OPTIMAL ESTIMATOR FOR LINEAR SYSTEMS

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Abstract. The observation problem along with the certain independent value plays a great role while carrying out control of dynamic systems in the conditions of uncertainty (Kalman, 1957, Krasovski, 1985, Leondes, 1976). A new approach on connection between the problems of control and observation is presented in (Gabasov, 1991). Developing it, we justify the solution of observation problem in the given paper that arises at optimization of linear dynamic systems. The paper consists of the two parts. In the part I the linear discrete system is investigated. In the part II the linear dynamic continuous system is considered.

Key words: observation, estimator, support plan, synthesis.

Introduction. The main directions of investigating optimal control problems are connected with construction of program controls or studing their characteristics. But always the problem of synthesis of optimal control system was supposed to be the central one. It was only for special-quadratic optimal control problems.

The results received by the authors in the field of extremal problems (R. Gabasov et al. "Constructive Methods Optimization". University Press, Minsk, 1984–1991) give the

possibility to construct finite algorithms of solution of optimal control problems both in program form and in feedback form.

Optimal estimators are an important part of constructing feedback optimal controls. Optimal estimator produces estimates of states and pertubations which are used by regulator to form the optimal control.

In the paper a new approach to construct algorithms of acting of estimators both for discrete and continuous dynamic systems is suggested. New estimators are based on principles of getting guaranteed (minimax) estimates. As for pertubations and errors of measurements accompaning the observation they are supposed to be taken from an arbitrary set given by linear restrictions-inequalities.

I. Optimal estimator for linear discrete system

1. Let on set $T(t^*) = \{t_*, t_* + h, t_* + 2h, \ldots, t^*\}$ the behaviour of dynamical system is described by equation

$$x(t + h) = A(t, h)x(t), \tag{1.1}$$

where $x(t) = (x_j(t), \ j \in J)$ is n-vector of state of system in the moment of time t; $J = \{1, 2, \ldots, n\}$; $h > 0$ is constant number, $A(t, h)$ is n×n matrix of parameters.

We shall consider that the discribe system (1.1) is obtained from the continuous

$$\dot{x} = A(t)x$$

after quantification of time with sufficiently small period $h > 0$. Then

$$A(t, h) = F(t + h, \ t), \quad \frac{\partial F(t, \tau)}{\partial t} = A(t)F(t, \tau), \quad F(t, \tau) = E.$$

The problem of observation appears any time when initial state $x(t_*)$ of system (1.1) is exactly unknown and a priori information

$$x(t_*) \in \check{X}_* = \{x \in R^n : Gx = f, \ d_* \le x \le d^*\},$$

$$(f \in R^m, \ G = G(I, J), \ I = \{1, 2, \ldots, m\}, \ \text{rank } G = m < n)$$

is so rough that it is unuseful for utility in control problems of system (1.1).

To clarify a priori distribution $\check{X}_*$ of initial states let us introduce the observation procedure by means of measuring device

$$y = c'(t)x + \xi(t), \tag{1.2}$$

where $\xi(t)$ is an error by means of which the measurement of linear combination $c'(t)x$ of coordinates of vector of state x is accessible. It is natural that the measurement errors are unknown but we shall consider that they satisfy unequalities

$$\xi_*(t) \le \xi(t) \le \xi^*(t), \quad t \in T(t^*). \tag{1.3}$$

Let measuring device (1.2) has registered signal $y(t)$, $t \in T(t_*)$ generated by some initial state $x(t_*) = z \in \check{X}_*$ and errors of measurements $\xi(t)$, $t \in T(t^*)$. Using this information clarify $\check{X}_*$.

DEFINITION. The set $\hat{X}_*(t^*)$ will be called a posteriori distribution of initial states corresponding to the observation process to the moment t^* if it consists of those and only those elements $z \in \check{X}_*$ that together with some errors of measurement $\xi(t)$, $t \in T(t^*)$, are able to generate the observed signal $y(t)$, $t \in T(t^*)$.

Evidently the set $\hat{X}_*(t^*)$ in full volume in control problems is used seldom. As a rule some its numerical characteristics (estimates) are needed. In linear control problems it is sufficient to know linear estimates

$$\hat{\alpha}(t^*) = \max p'(t^*)x(t^*|z), \quad z \in \hat{X}_* \, (t^*), \tag{1.4}$$

where $p(t^*)$ is a given n-vector, $x(t^*|z)$ is a state of system (1.1) in moment t^* under condition that in moment t_* it was in state z.

Problem (1.4) will be called the problem of optimal observation for discrete system (1.1) with the help of measuring device (1.2), (1.3).

2. Let $F_n(t, \tau)$, t, $\tau \in T(t^*)$ be a fundamental matrix of solution of equation (1.1):

$$F_h(t + h, \tau) = A(t, h)F_h(t, \tau), \quad F_h(\tau + h, \tau) = E,$$

$$F_h(t, \tau - h) = F_h(t, \tau)A(\tau, h), \quad F_h(t, t - h) = E,$$

where E is a unit diagonal n×n matrix.

The problem (1.1)–(1.4) can be written in the form

$$\widehat{a}(t^*) = \max p'(t^*)F_h(t^*, \ t_* - h)z,$$

$$\xi_*(t) \leqslant y(t) - c'(t)F_h(t, \ t_* - h)z \leqslant \xi^*(t), \quad t \in T(t^*),$$

$$Gz = f, \ d_* \leqslant z \leqslant d^*. \tag{1.5}$$

Designate $a'(t) = (a_1(t), a_2(t), \dots, a_n(t))' = -c'(t)F_h(t, t_* - h)$; $b'(t^*) = (b_1(t^*), b_2(t^*), \dots, b_n(t^*))' = p'(t^*F_h(t^*, t_* - h)$, after that problem (1.5) has the form

$$b'(t^*)z \to \max, \ \xi_*(t) \leqslant y(t) + a'(t)z \leqslant \xi^*(t), \quad t \in T(t^*),$$

$$Gz = f, \ d_* \leqslant x \leqslant d^*. \tag{1.6}$$

Since problem (1.6) is that of linear programming, it can be solved by finite methods. Let $\{z(t^*), \ S_{\text{sup}}(t^*)\}$ be an optimal support plan of problem (1.6). The support $S_{\text{sup}}(t^*) = \{J_{\text{sup}}(t^*), \ T_{\text{sup}}(t^*)\}$ presents a totality from set $J_{\text{sup}}(t^*) \subset J$ of support indices of optimal plan $z(t^*)$ and set $T_{\text{sup}}(t^*) \subset T(t^*)$ of support moments $t_* \leq \tau_1 = \tau_1(t^*) < \dots < \tau_l = \tau_l(t^*) \leq t^*$. In addition the correlations are fulfilled

$$m + |T_{\text{sup}}(t^*)| = |J_{\text{sup}}(t^*)|, \quad \det P \neq 0,$$

$$P = P(t^*) = P((T_{\text{sup}}(t^*), \ I),$$

$$J_{\text{sup}}(t^*)|t^*)) = \begin{pmatrix} a_j(t) \ : \ j \in J_{\text{sup}}(t^*) \\ t \in T_{\text{sup}}(t^*) \\ G(I, \ J_{\text{sup}}(t^*)) \end{pmatrix}$$

