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On System Identification Using the Closed-Loop Observations

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Abstract. The aim of the given paper is development of a joint input-output approach and its comparison with a direct one in the case of an additive correlated noise acting on the output of the system (Fig. 1), when the prediction error method is applied to solve the closed-loop identification problem by processing observations. In the case of the known regulator, the two-stage method, which belongs to the ordinary joint input-output approach, reduces to the one-stage method. In such a case, the open-loop system could be easily determined after some extended rational transfer function (25) is identified, including the transfer functions of the regulator and of the open-loop system, respectively, as additional terms. In the case of the unknown regulator, the estimate of the extended transfer function (27) is used to generate an auxiliary input. The form of an additive noise filter (36), that guarantees the minimal value of the mean square criterion (35), is determined. The results of numerical simulation and identification of the closed-loop system (Fig. 5) by computer, using the two-stage method and the direct approach are given (Figures 6–12, Table 1).

Key words: adaptive system, closed-loop, feedback, identification, observations.

1. Introduction

The closed-loop identification approaches can be divided into three main groups: the direct approach, the indirect approach, and the joint input-output, which are worked out to identify the open-loop system (Forsell and Ljung, 1999; Gevers, Ljung and Van den Hof, 2001). The direct approach is realized, using the input and noisy output observations when the feedback is ignored and, in such a case, the open-loop system is identified if the respective identifiability conditions are satisfied according to Isermann (1982). The indirect approach is used, first, to identify some closed-loop system transfer function and, second, to determine the open-loop system parameters, assuming that the regulator is known beforehand. The joint input-output approach regards the input and output both together as the output of some augmented system excited by some extra input or a set-point signal and noise. It determines the open-loop system parameters, applying the estimate of the transfer function of the augmented system (Forsell and Ljung, 1999). In this connection, in the case of linear feedback, the two-stage method (Van den Hof and Schrama,

1993) is proposed, provided that the system is in the same set of models that is considered. Recently a projection method for closed-loop identification has been worked out by Forsell and Ljung (2000), which belongs to the framework of the two stage method, too.

The joint input-output approach usually uses the well known ordinary prediction error method to solve the closed-loop identification problem (Ljung, 1978). Here, there arises a problem of well-grounded determination of the form of the input-output relashionship of some system transfer function, because many of its models, such as the finite impulse response (FIR), a high order autoregressive model with external input (ARX), a finite number of alternative orthogonal functions, such as Laguerre functions or generalized versions, or a noncausal FIR model could be used (Wahlberg, 1991; Van den Hof and Schrama, 1993; Forsell and Ljung, 2000) in order to create the auxiliary input of the system. It is also important here to analyse the effect of the correlated aditive noise on the accuracy of the estimates of unknown parameters, obtained by processing the observations (Tontiruttananon and Tugnait, 2001).

In this paper, a two-stage method, applying the prediction error model, will be analyzed in respect of the form of an additive correlated noise transfer function. We determine here the input-output system transfer function used to generate the auxiliary input in the case of the known regulator as well as in the opposite case. In this connection, the optimal structure of an additive noise filter, that ensures the minimal value of the minimized criterion, is obtained. The accuracy of current estimates of the parameters, determined by using the two-stage method and the direct approach, both based on the prediction error method, will be investigated by computer and the Matlab package applied to generate discrete-time systems and signals (V.K. Ingle and J.G. Proakis, 1997; Pupeikis, 2000a, b, c).

2. Statement of the Problem

Assume that a control system to be observed is causal, linear, and time-invariant (LTI) with one output $\{y(k)\}$ $k = 1, 2, \ldots$ and one input $\{u(k)\}$ $k = 1, 2, \ldots$ and given by the equation

$$\begin{aligned} y(k) &= G_0(q,\, \theta\,)u(k) + v(k), \\ v(k) &= H_0(q, \varphi)\xi(k), \end{aligned} \tag{1}$$

that consists of two parts: a process model $G_0(q, \theta)$ and a noise one $H_0(q, \varphi)$.

Here θ, φ are unknown parameter vectors, q is the time-shift operator (i.e., $q^{-1}u(k) = u(k-1)$), the initial signal $\{\xi(k)\}$ $k = 1, 2, \ldots$ used to generate unmeasurable noise $\{v(k)\}$ $k = 1, 2, \ldots$ is assumed to be statistically independent and stationary with

$$\begin{aligned} &E\{\xi(k)\} = 0, \\ &E\{\xi(k)\xi(k+\tau)\} = \sigma_\xi^2 \delta(\tau). \end{aligned} \tag{2}$$

$E\{\cdot\}$ is a mean value, σ_ξ^2 is the variance, $\delta(\tau)$ is the Kronecker delta function, $H_0(q,\varphi)$ is an inversely stable, monic filter, the input $\{u(k)\}$ $k=1,2,\ldots,N$ is given by

$$u(k) = \Psi\big(k, y^k, u^{k-1}, r(k)\big), \tag{3}$$

where $y^k = \big[y(1),\ldots,y(k)\big]$, $u^{k-1} = \big[u(1),\ldots,u(k-1)\big]$. The reference signal $\{r(k)\}$ $k=1,2,\ldots$ is a quasi-stationary signal, independent of the stochastic disturbance $\{v(k)\}$ $k=1,2,\ldots$, and Ψ is a given deterministic function such that the closed-loop system (1), (2) with the controller $G_R(q,\alpha)$ (see Fig. 1), which is designed for disturbance $\{v(k)\}$ $k=1,2,\ldots$ by minimizing a quadratic performance function

$$J = \lim_{N\to\infty} E\left\{\frac{1}{N}\sum_{k=0}^{N-1} y^2(k) + \rho\, u^2(k)\right\}, \tag{4}$$

is exponentially stable (Forsell and Ljung, 1999). Here α is the parameter vector of the controller, the factor $0 < \rho \leqslant 1$.

