

# On the Generative Capacity of Contextual Grammars with Catenation. Necessary Conditions

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**Abstract.** In this paper our attention is focused on the study of the properties for some families of contextual languages with catenation (see Fortiș, 1999, also Păun, 1982; Păun, 1997). Also, we are defining and analyzing some new properties characteristic to these families of languages. Using these properties we are able to establish some necessary conditions, and some pumping properties for families of contextual languages with catenation.

**Key words:** Marcus contextual grammar, contextual grammar, contextual grammar with choice, contextual grammar with  $F$ -selection.

## 1. Introduction

In the original definition for contextual grammars with choice,  $G = (V, A, C, \varphi)$ , the generated language is the smallest language satisfying the following requirements (see Marcus, 1969; Păun, 1982):

1. every word in  $A$  is also in  $L(G)$ ;
2. for every word  $x \in L(G)$  and for a word  $z \in A$ , and a context  $(u, v) \in \varphi(x)$ , the words  $uxv$ ,  $xz$  and  $zx$  are also in  $L(G)$ .

In this form, this definition has offered some support for modeling elaborate constructions in natural languages.

However, the construction of propositions, phrases or texts in a natural language can be done by using, together with the use of contexts, the operation of word catenation. Thus, we are able to obtain generative devices which extend the generative power of classical (Marcus) contextual grammars (see also Fortiș, 1999).

## 2. Basic Definitions

DEFINITION 1. A contextual grammar with choice is a construct

$$G = (V, A, C, \varphi)$$

where  $V$  is an alphabet,  $A$  is a finite language over  $V$ ,  $C$  is a finite subset of  $V^* \times V^*$ , and  $\varphi : V^* \rightarrow 2^C$ .

The strings of  $A$  are called axioms, the elements  $(u, v) \in C$  are called contexts,  $\varphi$  is the selection/choice mapping.

For a contextual grammar defined as above, we can define two relations over  $V^*$ : for  $x, y \in V^*$ , we write

$$\begin{aligned} x \Longrightarrow_{ex} y \quad \text{iff} \quad & y = uxv, \quad \text{for } (u, v) \in \varphi(x), \\ x \Longrightarrow_{in} y \quad \text{iff} \quad & x = x_1x_2x_3, y = x_1ux_2vx_3, \quad \text{for} \\ & x_1, x_2, x_3 \in V^*, \quad (u, v) \in \varphi(x). \end{aligned}$$

We say that  $\Longrightarrow_{ex}$  is an external direct derivation in  $G$ , and  $\Longrightarrow_{in}$  is an internal derivation step.

The language generated by a contextual grammar with choice,  $L_{ex}(G)$ , is the smallest language  $L \subseteq V^*$  such that:

1.  $A \subseteq L$ .
2. if  $x \in L$  and  $(u, v) \in \varphi(x)$ , then  $uxv \in L$ .

The language  $L_{in}(G)$  is the smallest language  $L \subseteq V^*$  such that:

1.  $A \subseteq L$ .
2. if  $x \in L$  and  $x = x_1x_2x_3$ , for  $x_1, x_2, x_3 \in V^*$ , and  $(u, v) \in \varphi(x_2)$ , then  $x_1ux_2vx_3 \in L$ .

Denote the family of contextual languages (with choice) externally generated by  $ECC$ , and the family of contextual languages (with choice) internally generated by  $ICC$ .

**DEFINITION 2.** A total contextual grammar is a construct

$$G = (V, A, C, \varphi),$$

where  $V$  is an alphabet,  $A$  is a finite subset of  $V^*$ ,  $C$  is a finite subset of  $V^* \times V^*$ , and  $\varphi : V^* \times V^* \times V^* \rightarrow 2^C$ .

We define the derivation relation as follows: for  $x, y \in V^*$ , we write

$$\begin{aligned} x \Longrightarrow_{in} y \quad \text{iff} \quad & x = x_1x_2x_3, y = x_1ux_2vx_3, \quad \text{for} \\ & x_1, x_2, x_3 \in V^*, (u, v) \in \varphi(x_1, x_2, x_3). \end{aligned}$$

Denote the family of languages generated by total contextual grammars by  $TC$ .

