INFORMATICA, 2001, Vol. 12, No. 2, 303–314

Mathematical Modeling of Metal Cutting Process

Jolanta JANUTĖNIENĖ

Klaipėda University
Bijūnų 17, 5800 Klaipėda, Lithuania
e-mail: mechanik@ku.jtf.lt

Donatas ŠVITRA

Klaipėda University
H. Manto 84, 5800 Klaipėda, Lithuania
e-mail: matkat@gmf.ku.lt

Received: November 2000

Abstract. In the practice of metal treatment by cutting it is frequently necessary to deal with self-excited oscillations of the cutting tool, treated detail and units of the machine tool. In this paper are presented differential equations with the delay of self-excited oscillations. The linear analysis is performed by the method of D-expansion. There is chosen an area of asymptotically stability and area D_2. It is prove that, in the area D_2 the stable periodical solution appears. The non-linear analysis is performed by the theory of bifurcation. The computational experiment of metal cutting process and results of these experiments are presented.

Key words: metal cutting, oscilations, stability.

1. Introduction

In the practice of metal treatment by cutting it is frequently necessary to deal with oscillations of the cutting tool, treated detail and units of the machine tool. These oscillations in many cases are an obstacle on the way to increase the productivity and quality of details treatment on a metal-cutting machines. Oscillations at metal cutting – phenomenon very diverse as owing to occurrence, and by the form of oscillatory movement.

The most difficult to eliminate and at the same time to investigate self-excited oscillations, i.e., oscillations, arising when there is no external periodic force. The self excited oscillations, stipulated by the process of cutting on metal-cutting machine tool can be divided into two kinds: the low frequency self excited oscillations and high-frequency self excited oscillations. The low-frequency self-excited oscillations is danger on break of the machine tool and the lathe tool. At high-frequency self excited oscillations, the machine tool work externally quiet, but on processed surface occurs small-sized inequality. The frequency of self-excited oscillations can reach up to 5000 Hz and higher. The stability of the process of chip formation is one of the main conditions, which should satisfy metal-cutting machine (Reshetov and Portman, 1986; De Bra, 1992).

2. Dynamics of Cutting Process

The stability of the process of chip formation is the main condition of metal cutting process. For the development of a theory of self excited oscillations at cutting it is necessary to use conformity with the law of the accompanying deformations of treated metal (Eljasberg, 1972). Specialty of the process of cutting is related to plastic properties of metal. Therefore delay cutting force, acting to lathe tool, in relation to coordinates of lathe tool. The self-excited oscillations at metal cutting are a result of delay of forces, which shake the system.

The reason of delay of forces at cutting of metal is the features of deformation process (Eljasberg, 1975). The edge a of lathe tool A (Fig. 1) does not constantly participates in deformation of the main chip, but only incises the layer of material and thus start the deformation process. At small oscillation of the system in direction x the oscillation of thickness of the chip and of the force P periodically detain.

The cutting force is variable (Ostholm, 1991). If to give for system some indignation in direction x, due to delay force P will get appropriate indignation, when lathe tool pass in direction y some path lp. An equation of delay of force P from coordinate

$$\Delta P(t) = B\Delta x(t - \tau_p), \tag{1}$$

where $\Delta P(t)$ – function (t), $\Delta x(t - \tau_p)$ – function $(t - \tau_p)$.

If the force of cutting will receive finite increase ΔP, than increase ΔQ of friction force Q will reach size $\Delta Q = f\Delta P$ only in some time τ_Q, necessary for moving of a chip on way l_Q. Similarly to the Eq. 1 equation of delay of friction force

$$\Delta Q(t) = f\Delta P(t - \tau_Q). \tag{2}$$

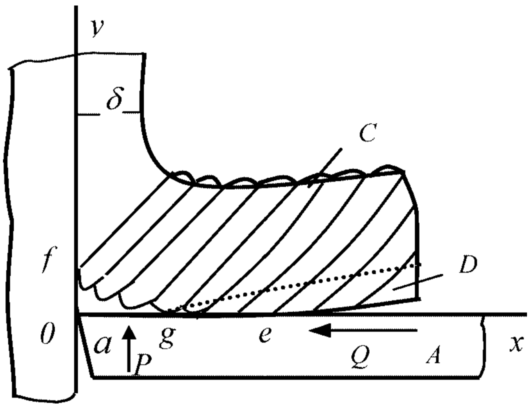

Fig. 1. The process of the chip formation.