The basis of identification is the data set

$$Z^N = \{r(1),\ldots,r(N), u(1),\ldots,u(N)\},$$

when the regulator $G_R(q,\alpha)$ is known, and the data set

$$\tilde{Z}^N = \big\{r(1),\ldots,r(N), u(1),\ldots,u(N), y(1),\ldots,y(N)\big\}, \tag{5}$$

in the opposite case. The first data set consists of measured observations of the reference signal $\{r(k)\}$ and input $\{u(k)\}$ $k=1,2,\ldots,N$, while the other one consists of the same observations, including the measurements of the noisy output $\{y(k)\}$ $k=1,2,\ldots,N$, too. The aim of the given paper is to investigate the two-stage approach in the case of additive correlated noise $\{\nu(k)\}$ $k=1,2,\ldots,N$, acting on the output of the system $G_0(q,\theta)$ to be identified.

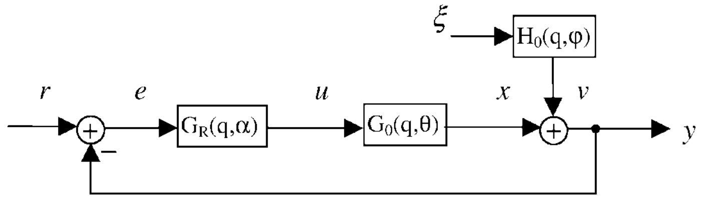

Fig. 1. A closed-loop system to be observed.

3. The Joint Input-Output Approach

An ordinary input-output approach is worked out for a closed-loop system, that, in comparison to the system shown in Fig. 1, has one difference: the controller has to be in the feedback. Therefore the augmented system according to Forssell and Ljung, 1999, is

$$\begin{bmatrix} y(k) \\ u(k) \end{bmatrix} = G_0(q)r(k) + H_0(q) \begin{bmatrix} e(k) \\ d(k) \end{bmatrix}, \tag{6}$$

supposing that the regulator is linear and of the form

$$u(k) = r(k) - G_R(q)y(k) + d(k). \tag{7}$$

Here

$$G_0(q) = \begin{bmatrix} G_0^c(q) \\ S_0^i(q) \end{bmatrix}, \quad H_0(q) = \begin{bmatrix} S_0(q)H_0(q) & G_0(q)S_0^i(q) \\ -G_R(q)S_0(q)H_0(q) & S_0^i(q) \end{bmatrix}, \tag{8}$$

$$\begin{aligned} G_0^c(q) &= S_0(q)G_0(q), \quad S_0(q) = \big(1 + G_0(q)G_R(q)\big)^{-1}, \\ S_0^i(q) &= \big(1 + G_R(q)G_0(q)\big)^{-1}, \end{aligned} \tag{9}$$

$e(k), d(k)$ are independent noise sources ($d(k)$ does not necessarily have to be white). The idea is to identify the augmented system (8) using a model of the form (Forsell and Ljung, 1999)

$$\begin{bmatrix} y(k) \\ u(k) \end{bmatrix} = G(q,\theta)r(k) + H(q,\varphi) \begin{bmatrix} e(k) \\ d(k) \end{bmatrix}, \tag{10}$$

where parametrizations of the transfer functions $G(q,\theta)$ and $H(q,\varphi)$ are not further specified, and to compute open-loop estimate $\hat{G}_N$ and $\hat{H}_N$. It is shown in (Forsell and Ljung, 1999) that different parametrizations lead to different methods. There are methods in which the reference signal $\{r(k)\}$ for $k = 1, 2, \ldots, N$ is assumed to be equal to zero and where $y^k = [y(1), \ldots, y(k)]$, $u^{k-1} = [u(1), \ldots, u(k-1)]$ are modelled jointly as time series. Then the open-loop system is reconstructed from the estimate of H_0 (Gustavsson *et al.*, 1977).

If the transfer function $G(q,\theta)$ in equation (10) is parametrized as

$$G(q,\theta) = \begin{bmatrix} G^{yr}(q,\theta) \\ G^{ur}(q,\theta) \end{bmatrix}, \tag{11}$$

and the parameter vector α is estimated using a fixed noise model $H(q,\varphi) = H_*(q)$, then the straightforward approach, according to the formula

$$\hat{G}_N(q) = \hat{G}_N^{yr}(q) \left(\hat{G}_N^{ur}(q)\right)^{-1}, \tag{12}$$

that simply divides the estimates $\hat{G}_N^{yr}$ and $\hat{G}_N^{ur}$ could be used in order to obtain an open-loop estimate $\hat{G}_N$. A prefilter could also be introduced here for shaping the bias distribution of the resulting models.

Recently, as noted in (Forsell and Ljung, 1999), the two-stage method (Van den Hof and Schrama, 1993) and the related projection method (Forsell and Ljung, 2000), which fall in the framework of the joint input-output approach, have been proposed. Both of them could be explained by using the following two steps (Forsell and Ljung, 1999):

(1) estimate the $\mu-$parameters in the model

$$u(k) = S(q,\mu)r(k) + H_1(q)e(k) \tag{13}$$

and generate a signal $\hat{u} = \hat{S}_N r$, where $\hat{S}_N$ is the estimate of the sensitivity function $S(q,\mu)$ of the augmented system determined by using N pairs of observations of the reference signal $\{r(k)\}$ and input $\{u(k)\}$ $k = 1, 2, \ldots, N$;

(2) identify the open-loop system using the model

$$y(k) = G(q,\lambda)\hat{u}(k) + H_2(q)e(k). \tag{14}$$

These methods could be seen as joint input-output methods where the correlation between the noise sources in equations (13), (14) is ignored. It could be mentioned that the two-stage method will fail in the case of the nonlinear controller. In such a case, this problem could be avoided if the projection method is used (Forssell and Ljung, 2000).