Starting from the definition of a (simple) contextual grammar, one can get the following definition

**DEFINITION 3.** A contextual grammar with catenation is a construct

$$G = (V, A, C),$$

where  $V$  is an alphabet,  $A$  is a finite language over  $V$  and  $C$  is a finite set of contexts over  $V$ .

The language generated by a contextual grammar with catenation is the smallest language  $L$  satisfying the following requirements:

1.  $A \subseteq L$ ;
2. for a word  $x \in L$  and a context  $(u, v) \in C$ , the word  $uxv$  is in  $L$ ;
3. for the words  $x, y \in L$ , the word  $xy$  is in  $L$ .

Denote the family of contextual languages with catenation by  $CCON$ .

Related to a contextual grammar with catenation one can define the following relation of derivation: for  $x, y \in V^*$ , we write

$$x \Longrightarrow y \quad \text{iff} \quad \begin{array}{l} y = uxv, \quad \text{where } (u, v) \in C \quad \text{or} \\ y = xz \quad \text{or} \quad y = zx, \quad \text{where } z \in L(G). \end{array}$$

Using the notation  $\Longrightarrow^*$  for the reflexive and transitive closure of the relation  $\Longrightarrow$  one can associate with a contextual grammar with catenation,  $G$ , the following language

$$L(G) = \{x \in V^* \mid a \Longrightarrow^* x, \text{ for every } a \in A\}.$$

A natural idea is to restrict the use of contexts and catenation in order to generate a word for a contextual grammar with catenation. This idea is leading us to the following

**DEFINITION 4.** A contextual grammar with catenation with choice is a construct

$$G = (V, A, C, \varphi),$$

where  $V$  is an alphabet,  $A$  is a finite language over  $V$ ,  $C$  is a finite set of contexts over  $V$ , and  $\varphi : V^* \rightarrow 2^C$  is the function of selection (the selector).

The language generated by a contextual grammar with catenation, with choice is the smallest language  $L$  satisfying the following requirements:

1.  $A \subseteq L$ ;
2. for  $x \in L$  and  $(u, v) \in \varphi(x)$ ,  $uxv \in L$ ;
3. for  $x, y \in L$  and  $(\lambda, \lambda) \in \varphi(xy)$ ,  $xy \in L$ .

Denote the family of contextual languages with catenation, with selection by  $CSEL$ .

We can define now the relation of derivation for contextual grammars with catenation, with choice, as follows: for  $x, y \in V^*$ , we write

$$x \Longrightarrow y \quad \text{iff} \quad \begin{array}{l} y = uxv \quad \text{with } (u, v) \in \varphi(x) \quad \text{or} \\ y = xz, \quad \text{with } (\lambda, \lambda) \in \varphi(xz) \\ (y = zx, \quad \text{with } (\lambda, \lambda) \in \varphi(zx), \text{ respectively}). \end{array}$$

Using the notation  $\Longrightarrow^*$  for the reflexive and transitive closure of the relation  $\Longrightarrow$  one can associate with a contextual grammar with catenation, with choice,  $G$ , the following language

$$L(G) = \{x \in V^* \mid a \Longrightarrow^* x, \text{ for every } a \in A\}.$$

**DEFINITION 5.** A contextual grammar with catenation, in modular presentation, is a system

$$G = (V, A, P),$$

where  $V$  is an alphabet,  $A$  is a finite language over  $V$  and  $P$  is a finite set of pairs of the form  $(S, C)$ , with  $S \subseteq V^*$ ,  $S \neq \emptyset$  and  $C$  is a finite subset of  $V^* \times V^*$ .

The pairs  $(S, C)$  are called productions,  $S$  is the selector of a production and  $C$  is its set of contexts.

