo metodas. Empirinis teisingumas patvirtinamas remiantis trisdešimties Ispanijos akcijų rinkos kompanijų duomenų imtimi.

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