4. The Two-Stage Approach for the Prediction Error Model

The input signal $\{u(k)\}$and the output signal $\{y(k)\}$ $k = 1, 2, \ldots$ of the closed-loop system given in Fig. 1 are determined according to

$$u(k) = \big[r(k) - y(k)\big]G_R(q,\alpha) \tag{15}$$

and

$$y(k) = G_0(q,\theta)u(k) + H_0(q,\varphi)\xi(k), \tag{16}$$

respectively. By combining (15) and (16) we obtain the closed-loop relations

$$y(k) = \Phi_0(q,\beta)G_0(q,\theta)G_R(q,\alpha)r(k) + \Phi_0(q,\beta)H_0(q,\varphi)\xi(k), \tag{17}$$

$$u(k) = W_0(q,\beta)G_R(q,\alpha)r(k) - W_0(q,\beta)G_R(q,\alpha)H_0(q,\varphi)\xi(k), \tag{18}$$

with the output and input sensitivity functions

$$\Phi_0(q,\beta) = \big[1 + G_0(q,\theta)G_R(q,\alpha)\big]^{-1}, \tag{19}$$

$$W_0(q,\beta) = \big[1 + G_R(q,\alpha)G_0(q,\theta)\big]^{-1}, \tag{20}$$

correspondingly. Rewriting relation (18) in such a form

$$u(k) = W_0(q,\beta)G_R(q,\alpha)\big[r(k) - H_0(q,\varphi)\xi(k)\big], \tag{21}$$

we get

$$r(k) = C(q,c)u(k) + H_0(q,\varphi)\xi(k), \tag{22}$$

or

$$r^*(k) = C(q,c)u^*(k) + \xi(k), \tag{23}$$

where

$$r^*(k) = H_0^{-1}(q,\varphi)r(k), u^*(k) = H_0^{-1}(q,\varphi)u(k), \tag{24}$$
$$C(q,c) = W_0^{-1}(q,\beta)G_R^{-1}(q,\alpha) = G_R^{-1}(q,\alpha) + G_0(q,\theta). \tag{25}$$

The system, corresponding to expression (25), is presented in Fig. 2.

Introducing

$$\begin{aligned} G_R(q,\alpha) &= \frac{g_0 + g_1 q^{-1} + \ldots + g_\nu\, q^{-\nu}}{r_0 + r_1 q^{-1} + \ldots + r_v q^{-v}}, \\ G_0(q,\theta) &= \frac{b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots + a_m q^{-m}}, \end{aligned} \tag{26}$$

we could rewrite (25) in such an extended form

$$\begin{aligned} C(q,c) &= \frac{r_0 + r_1 q^{-1} + \ldots + r_v q^{-v}}{g_0 + g_1 q^{-1} + \ldots + g_\nu q^{-\nu}} + \frac{b_0 + b_1 q^{-1} + \ldots + b_m q^{-m}}{1 + a_1 q^{-1} + \ldots + a_m q^{-m}} \\ &= \frac{p_0 + p_1 q^{-1} + \ldots + p_n q^{-n}}{d_0 + d_1 q^{-1} + \ldots + d_w q^{-w}} = \frac{P(q,p)}{D(q,d)}. \end{aligned} \tag{27}$$

The estimate

$$\begin{aligned} \hat{c}(N) &= \big(\hat{p}(N), \hat{d}(N)\big)^T \\ &= \big(\hat{p}_0(N), \hat{p}_1(N), \ldots, \hat{p}_n(N), \hat{d}_0(N), \hat{d}_1(N), \ldots, \hat{d}_w(N)\big)^T \end{aligned} \tag{28}$$

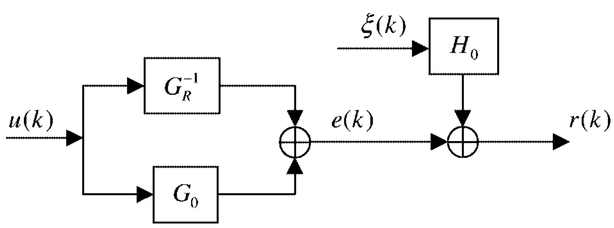

Fig. 2. The block scheme of the system, when the regulator is known.

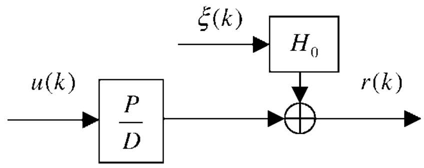

Fig. 3. The block scheme of the system, when the regulator is unknown.

of the vector

$$c = (p, d)^T = (p_0, p_1, \ldots, p_n, d_0, d_1, \ldots, d_w)^T \tag{29}$$

of parameters of the system with the transfer function $C(q, c)$, shown in Fig. 3, could be found as follows

$$\hat{c}_N = \arg\min_{c \in \Omega} Q_N(c), \tag{30}$$

by minimizing the criterion

$$Q_N(c) = \frac{1}{N} \sum_{k=1}^{N} \varepsilon^2(k, c). \tag{31}$$

Here

$$\varepsilon(k, c) = D(q, d) r^*(k) - P(q, p) u^*(k) \tag{32}$$

is a prediction error (Fig. 4), Ω is the area of permissible parameter values, restricted by the stability conditions of the respective linear difference equation.