For a contextual grammar with catenation, in modular presentation, the relation of derivation can be defined as follows: for  $x, y \in V^*$  and the production  $(S, C) \in P$  we write

$$\begin{aligned} x \Longrightarrow_{ex} y \text{ iff } & y = uxv, \text{ with } (x, (u, v)) \in (S, C), \text{ or} \\ & y = xz, \text{ with } (xz, (\lambda, \lambda)) \in (S, C) \\ & (y = zx, \text{ with } (zx, (\lambda, \lambda)) \in (S, C), \text{ respectively}). \end{aligned}$$

We say that a contextual grammar with catenation, in modular presentation,

$$G = (V, A, (S_1, C_1), \dots, (S_n, C_n)),$$

is of type  $F$  (or with  $F$  selection), if every selector  $S_i$ , for  $1 \leq i \leq n$ , is from the family  $F$ . Denote by  $CCON(F)$  the family of contextual languages with catenation, in modular presentation, of type  $F$ .

Also, we can consider the following relation of derivation: for  $x, y \in V^*$  and  $(S, C) \in P$  we write

$$\begin{aligned} x \Longrightarrow_{in} y \text{ iff } & y = x_1ux_2vx_3, \quad \text{where } (x_2, (u, v)) \in (S, C), \quad \text{or} \\ & y = x_1x_2zx_3, \quad \text{with } (x_2z, (\lambda, \lambda)) \in (S, C) \\ & (y = x_1zx_2x_3, (zx_2, (\lambda, \lambda)) \in (S, C), \quad \text{respectively}), \\ & \text{and } x = x_1x_2x_3, z \in L(G). \end{aligned}$$

Denote by  $CCON_{in}$  the family of contextual languages with catenation, with internal derivation and by  $CCON_{in}(F)$ , the family of contextual languages with catenation, in modular presentation, of type  $F$ , with internal derivation.

### 3. Generative Capacity. Necessary Conditions

In what follows, we shall review some of the properties presented in (Păun, 1997). Also, we define new properties, characteristic to contextual languages with catenation.

**DEFINITION 6.** We say that a language  $L \subseteq V^*$  has the *EBS* (external bounded step) property if there is a constant  $p$  such that for each  $x \in L$ ,  $|x| > p$ , there is  $y \in L$  such that  $x = uyv$  and  $0 < |uv| \leq p$  (Păun, 1997).

As proved in (Păun, 1997), a language is *ECC* if and only if it possesses the *EBS* property. For the families of contextual languages with catenation, the following results hold:

**Lemma 1.** *The family  $CCON$  does not possess the *EBS* property.*

*Proof.* It is easy to verify that the language  $D_{\{a,b\}}$  (the Dyck language over  $\{a, b\}$ ) is *CCON* and it is not *ECC*.

By using this result, we can easily establish the following

**Lemma 2.** *None of the families  $CCON_\omega$ ,  $CSEL_\omega$  with  $\omega \in \{in, ex\}$  does possess the *EBS* property.*

Notice that the negative results regarding the *EBS* property occurs because of the operation of catenation involved in the definition of a contextual grammar with catenation (with internal or external derivation). Thus, we can consider the following properties:

**DEFINITION 7.** We say that a language  $L \subseteq V^*$  has the *EBSC* (external bounded step with catenation) property if there is a constant  $p$  such that for each  $x \in L$ ,  $|x| > p$ , there are  $y, z \in L \cup \{\lambda\}$  such that  $x = uyzv$  ( $uzyv$ , respectively),  $|uzv| > 0$ ,  $|uv| \leq p$  and  $|yz| > 0$ .

**DEFINITION 8.** We say that a language  $L \subseteq V^*$  has the *LEBSC* (external bounded step with limited catenation) property if there is a constant  $p$  such that for each  $x \in L$ ,  $|x| > p$ , there are  $y, z \in L \cup \{\lambda\}$  such that  $x = uyzv$  ( $uzyv$ , respectively),  $0 < |uzv| \leq p$  and  $|yz| > 0$ .