Equations of the small oscillation of dynamic system:

$$\ddot{x}(t) + \frac{b_x}{m_x}\dot{x}(t) + \omega_x^2 x(t) = -\frac{fB}{m_x}x(t - \tau_p - \tau_Q), \tag{3}$$

$$\ddot{y}(t) + \frac{b_y}{m_y}\dot{y}(t) + \omega_y^2 y(t) = -\frac{B}{m_y}x(t - \tau_p), \tag{4}$$

where $\omega_x^2 = \frac{c_x}{m_x}$; m_x and m_y – masses, c_x and c_y – coefficients of elasticity.

In the system of Eqs. (3)–(4) the time of delay depends on the functions x and y

$$\tau_p = \frac{l_p}{v_s + \dot{y}}, \quad \tau_Q = \frac{l_Q}{v_s + \dot{y} + \zeta\dot{x}}\,. \tag{5}$$

There l_p and l_Q – the path of delay, v_s – cutting speed.

$B = kb_c\mu\delta^{\mu\varepsilon - 1}$ – relative cutting force, δ – the thickness of the chip, μ – the power estimating the characteristic of metals and the form of lathe tool, b_c – the width of the chip , k – relative pressure.

The process of the chip formatting is presented more detail in (Eljasberg, 1972; Eljasberg, 1975) and there the linear analysis of the system of differential Eqs. (1)–(2) is conducted only. Our purpose will be the complete detailed linear and nonlinear analysis of system (3)–(4) by the methods of bifurcation theory, advanced in work (Kolesov and Švitra, 1979).

3. Linear Analysis

After linearising of Eq. 3, we get linear differential equation

$$\ddot{x}(t) + \frac{b_x}{m_x}\dot{x}(t) + \omega_x^2 x(t) + \frac{fB}{m_x}x\left(t - \frac{l_p + l_Q}{v_s}\right) = 0. \tag{6}$$

A characteristic quasi-polynomial of the Eq. 6 is

$$P(\lambda) = \lambda^2 + \alpha_1\lambda + \alpha_2 + k_1 \mathrm{e}^{-\lambda h_Q}, \tag{7}$$

where $\alpha_1 = \frac{b_x}{m_x}$; $\alpha_2 = \omega_x^2$; $k_1 = \frac{fB}{m_x}$; $h_Q = \frac{l_p + l_Q}{v_s}$.

We will look at the distribution of radicals of Eq. 6 in the plane of parameters k_1 and α_2 using the method of D-expansion. The Eq. 7 has the radical equal zero when $\lambda = 0$ and when $\alpha_1 = -\alpha_2$. We will look when Eq. 7 have only imaginary radicals.

Let's put $\lambda = i\sigma$, then

$$(i\sigma)^2 + \alpha_1 i\sigma + \alpha_2 + k_1 \mathrm{e}^{-i\sigma h_Q} = 0$$

and then let us separate real and imaginary parts

$$\begin{cases} -\sigma^2 + \alpha_2 + k_1\cos(\sigma h_Q) = 0, \\ \alpha_1\sigma - k_1\sin(\sigma h_Q) = 0. \end{cases}$$