Let us introduce the optimal solution

$$c^* = (p^*, d^*)^T = (p_0^*, p_1^*, \ldots, p_n^*, d_0^*, d_1^*, \ldots, d_w^*)^T, \tag{33}$$

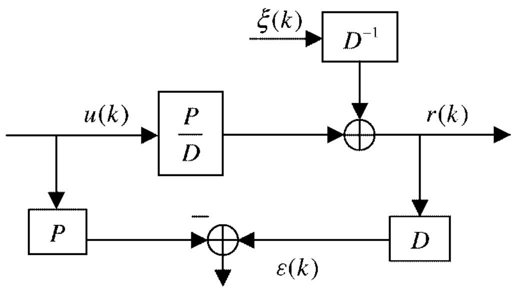

Fig. 4. The prediction error model for the system shown in Fig. 3.

consisting of the true values of the parameters. Then we get

$$\varepsilon(k, c^*) = D(q, d^*)r(k) - P(q, p^*)u(k) = \xi(k) \tag{34}$$

and,

$$Q_N(c^*) = \frac{1}{N}\sum_{k=1}^{N} \varepsilon^2(k, c^*) = \sigma_\xi^2, \tag{35}$$

respectively, if and only if

$$H_0(q, \varphi) = D^{-1}(q, d^*). \tag{36}$$

In such a case, the prediction error (34) has the zero mean $E\{e(k, c^*)\} = 0$, is noncorrelated and its correlation function could be expressed by the formula

$$K_{\varepsilon\varepsilon}(\tau) = \sigma_\varepsilon^2 \delta(\tau), \quad \delta(\tau) = \begin{cases} 1 & \text{for} \quad \tau = 0, \\ 0 & \text{for} \quad |\tau| \neq 0. \end{cases} \tag{37}$$

It follows, that the prediction error model, presented in Fig. 4, gives the minimal value of criterion (35) for $c = c^*$ when $H_0(q, \varphi)$ is of the form (36). It also means that the stochastic disturbance $\{v(k)\}k = 1, 2, \dots$ is an autoregressive (AR) process that is generated by filtering a white noise sequence $\{\xi(k)\}$ $k = 1, 2, \dots$ by a filter of the form (36).

5. The Recursive Estimation Procedure

For the estimation of unknown parameters, the ordinary prediction error method, based on the RLS of the form

$$\hat{c}(k) = \hat{c}(k-1) + \frac{P(k-1)z(k-1)}{1 + z^T(k)P(k-1)z(k)}\big[r(k) - z^T(k)\hat{c}(k-1)\big], \tag{38}$$

$$P(k) = P(k-1) - \frac{P(k-1)z(k)z^T(k)P(k-1)}{1 + z^T(k)P(k-1)z(k)}, \quad k = 1, 2, \dots, N \tag{39}$$

could be used with the vector of observations

$$z^T(k) = \big(r(k-1), \dots, r(k-w), u(k-1), \dots, u(k-n)\big) \tag{40}$$

and some initial values of the vector $\hat{c}(0)$ and matrix $P(0)$.

Here

$$\hat{c}^T(k) = \big(\hat{p}_0(k), \hat{p}_1(k), \dots, \hat{p}_n(k), \hat{d}_1(k), \hat{d}_2(k), \dots, \hat{d}_w(k)\big) \tag{41}$$

is a current estimate of the parameter vector

$$c^T = (p_0, p_1, \dots, p_n, d_1, d_2, \dots, d_w), \tag{42}$$

assuming that $d_0 = 1$. The next step is to calculate the current k-th value of the auxiliary input according to the formula

$$\hat{u}(k) = C^{-1}\big(q, \hat{c}(k)\big) r(k), \tag{43}$$

assuming that $C(q, \hat{c}(k))$ is inversely stable on each k-th iteration. Then the current estimates of the parameter vector

$$\theta^T = (a^T, b^T) = (a_1, a_2, \dots, a_m, b_0, b_1, \dots, b_m) \tag{44}$$

could be determined by the next RLS of the form

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\Gamma\,(k-1)\tilde{z}(k-1)}{1 + \tilde{z}^T(k)\Gamma\,(k-1)\tilde{z}(k)}\big[y(k) - \tilde{z}^T(k)\hat{\theta}(k-1)\big], \tag{45}$$

$$\Gamma(k) = \Gamma(k-1) - \frac{\Gamma(k-1)\tilde{z}(k)\tilde{z}^T(k)\Gamma(k-1)}{1 + \tilde{z}^T(k)\Gamma(k-1)\tilde{z}(k)}, \quad k = 1, 2, \dots, N \tag{46}$$

with the vector of observations

$$\tilde{z}^T(k) = \big(y(k-1), \dots, y(k-m), \hat{u}(k-1), \dots, \hat{u}(k-m)\big) \tag{47}$$

and some initial values of the vector $\hat{\alpha}(0)$ and matrix $\Gamma(0)$.

Here

$$\hat{\theta}^T(k) = \big(\hat{a}_1(k), \hat{a}_2(k), \dots, \hat{a}_m(k), \hat{b}_0(k), \hat{b}_1(k), \dots, \hat{b}_m(k)\big) \tag{48}$$

is an estimate of the parameter vector (44).