**Lemma 3.** *Every  $CSEL$ ,  $CCON(F)$  languages possess the *EBSC* property.*

*Proof.* Let  $L$  be a *CSEL* language,  $x$  a word in  $L$  and the constant  $p$  defined as follows

$$p = 2 \cdot \max\left\{\max_{a \in A}\{|a|\}, \max_{(u,v) \in C}\{|uv|\}\right\},$$

such that  $|x| > p$ . Then  $x$  is of one of the following forms

1.  $x = ux_1v$ , with  $(u, v) \in C$ ,  $x_1 \in L$ .

In this situation, choose  $y = x_1$ ,  $z = \lambda$ .

2.  $x = x_1x_2$ , with  $x_1, x_2 \in L$ .

Now, choose  $y = x_1$ ,  $z = x_2$  and  $(u, v) = (\lambda, \lambda)$ .

In both cases, the conditions of the property *EBSC* are verified, thus the language  $L$  posses the *EBSC* property.

REMARK 1. Every language  $L$  that posses the *EBS* property also posses the *EBSC* property. The reverse does not hold.

It is enough to observe that the language  $D_{\{a,b\}}$  has the *EBSC* property (this language is *CSEL*, so we can use the result from Lemma 3).

Also, for the families *ICC* and *CSEL<sub>in</sub>*, the following result holds

**Lemma 4.** *None of the families ICC, CSEL<sub>in</sub> does posses the EBSC property.*

*Proof.* Let us consider the following *ICC* language (observe that this language is also *CSEL<sub>in</sub>*),

$$L = \{a^n b^n c | n \geq 0\}.$$

The language  $L$  does not posses the *EBS* property, so it does not posses the *EBSC* property either.

Next, we can establish the following result

**Lemma 5.** *A language is CSEL if and only if it posses the EBSC property.*

*Proof.* Let  $L \subseteq V^*$  a language possessing the *EBSC* property for a constant  $p$ . Construct the following *CSEL* grammar,  $G = (V, A, C, \varphi)$ , where

$$\begin{aligned} A &= \{x \in L | |x| \leq p\} \\ C &= \{(u, v) | u, v \in V^*, 0 < |uv| \leq p\}, \\ \varphi(x) &= \{(u, v) \in C | u xv \in L\}, x \in V^*. \end{aligned}$$

Next, for every word of the form  $xz$ , with  $z \in L$ , we add to the set  $\varphi(xz)$ , constructed as above, the set  $\{(\lambda, \lambda) | \text{if } xz \in L\}$ , and to the set  $\varphi(zx)$ , the set  $\{(\lambda, \lambda) | \text{if } zx \in L\}$ , with  $x \in V^*$ .

The equality  $L = L(G)$  results immediately, from the definitions of  $A$ ,  $C$  and  $\varphi$ .

DEFINITION 9. A language  $L \subseteq V^*$  has the *IBS* (*internal bounded step*) property if there is a constant  $p$  such that for each  $x \in L$  with  $|x| > p$ , there is  $y \in L$  such that  $x = x_1 u x_2 v x_3$ ,  $y = x_1 x_2 x_3$  and  $0 < |uv| \leq p$  (Păun, 1997).

A language  $L \subseteq V^*$  has the *BLI* (bounded length increase) if there is a constant  $p$  such that for each  $x \in L$  with  $|x| > p$  there is  $y \in L$  with  $0 < |x| - |y| \leq p$  (Păun, 1997).