Let us introduce correlations

$$Q = Q(t^*) = Q(J_{\text{sup}}(t^*), \ (T_{\text{sup}}(t^*), \ I)|t^*) = P^{-1}(t^*)$$

$$= \left(\begin{array}{c} ((q_j(t) \ : \ t \in T_{\text{sup}}(t^*)); \ (q_{ji} \ : \ i \in I))' \\ j \in J_{\text{sup}}(t^*) \end{array} \right),$$

$$A(T_{\text{sup}}(t^*), \ J) = \left(\begin{array}{c} a'(t) \\ t \in T(t^*) \end{array} \right)$$

Construct sets $T_N = T_N(t^*) = T(t^*) \backslash T_{\text{sup}}(t^*); \ J_N(t^*) = j \backslash J_{\text{sup}}(t^*)$. We shall ascribe to each moment of time $t \in T(t^*)$ and indices $j \in J, \ i \in I$ the numbers

$$v(t) = v(t|t^*), \ \Delta_j = \Delta_j(t^*), \ \mu_i = \mu_i(t^*) : \quad \cdot$$

$$v(t) = 0, \ t \in T_N(t^*);$$

$$\Delta_j(t^*) = 0, \ j \in J_{\text{sup}}(t^*);$$

$$\mu = \mu(t^*) = (\mu_i(t^*), \ i \in I);$$

$$v_{\text{sup}} = v(T_{\text{sup}}(t^*)) = (v(\tau_1(t^*)), \ v(\tau_2(t^*)), \dots,$$

$$v(\tau_l(t^*)));$$

$$v_N = v(T_N(t^*)) = (v(t), \ t \in T_N(t^*));$$

$$u_{\text{sup}} = (v(T_{\text{sup}}(t^*)), \ \mu(t^*)) = (v_{\text{sup}}, \ \mu);$$

$$u'_{\text{sup}} = b'_{\text{sup}}Q(t^*), \ b_{\text{sup}} = (b_j(t^*), \ j \in J_{\text{sup}}(t^*));$$

$$u_N = u(T_N(t^*)) = (v(T_N(t^*)), \ \mu(t^*));$$

$$u = u(T(t^*)) = (u_{\text{sup}}, \ u_N);$$

$$\Delta'(J_N) = \Delta'(t^*|J_N(t^*)) = (\Delta_j(t^*),$$

$$j \in J_N(t^*))' = v'_{\text{sup}}A(T_{\text{sup}}(t^*), \ J_N(t^*))$$

$$+ \mu'G(I, \ J_N(t^*)) - b'_N(t^*),$$

$$b_N(t^*) = (b_j(t^*), \ j \in J_N(t^*)).$$

The plan $z(t^*)$ is optimal if and only if there exists the support $S_{\text{sup}}(t^*)$ wherein

The plan $z(t^*)$ is optimal if and only if there exists the support $S_{\text{sup}}(t^*)$ wherein

$$\Delta_j(t^*) \leq 0 \quad \text{if } z_j(t^*) = d_j^*;$$
$$\Delta_j(t^*) \geq 0 \quad \text{if } z_j(t^*) = d_j^*;$$
$$\Delta_j(t^*) = 0 \quad \text{if } d_{*j} < z_j(t^*) < d_j^*;$$
$$j \in J_N(t^*);$$
$$v(\tau_i(t^*)) \geq 0 \quad \text{if } y(\tau_i(t^*)) + a'(\tau_i(t^*))z(t^*) = \xi^*(\tau_i(t^*));$$
$$v(\tau_i(t^*)) \leq 0 \quad \text{if } y(\tau_i(t^*)) + a'(\tau_i(t^*))z(t^*) = \xi_*(\tau_i(t^*));$$
$$v(\tau_i(t^*)) = 0$$
$$\text{if } \xi_*(\tau_i(t^*)) < y(\tau_i(t^*)) + a'(\tau_i(t^*))z(t^*) < \xi^*(\tau_i(t^*));$$
$$i = \overline{1, \, l}. \qquad (1.7)$$

3. The observation problem considered in the previous paragraph is a program one in that sense that the observed signal $y(t)$ in it is supposed to be known to the moment t_* all over the interval $T(t^*)$. While constructing optimal regulators for (1.1) (in other terminology while carrying out synthesis of optimal systems of control) the information on system state should enter in the mode of real time. In this connection there arises the problem of synthesis of optimal estimator designated for calculation of necessary estimates in the mode of real time.

Let for some $\theta \in T(t^*)\backslash t^*$ the process of observation was carried out on set $T(\theta) = \{t_*, t_*+h, \ldots, \theta\}$ and $\{z(\theta), S_{\text{sup}}(\theta)\}$ its result (solution of problem (1.6) wherein moment t^* is changed for θ.) Assume that at the next moment $t = \theta + h$ the device (1.2) registered signal $y(\theta + h)$ with error of measurement $\xi(\theta + h)$ satisfying unequalities (1.3).

Construction of optimal support plan $\{z(\theta+h), S_{\text{sup}}(\theta + h)\}$ of problem (1.6) for any values $y(\theta + h)$ will be called the synthesis of optimal estimator of discrete system (1.1) with measuring device (1.2), (1.3).

By information obtained to moment θ we shall calculate the value

$$\omega(\theta) = y(\theta + h) + a'(\theta + h)z(\theta). \qquad (1.8)$$

If $\xi_*(\theta + h) \leq \omega(\theta) \leq \xi^*(\theta + h)$, then $\{z(\theta + h), \ S_{\mathrm{sup}}(\theta + h)\} = \{z(\theta), \ S_{\mathrm{sup}}(\theta)\}$. Therefore the problem of synthesis of optimal estimator in moment θ does not arise for $\omega(\theta) \in [\xi_*(\theta + h), \ \xi^*(\theta + h)]$. Let (for the definiteness sake) $\omega(\theta) > \xi^*(\theta + h)$. Immerse problem (1.6) into the family of extreme problems depending on parameter ρ:

$$b'z \to \max, \ \xi_*(t) \leq y(t) + a'(t)z \leq \xi^*(t), \quad t \in T(\theta),$$
$$\xi_*(\theta + h) \leq y(\theta + h) + a'(\theta + h)z \leq \rho, \quad (1.9)$$
$$Gz = f, \quad d_* \leq z \leq d^*.$$

Problem (1.9) for $\rho = \omega(\theta)$ has the solution $\{z(\theta), S_{\mathrm{sup}}(\theta)\}$. To find $\{z(\theta + h), \ S_{\mathrm{sup}}(\theta + h)\}$ we shall decrease iteratively parameter ρ : $\omega(\theta) = \rho_0 > \rho_1 > \ldots > \rho_N = \xi^*(\theta + h)$ constructing at the same time solutions $\{z^k, \ S_{\mathrm{sup}}^k\} = \{z(\theta|\rho_k), \ S_{\mathrm{sup}}(\theta| \rho_k)\}$ of problem (1.9). Then assume $z(\theta + h) = z^N, \ S_{\mathrm{sup}}(\theta + h) = S_{\mathrm{sup}}^N$.