In the case of the known regulator the system is acting according to the scheme shown in Fig. 2. Then the estimation procedure is completely simplified, because the problem is only in determining the parameters $\theta^T = (a^T, b^T) = (a_1, a_2, \dots, a_m, b_0, b_1, \dots, b_m)$, using the RLS of the form

$$\hat{\theta}(k) = \hat{\theta}(k-1) + \frac{\Lambda\,(k-1)\bar{z}(k-1)}{1 + \bar{z}^T(k)\Lambda\,(k-1)\bar{z}(k)}\big[r^*(k) - \bar{z}^T(k)\hat{\theta}(k-1)\big], \tag{49}$$

$$\Lambda(k) = \Lambda(k-1) - \frac{\Lambda\,(k-1)\bar{z}(k)\bar{z}^T(k)\Lambda\,(k-1)}{1 + \bar{z}^T(k)\Lambda\,(k-1)\bar{z}(k)}, \quad k = 1, 2, \dots, N, \tag{50}$$

with the vector of observations

$$\bar{z}^T(k) = \big(r^*(k-1), \dots, r^*(k-m), u(k-1), \dots, u(k-m)\big), \tag{51}$$

$$r^*(k) = r(k) - G_R^{-1}(q, \alpha) u(k) \tag{52}$$

and some initial values of the vector $\hat{\theta}(0)$ and matrix $\Lambda(0)$.

6. Numerical Simulation

The closed-loop system to be simulated is shown in Fig. 5 and is described by the linear difference equation of the form

$$y(k) = 0.75u(k-1) + 0.985y(k-1) + \lambda_0\xi(k), \quad k = 0, 1, 2, \ldots, 200 \tag{53}$$

where $\xi(k)$ is a sequence of independent identically distributed variables with (2), λ_0 is a constant that determines the intensity of additive noise $\{v(k)\}$.

The controller design equation is

$$u(k) = e(k) + 0.1005u(k-1) - 0.1016u(k-2), \quad k = 0, 1, 2, \ldots, 200, \tag{54}$$

where

$$e(k) = r(k) - y(k). \tag{55}$$

In Fig. 6, the simulated input $\{u(k)\}$, noisy output $\{y(k)\}$ and the reference signal $\{r(k)\}$ of the closed-loop system, shown in Fig. 5, are presented. The sequences, including that of the noise $\{\xi(k)\}$ $k = 0, 1, 2, \ldots, 200$ with $\lambda_0 = 0.1$ and $\lambda_0 = 0.5$, were generated using the respective Matlab functions. The signals $\{r(k)\}$ and $\{u(k)\}$, presented in Fig. 7a, were processed by the RLS of the form (38)–(40) in order to obtain the estimates $\hat{p}_0(k), \hat{p}_1(k), \hat{p}_2(k), \hat{p}_3(k), \hat{d}_1(k)$ of the parameters p_0, p_1, p_2, p_3, d_1 of the transfer function

$$C(q, c) = \frac{p_0 + p_1q^{-1} + p_2q^{-2} + p_3q^{-3}}{1 + d_1q^{-1}}. \tag{56}$$

The estimates $\hat{p}_0(k), \hat{p}_1(k), \hat{p}_2(k), \hat{p}_3(k), \hat{d}_1(k)$ and the true values of the parameters p_0, p_1, p_2, p_3, d_1 (dotted lines) are shown in Fig. 7b–f, respectively. Here the true values of parameters are: $p_0 = 1$, $p_1 = -0.3355$; $p_2 = 0.2006$, $p_3 = -0.10011$, $d_1 = -0.985$. The estimates $\hat{p}_0(k), \hat{p}_1(k), \hat{p}_2(k), \hat{p}_3(k), \hat{d}_1(k)$, calculated for $k = 200$, were substituted into (56) instead of the true values of parameters p_0, p_1, p_2, p_3, d_1, which really are unknown. Afterwards, the auxiliary input sequence $\{\hat{u}(k)\}$ for $k = 0, 1, 2, \ldots, 200$ was

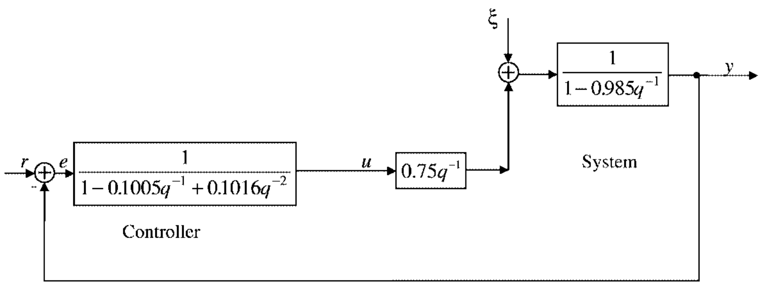

Fig. 5. Simulated closed-loop system with an additive correlated noise and the reference signal.

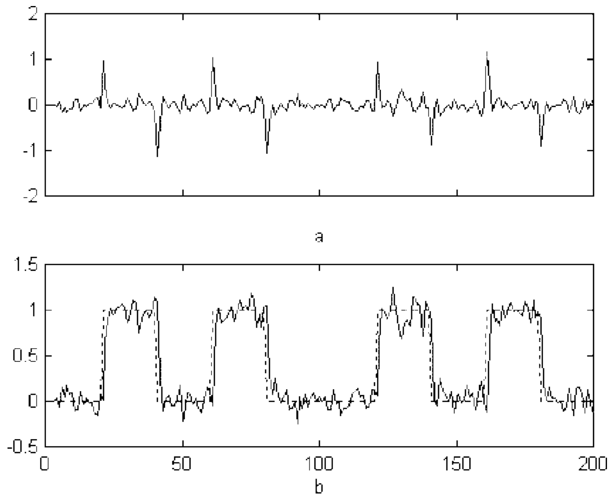

Fig. 6. Signals of the closed-loop sysstem (53)–(55) in the presence of additive noise on the output: x-axis – numbers of observations, y-axis – amplitudes, input $\{u(k)\}$ – a, noisy output $\{y(k)\}$, and the reference signal $\{r(k)\}$ (dotted line) – b, $\lambda_0 = 0.1$.