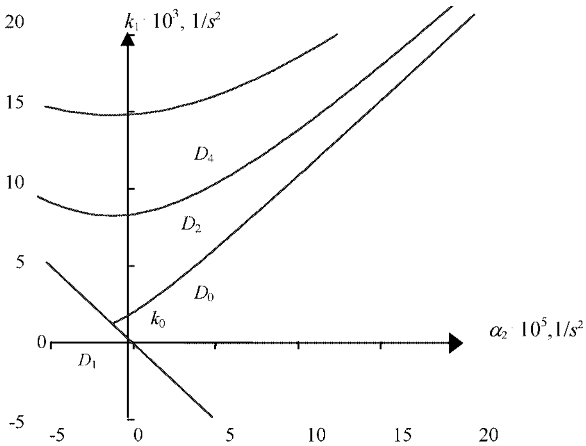

Fig. 2. D-expansion in the plane of parameters k_1 and α_2.

After rearrangement we get equations of remaining curves of D-expansion in the following parametrical forms

$$\begin{cases} k_1 = \dfrac{\alpha_1 \sigma}{\sin(\sigma h_Q)}, & (8) \\ \alpha_2 = \sigma^2 - \alpha_1 \sigma \mathrm{ctg}\,(\sigma h_Q). & (9) \end{cases}$$

When is $\sigma \to 0$, from (8) and (9) equations we define the return point with the coordinates

$$\lim_{\sigma \to 0} \alpha_2 = -\frac{\alpha_1}{h_Q}; \quad \lim_{\sigma \to 0} k_1 = \frac{\alpha_1}{h_Q}.$$

According the experimental results (Eljasberg, 1975; Eljasberg and Binder, 1989; Ostholm, 1991) we can calculate the values of the coefficient α_1 and time of delay h_Q. Let choose the speed of cutting $v_s = 140$ m/min, the delay path of cutting force $P - l_p = 0.35$ mm, the delay path of friction force $Q - l_Q = 0.32$ mm, $c_x = 40000$ N/mm, $m_x = 4.64 \cdot 10^{-3}$ Ns^2/mm, $b_x = 0.0118$ Ns/mm.

We get $\alpha_1 = 25$ s^{-1}, $h_Q = 2.87\ 10^{-4}$ s. When we put those values in the equations (8) and (9) then we get D-expansion in the plane parameters k_1 and α_2. We can see D-expansion in the Fig. 2.

We have to emphasize that in the real cutting process only positive values of parameters α_2 and k_1 are important. From the separated areas of D-expansion we are interested in the area D_0 of asymptotical stability and areas D_2 and D_4 which describe self excited oscillations arising during process of cutting.

1. Let us say $k_1 = 0$, $\alpha_1 > 0$, $\alpha_2 > 0$. Then quasi-polynomial (7) has two real negative radicals.
2. Let us say $k_1 = 0$, $\alpha_1 > 0$, $\alpha_2 < 0$. Then quasi-polynomial (7) has one real positive and one real negative radical.

Thus in the plane of parameters k_1 and α_2 we can separate the area D_0 of asymptotical stability and area D_1, where quasi-polynomial (7) has one radical with the positive real part. The real parts of other radicals are negative (Fig. 2). This conclusion follows from the essence of the method of D-expansion.

3. Let us say $\alpha_1 > 0$, $\alpha_2 = 0$. When

$$k_0 = \sigma_0 \sqrt{\alpha_1^2 + \sigma_0^2} \tag{10}$$

and when σ_0 is the only radicals in the interval $(0, \pi/2h_Q)$ of the equation

$$\operatorname{ctg} \sigma h_Q = \frac{\sigma}{\alpha_1}, \tag{11}$$

then quasi-polynomial (7) has a couple solely imaginary radicals $\pm i\sigma_0$. The real parts of other radicals are negative.

