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The Solution of Two-Dimensional Neutron Diffusion Equation with Delayed Neutrons

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Abstract. The distribution of neutron population in nuclear reactor is described by using transport equations. One of possible approximations of neutron transport equation is given by the neutron diffusion equation. The paper presents numerical solution method of one group neutron diffusion equation with one group of delayed neutrons.

Key words: neutron diffusion equation, the method of summary approximation, double sweep method.

1. Introduction

Nuclear fission discovered in 1939, showed great promise as a new source of energy, which could be converted into inexpensive electricity. In a nuclear reactor power, which is generally utilized to produce steam for the generation of electricity, appears due to nuclear fission. The fission process is associated with the release of a large amount of energy per unit mass of nuclear fuel. The fission reaction is initiated by neutrons and is accompanied with the liberation of neutrons. This phenomenon is implemented by the design of a nuclear reactor. Once the fission reaction has been started in a few nuclei by means of an external source of neutrons, it can be maintained in other nuclei by the help of neutrons produced in the reaction. The neutron population distribution in nuclear reactor is described by transport equations.

One of the simplest approximations to neutron transport that has been widely used in research and practice is the approximation given by diffusion theory. The diffusion equation describes the behavior of a large amount of neutrons when individual properties of separate neutron trajectories in matter are "averaged". Neutrons released in fission, can be divided into two categories namely, prompt neutrons and delayed neutrons. Prompt neutrons are released within about 10^{-14} s of the instant of fission. The emission of prompt neutrons ceases immediately after fission has occurred, and the delayed neutrons continue to be expelled over a period of several minutes.

2. Statement of the Problem

The purpose of this paper is to present a numerical solution method of one group neutron diffusion equation when delayed neutrons are averaged by one group of delayed neutrons (Almenas and Lee, 1992):

$$\begin{aligned} \frac{1}{v_c}\frac{\partial\,\varphi\,(x,t)}{\partial\,t} &= D\nabla^2\varphi(x,t) + (\gamma\Sigma_f - \Sigma_a)\varphi(x,t) + \lambda C(x,t), \\ \frac{\partial\,C(x,t)}{\partial\,t} &= \beta\gamma\Sigma_f\varphi(x,t) - \lambda C(x,t), \end{aligned} \tag{1}$$

with the initial $\varphi(x,0) = \varphi_0(x)$, $C(x,0) = C_0(x)$ and boundary conditions $\varphi(x_b,t)=0$.

Here v_c – neutron velocity, $\varphi = \varphi(x,t)$ – neutron flux, $C(x,t)$ – the density of precursors, D – neutron diffusion coefficient, γ – average number of neutrons produced per fission, Σ_f – macroscopic fission cross section, Σ_a – macroscopic absorption cross section λ-radioactive decay constant, β – fraction of the fission neutrons which are delayed, $x = (x_1, x_2)$ – point in the region V, x_b – point of V boundary.

Neutron diffusion equation is obtained under assumptions that scattering is isotropic in the laboratory system of coordinates, all neutrons have the same energy and region V is homogeneous. This leads to diffusion coefficient to be independent from the position in space.

3. Algorithm Description

To obtain the solution of system of equations (1) we will use the method of summary approximation. The aim of method is to replace multi-dimensional differential equation with partial derivatives by the system of one-dimensional differential equations (Samarskij, 1983).

Denoting $\Sigma = \gamma\Sigma_f - \Sigma_a$, $B = \beta\gamma\Sigma_f$, from the system of equations (1) we can obtain a parabolic type equation

$$\frac{1}{v_c}\frac{\partial\,\varphi\,(x,t)}{\partial\,t} = D\nabla^2\varphi(x,t) + \Sigma\varphi(x,t) + \lambda \mathrm{e}^{-\lambda\,t}\int\limits_0^t B\mathrm{e}^{\lambda\,t_1}\varphi\,(x,t_1)\mathrm{d}\,t_1,$$

which is a partial case of equation with the memory:

$$\frac{\partial u}{\partial t}(x,t) + Au(x,t) = \int\limits_0^t f(t,s,x,u(x,s))\mathrm{d}\,s,$$

where A is self-adjoint elliptic type operator.