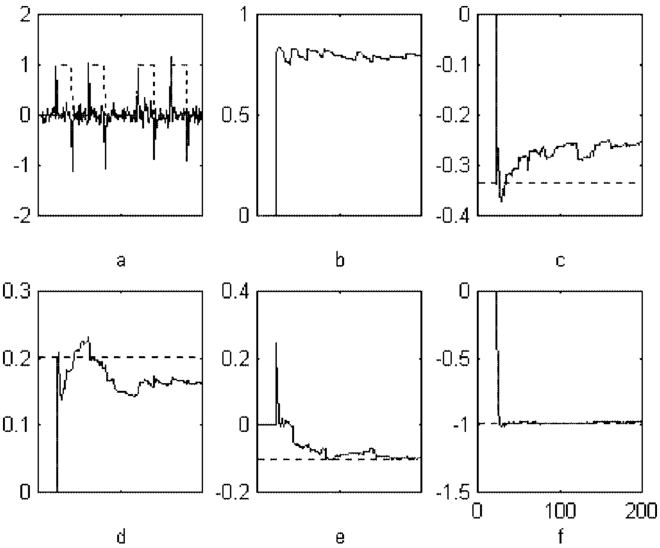

Fig. 7. Processed signals $\{u(k)\}$, $\{r(k)\}$ (dotted line) (a), the estimates $\hat{p}_0, \hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{d}_1$ and the true values of the parameters p_0, p_1, p_2, p_3, d_1 (b, c, d, e, f), respectively, x-axis – numbers of observations, y-axis – amplitudes of signals (7a) and the values of parameters (7b–7f), $\lambda_0 = 0.1$.

generated by filtering the reference signal $\{r(k)\}$ by the filter $C^{-1}(q, \hat{c})$, which was assumed to be inversely stable. Then we obtain

$$\hat{u}(k) = C^{-1}(q, \hat{c})r(k) = \frac{1 + \hat{d}_1 q^{-1}}{1 + \hat{p}_1 q^{-1} + \hat{p}_2 q^{-2} + \hat{p}_3 q^{-3} + \hat{p}_4 q^{-4}} r(k). \tag{57}$$

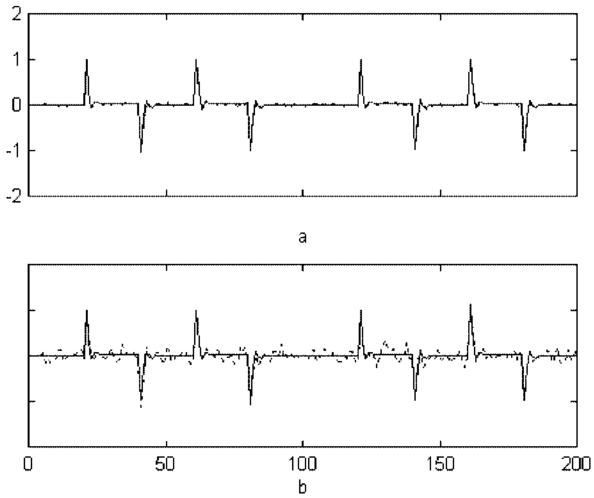

Fig. 8. The input sequences of the closed-loop system (53), (54), which are generated in an absence of $\{\xi(k)\}$ (continuous line), according to the formula (57) (dotted line in 8a) and according to the formula (54) in a presence of $\{\xi(k)\}$ (dotted line in 8b), $\lambda_0 = 0.1$.

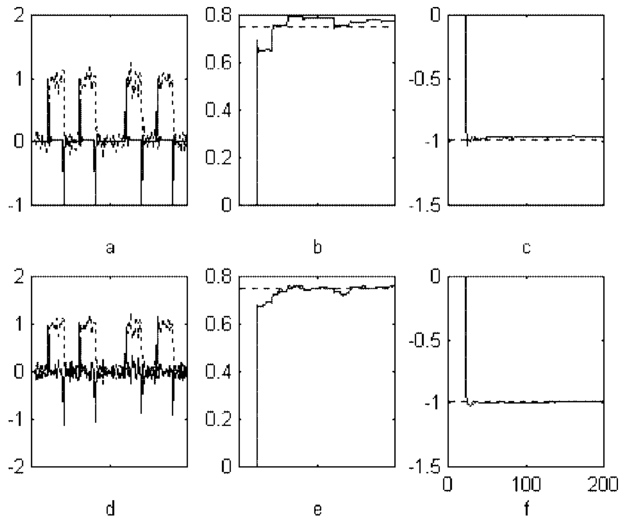

Fig. 9. Processed signals $\{\hat{u}(k)\}$, $\{y(k)\}$ (a), $\{u(k)\}$, $\{y(k)\}$ (d), the estimates $\hat{b}_0, \hat{a}_1$ and the true values b_0, a_1 (dotted lines) (b,c,e,f), respectively, x-axis – numbers of observations, y-axis – amplitudes of sugnals (a, d) and the values of parameters (b, c, e, f), $\lambda_0 = 0.1$.