Lemma 1. *When, $\alpha_1 > 0$, $\alpha_2 = 0$ and $k_1 > k_{10}$ quasi-polynomial has a couple of complex joint radicals with the positive real part while real parts of other radicals are negative.*

Proof. Let say $k_1 = k_1 + \varepsilon$ and $\lambda(\varepsilon) = \tau(\varepsilon) \pm i\sigma(\varepsilon)$. Then $\tau(0) = 0$, $\sigma(0) = \sigma_0$.
We have to display, that

$$\tau_0' = \frac{\mathrm{d}}{\mathrm{d}\,\varepsilon} \tau(\varepsilon) \mid_{\varepsilon=0} > 0. \tag{12}$$

From the identity $P[\lambda(\varepsilon); \varepsilon] = 0$, where $P(\lambda)$ is quasi-polynomial (7), we get

$$\tau_0' = -\mathrm{Re} \frac{P_{0\varepsilon}'}{P_{0\lambda}'}, \tag{13}$$

where $P_{0\varepsilon}' = P_\varepsilon'(\lambda; \varepsilon)$, $P_{0\lambda}' = P_\lambda'(\lambda; \varepsilon)$, $\varepsilon = 0$, $\lambda = i\sigma_0$.
As $i\sigma_0$ is a radical of quasi-polynomial (7) when $k_1 = k_{10}$ and $\alpha_2 = 0$, then we get

$$\tau_0' = \frac{\sigma_0^2 \left\lfloor \alpha_1 + h_Q(\sigma_0^2 + \alpha_1^2) \right\rfloor}{k_{10} \left| P_{0\lambda}' \right|^2}. \tag{14}$$

Obviously, that $\tau_0' > 0$. Lemma is proved.

Thus we separated the area D_2. We can separate the areas D_3, D_4 and so on. Accordingly it was done D-expansion in the plane of parameters k_1 and α_1 .

4. Nonlinear Analysis

Let's investigate system of differential equations with delay, depending of searching function:

$$\ddot{x}(t) + \alpha_1 \dot{x}(t) + \alpha_2 x(t) + k_1(\varepsilon) x(t - \tau_p - \tau_Q) = 0, \tag{15}$$
$$\ddot{y}(t) + \beta_1 \dot{y}(t) + \beta_2 x(t) + k_2 x(t - \tau_p) = 0. \tag{16}$$

Let take in to linear part a small parameter ε and

$$k_1(\varepsilon) = k_{10} + \varepsilon.$$

We change time

$$t = (1 + c)\tau, \tag{17}$$

and get

$$x''(\tau) + \alpha_1 x'(\tau)(1+c) + \alpha_2(\varepsilon) x(\tau)(1+c)^2 = -(1+c)^2 (k_1 + \varepsilon) x(\tau - h_Q + W), \tag{18}$$

where $W = h_Q c + \Delta_H - \Delta_H c$.

$$\Delta_H = (p_2 + q_2)\, y' + q_2 x' - (p_3 + q_3)\, y'^2 - 2q_3 x' y' - q_3 x'^2,$$

where $p_2 = \frac{l_p}{v_s^2}$, $q_2 = \frac{l_Q}{v_s^2}$, $p_3 = \frac{l_p}{v_s^3}$, $q_3 = \frac{l_p}{v_s^3}$.

We will expend the functions in power series by ξ

$$x\,(\tau) = \xi \cos \sigma_0 \tau + \xi^2 x_2\,(\tau) + \xi^3 x_3\,(\tau) + \ldots, \tag{19}$$
$$y\,(\tau) = \xi y_1\,(\tau) + \xi^2 y_2\,(\tau) + \xi^3 y_3\,(\tau) + \ldots, \tag{20}$$
$$c = \xi^2 c_2 + \xi^4 c_4 + \ldots, \tag{21}$$
$$\varepsilon = \xi^2 b_2 + \xi^4 b_4 + \ldots. \tag{22}$$

We will expend the function $(\tau - h_+ W)$ by the series of Taylor:

$$x\,(\tau\ - h_Q + W) = x\,(\tau\ - h_q) + x'\,(\tau\ - h_q)\, W + \frac{1}{2}\ddot{x}\,(\tau\ - h_q)\, W^2 + \ldots. \tag{23}$$