4. Passing to description of algorithm of optimal estimator operation we shall designate T_{sup}^k, J_{sup}^k the set of support moments and indices from J on k-th iteration and assume

$$T_N^k = [(\{0\} \cup \{h\} \cup \{t + h, \ t \in T_{\mathrm{sup}}^k\}) \cap T(\theta + h)]. \quad (1.10)$$

Let $L_{\mathrm{sup}}^k = \{T_{\mathrm{sup}}^k, \ I\}$.
The totality

$$C^k(\theta) = \{z^k, \ S_{\mathrm{sup}}^k, \ y(T_{\mathrm{sup}}^k), \ Q^k = Q^k(J_{\mathrm{sup}}^k, \ L_{\mathrm{sup}}^k), \ \rho_k\}$$

will be called the state of algorithm on k-th iteration in moment of time θ.

The zero state of algorithm $C^0(\theta)$ compose of components:

$$z^0 = z(\theta); \quad S^0_{\mathrm{sup}} = S_{\mathrm{sup}}(\theta); \quad y(T^0_{\mathrm{sup}}) = y(T_{\mathrm{sup}}(\theta));$$

$$Q^0 = Q(\theta); \quad \rho = \omega(\theta).$$

Iteration of algorithm $C^k(\theta) \to c^{k+1}(\theta)$ consists of the following steps:

Step 1. Check condition: $\theta + h \in T^k_{\mathrm{sup}}$;

If it is fulfilled, pass to step 2. Otherwise pass to step 5.

Step 2. Let $q^k(\theta + h) = Q^k(J^k_{\mathrm{sup}}, \theta + h) = q^k(\tau_l(\theta)) = q^k_l = (q^k_{jl}, \ j \in J^k_{\mathrm{sup}})$.

Count:

$$\beta^k_j = \begin{pmatrix} (z^k_j - d^*_j)/q^k_{jl} & \text{for } q^k_{jl} < 0, \\ (z^k_j - d_{*j})/q^k_{jl} & \text{for } q^k_{jl} > 0, \ \ j \in J^k_{\mathrm{sup}}; \\ \infty & \text{for } q^k_{jl} = 0, \end{pmatrix} \quad (1.11)$$

$$\beta^k(t) = \begin{cases} \left[(y(t) + a'(t)z^k) - \xi^*(t)\right]/a'_{\mathrm{sup}}(t)q^k_l, \\ \qquad\qquad \text{for } a'_{\mathrm{sup}}(t)q^k_l < 0; \\ \left[(y(t) + a'(t)z^k) - \xi_*(t)\right]/a'_{\mathrm{sup}}(t)q^k_l, \\ \qquad\qquad \text{for } a'_{\mathrm{sup}}(t)q^k_l > 0; \\ \infty \qquad \text{for } a'_{\mathrm{sup}}(t)q^k_l = 0; \ \ t \in T^k_N. \end{cases} \quad (1.12)$$

$$\begin{aligned} \beta^k(\theta + h) &= \rho_k - \xi^*(\theta + h) \\ \beta^k_{j_0} &= \min \beta^k_j, \ \cdot j \in J^k_{\mathrm{sup}}; \\ \beta^k(t^0) &= \min \beta^k(t), \ t \in T^k_N; \\ \beta^k_0 &= \min\{\beta^k_{j_0}, \ \beta^k(t^0), \ \beta^k(\theta + h)\}. \end{aligned} \quad (1.13)$$

Assume

$$z^{k+1}_{\mathrm{sup}} = (z^{k+1}_j, \ j \in J^k_{\mathrm{sup}}) = z^k_{\mathrm{sup}} - \beta^k_0 q^k_{\mathrm{sup}} \ l;$$

$$\rho_{k+1} = \rho_k - \beta^k_0.$$

Here $q_{\mathrm{sup}\, l}^k = (q_{jl}^k,\ j \in J_{\mathrm{sup}}^k);\ z_{\mathrm{sup}}^k = (z_j^k,\ j \in J_{\mathrm{sup}}^{\prime k});$

$$z^{k+1} = (z_{\mathrm{sup}}^{k+1},\ z_N^k);\ z_N^k = (z_j^k,\ j \in J_N^k).$$

The following cases are possible:

$$\text{a)}\beta_0^k = \beta_{j_0}^k;\quad \text{b)}\beta_0^k = \beta^k(t^0);\quad \text{c)}\beta_0^k = \beta^k(\theta + h).$$

If the case a) is realized we pass to step 3. In the case b) pass to step 4. In the case c) pass to correlation (1.31) of step 6.

Step 3. Calculate

$$\Delta u^{k'} = (\Delta v^k,\ \Delta \mu^k)' = (\Delta v^k (T_{\mathrm{sup}}^k),$$

$$(\Delta \mu_i,\ i = \overline{1,m}))' = e_{j_0}' Q^k (J_{\mathrm{sup}}^k,\ L_{\mathrm{sup}}^{k \bullet}) \mathrm{sign}\ q_{j_0 l}^k;$$

$$\Delta \delta^{k'}(J) = \Delta u^{k'} P^k(L_{\mathrm{sup}}^k,\ J);$$

$$\sigma^k(t) = \begin{pmatrix} -v^k(t)/\Delta v^k(t) & \text{for } \Delta^k(t)\Delta v^k(t) < 0, \\ \infty & \text{for } \Delta^k(t)\Delta v^k(t) \geq 0, \\ t \in T_{\mathrm{sup}}^k; \end{pmatrix} \quad (1.14)$$

$$\sigma_j^k = \begin{pmatrix} -\Delta_j/\Delta \delta_j^k & \text{for } \Delta_j^k \Delta \delta_j^k < 0, \\ \infty & \text{for } \Delta_j^k \Delta \delta_j^k \geq 0, \end{pmatrix} \quad j \in J_{\mathrm{sup}}^k; \quad (1.15)$$

$$\sigma^k(t^0) = \min \sigma^k(t),\ t \in T_{\mathrm{sup}}^k;$$

$$\sigma_{j*}^k = \min \sigma_j^k,\ j \in J_N^k; \quad (1.16)$$

$$\sigma_0^k = \min\{\sigma^k(t^0),\ \sigma_{j*}^k\}.$$

Assume

$$S_{\mathrm{sup}}^{k+1} = \{T_{\mathrm{sup}}^{k+1},\ J_{\mathrm{sup}}^{k+1}\},$$

$$T_{\mathrm{sup}}^{k+1} = T_{\mathrm{sup}}^k \backslash t^0;\ J_{\mathrm{sup}}^{k+1} = J_{\mathrm{sup}}^k \backslash j_0,\ \text{if } \sigma_0^k = \sigma^k(t^0); (1.17)$$

$$T_{\mathrm{sup}}^{k+1} = T_{\mathrm{sup}}^k;\ J_{\mathrm{sup}}^{k+1} = (J_{\mathrm{sup}}^k \backslash j_0), \cup j_*,\ \text{if } \sigma_0^k = \sigma_{j*}^k \quad (1.18)$$