The auxiliary input of the form (57) and inputs of the form (54), generated both in the presence and in the absence of additive noise $\{v(k)\}$, are shown in Fig. 8. It follows (Fig. 8a) that the auxiliary input (dotted line) approximates the true input (solid line) more exactly than the input, generated in the presence of $\{v(k)\}$ (Fig. 8b, dotted line).

The next step is estimation of the parameters $\theta^T = (a_1, b_0)$, when $a_1 = -0.985$ and $b_0 = 0.75$ using the two-stage method and the direct approach. The current pairs of signals $\{\hat{u}(k)\}$, $\{y(k)\}$ (Fig. 9a) and $\{u(k)\}$, $\{y(k)\}$ (Fig. 9d) were used in the RLS of the form (45)–(47) to calculate the estimates $\hat{a}_1(k), \hat{b}_0(k)$ by the two above mentioned techniques, respectively. The calculated estimates of b_0 were shown in Fig. 9 b, e while the estimates of a_1 – in Fig. 9c, f. In this case, the current estimates presented in Fig. 9b, c were calculated, using the two-stage method and signals shown in Fig. 9a while the ones presented on Fig. 9e, f – using the direct approach and signals shown in Fig. 9d. Similar results of parameter $\theta^T = (a_1, b_0)$ estimation by the same two approaches but only for $\lambda_0 = 0.5$ are shown in Fig. 11. It follows (Fig. 9) that for $\lambda_0 = 0.1$, the accuracy of estimates $\hat{a}_1(k), \hat{b}_0(k)$, calculated by both approaches, is more or less similar. For $\lambda_0 = 0.5$, the accuracy of estimates $\hat{a}_1(k)$, calculated using the direct approach (Fig. 11c) is higher as compared to the estimates, obtained by the two-stage method (Fig. 11f). On the other hand, in spite of this, the estimates $\hat{a}_1(k)$, determined by the direct approach for a large enough set of processed observations, are outside of stability area of the difference equation (53).

10 experiments with different realizations of additive noise $\{v(k)\}$ and different levels of its intensity were carried out in order to investigate more precisely and to compare the accuracy of estimates of the parameters $\theta^T = (a_1, b_0)$ obtained using the two-stage method and the direct approach. In each i-th experiment the estimates of parameters $a_1 = -0.985$ and $b_0 = 0,75$ were calculated, using the RLS of the form (45)–(47) and the above mentioned techniques. Figures 10, 12 and Table 1 illustrate the values $\bar{a}_1, \bar{b}_0$ of estimates $\hat{a}_1(k), \hat{b}_0(k)$, (averaged by 10 experiments), and their confidence intervals

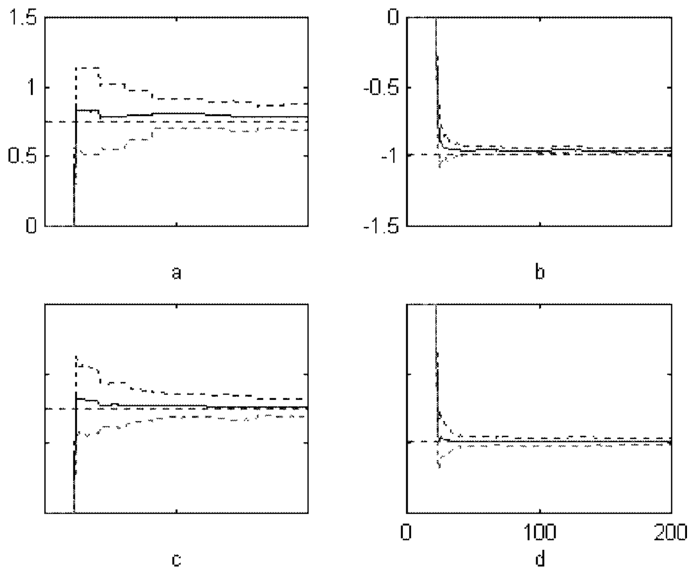

Fig. 10. Averaged values $\bar{b}(k), \bar{a}(k)$ (continuous line) and $\bar{a}(k) \pm \Delta_1, \bar{b}(k) \pm \Delta_2$ (dotted lines); $\bar{b}(k)$ – (a, c), $\bar{a}(k)$ – (b, d), $\lambda_0 = 0.1$.

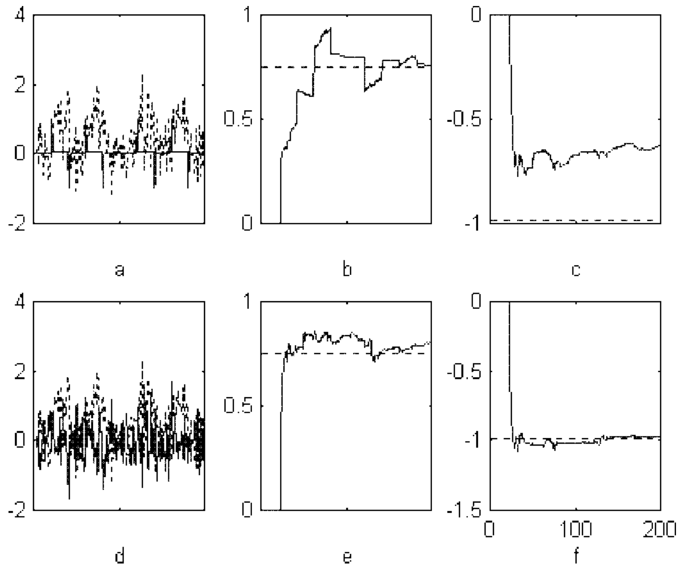

Fig. 11. The values and markings are the same as in Fig. 9, $\lambda_0 = 0.5$.