After expansion of right and left parts in to series in accordance with ξ and after sorting coefficients nearby the same powers of ξ, we get the sequence of linear nonhomogeneous differential equations with the period $2\pi/\sigma_0$

$$x_1''\,(\tau) + \alpha_1 x_1'\,(\tau) + \alpha_2 x_1\,(\tau) + k_1 x_1\,(\tau\ - h_Q) = 0, \tag{24}$$
$$\begin{aligned} &x_2''\,(\tau) + \alpha_1 x_2'\,(\tau) + \alpha_1 x_2\,(\tau) + k_1 x_2\,(\tau\ - h_Q) \\ &\quad = -\left[k_1 x_1'\,(\tau\ - h_Q)\,(p_2 + q_2)\, y_1' + k_1 x_1'\,(\tau\ - h_Q)\, q_2 x_1'\right], \end{aligned} \tag{25}$$

$$
\begin{aligned}
& x_3''(\tau) + \alpha_1 x_3'(\tau) + \alpha_1 x_3(\tau) + k_1 x_3(\tau - h_Q) \\
& \quad = -b_2 x_1'(\tau - h_Q) - c_2 \Big[\alpha_1 x_1'(\tau) + 2\alpha_1 x_1(\tau) \\
& \qquad + 2k_1 x_1(\tau - h_Q) + k_1 h_Q x_1'(\tau - h_Q)\Big] - \varphi(\tau) \qquad (26) \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

$$
\begin{aligned}
& y_1''(\tau) + \beta_1 y_1'(\tau) + \beta_2 y_1(\tau) = -k_2 x_1(\tau - h_P), \\
& y_2''(\tau) + \beta_1 y_2'(\tau) + \beta_2 y_2(\tau) = -\lfloor k_2 x_2(\tau - h_p) + k_2 p_2 x_1'(\tau - h_p)\, y_1' \rfloor, \qquad (27) \\
& y_3''(\tau) + \beta_1 y_3'(\tau) + \beta_1 y_3(\tau) \\
& \quad = -k_1 x_3(\tau - h_p) - b_2 x_1'(\tau - h_p) - c_2 \Big[\beta_1 y_1'(\tau) + 2\beta_1 y_1(\tau) \\
& \qquad + 2k_2 x_1(\tau - h_p) + k_2 h_p x_1'(\tau - h_p)\Big] - \phi(\tau), \qquad (28) \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

here

$$
\begin{aligned}
\varphi(\tau) = {} & k_1 x_1'(\tau - h_Q)\left[(p_2 + q_2)\, y_2' - (p_3 + q_3) y_2'^2 + q_2 x_2' - 2q_3 y_1' x_1' - q_3 x_2'^2\right] \\
& + \frac{1}{2} k_1 x_1''(\tau - h_Q)\left[\left(p_2^2 + q_2^2\right) y_1'^2 + 2p_2 q_2\left(y_1'^2 + y_1' x_1'\right) + q_2^2\left(y_1' x_1' + x_1'^2\right)\right] \\
& + k_1 x_2'(\tau - h_Q)\left[\left(p_2^2 + q_2^2\right) y_1' + q_2 x_1'\right] \qquad (29)
\end{aligned}
$$

and

$$
\begin{aligned}
\phi(\tau) = {} & k_2 \Big[x_1'(\tau - h_p)(p_2 y_2' + y_1'^2) + x_2'(\tau - h_p) p_2 y_1' \\
& + \frac{1}{2} x_1''(\tau - h_p)\, p_2^2 y_1'^2\Big], \qquad (30)
\end{aligned}
$$

where

$$
\begin{aligned}
y_1 &= A_s \sin \sigma_0 \tau + A_c \cos \sigma_0 \tau, \\
x_2 &= B_0 + B_s \sin 2\sigma_0 \tau + B_c \cos 2\sigma_0 \tau, \\
y_2 &= D_0 + D_s \sin 2\sigma_0 \tau + D_c \cos 2\sigma_0 \tau.
\end{aligned}
$$

Calculation of this members is presented in (Janutėnienė and Švitra, 2000).