$$L_{\mathrm{sup}}^{k+1} = \{T_{\mathrm{sup}}^{k+1},\ I\}.$$

Let situation (1.17) be realized. Then

$$Q^{k+1} = Q^{k+1}(J_{\mathrm{sup}}^{k+1},\ L_{\mathrm{sup}}^{k+1}) = Q^k(J_{\mathrm{sup}}^k\backslash j_0,\ L_{\mathrm{sup}}^k\backslash t^0)$$
$$- Q^k(J_{\mathrm{sup}}^k\backslash j_0,\ t^0) \times Q^k(j_0,\ L_{\mathrm{sup}}^k\backslash t^0)/q_{j_0 i_0}^k,$$
$$q_{j_0 i_0}^k = Q^k(j_0,\ t^0); \tag{1.19}$$

$$\begin{cases} v^{k+1}(T_{\mathrm{sup}}^{k+1}) = v^k(T_{\mathrm{sup}}^k\backslash t^0) + \sigma_0^k \Delta v^k(T_{\mathrm{sup}}^k\backslash t^0), \\ \mu^{k+1}(I) = \mu^k + \sigma_0^k \Delta\mu^k, \\ \Delta^{k+1}(J_N^{k+1}\backslash j_0) = \Delta^k(J_N^k) + \sigma_0^k \Delta\delta^k(J_N^k), \\ \Delta_j^{k+1} = \sigma_0^k \Delta\delta_0^k. \end{cases} \tag{1.20}$$

If situation (1.18) is realized then

$$Q^{k+1} = Q^{k+1}(J_{\mathrm{sup}}^{k+1},\ L_{\mathrm{sup}}^{k+1})$$
$$= Q^k(J_{\mathrm{sup}}^k,\ L_{\mathrm{sup}}^k) - Q^k(J_{\mathrm{sup}}^k,\ L_{\mathrm{sup}}^k)$$
$$\times [P^k(L_{\mathrm{sup}}^k,\ j_*) - P^k(L_{\mathrm{sup}}^k,\ j_0)] \times Q^k(j_0, L_{\mathrm{sup}}^k)$$
$$/[Q^k(j_0,\ L_{\mathrm{sup}}^k) \times P^k(L_{\mathrm{sup}}^k,\ j_*)]. \tag{1.21}$$

$$\begin{cases} v^{k+1}(T_{\mathrm{sup}}^{k+1}) = v^k(T_{\mathrm{sup}}^k) + \sigma_0^k \Delta v^k(T_{\mathrm{sup}}^k), \\ \mu^{k+1} = \mu^k + \sigma_0^k \Delta\mu^k, \\ \Delta^{k+1}(J_N^{k+1}\backslash j_0) = \Delta^k(J_N^k\backslash j_*) + \sigma_0^k \Delta\delta^k(J_N^k\backslash j_*), \\ \Delta_{j_0}^{k+1} = \sigma_0^k \Delta\delta_{j_0}^k. \end{cases} \tag{1.22}$$

Pass to step 6.

Step 4. Calculate

$$\Delta u^{k'} = (\Delta v^k,\ \Delta\mu^k)' = (\Delta v^k,\ \Delta\mu^k)'$$
$$= (\Delta v^k(T_{\mathrm{sup}}^k),\ (\Delta\mu_i,\ i \in I))'$$
$$= a'_{\mathrm{sup}}(t^0)Q^k \times \mathrm{sign}(a'_{\mathrm{sup}}(t^0)q_l^k),\ \Delta\delta^{k'} = \Delta\delta^{k'}(J)$$
$$= \Delta u^{k'} P^k(L_{\mathrm{sup}}^k, J) - a'(t^0) \times \mathrm{sign}(a'_{\mathrm{sup}}(t^0)q_l^k). \tag{1.23}$$

Following from (1.14–1.16), (1.23) find σ_0^k.

Change the support $S_{\text{sup}}^k \to S_{\text{sup}}^{k+1}$:

$$T_{\text{sup}}^{k+1} = (T_{\text{sup}}^k \backslash t^*) \cup t^0; \quad J_{\text{sup}}^{k+1} = J_{\text{sup}}^k \text{ if } \sigma_0^k = \sigma^k(t^*);(1.24)$$

$$T_{\text{sup}}^{k+1} = T_{\text{sup}}^k \cup t^0; \quad J_{\text{sup}}^{k+1} = (J_{\text{sup}}^k \cup j_* \text{ if } \sigma_0^k = \sigma_{j_*}^k \quad (1.25)$$

$$L_{\text{sup}}^{k+1} = \{T_{\text{sup}}^{k+1}, I\}.$$

If situation (1.24) is realized then

$$
\begin{aligned}
Q^{k+1} &= Q^{k+1}(J_{\text{sup}}^{k+1}, L_{\text{sup}}^{k+1}) \\
&= Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k) - Q^k(J_{\text{sup}}^k, t^*) \\
&\quad \times [P^k(t^*, J_{\text{sup}}^k) - P^k(t^0, J_{\text{sup}}^k)] \times Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k) \\
&\quad /[-P^k(t^0, J_{\text{sup}}^k) \times Q^k(J_{\text{sup}}^k, t^*)];
\end{aligned}
\qquad (1.26)
$$

$$
\begin{cases}
v^{k+1}(T_{\text{sup}}^{k+1} \backslash t^0) = v^k(T_{\text{sup}}^k \backslash t^*) + \sigma_0^k \Delta v^k(T_{\text{sup}}^k \backslash t^*), \\
v^{k+1}(t^0) = -\sigma_0^k \text{sign}(a'_{\text{sup}}(t^0)q_l^k), \\
\mu^{k+1} = \mu^k + \sigma_0^k \Delta \mu^k, \\
\Delta^{k+1}(J_N^{k+1}) = \Delta^k(J_N^k) + \sigma_0^k \Delta \delta^k(J_N^k),
\end{cases}
\qquad (1.27)
$$

Let situation (1.25) be realized. Then

$$Q^{k+1} = Q^{k+1}(J_{\text{sup}}^{k+1}, L_{\text{sup}}^{k+1}) =$$

$$
= \left(
\begin{aligned}
&Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k) + Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k) \times (P^k(L_{\text{sup}}^k, j_*) \\
&\quad \times P(t^0, J_{\text{sup}}^k) \times Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k)/W, \\
&- P^k(t^0, J_{\text{sup}}^k) \times Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k)/W, \\
&- Q^k(J_{\text{sup}}^k, L_{\text{sup}}^k) \times P^k(L_{\text{sup}}^k, j_*)/W
\end{aligned}
\right)
$$

$$1/W \qquad (1.28)$$

$$W = P^k(t^0, j_*) - P^k(t^0, J^k_{\text{sup}})$$

$$\times Q^k(J^k_{\text{sup}}, L^k_{\text{sup}}) \times P^k(L^k_{\text{sup}}, j_*);$$

$$\begin{cases} v^{k+1}(T^{k+1}_{\text{sup}} \backslash t^0) = v^k(T^k_{\text{sup}} \backslash t^*) + \sigma^k_0 \Delta v^k(T^k_{\text{sup}}), \\ v^{k+1}(t^0) = -\sigma^k_0 \text{sign}(a'_{\text{sup}}(t^0) q^k_l), \\ \Delta^{k+1}(J^{k+1}_N) = \Delta^k(J^k_N \backslash j_*) + \sigma^k_0 \Delta \delta^k(J^k_N \backslash j_*), \\ \mu^{k+1} = \mu^k + \sigma^k_0 \Delta \mu^k. \end{cases} \qquad (1.29)$$

Pass to step 6.