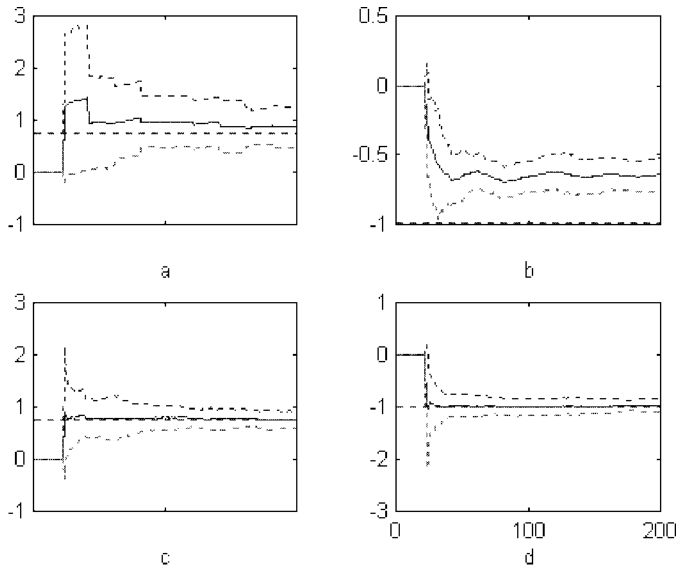

Fig. 12. The values and markings are the same as in Fig. 10, $\lambda_0 = 0.5$

obtained by the formulas

$$\Delta_1 = \pm t_\alpha \frac{\hat{\sigma}_a}{\sqrt{L}}, \quad \Delta_2 = \pm t_\alpha \frac{\hat{\sigma}_b}{\sqrt{L}}. \tag{58}$$

Table 1

Averaged values $\bar{a}_1, \bar{b}_0$ of estimates $\hat{a}_1(k), \hat{b}_0(k)$ and their confidence intervals (58) for different k and different intensity of additive noise

k	$\bar{a}_1 \pm \Delta_1$	$\hat{b}_0 \pm \Delta_2$
	$\lambda_0 = 0.1$	
50	-0.9564 ± 0.0318	0.7884 ± 0.2350
	-0.9830 ± 0.0389	0.7791 ± 0.1526
100	-0.9569 ± 0.0207	0.8083 ± 0.1085
	-0.9878 ± 0.0315	0.7730 ± 0.0870
150	-0.9575 ± 0.0222	0.7887 ± 0.1056
	-0.9853 ± 0.0284	0.7605 ± 0.0882
200	-0.9567 ± 0.0223	0.7868 ± 0.0915
	-0.9809 ± 0.0232	0.7579 ± 0.0591
	$\lambda_0 = 0.5$	
50	-0.6933 ± 0.1890	0.8920 ± 0.9324
	-1.0221 ± 0.2498	0.8295 ± 0.3575
100	-0.6729 ± 0.1361	0.8253 ± 0.5218
	-0.9867 ± 0.1477	0.7858 ± 0.1886
150	-0.6739 ± 0.0935	0.7600 ± 0.3281
	-0.9982 ± 0.1122	0.7807 ± 0.1889
200	-0.6815 ± 0.0629	0.7256 ± 0.3709
	-1.0077 ± 0.1015	0.7756 ± 0.2111

Here $\hat{\sigma}_a, \hat{\sigma}_b$ are estimates of the variances σ_a and σ_b; respectively, $\alpha = 0.05$ is the significance level; $t_\alpha = 2.26$ is the $100(1-\alpha)\%$ point of Student's distribution with $\nu = L - 1$ degrees of freedom; $L = 10$ is the number of experiments.

In this connection, in Table 1, the first line for each number k of the processed observations corresponds to the estimates, calculated using the two-stage method and the signals $\{\hat{u}(k)\}$, $\{y(k)\}$, while the second line – to the estimates, obtained using the direct approach and the signals $\{u(k)\}$, $\{y(k)\}$, respectively. It should be noted that from the simulation results, presented in Figures 10, 12 and Table 1, imply that the accuracy of the estimates $\bar{a}_1, \bar{b}_0$, calculated by the direct approach, is higher for both values λ_0. On the other hand, in this case, there is a possibility to obtain the unstable model of system (53) because of the current values of $\hat{a}_1(k)$. It appears that with an increase of λ_0 the instability grows and may become almost real.

7. Conclusions

The additive correlated noise in observations to be processed strongly influences the quality of the closed-loop identification. Therefore the simplicity of the direct approach and the accuracy of estimates calculated by it could be the main advantage in comparison with the joint input-output approach if not for one problem: with an increase of intensity of additive noise, the estimates of the parameter in the denominator of the open-loop system transfer function take the values outside of the stability area of parameters. On the other hand, the estimates obtained by the two-stage method are less accurate but they satisfy the stability conditions even under intensive noise.

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Apie sistemos identifikavimą, taikant uždarojo ciklo stebėjimus

Rimantas PUPEIKIS

Straipsnyje plėtojamas jungtinis įėjimo-išėjimo metodas sistemoms su grįžtamuoju ryšiu (1)–(4), kai sistemos išėjime veikia adityvusis koreliuotasis triukšmas. Nustatytos uždarojo ciklo bei adityviojo triukšmo sistemų perdavimo funkcijos (27), (36), suteikiančios kriterijui (35) minimalią reikšmę. Gauti dinaminės sistemos (53)–(55) (5 pav.) signalų bei parametrų skaičiavimų rezultatai (6–12 pav., 1 lentelė), taikant jungtinio įėjimo-išėjimo ir tiesioginio metodų algoritmus.