The obtained values we put in to Eq. 26. It is known that we can solve the periodical function with the period $2\pi/\sigma_0$ only is case if the function meet the following condition

$$
\begin{cases}
\displaystyle\int_0^{\frac{2\pi}{\sigma_0}} x(\tau, \xi) \sin \sigma_0 \tau \,\mathrm{d}\,\tau = 0, \\
\displaystyle\int_0^{\frac{2\pi}{\sigma_0}} x(\tau, \xi) \cos \sigma_0 \tau \,\mathrm{d}\,\tau = 0.
\end{cases} \qquad (31)
$$

After estimation of those conditions from the Eq. 26 we get two equations. We will get b_2 and c_2 from those two equations

$$\begin{cases} c_2\Big(-\frac{2}{\sigma_0}\alpha_2-\frac{2}{\sigma_0}k_1\cos\sigma_0 h_Q-k_1 h_Q\sin\sigma_0 h_Q\Big)-b_2\frac{1}{\sigma_0}\cos\sigma_0 h_Q=k_1\sigma_0 S_1, & (33)\\ c_2\Big(\alpha_1-\frac{2}{\sigma_0}k_1\sin\sigma_0 h_Q+k_1 h_Q\cos\sigma_0 h_Q\Big)-b_2\frac{1}{\sigma_0}\sin\sigma_0 h_Q=k_1\sigma_0 S_2, & (34)\end{cases}$$

where

$$S_1 = E\cos\sigma_0 h_Q + F\sin\sigma_0 h_Q + G\cos 2\sigma_0 h_Q + H\sin 2\sigma_0 h_Q,$$
$$S_2 = L\cos\sigma_0 h_Q - M\sin\sigma_0 h_Q - H\cos 2\sigma_0 h_Q + G\sin 2\sigma_0 h_Q.$$

We get forms of b_2 and c_2

$$c_2 = \frac{k_1\sigma_0^2\,(S_2 - S_1 tg\sigma_0 h_Q)}{I_2 - I_1 tg\sigma_0 h_Q}, \tag{35}$$
$$b_2 = \frac{c_2 I_1 - k_1\sigma_0^2 S_1}{\cos\ \sigma_0 h_Q}. \tag{36}$$

From (22) we can consider, that

$$\xi_* = \sqrt{\frac{\varepsilon}{b_2}} + \mathrm{O}(\varepsilon),$$

then $\tau \approx \dfrac{t}{1+\dfrac{c_2}{b_2}\varepsilon}$.

We get the periodical solution of system differential equations (15)–(16):

$$\begin{cases} x(t) = \sqrt{\frac{\varepsilon}{b_2}}\cos\dfrac{\sigma_0 t}{1+\dfrac{c_2}{b_2}\varepsilon} + \mathrm{O}(\varepsilon),\\ y(t) = \sqrt{\frac{\varepsilon}{b_{2*}}}\left(A_s\sin\ \sigma_0\tau + A_c\cos\ \sigma_0\tau\right) + \mathrm{O}(\varepsilon).\end{cases}$$

The approximate values obtained, when $t = 1.5$ mm, $v_s = 120$ m/min, $s = 0.25$ mm/rev (Fig. 3). The periodical solution of system (15)–(16) differential equations is stable, then $\tau_0'\varepsilon > 0$.

5. Computational Experiments

We will solve the system of differential equations (3) and (4) by the method of Oiler.