Step 5. Introduce moment $\theta + h$ into the support. For this purpose calculate

$$\Delta u^{k'} = (\Delta v^k, \ \Delta \mu^k)' = (\Delta v^k(T^k_{\text{sup}}),$$

$$(\Delta \mu^k_i, \ i \in I))' = a'_{\text{sup}}(\theta + h) \times Q^k \times \text{sign}(\xi^*(\theta + h) - \omega(\theta)),$$

$$\Delta \sigma^{k'} = \Delta \sigma^{k'}(J) = \Delta u^{k'} P^k(L^k_{\text{sup}}, J)$$

$$- a'(\theta + h) \text{sign}(\xi^*(\theta + h) - \omega(\theta)). \qquad (1.30)$$

According rules (1.24), (1.25) change the support $S^k_{\text{sup}} \rightarrow S^{k+1}_{\text{sup}}$.

Following (1.26)–(1.29) construct Q^{k+1}, v^{k+1}, μ^{k+1}, Δ^{k+1}_N. Assume $S^k_{\text{sup}} = S^{k+1}_{\text{sup}}$; $Q^k = Q^{k+1}$; $P^k = P^{k+1}$, $v^k = v^{k+1}$; $\mu^k = \mu^{k+1}$; $\Delta^k_N = \Delta^{k+1}_N$ and pass to step 2.

Step 6. If $\rho_{k+1} > \xi^*(\theta + h)$ then the k-th iteration of algorithm $C^k(\theta) \rightarrow C^{k+1}(\theta)$ in moment θ is fulfilled.
For

$$\rho_{k+1} \leq \xi^*(\theta + h), \qquad (1.31)$$

operation of optimal estimator in moment θ is fulfilled $(k + 1, N)$. The zero state of algorithm in moment $\theta + h$ as follows:

$$C^0(\theta + h) = (C^{k+1}(\theta) \backslash \rho_{k+1}) \cup (\rho_0 = \omega(\theta + h)).$$

The algorithm operation for case $\omega(\theta) > \xi^*(\theta + h)$ is described completely. Case $\omega(\theta) < \xi_*(\theta + h)$ is investigated similarly.

Remark. While realizing step 5 in formulas of recalculation of potentials and estimates (1.27), (1.29) we assume that

$$\text{sign}(a'_{\text{sup}}(t^0)q_l^k) =$$

$$= \begin{pmatrix} \text{sign}(\xi^*(\theta + h) - \omega(\theta)) & \text{if } \omega(\theta) > \xi^*(\theta + h), \\ \text{sign}(\xi_*(\theta + h) - \omega(\theta)) & \text{if } \omega(\theta) < \xi_*(\theta + h). \end{pmatrix} \quad (1.32)$$

5. Example. As an illustration of the obtained results we consider a discrete model of problem of estimator synthesis for a dynamical system describing the motion of materiel point on rectangular section of path under the action of a constant force.

The mathematical model of system under study as follows:

$$T(t^*) = [0,\, t^*]; \quad \dot{x}_1 = x_2, \quad x_1(0) = z_1, \quad |z_1| \le 1$$
$$\dot{x}_2 = x_3, \quad x_2(0) = z_2, \quad |z_2| \le 1$$
$$\dot{x}_3 = 0, \quad x_3(0) = 2z_3, \quad |z_3| \le 1$$
$$\widehat{\alpha}(\theta) = x_2(\theta) = z_2 + 2\theta z_3 \to \max, \theta \ge 0.$$

The discrete system the states of which in moments $0,\ h,\ 2h,\ldots,t^*$ coincide with those of the continuous one is described by the following equations:

$$x(t + h) = A(h)x(t), \qquad (1.33)$$

where

$$A(h) = F(h,\ 0) = \begin{bmatrix} 1 & h & h^2/2 \\ 0 & 1 & h \\ 0 & 0 & 1 \end{bmatrix}$$

Evidently a priori estimate of component $x_2(\theta)$ equals to $\widehat{\alpha}(\theta) = 1 + 2\theta$, $\theta \in \{0,\ h,\ 2h,\ldots,t^*\}$. It is understood, that for large $\theta > 0$ this estimate may occur too rough.

Supplement system (1.33) by measuring device

$$y = x_1 + \xi,$$

that is able to register in moment 0, h, $2h, \ldots, t^*$ the component x_1 with error ξ satisfying the unequality

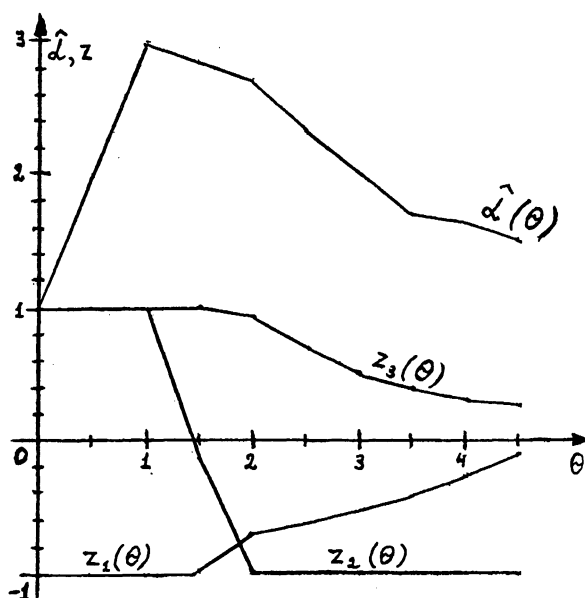$$|\xi(t)| \le 1, \quad t \ge 0.$$

Fig. 1. Laws of initial states and estimate change.

Let the output signal $y(t) \equiv 0$, $t \ge 0$, be known. Then we come to the program problem of observation:

$$\widehat{\alpha}(\theta) = x_2 + 2\theta z_3 \to \max, \quad |z_1 + t z_2 + t^2 z_3| \le 1,$$
$$t \in \{0, \ h, \ 2h, \ldots, \theta\}, \quad |z_i| \le 1, \quad i = \overline{1,3}. \tag{1.34}$$

In real processes of control the output signal of the measuring device is not known in advance. It enters as the control

process (in mode of real time) develops. It is too ineffective to solve the program problem (1.34) directly for each current moment θ.

The problem of synthesis consists in construction of procedure of estimate correction as the measuring device output signals enter in real time.

Let $\theta = 0$. It is easy to see, that an optimal solution of problem (1.34) in this case is as follows

$$z_1(0) = -1; \quad z_2(0) = 1; \quad z_3(0) = 1; \quad S_{\mathrm{sup}}(0) = \varnothing.$$

Problem (1.34) solved according to the algorithm described up to the moment $t^* = 8.5$. The results of solution are presented in the Table. A posteriori estimate values for continuous problem (1.32) are given in column $\widehat{\alpha}_c(\theta)$. The laws of change of optimal plan $z(\theta)$ components and the estimates $\widehat{\alpha}(\theta)$ for problem (1.34) are given in Fig. 1.