Using the method of summary approximation we obtain:

$$\begin{aligned}
&\frac{1}{v_c}\frac{\partial\,\varphi_{\,(1)}}{\partial\,t} - D\frac{\partial^{\,2}\varphi_{\,(1)}}{\partial\,x_1^2} - \tfrac{1}{2}\Sigma\varphi_{(1)} = \tfrac{1}{2}\lambda\mathrm{e}^{-\lambda\,t}\int\limits_0^t B\mathrm{e}^{\lambda\,t_1}\varphi_{\,(1)}\mathrm{d}\,t_1,\\
&\frac{1}{v_c}\frac{\partial\,\varphi_{\,(2)}}{\partial\,t} - D\frac{\partial^{\,2}\varphi_{\,(2)}}{\partial\,x_2^2} - \tfrac{1}{2}\Sigma\varphi_{(2)} = \tfrac{1}{2}\lambda\mathrm{e}^{-\lambda\,t}\int\limits_0^t B\mathrm{e}^{\lambda\,t_1}\varphi_{\,(2)}\mathrm{d}\,t_1,\\
&\varphi_{(1)}(x,0) = \varphi_0(x),\\
&\varphi_{(1)}(x,t_j) = \varphi_{(2)}(x,t_j), \quad t_j \leqslant t \leqslant t_{j+1},\\
&\varphi_{(2)}(x,t_j) = \varphi_{(1)}(x,t_{j+1}),
\end{aligned} \tag{2}$$

where $\varphi_{(2)}(x,t_{j+1})$ is the solution of this system at the time moment t_{j+1}.

Let τ be a time step and h – a space grid step. Applying the Euler scheme for time discretization and choosing the approximation by spatial coordinates

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\varphi(x+h,t) - 2\varphi(x,t) + \varphi(x-h,t)}{h^2},$$

we obtain a system of equations

$$\begin{aligned}
&\frac{1}{v_c}\,\frac{\varphi_{(1)}(x,t) - \varphi_{(1)}(x,t-\tau\,)}{\tau}\\
&\quad -D\frac{\varphi_{(1)}(x+h_1,t) - 2\varphi_{(1)}(x,t) + \varphi_{(1)}(x-h_1,t)}{h_1^2} - \tfrac{1}{2}\Sigma\varphi_{(1)}(x,t)\\
&= \tfrac{1}{2}\lambda\mathrm{e}^{-\lambda t}\tau\sum_{j=0}^{n-1} B\mathrm{e}^{\lambda\,j\tau}\varphi_{\,(1)}(x,j\tau\,),\\
&\frac{1}{v_c}\,\frac{\varphi_{(2)}(x,t) - \varphi_{(2)}(x,t-\tau)}{\tau}\\
&\quad -D\frac{\varphi_{(2)}(x+h_2,t) - 2\varphi_{(2)}(x,t) + \varphi_{(2)}(x-h_2,t)}{h_2^2} - \tfrac{1}{2}\Sigma\varphi_{(2)}(x,t)\\
&= \tfrac{1}{2}\lambda\mathrm{e}^{-\lambda t}\tau\sum_{j=0}^{n-1} B\mathrm{e}^{\lambda j\tau}\varphi_{(2)}(x,j\tau),\\
&\varphi_{(1)}(x,0) = \varphi_0(x),\\
&\varphi_{(1)}(x,t) = \varphi_{(2)}(x,t),
\end{aligned} \tag{3}$$

$$\varphi_{(2)}(x,t-\tau) = \varphi_{(1)}(x,t). \tag{4}$$

This system of equations (3) approximates that of (2) with accuracy $\mathrm{O}(\tau + h^2)$ (Sapagovas and Vileiniškis, 1998).

Since the matrix of equation system (3) is tridiagonal, we use a double sweep method (Samarskij, 1983) for its solution.

4. Numerical Results

The described method has been tested to calculate the neutron flux in two-dimensional rectangular region 50×50 cm, placed in a vacuum (i.e., $\varphi(x_b, t) = 0$). In all cases $h_1 = h_2 = h$. Macroscopic cross sections and other initial data for thermal neutrons were taken from (Langenbuch *et al.*, 1977). Results are presented in Tables 1–4. These tables show maximal value of neutron flux reached in a center of region. In all cases initial conditions $\varphi(x, 0) = 1.0$.