**Lemma 6.** *The family CCON has the IBS property.*

*Proof.* For a language  $L \in CCON$  it is enough to choose a value

$$p = 3 \cdot \max\{\max_{a \in A}\{|a|\}, \max_{(u,v) \in C}\{|uv|\}\}.$$

Then a word  $x \in L$  with  $|x| > p$  is of one of the following forms

1.  $x = u_1 u_2 \dots u_n a v_n \dots v_2 v_1$ . Choose  $u = u_i$ ,  $v = v_i$  with  $1 \leq i \leq n$ . Then the word  $y = u_1 u_2 \dots u_{i-1} u_{i+1} \dots u_n a v_n \dots v_{i+1} v_{i-1} \dots v_2 v_1$  is also in  $L$  and  $0 < |uv| \leq p$
2.  $x = u_1 x_1 x_2 v_1$ , with  $u_1 x_1 v_1, x_2 \in L$ . Next we analyze the word  $x_2 \in L$ . If the form of this word is the form from 1, it is easy to find the words  $u, y, v \in V^*$  satisfying the conditions of the property *IBS*; otherwise we shall continue our analysis with a subword of  $x_2$ , let this word be  $x_2^{(1)} \in L$ . After a finite number of steps we can, eventually, identify the words  $x_2^{(k)}, x_2^{(k+1)} \in L$ , with  $x_2^{(k+1)} \in \text{Sub}(x_2^{(k)})$  and  $0 < |x_2^{(k+1)}| \leq p$ . In this situation choose  $u = x_2^{(k+1)}$  and  $v = \lambda$ . The word  $y$  obtained by eliminating the substring  $x_2^{(k+1)}$  from the word  $x$  is also in  $L$ , and  $0 < |uv| \leq p$ .

For the *CSEL* family, the result is negative:

**Lemma 7.** *The family CSEL does not possess the IBS property.*

*Proof.* In order to prove this result, we shall consider the language

$$L = \{a^{2^n} \mid n \geq 0\}.$$

This language is generated by the following CSEL grammar

$$G = (\{a\}, \{a\}, \varphi),$$

where  $\{(\lambda, \lambda)\} \subseteq \varphi(a^{2^m})$  for  $m \geq 1$ . This language is not with bounded length increase, because the difference between two consecutive words in this language is  $2^k - 2^{k-1}$ , for  $k > 0$ . Consequently, *CSEL* does not possess the *IBS* property.

REMARK 2. From the result in Lemma 7 it follows that there are *CSEL* languages that are not *TC*. In fact, the families *TC* and *CSEL* are incomparable.

Also, from the proof for Lemma 7, one can establish the following Lemma:

**Lemma 8.** *The family CSEL does not possess the BLI property.*

*Proof.* It is enough to observe that the proof for Lemma 7 is based on the fact that the property IBS does imply the property BLI.

Now it is not hard to establish a similar result for  $CCON_{in}$ ,

**Lemma 9.** *The family  $CCON_{in}$  does possess the IBS property.*

As for the case of the EBS property, we can introduce the following properties

**DEFINITION 10.** A language  $L \subseteq V^*$  has the *IBSC (internal bounded step with catenation)* property if there is a constant  $p$  such that for each  $x \in L$ , with  $|x| > p$ , there are  $y, z \in L \cup \{\lambda\}$ , where  $x = x_1ux_2zvz_3$  (or  $x = x_1uzx_2vx_3$ ),  $y = x_1x_2x_3$ ,  $|uzv| > 0$  and  $|uv| \leq p$ .

**DEFINITION 11.** A language  $L \subseteq V^*$  has the *LIBSC (limited internal bounded step with catenation)* property if there is a constant  $p$  such that for each  $x \in L$ , with  $|x| > p$ , there are  $y, z \in L \cup \{\lambda\}$ , where  $x = x_1ux_2zvz_3$  (or  $x = x_1uzx_2vx_3$ ),  $y = x_1x_2x_3$ , and  $0 < |uzv| \leq p$ .