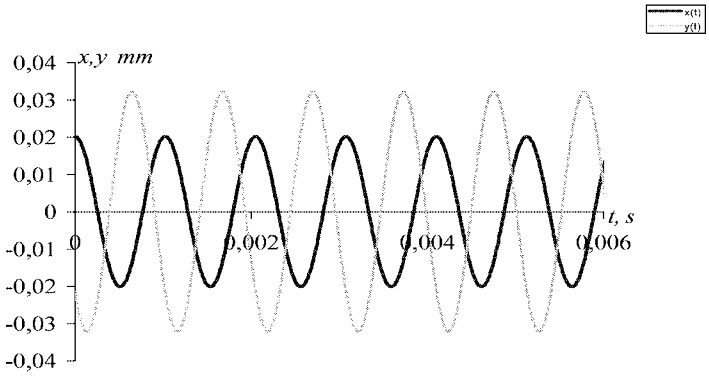

Fig. 3. The self excited oscillation by coordinates x and y.

We know, that acceleration – $\ddot{x} = \frac{\mathrm{d}\, v}{\mathrm{d}\, t}$ and speed – $\dot{x} = v = \frac{\mathrm{d}\, x}{\mathrm{d}\, t}$.

$$\begin{cases} \mathrm{d}\, v_x = (-\alpha_1 v_x - \alpha_2 x - k_1 x(t - \tau_p - \tau_Q))\, \mathrm{d}\, t, \\ \mathrm{d}\, v_y = (-\beta_1 v_y - \beta_2 y - k_2 x(t - \tau_p))\, \mathrm{d}\, t, \\ \mathrm{d}\, x = v_x \mathrm{d}\, t, \\ \mathrm{d}\, y = v_y \mathrm{d}\, t. \end{cases}$$

In accordance with initial conditions $x(0) = 0$, $y(0) = 0$, $v_x(0) = v_{x0}$, $v_y(0) = 0$ we calculate:

$$\begin{cases} x_n = x_{(n-1)} + v_{x(n-1)} \mathrm{d}\, t, \\ y_n = y_{(n-1)} + v_{y(n-1)} \mathrm{d}\, t, \\ v_{xn} = v_{x(n-1)} + \big(-\alpha_1 v_{x(n-1)} - \alpha_2 x_{(n-1)} - k_1 x(t_1 - \tau_p - \tau_Q)\big)\, \mathrm{d}\, t, \\ v_{yn} = v_{y(n-1)} + \big(-\beta_1 v_{y(n-1)} - \beta_2 y_{(n-1)} - k_2 x(t_1 - \tau_p)\big)\, \mathrm{d}\, t, \end{cases}$$

$\mathrm{d}\, t$ – step of the time; let's select $\mathrm{d}\, t = 0.00001$ s.

The members $x(t_n - \tau_p - \tau_Q)$ and $x(t_n - \tau_p)$ depends of searching function. We find them by the computation program presented in Fig. 5

Results of computational experiment present in Fig. 4.

6. Conclusions

Results of computational experiment correspond with theoretical solution of system differential equation (3)–(4). We can model the metal cutting process, when we change the parameter k_1. Thus, when coefficients $\alpha_1, \alpha_2, \beta_1$ and β_2 have a different values, we can find conditions, when the system of differential equations have a stable periodical solution or assymptotical stable solution.