Table 1 shows the results for time moment $t = 0.003$ sec obtained by employing several different time step sizes when $D = 0.356$ cm, $B = 0.000735$ cm^{-1}, $\lambda = 0.08$ s^{-1}, $h = 2.5$ cm. We can see from Table 1 that results agree well when time step is less than $2.0\cdot 10^{-05}$ in both cases, for negative Σ values and for positive ones. Therefore we used the time step equal to $1.0\cdot 10^{-05}$, for the analysis of mesh size influence.

Table 1

Dependence of solution on time-step size

$\Sigma = 0.005$ cm^{-1}		$\Sigma = -0.005$ cm^{-1}	
τ (s)	$\varphi_{\max}$	τ (s)	$\varphi_{\max}$
$2.5\cdot 10^{-06}$	8.3778	$2.5\cdot 10^{-06}$	$0.4664\cdot 10^{-02}$
$5.0\cdot 10^{-06}$	8.3825	$5.0\cdot 10^{-06}$	$0.4697\cdot 10^{-02}$
$1.0\cdot 10^{-05}$	8.3919	$1.0\cdot 10^{-05}$	$0.4764\cdot 10^{-02}$
$2.0\cdot 10^{-05}$	8.4109	$2.0\cdot 10^{-05}$	$0.4899\cdot 10^{-02}$
$4.0\cdot 10^{-05}$	8.4491	$4.0\cdot 10^{-05}$	$0.5176\cdot 10^{-02}$

Table 2 presents results for the time moment $t = 0.003$ sec, when the time step is equal to $1.0\cdot 10^{-05}$ and $D = 0.356$ cm, $B = 0.000735$ cm^{-1}, $\lambda = 0.08$ s^{-1}, $\Sigma = 0.005$ cm^{-1}.

We can see from Table 2 that neutron flux differs slightly in all cases calculated with different mesh sizes. For calculations, presented in Tables 3 and 4 therefore, mesh size is set to be 2.5 cm.

Table 2

Dependence of solution on mesh size

h (cm)	$\varphi_{\max}$
1.0	8.3906
1.25	8.3907
2.5	8.3919
5.0	8.3958
8.33	8.3975

Table 3
Dependence of solution on physical properties

Σ (cm^{-1})	$\varphi_{\max}$
0.0025	1.2841
0.005	8.3919
-0.0025	$0.3060\cdot 10^{-01}$
-0.005	$0.4764\cdot 10^{-02}$

Table 3 presents neutron flux for the time moment $t = 0.003$ sec and $D = 0.356$ cm, $B = 0.000735$ cm^{-1}, $\lambda = 0.08$ s^{-1}, $\tau = 1.0\cdot 10^{-05}$ s, $h = 2.5$ cm, and for different Σ values.

We can see from Table 3 that maximal neutron flux variation depends on region physical characteristics. For positive Σ values (i.e., more neutrons are generated than absorbed), flux value increases when Σ value increases. For negative Σ value, flux value decreases when absolute Σ value increases.

Table 4 shows the influence of delayed neutrons for the time moment $t = 0.003$ sec when $D = 0.356$ cm, $B = 0.000735$ cm^{-1}, $\Sigma = 0.005$ cm^{-1}, $\tau = 1.0\cdot 10^{-05}$ s, $h = 2.5$ cm.

Influence of delayed neutrons to the neutron flux is less than 1% as expected (Table 4).

Table 4
Influence of delayed neutrons

λ (s^{-1})	$\varphi_{\max}$
0.08	8.3919
0.0	8.3205

5. Conclusion

Nuclear reactor will be safe only if we control the reactor operation. We must understand processes in the reactor core therefore. The neutron flux is one of the main reactor parameters. The aim of this paper is to present numerical algorithm for solution of two-dimensional neutron diffusion equation, which describes the neutron transport in nuclear reactor. Our suggestion is to replace a system of differential equations by one integro-differential equation and use method of summary approximation for numerical solution of this integrodifferential equation. The proposed method has been tested for rectangular area, placed in vacuum. Numerical solution results received agree well with theoretical neutron flux behavior in rectangular region placed in vacuum.

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Dvimatės neutronų difuzijos lygties su vėluojančiais neutronais sprendimas

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Neutronų sklaidos pasiskirstymas atominiame reaktoriuje aprašomas difuzijos lygtimi, atsižvelgiant į vėluojančių neutronų įtaką. Dviejų diferencialinių lygčių sistema sprendžiama baigtinių skirtumų metodu. Aprašomas sprendimo algoritmas, pateikti skaičiavimo rezultatai bei išvados.