We note that the estimate (1.4) have the more general form $\widehat{\alpha}(\theta) = \max\limits_{z \in \widehat{X}_*(\theta)} p'(\theta)x(\theta|z)$. The algorithm changes in this situation in the following way. The vector $b(\theta)$ is replaced by the vector $b(\theta + h)$.

II. Optimal Estimator of Dynamic System

1. Consider n-dimensioned dynamic system, the behavior of which on interval of time $T = [t_*, t^*]$

$$\dot{x} = A(t)x \qquad (2.1)$$

together with piece-wise continuous $n \times n$-matrix function $A(t)$, $t \in T$.

Assume that the exact initial state $x(t_*)$ of system (2.1) is not known. A priori information about it has the form

$$x(t_*) = \check{X}_* = \{x \in R^n : Gx = f, \, d_* \leqslant x \leqslant d^*\}, \qquad (2.2)$$
$$(f \in R^m).$$

A priori distribution $\check{X}_*$ of initial states generates a priori motion $\check{X}(t) = \{x(t|z), \ z \in \check{X}_*\}$, $t \in T$, of system (2.1), composed of different trajectories $x(t|z)$, $t \in T$, of system (2.1) outgoing from points $x(t_*) = z \in \check{X}_*$ in moment t_*.

In control problems the a priori distribution $\check{X}(t^*)$ of terminal states often due the excessive uncertainty does not allow to construct effective controls. The decrease uncertainties of terminal states we introduce the procedure of observation. Assume that there is a measuring device

$$y = C(t)x + \xi, \ (y \in R^k),$$

that registers linear combinations $C(t)x(t)$ of components of state $x(t)$ in each moment $t \in T$ with errors $\xi(t)$.

Table. Synthesis estimator example.

θ	$z_1(\theta)$	$z_2(\theta)$	$z_3(\theta)$
0.0	-1.0	1.0	1.0
0.5	-1.0	1.0	1.0
1.0	-1.0	1.0	1.0
1.5	-1.0	-0.166667	1.0
2.0	-0.7333333	-1.0	0.933333
2.5	-0.6666667	-1.0	0.6666667
3.0	-0.5	-1.0	0.5
3.5	-0.4	-1.0	0.4
4.0	-0.2363636	-1.0	0.327278
4.5	-0.1050949	-1.0	0.276923
5.0	0.0507108	-1.0	0.2379716
5.5	0.1979162	-1.0	0.2083335
6.0	0.3445372	-1.0	0.184874
6.5	0.511278	-1.0	0.1654137
7.0	0.666667	-1.0	0.149660
7.5	0.8295468	-1.0	0.1363636
8.0	1.0	-1.0	0.125
8.5	1.0	-0.9444443	0.1111111

Table (continuation)

θ	$J_{\text{sup}}(\theta)$	$T_{\text{sup}}(\theta)$	$\widehat{\alpha}(\theta)$	$\widehat{\alpha}_c(\theta)$
0.0	$\varnothing$	$\varnothing$	1.0	1.0
0.5	$\varnothing$	$\varnothing$	2.0	2.0
1.0	$\varnothing$	$\varnothing$	3.0	3.0
1.5	2	1.5	2.833333	2.828
2.0	1;3	0.5;2.0	2.733333	2.732
2.5	1;3	1.0;2.5	2.333333	2.295
3.0	1;3	1.0;3.0	2.0	2.0
3.5	1;3	1.5;3.5	1.8	1.785
4.0	1;3	1.5;4.0	1.618182	1.618
4.5	1;3	2.0;4.5	1.492307	1.487
5.0	1;3	2.0;5.0	1.379716	1.379
5.5	1;3	2.5;5.5	1.291667	1.289
6.0	1;3	2.5;6.0	1.218488	1.215
6.5	1;3	3.0;6.5	1.150378	1.148
7.0	1;3	3.5;7.0	1.0955240	1.094
7.5	1;3	3.5;7.5	1.045454	1.044
8.0	1;3	4.0;8.0	1.0	1.0
8.5	2;3	4.5;8.5	0.944444	0.941

The matrix function $C(t)$, $t \in T$, is considered to be piece-wise continuous. Any piece-wise continuous functions $\xi(t)$, $t \in T$, satisfying unqualities

$$\xi_* \leqslant \xi(t) \leqslant \xi^*, \quad t \in T, \quad (\xi_*, \ \xi^* \in R^k) \qquad (2.3)$$

can be realized as errors of measurement.

Let we carry out observation on interval $T_\theta = [t_*, \theta]$ and register signal $y(t)$, $t \in T_\theta$. Information obtained from signal $y(t)$, $t \in T_\theta$, allows to decrease the uncertainty of the system at any moment of time.

The set $\hat{X}_*^\theta$ will be called a posteriori distribution of initial states of system (2.1) if it is composed of those and only those initial states $z = x(t_*) \in \check{X}_*$, that together with some errors of measurement $\xi(t)$, $t \in T_\theta$, are able to generate the observed signal $y(t)$, $t \in T_\theta$.

A posteriori motion $\hat{X}^\theta(t) = \{x(t|z), \ z \in \hat{X}_*^\theta\}$, $t \in T_\theta$, and a posteriori distribution $\hat{X}^\theta(t^*)$ of terminal states of system (2.1) correspond to a posteriori distribution of initial states $\hat{X}_*^\theta$.

As a rule, full information about sets $\check{X}_*$, $\hat{X}_*^\theta$, $\check{X}(t^*)$, $\hat{X}^\theta(t^*)$, is not used in problems of control. In a special case it is enough to know the certain numerical characteristics (estimates) of these sets. Linear problems of control operate with linear estimates of the form

$$\hat{\alpha}^\theta = \max_{z \in \hat{X}^\theta} h' x(t^*|z). \tag{2.4}$$

Calculation of estimates of type (2.4) will be called a linear problem of observation.

Let us introduce functions

$$h'(t) = h' F(t, t_*), \ M(t) = C(t) F(t, t_*), \ t \in T,$$

where $F(t, \tau)$, $t, \tau \in T$, is a fundamental matrix of solution of system (2.1):

$$\frac{\partial F}{\partial t} = A(t)F, \ F(t_*, t_*) = E;$$

E is a unit diagonal $n \times n$-matrix.

In terms of these functions the problem (2.1)–(2.4) takes the form

$$\hat{\alpha}^\theta = \max h'(t^*)z, \ \xi_* \leqslant y(t) - M(t)z \leqslant \xi^*, \ t \in T_\theta, \tag{2.5}$$
$$Gz = f, \ d_* \leqslant z \leqslant d^*.$$

The constructive theory of linear semifinite extreme problems of type (2.5) is presented in (Gabasov, 1986). The results of numerical experiments with respect to finite algorithms developed on the base of this theory are given in (Kirillova, 1986).

2. While constructing optimal controls of feedback type the estimates of a posteriori distribution of terminal states are to be calculated in the mode of real time. It's clear that to do it directly solving problem (2.5) for various $\theta \geqslant t_*$ is unreasonable and practically impossible due to the excessive demands to computer high performand.