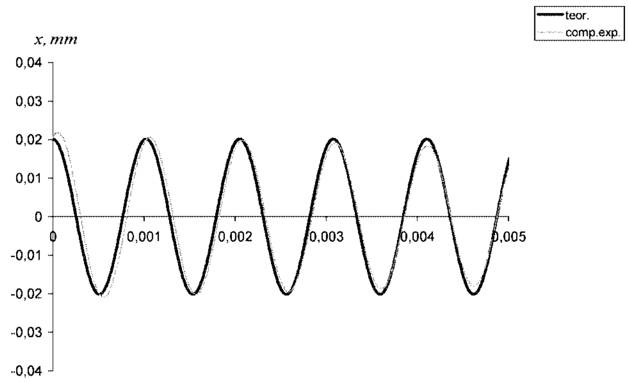

Fig. 4. The oscillations by coordinate x, when $s = 0.25$ mm, $t = 1.5$ mm, $v_s = 120$ m/min.

```
Function FindRow(FirstRow, AValue)
Dim i As Integer
Dim Difference
i =FirstRow
Do
Difference=Abs(AValue−Cells(i, 1).Value)
i = i + 1
Loop Until (Difference <= Abs(AValue−Cells(i, 1).Value))
FindRow= i − 1
End Function
Sub Macro2()
Dim r, RowNo
r = 10
For RowNo = 10 To 1000
r =FindRow(r, Cells(RowNo, 4).Value)
Cells(RowNo, 8).Value=Cells(r, 3).Value
Next RowNo
End Sub
```

Fig. 5. The algorithm of computation program for find $x(t_n - \tau_p - \tau_Q)$.

References

De Bra, D. (1992). Vibration isolation of precision Machine Tools and instruments. *CIRP*, **41**(2), 711–716.

Eljasberg, M.E. (1971). The theory and calculation of stability on metal cutting process. *Stanki i Instrument*, **11**, 6–11 (in Russian).

Eljasberg, M.E. (1972). The theory and calculation of stability on metal cutting process. *Stanki i Instrument*, **1**, 3–7 (in Russian).

Eljasberg, M.E. (1975). About calcutalion of stability of metall cutting process. *Stanki i Instrument*, **2**, 19–27 (in Russian).

Eljasberg, M.E., M.G. Binder (1989). Stable of oscillation system of metal cutting machine. *Stanki i Instrument*,

10, 19–22 (in Russian).

Janutėnienė, J., D. Švitra (1998). Linear analysis of differecial equations of metall cutting process. *LMD mokslo darbai*, Vol 2. Vilnius, Technika, pp. 470–474.

Janutėnienė, J., D. Švitra (2000). Investigation of self excited oscillation in metal cutting process. *Mechanika*, **26**(6), 69–74.

Kolesov, Yu., D. Švitra (1979). *Oscillation in Systems with the Delay*. Vilnius, Mokslas.

Ostholm, S. (1991). *Simulering och Identifiering av Skareggars Mekaniska Belastringsbild.* Institutionen for mekanisk teknologi och verktysmaskiner lunds tekniska horskola.

Reshetov, D.N., V.T. Portman (1986). *Precision of Machine Tools.* Mashinostroenije, Moscow (in Russian).

J. Janutėnienė received the Dipl.Eng. degree from Kaunas Polytechnic Institute, Lithuania, in 1990. Currently she is a person maintaining a doctor's thesis in Klaipėda University. Her research interents include mathematical modelling of technical system.

D. Švitra has received the degree of habil.doctor of mathematics from the Academy of Sciences of USSR in 1989. He is a professor of mathematics at the Klaipėda University. His research interests include nonlinear modelling in biology and ecology.

Metalų pjovimo proceso matematinis modeliavimas

Jolanta JANUTĖNIENĖ, Donatas ŠVITRA

Straipsnyje nagrinėjamos metalų pjovimo proceso metu atsirandančius autosvyravimus aprašančios diferencialinės lygtys. Atlikta diferencialinių lygčių su vėlavimu, priklausančiu nuo ieškomos funkcijos, tiesinė analizė. D- suskaidymo metodu išskirta asimptotinio stabilumo sritis, kurioje autosvyravimai nesusižadina bei sritis, kurioje atsiranda stabilus periodinis sprendinys. Taikant bifurkacijų teoriją atlikta diferencialinių lygčių netiesinė analizė ir gauta stabilaus periodinio sprendinio išraiška. Atliktas diferencialinių lygčių skaitinis eksperimentas ir jo rezultatai palyginti su teorinio sprendinio reikšme. Naudojantis gautais rezultatais galima matematiškai modeliuoti metalų pjovimo procesą.