Now we can prove that

**Lemma 10.** *The families  $CCON_{in}$ ,  $CSEL_{in}$  does possess the IBSC property.*

*Proof.* Indeed, if  $x \in L$  with  $|x| > p$ , where  $p$  is an integer convenient chosen for a language  $L \in CSEL_{in}$ , it follows that

$$\begin{aligned} x = x_1ux_2vx_3, & \quad \text{with} \quad x_1x_2x_3 \in L, (u, v) \in C \text{ or} \\ x = x_1x_2zx_3, & \quad \text{with} \quad x_1x_2x_3 \in L, z \in L \text{ or} \\ x = x_1zx_2x_3, & \quad \text{with} \quad x_1x_2x_3 \in L, z \in L. \end{aligned}$$

so the language  $L \in CSEL_{in}$  possesses the IBSC property. One can choose the value  $p$  as follows:

$$p = \max\left\{\max_{(u,v) \in C}\{|uv|\}, \max_{a \in A}\{|a|\}\right\}.$$

**Lemma 11.** *Each of the LIBSC and IBSC properties imply the IBS property.*

**REMARK 3.** Every  $CSEL_{in}$  language satisfying the LIBSC property is ICC.

*Proof.* Let  $L$  be a  $CSEL_{in}$  language satisfying the LIBSC property and  $G = (V, A, C, \varphi)$  be a  $CSEL_{in}$  grammar generating the language  $L$ ,  $L = L(G)$ . Then in every



derivation of a word  $x \in L$  we can use only catenation with words from the language  $\{z \in V^* \mid |z| \leq p\}$ .

This language being finite, we can add contexts of the form  $(\lambda, z)$  and  $(z, \lambda)$ , where  $z \in V^*$ , with  $|z| \leq p$ , to the set of contexts,  $C$ .

Also, we can modify the selector  $\varphi$  (and construct a selector  $\varphi'$ ) such that if  $\varphi(xz) = \{(\lambda, \lambda)\}$  in the original  $CSEL_{in}$  language, we have that  $\{(\lambda, z)\} \in \varphi'(x)$ .

**DEFINITION 12.** A language  $L \subseteq V^*$  has the *IAP* property if the length set of  $L$  contains infinite arithmetic progression (Păun, 1997).

**Lemma 12.** *The family CSEL does not possess the IAP property.*

*Proof.* We shall consider the following sets

$$L_k = \left\{ a^i \mid 2^{2k+1} \leq i \leq 2^{2(k+1)} \right\}.$$

The language  $L = \bigcup_{k \geq 0} L_k$  is *CSEL*, but does not have the *IAP* property.

Indeed, the following  $CSEL_{in}$  grammar,

$$G = (\{a\}, \{a, a^2\}, \{(\lambda, \lambda), (\lambda, a)\}, \varphi),$$

where

$$\varphi(a^i a^j) = \begin{cases} \{(\lambda, \lambda)\}, & \text{if } 2^{2(k+1)} \geq i + j \geq 2^{2k+1}, k \geq 0 \\ \{(\lambda, a)\}, & \text{if } 2^{2(k+1)} > i + j \geq 2^{2k+1}, k \geq 0 \end{cases}$$

generates the language  $L(G) = \{a^k \mid 2^{2(k+1)} \geq k \geq 2^{2k+1}, k \geq 0\}$ .

**Lemma 13.** *For languages in CSEL there are no pumping properties.*

*Proof.* Consider a language  $L \subseteq V^*$  and  $c \notin V$ . Then the language  $L_1 = V^* \cup L\{c\}$  is *CSEL*. On the other hand, consider the language  $L$  from the proof of Lemma 12. Then no substring of a word  $a^i c$ , with  $a^i \in L$ ,  $i \geq 0$  can be pumped.

As expected, the following Lemma holds

**Lemma 14.** *The families CCON, CCON<sub>in</sub> have the IAP property. The family CSEL<sub>in</sub> has the IAP property.*

*Proof.* It is enough to consider a word  $x \in L$ , where  $L$  is a language from *CCON*, *CCON<sub>in</sub>*. Then each of the words  $\underbrace{x \dots x}_m$ , with  $m \geq 1$ , is in  $L$ .