The purpose of the present paper is to obtain equations that describe the lows of change of elements for solution of problem (2.5) in time θ. To simplify calculations we shall limit ourselves by the case $G = 0$, $f = 0$, $k = 1$. According to (Gabasov, 1986) an optimal support plan $\{z(\theta), S_{\mathrm{sup}}(\theta)\}$ is the solution of simple problem (2.5). Support $S_{\mathrm{sup}}(\theta)$ consist of two components $\{\tau_{\mathrm{sup}}(\theta), I_{\mathrm{sup}}(\theta)\}$. The first component $\tau_{\mathrm{sup}}(\theta)$ is the totality of moments of time $t_* \leqslant \tau_1(\theta) < \tau_2(\theta) < \ldots < \tau_l(\theta) \leqslant \theta$. The second component $I_{\mathrm{sup}}(\theta)$ contains l-indices from the set $I = \{1, 2, \ldots, n\}$ of indices of plan z. According to the definition the non-generated support $l \times l$ matrix

$$M_{\mathrm{sup}}(\theta) = M(\tau_{\mathrm{sup}}(\theta), \; I_{\mathrm{sup}}(\theta)) = \begin{pmatrix} M_j(\tau_i(\theta)) : j \in I_{\mathrm{sup}} \\ i = \overline{1, l} \end{pmatrix},$$

correspond to support $S_{\mathrm{sup}}(\theta)$, where $M_j(t)$, is the j-th column of matrix M(t).

Support $S_{\mathrm{sup}}(\theta)$ is accomponied by a vector of potentials $\nu = \nu(\tau_{\mathrm{sup}}(\theta)) = (\nu(\tau_1(\theta)), \ldots, \nu(\tau_l(\theta)))$, calculated by formula

$$\nu' = h'_{\mathrm{sup}}(t^*) M_{\mathrm{sup}}^{-1}(\theta)$$

where

$$h_{\mathrm{sup}}(t^*) = (h_j(t^*), \quad j \in I_{\mathrm{sup}}(\theta)).$$

A vector of potentials ν together with the vector of estimates

$$\Delta'(\theta) = \Delta'(\theta|I_N) = (\Delta_j(\theta), \ j \in I_N)'$$
$$= \nu' M(\tau_{\mathrm{sup}}(\theta), I_N) - h'_N(t^*), \ I_N = I\backslash I_{\mathrm{sup}},$$

composes the criteria base of optimally (Gabasov, 1986):

$$\Delta_j(\theta) \geqslant 0 \text{ if } z_j(\theta) = d_{*j}; \ \Delta_j(\theta) \leqslant 0 \text{ if } z_j(\theta) = d^*_j;$$

$$\Delta_j(\theta) = 0 \text{ if } d_{*j} < z_j(\theta) < d^*_j; \ j \in I_N;$$

$$\nu(\tau_i(\theta)) \leqslant 0 \text{ if } y(\tau_i(\theta)) - M(\tau_i(\theta))z(\theta) = \xi^*;$$

$$\nu(\tau_i(\theta)) \geqslant 0 \text{ if } y(\tau_i(\theta)) - M(\tau_i(\theta))z(\theta) = \xi_*;$$

$$\nu(\tau_i(\theta)) = 0 \text{ if } \xi_* < y(\tau_i(\theta)) - M(\tau_i(\theta))z(\theta) < \xi^*;$$

$$i = \overline{1, l}.$$

Let θ be such moment of time that:

1) Functions $A(t)$, $C(t)$, $y(t)$, $t \in T_\theta$, are continuous together with the second derivatives in the neighborhood of points $\tau_i(\theta)$, $i = \overline{1, l}$; where

$$\ddot{y}(\tau_i(\theta)) - \ddot{M}(\tau_i(\theta))z(\theta) \neq 0;$$

2) $\quad d_{*\,\mathrm{sup}} < z_{\mathrm{sup}}(\theta) < d^*_{\mathrm{sup}};$
3) $\quad \xi_* < y(t) - M(t)z(\theta) < \xi^*, \ t \in T_\theta\backslash\tau_{\mathrm{sup}}(\theta);$
4) $\quad \nu(\tau_{\mathrm{sup}}(\theta)) \neq 0;$
5) $\quad \Delta(\theta\backslash I_N) \neq 0;$
6) $\quad \tau_l(\theta) = \theta, \quad \dot{y}(\theta) - \dot{M}(\theta)z(\theta) \neq 0.$

Then in the neighborhood of point θ the component $I_{\mathrm{sup}}(\theta)$ of optimal support $S_{\mathrm{sup}}(\theta)$ is constant, and the component $\tau_{\mathrm{sup}}(\theta)$ and support components $z_{\mathrm{sup}}(\theta)$ of optimal plan $z(\theta)$ satisfy equations:

$$\begin{cases} M_{\mathrm{sup}}(\tau_i)\dot{z}_{sup} = 0, \ i = \overline{1, l-1} \\ M_{\mathrm{sup}}(\theta)\dot{z}_{\mathrm{sup}} = \dot{y} - \dot{M}z, \\ (\ddot{y}(\tau_i) - \ddot{M}(\tau_i)z)\dot{\tau}_i = \dot{M}_{\mathrm{sup}}(\tau_i)\dot{z}_{\mathrm{sup}}, \ i = \overline{1, l-1}, \\ \dot{\tau}_l = 1. \end{cases} \quad (2.6)$$

Correlations (2.6) will be called equations of an optimal estimator. Initial conditions for $\theta = t_*$ for equations (2.6) are obtained from consideration of a priori distribution.

Equations (2.6) are integrated as the results of measurement $y(\theta$ enter. Their form changes together with the change of component $I_{\text{sup}}(\theta)$ of optimal support $S_{\text{sup}}(\theta)$.

The rules of change $I_{\text{sup}}(\theta)$ are obtained from those of change of the support of adaptive method (Gabasov, 1980).

3. As illustration of the cited results consider a problem of observation for the motion of material point on a rectilinear section of path under the action of constant force. The mathematical model of the studied system

$$\ddot{x} = w. \tag{2.7}$$

Assume that the initial moment $t_* = 0$ the points was found somewhere in the neighborhood of point $x = 0 : |x(0)| \leqslant 1$. Let at this moment its unknown velocity $\dot{x}(0)$ satisfied the unequality $|\dot{x}(0)| \leqslant 1$. It is also known that the force acting onto the point can take any value from the set

$$\check{W} = \left\{ w \in R^1 : |w| \leqslant 2 \right\}.$$

It is required to estimate the maximal possible value of velocity of the point at the current moment θ.

Evidently a priori estimate of velocity equals to $\check{\alpha}^\theta = 1 + 2\theta$. It is understood that for large $\theta > 0$ this estimate may occur to be extremely gross.

Supplement system (2.7) by a measuring device

$$y = x + \xi,$$

which is able instantly (without inertial) to fix the position of point with error ξ, satisfying unequality

$$|\xi(t)| \leqslant 1, \quad t \in [0, \theta].$$

Let us study the situation when the measuring device registered signal $y(t) \equiv 0$, $t \geqslant 0$.