For the family  $CSEL_{in}$  we can construct the following result

**Lemma 15.** *If  $L \subseteq V^*$ ,  $L \in CCON_{in}$ , there are two constants  $p, q$  such that every  $z \in L$ ,  $|z| > p$ , can be written in the form  $z = uvwxy$  with  $u, v, w, x, y \in V^*$ ,  $0 < |vx| \leq q$ , and  $uv^iwx^iy \in L$  for all  $i \geq 0$ .*

*Proof.* Let us consider the following values for  $p$  and  $q$

$$q = 2 \cdot \max\{\max\{|uv|, (u, v) \in C\}, \max\{|z|, z \in A\}\},$$

$$p = 3 \cdot \max\{\max\{|uv|, (u, v) \in C\}, \max\{|z|, z \in A\}\}.$$

Let  $z \in L$ , with  $|z| > p$ . Then the word  $z \in L$  can be written in one of the following forms:

1.  $z = z_1t_1z_2t_2z_3$ , with  $(t_1, t_2) \in C$ ,  $z_1z_2z_3 \in L$ . Choose  $u = z_1$ ,  $v = t_1$ ,  $w = z_2$ ,  $x = t_2$  and  $y = z_3$ . Then  $0 < |vx| \leq q$ . Also, observe that there exist the derivation

$$a \Longrightarrow^* z_1z_2z_3 \Longrightarrow z_1t_1z_2t_2z_3$$

$$\Longrightarrow^* z_1t_1^i z_2t_2^i z_3, \text{ with } a \in A, i > 0.$$

so  $uv^iwx^iy \in L$  for all  $i \geq 0$ .

2.  $z = z^{(0)} = z_1z_2z^{(1)}z_3$ , with  $|z^{(1)}| < |z_1z_2z_3|$  ( $z = z_1xz_2z_3$ , with  $|z^{(1)}| < |z_1z_2z_3|$ , respectively). In this situation, we focus our attention on the subword  $z^{(1)}$  and repeat the analysis for this subword of  $z$ . If this subword is of the form 1, then it is easy to finish our proof for this case also. Presume that  $z^{(1)}$  is of the same form as  $z^{(0)}$ . Then, using mathematical induction, we can identify either an axiom  $a \in A$  such that  $z = z'_1z'_2az'_3$  ( $z = z'_1az'_2z'_3$ , respectively), and  $z'_1z'_2z'_3 \in L$ ; or a context  $(t_1, t_2) \in C$ , such that  $z = z'_1t'_1z'_2t'_2z'_3$ . Then, for the first situation, choose  $u = z'_1$ ,  $v = \lambda$ ,  $w = z'_2$ ,  $x = a$ ,  $y = z'_3$ . Then  $0 < |vx| \leq q$ . Also, observe that there exist the derivation

$$a_1 \Longrightarrow^* z'_1z'_2z'_3 \Longrightarrow z'_1z'_2az'_3$$

$$\Longrightarrow^* z'_1z'_2a^iz'_3, \text{ with } a \in A, i > 0.$$

For the family  $CCON$ , since the inclusion  $CCON \subseteq CF$  holds, it follows that we can apply the Bar-Hillel Lemma.

Using a similar reasoning, we can produce a similar result for  $CSEL_{in}$  languages. Thus, we obtain that exist some pumping properties for the family  $CSEL_{in}$  also.

**REMARK 4.** For  $CSEL$  languages, there is no pumping properties, as established in Lemma 13. However, we can define a structure for the words generated by this family of languages by imposing more control in derivation. In this way we can obtain new families of languages, offering more properties (see Fortiș, 1998; Fortiș, 2000 and Vide-Păun, 1998).

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**Apie grandinių konteksto gramatikų generatyvinę galią. Būtinios sąlygos**

Teodor Florin FORTIŠ

Straipsnyje nagrinėjamos tam tikro grandinių konteksto gramatikų poklasio savybės. Išskiriamos ir analizuojamos kai kurios naujos šio poklasio gramatikomis nusakomų kalbų savybės. Naudojantis šiomis savybėmis yra apibrėžiamos būtinios generavimo sąlygos ir eilučių išsiurbos sąlygos.