If we introduce phase variables

$$x_1 = x, \quad x_2 = \dot{x}, \quad x_3 = w,$$

vectors $h = (0, 1, 0)$, $c = (1, 0, 0)$, matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$, then

the problem considered in the given item becomes a special case of problem from item 1 for $n = 3$, $m = 1$, $G = 0$, $f = 0$, $d_* = (-1, -1, -2)$, $d^* = (1, 1, 2)$, $\xi_* = -1$, $\xi^* = 1$, $t^* = \theta$.

The observation problem (2.5) for $\theta = 4$ takes the form

$$x_2 + 8x_3 \to \max, \quad |x_1 + tx_2 + t^2 x_3| \leqslant 1 \atop t \in [0, 4], \ |x_i| \leqslant 1, \ i = \overline{1, 3};} \tag{2.8}$$

where $x_3 = w/2$.

The solution of problem (8) is given in (Kirillova, 1986):

$$x^0 = (-0.236068; \ -1.0; \ 0.327254); \quad \tau_1(4) = 1.527864; \atop \tau_2(4) = 4; \ I_{\sup}(4) = \{1, 3\}.} \tag{2.9}$$

Equation of optimal estimator for $\theta \in [\sqrt{2} + 1/2, 8]$:
$I_{\sup}(\theta) = \{1, 3\}$; $\dot{x}_1 + \tau_1 \dot{x}_3 = 0$; $x_1 + \theta^2 x_3 = 1 + \theta$; $\dot{\tau}_1 = -\tau_1 \dot{x}_3/x_3$; $\tau_2 = \theta$, with initial conditions (2.9).

Optimal estimator for $\theta \in [8, \infty]$: $I_{\sup}(\theta) = \{2, 3\}$; $\dot{x}_2 + \tau_1 \dot{x}_3 = 0$; $x_2 + \theta x_3 = 1$; $\dot{\tau}_1 = -(\dot{x}_2 + 2\tau_1 \dot{x}_3)/2x_3$; $\tau_2 = \theta$. Initial conditions: $x_2(8) = -1$; $x_3(8) = 1/8$; $\tau_1(8) = 4$; $\tau_2(8) = 8$. In the given case the solutions are found in the explicit form: $x_2(\theta) = -8/\theta$; $x_3(\theta) = 8/\theta^2$; $\tau_1(\theta) = \theta/2$; $\tau_2(\theta) = \theta$. For $\theta \in [\sqrt{2}, \sqrt{2} + 1/2]$ have: $I_{\sup}(\theta) = \{1, 2\}$; $\dot{x}_1 + \tau_1 \dot{x}_2 = 0$; $x_1 + \theta x_2 = 1 - \theta^2$; $\dot{\tau}_1 = -\dot{x}_2/2$; $\tau_2 = \theta$; $x_1(\sqrt{2} + 1/2) = -3/4$; $x_2(\sqrt{2} + 1/2) = -1$; $\tau_1(\sqrt{2} + 1/2) = 1/2$; $\tau_2(\sqrt{2} + 1/2) = \sqrt{2} + 1/2$.
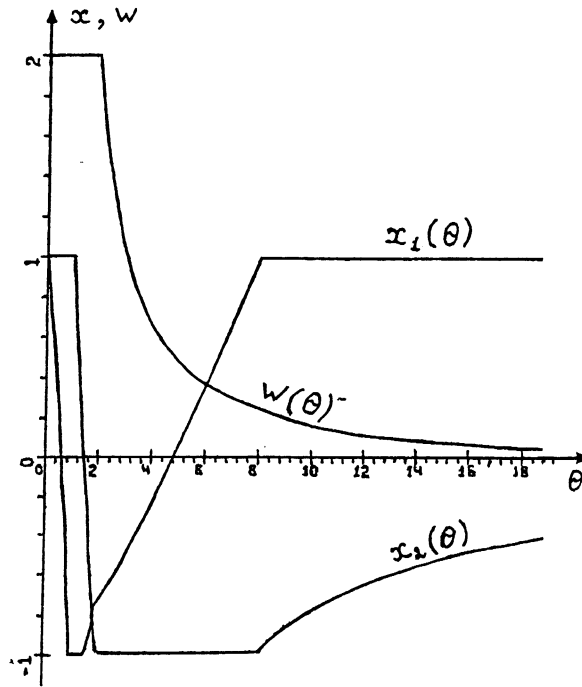
Fig. 2. Laws of initial states and force change.

For $\theta \in [1, \sqrt{2}]$: $I_{\text{sup}}(\theta) = \{2\}$; $x_2(\theta) = (2 - \theta^2)/\theta$; $\tau_1(\theta) = \theta$.
For $\theta \in [0, 1]$: $I_{\text{sup}}(\theta) = \{1\}$; $x_1(\theta) = 1 - \theta - \theta^2$; $\tau_1(\theta) = \theta$.

The laws of change in time of optimal plan $x(\theta)$ are presented in Fig. 2. The law of change of estimate $\hat{a}^\theta$ of maximal possible velocity and the law of change in time of optimal support moments $\tau_{\text{sup}}(\theta)$ are presented in Fig. 3. It's shown that on interval [0,1] a posteriori estimate coincides with that a priori i.e. due to the measurement errors on this interval the uncertainty of problem with respect to the accessible signal is failed to be decreased. after moment $\theta = 8$ an optimal estimator is structurally stabilized ($I_{\text{sup}}(\theta) = \{2, 3\}$ for $\theta \geqslant 8$).

We note that estimate (2.4) have the more general form $\hat{a}^\theta = \max\limits_{z \in \hat{X}^\theta_*} h'x(\theta|z)$ in example. The equation of estimator
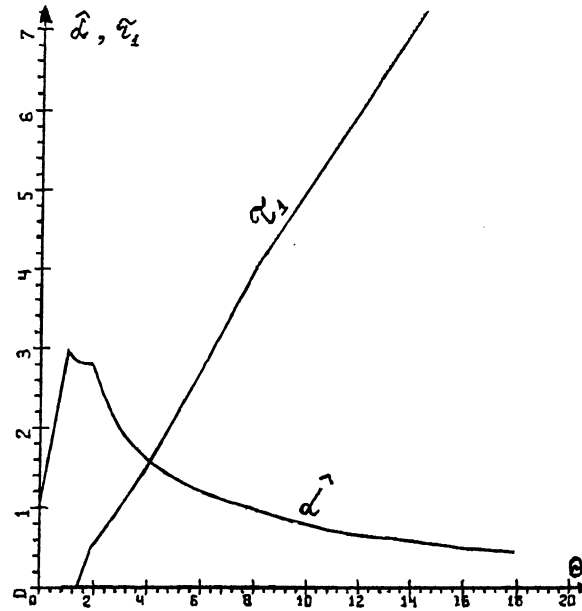
Fig. 3. Laws of estimate and suport moment change.

(2.6) is fulfilled also for it.

Conclusion. Using constructive methods of solving extremal problems created by authors before, algorithms of acting optimal estimators for linear discrete and continuous systems in conditions of uncertainty are constructed. These algorithms are oriented on using microprocessor's elements.

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